# PHILOSOPHICAL

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# ROYAL SOCIETY

OF

# LONDON.

FOR THE YEAR MDCCCLXVII.

VOL. 157.

LONDON:

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MDCCCLXVIII.

# ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the



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thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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The Meteorological Journal hitherto kept by the Assistant Secretary at the Apartments of the Royal Society, by order of the President and Council, and published in the Philosophical Transactions, has been discontinued. The Government, on the recommendation of the President and Council, has established at the Royal Observatory at Greenwich, under the superintendence of the Astronomer Royal, a Magnetical and Meteorological Observatory, where observations are made on an extended scale, which are regularly published. These, which correspond with the grand scheme of observations now carrying out in different parts of the globe, supersede the necessity of a continuance of the observations made at the Apartments of the Royal Society, which could not be rendered so perfect as was desirable, on account of the imperfections of the locality and the multiplied duties of the observer.

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## Adjudication of the Medals of the Royal Society for the year 1867 by the President and Council.

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The Copley Medal to Karl Ernst von Baer, of St. Petersburg, For. Mem. R.S., for his discoveries in Embryology and Comparative Anatomy, and for his Contributions to the Philosophy of Zoology.

A Royal Medal to Messrs. John Bennet Lawes, F.R.S., and Dr. Joseph Henry Gilbert, F.R.S., for their Researches in Agricultural Chemistry.

A Royal Medal to Sir William Edmond Logan, F.R.S., for his Geological Researches in Canada, and the Construction of a Geological Map of that Colony.

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The Bakerian Lecture was delivered by Frederick Augustus Abel, F.R.S.; it was entitled "Researches on Gun-cotton.—Second Memoir. On the Stability of Gun-cotton."

The Croonian Lecture was delivered by Dr. J. Burdon Sanderson; it was entitled "On the Influence exercised by the Movements of Respiration on the Circulation of the Blood."

A List of Public Institutions and Individuals, entitled to receive a Copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

*Observatories.*

Armagh.
Cape of Good Hope.
Dublin.
Edinburgh.
Greenwich.
Kew.
Liverpool.
Madras.
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*Institutions.*

Barbadoes ..... Library and Museum.
Calcutta ..... Asiatic Society.
Geological Museum.
Cambridge ..... Philosophical Society.
Cape Town ..... South African Library.
Dublin ..... Royal Dublin Society.
Royal Irish Academy.
Edinburgh ..... Royal Society.
London ..... Admiralty Library.
Chemical Society.
Entomological Society.
Geological Society.
Geological Survey of Great Britain.
Institution of Civil Engineers.
Linnean Society.
London Institution.
Royal Asiatic Society.
Royal Astronomical Society.
Royal College of Surgeons.
Royal College of Physicians.
Royal Geographical Society.
Royal Horticultural Society.
Royal Institute of British Architects.
Royal Institution of Great Britain.
Royal Medical and Chirurgical Society.
Royal Society of Literature.
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The Queen's Library.
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*Denmark.*

Copenhagen ..... Kongelige Danske Videnskabernes Selskab.

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Montpellier ..... Académie des Sciences et Lettres.
Faculté de Médecine.
Paris ..... Académie des Sciences de l'Institut.
Dépôt de la Marine.
Ecole des Mines.
Ecole Normale Supérieure.
Jardin des Plantes.
Société Entomologique.
Société de Géographie.
Société Géologique.
Société d'Encouragement pour l'Industrie Nationale.
Toulouse ..... Académie Impériale des Sciences.

*Germany.*

Altona ..... Die Sternwarte.
Berlin ..... Königliche Akademie der Wissenschaften.
Physikalische Gesellschaft.
Brünn ..... Naturforschender Verein.
Danzig ..... Naturforschende Gesellschaft.
Dresden ..... Kaiserliche Leopoldino-Carolinische deutsche Akademie der Naturforscher.
Frankfort ..... Senckenbergische naturforschende Gesellschaft.
Giessen ..... Grossherzogliche Universität.
Göttingen ..... Königliche Gesellschaft der Wissenschaften.
Gratz ..... Naturwissenschaftliche Verein für Steiermark.
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Munich ..... Königliche Bayerische Akademie der Wissenschaften.
Prague ..... Königliche böhmische Gesellschaft der Wissenschaften.
Vienna ..... Kaiserliche Akademie der Wissenschaften.
Geologische Reichsanstalt.
Würzburg ..... Physikalisch-medicinische Gesellschaft.

*Hungary.*

Pesth ..... Á Magyar Tudós Társaság—Die Ungarische Akademie der Wissenschaften.

*Italy.*

Bologna ..... Accademia delle Scienze dell' Istituto.
Catanea ..... Accademia Gioenia di Scienze Naturali.
Florence ..... Reale Museo di Fisica.
Milan ..... Reale Istituto Lombardo di Scienze, Lettere ed Arti.
Modena ..... Società Italiana delle Scienze.
Naples ..... Società Reale, Accademia delle Scienze.
Palermo ..... Accademia di Scienze e Lettere.
Rome ..... Accademia Pontificia de' Nuovi Lincei.
Osservatorio del Collegio Romano.
Turin ..... Reale Accademia delle Scienze.
Venice ..... I. R. Istituto Veneto di Scienze, Lettere ed Arti.

*Java.*

Batavia ..... Bataviaasch Genootschap van Kunsten en Wetenschappen.

*Netherlands.*

Amsterdam ..... Koninklijke Akademie van Wetenschappen.
Haarlem ..... Hollandsche Maatschappij der Wetenschappen.
Rotterdam ..... Bataafsch Genootschap der Proefondervindelijke Wijsbegeerte.

*Portugal.*

Lisbon ..... Academia Real das Sciencias.

*Russia.*

Kazan ..... Imperatorsky Kazansky Universitet.
Moscow ..... Société Impériale des Naturalistes.
Le Musée Publique.
Pulkowa ..... Nikolai-Haupsternwarte.
St. Petersburg .... Académie Impériale des Sciences.

*Spain.*

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Madrid ..... Real Academia de Ciencias.

*Sweden and Norway.*

Christiania ..... Kongelige Norske Frederiks Universitet.
Trondhjem ..... Kongelige Norske Videnskabers Selskab.
Gottenburg ..... Kongl. Vetenskaps och Vitterhets Samhälle.
Stockholm ..... Kongliga Svenska Vetenskaps-Akademie.

*Switzerland.*

Bern ..... Allg. Schweizerische Gesellschaft.
Geneva ..... Société de Physique et d'Histoire Naturelle.
Institut National Genevois.

*Transylvania.*

Klausenburg ..... Az Erdélyi Muzeum—Das siebenbürgisches Museum.

*United States.*

Albany ..... New York State Library.
Boston ..... American Academy of Sciences.
Newhaven (Conn.) . The Editors of the American Journal.
Cambridge ..... Harvard University.
Philadelphia ..... Academy of Natural Sciences.
American Philosophical Society.
Washington ..... Smithsonian Institution.
U. S. Naval Observatory.

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The *fifty* Foreign Members of the Royal Society.

A List of Public Institutions and Individuals, entitled to receive a Copy of the Astronomical Observations (including Magnetism and Meteorology) made at the Royal Observatory at Greenwich, on making application for the same directly or through their respective agents, within two years of the date of publication.

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*Observatories.*

Altona.
Armagh.
Berlin.
Breslau.
Brussels.
Cadiz.
Cambridge.
Cape of Good Hope.
Coïmbra.
Copenhagen.
Dorpat.
Dublin.
Edinburgh.
Helsingfors.
Königsberg.
Madras.
Mannheim.
Marseille.
Milan.
Munich.
Oxford.
Palermo.
Paris.
Seeberg.
Tübingen.
Turin.
Vienna.
Wilna.

*Institutions.*

Aberdeen ..... University.
Berlin ..... Academy of Sciences.
Boston ..... American Academy of Sciences.
Brunswick, U.S. ..... Bowdoin College.
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Pesth ..... Hungarian Academy of Sciences.
Philadelphia ..... American Philosophical Society.
St. Andrews ..... University.
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Lowndes' Professor of Astronomy ..... Cambridge.
Plumian Professor of Astronomy ..... Cambridge.
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South, Sir James ..... Kensington.
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## A List of Observatories, Institutions and Individuals, entitled to receive a Copy of the Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich.

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| | |
|---|---|
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# PHILOSOPHICAL TRANSACTIONS.

I. *Discussion of Tide Observations at Bristol.* *By* T. G. Bunt, *Bristol.* *Communicated by the Astronomer Royal.*

Received October 24,—Read December 6, 1866.

This paper contains the result of an attempt to discover empirically the Laws of the Diurnal Inequality of the Times and Heights, and of the Solar Inequality of the Times of High Water at the Port of Bristol.

The observations employed in this discussion are those that have been taken by the Bristol Self-registering Tide-Gauge, which has been kept steadily at work, with a few occasional interruptions, from the period of its erection in 1837 to the present time. This instrument consists essentially of a Clock, a Cylinder, a Float, and a Pencil, by means of which every tide marks a curve on a sheet of paper, from which the time and height of high water are ascertained. Its details are described, and an engraving of it given, both in the Philosophical Transactions for 1838, Part II., p. 249, and in the Article "Tides and Waves" in the Encyclopædia Metropolitana, written by the present Astronomer Royal.

During the far greater part of these twenty-nine years, the Tide-Gauge has been at work under my own continual inspection; and it may be important to remark that the clock has, from the commencement, been carefully adjusted to Bristol mean time by transit observations: viz. during the first fourteen years by comparison with the Transit Clock of Messrs. Muston and Gath, chronometer makers in this city, besides frequent sextant altitudes of the sun taken by myself, and for the last fifteen years by a transit instrument in my own house.

Soon after the Meeting in 1836 of the British Association in Bristol, I was employed under its auspices, in assisting the late Dr. Whewell in his discussions of the Bristol Tides; the results of which appeared in a succession of papers in the Philosophical Transactions, extending from 1837 to 1840.

The Diurnal Inequality of the Tides was that branch of the subject to which Dr. Whewell's attention was specially directed. In explanation of this term, if any such be necessary, it may be stated briefly that successive tides do not increase or decrease by a

regular progression, but are alternately higher and lower than such a progression would require. The intervals also between the times of high water and the preceding lunar transit to which they are referred, are found to be alternately longer and shorter, so as to present a similar irregularity.

The Diurnal Inequality of Height in the Tides of Bristol, though of no great magnitude, averaging only $2\frac{1}{2}$ inches up and down, is at once apparent on the most cursory examination; the inequality of time is not quite so easily detected. It was indeed stated by Sir JOHN LUBBOCK so lately as in 1839, in his 'Elementary Treatise on the Tides,' p. 39, that "the diurnal inequality in the interval is inappreciable on our coasts;" and again, at p. 40, "the diurnal inequality in the time of high water on our coasts is too minute to be detached from the inevitable errors of observation."

There will, I apprehend, be little difficulty in showing that this remark of Sir JOHN LUBBOCK's is not applicable to the port of Bristol. For although the diurnal inequality in the times of high water is not large, its average magnitude, on comparing two successive intervals, being little more than four minutes between both, or two minutes for the single inequality in each, yet it is almost always perceptible in the Tide-Gauge Observations in tolerably calm weather. In the accompanying Plates of six months' observations in 1865, the diurnal inequality of both times and heights is everywhere apparent.

At Dr. WHEWELL's request, I bestowed much time and labour from the first, in endeavouring to trace out the laws of both these inequalities, and especially that of the times, but for several years with but little success, chiefly for want of a greater number of observations. After several modes of arrangement which Dr. WHEWELL suggested had been tried, he at length arrived at the conclusion that, in order to succeed, we must divide our observations, first into twenty-four groups for the twenty-four half months, and then each of these into twenty-four smaller groups for the twenty-four hours of lunar transit. It thus became evident that little progress could be made in this investigation, until a large mass of observations had been accumulated.

These being now obtained, I have again taken the subject in hand, and embodied the results in the accompanying Plates. For the Inequality of the Times I have taken nearly the whole of the observations. The anterior epoch employed throughout is that of the third preceding lunar transit, averaging an interval of about forty-four hours. The time of every high water had been already calculated (for my annual tide table) by adding to the time of moon's transit the corresponding semimenstrual interval, with the corrections for lunar and solar parallax and declination. From this computed time, the observed time of high water had been subtracted, and the residue, or error, with its proper sign, recorded; being supposed to consist mainly of the uncorrected diurnal inequality. The amount of the inequality was taken thus. Let $a$ be the error of a time of high water computed from a *south* transit of the moon, and $b$, $b'$ the preceding and following error, then $(a-b)+(a-b')$ will be $=4$ times the inequality of the high water which has the error $a$. The quadruple inequality of every *south* transit observation was thus taken, and inserted in its proper group. The $(24\times24=)576$ groups thus

formed being completed were then cast up, averaged, and divided by 4: the quotients were the terms out of which the fig 1, Plate I. was constructed.

Fig. 2, the diurnal inequality of height, was obtained in a precisely similar manner, except that only about fourteen years' observations were employed in its construction.

The tide to which both these figures refer, is that which follows the *south* transit at an interval of about forty-four hours. In each figure the twenty-four vertical lines represent the twenty-four hours of this south transit, *apparent* time. The twenty-four horizontal lines represent the half months, and are the axes from which the curves are measured, upwards and downwards, the scale being $\frac{1}{15}$ of an inch to a minute of time in fig. 1, and $\frac{1}{15}$ of an inch to an inch of height in fig. 2. When the curve is above the axis, it indicates in fig. 1 that the time is *later*, in consequence of the inequality; and in fig. 2 that the height is *greater* than it would have otherwise been, and *vice versâ*. The distances between these horizontal lines, or axes, are arbitrary, being merely a matter of convenience.

Both these sets of curves were laid down exactly according to the averages of the groups, without any arbitrary alteration whatever. In forming the groups, a very few enormous residues, evidently the effects of storms, or other accidental causes, were excluded. Their number was, I believe, much less than 1 per cent. Of a still less number of residues, one half of their amount only was taken, instead of the whole.

My next attempt was to improve the solar inequality correction, or that which is due to the variations of the solar parallax and declination. This was a more laborious affair, requiring the recalculation of the 19,000 observed times of high water, in order that the same formulæ, or curves of lunar correction might be employed throughout. The residues found after these new calculations were arranged, as those for the diurnal inequality had been, for each half month, and each of the twenty-four hours of transit; as I wanted to see whether anything would be gained by keeping the observations taken during the first half lunation separate from those taken during the second. I have never made this separation in arrangements for finding or improving any of the lunar curves, nor ever until now in those for the solar inequality; nor am I aware that it has ever been done by any other person. These results are contained in Plate II. fig. 3.

For the purpose of more easily comparing together these two portions of the curves, I have dotted in a copy of the curves lying between the hours $12^h\ 54^m$ and $0^h\ 54^m$ upon those of the first twelve hours of transit. The difference in many places is considerable, and evidently systematic.

The circumstance which caused the average of each group to fall on the fifty-fourth minute of the hour, was, that the residues were not posted singly, but in pairs, the error of the term computed from a *south* transit of the moon being in every case combined with the following one.

In order to separate, at least approximately, the effects of the solar parallax and declination from each other, the following method was adopted. The curves of June 8th and June 23rd were combined in one pair, and those of December the 8th and 23rd in

another. Their mean parallaxes and declinations, according to the Nautical Almanac for 1866, became as follows:—

| | ⊙'s Decl. | ⊙'s H. Par. |
|---|---|---|
| | ° ′ | ″ |
| June 8 . . . | 22 51 | 8·45 |
| June 23 . . . | 23 26 | 8·44 |
| Mean . . | 23 8 | 8·445 |
| Dec. 8 . . . | 22 44 | 8·71 |
| Dec. 23 . . . | 23 27 | 8·72 |
| Mean . . | 23 5 | 8·715 |

Here we have a difference of 0″·27 of parallax, with only 0° 3′ of declination.

From these pairs of curves are obtained the solar parallax curves in Plate II. fig. 4.

In like manner, by combining eight curves in two sets of four curves in each, we obtain the following means:—

| | ⊙'s Decl. | ⊙'s H. Par. |
|---|---|---|
| | ° ′ | ″ |
| March 23 . . | 1 34 | 8·60 |
| April 8 . . | 7 14 | 8·56 |
| Sept. 23 . . | 0 5 | 8·555 |
| Oct. 8 . . | 5 54 | 8·59 |
| Mean (from squares of the declinations) | 4 40 | 8·576 |
| June 8 . . . | 22 51 | 8·45 |
| June 23 . . . | 23 26 | 8·44 |
| Dec. 8 . . . | 22 44 | 8·71 |
| Dec. 23 . . | 23 27 | 8·72 |
| Mean . . | 23 7 | 8·58 |

Here the mean declinations are 4° 40′, 23° 7′; while the parallaxes differ only 0″·004. These give the solar declination curves in Plate II. fig. 5.

During the remarkably fine weather of the summer of 1865, I found the curves drawn by the pencil of the tide-gauge on the sheet of paper wrapped round the cylinder, more symmetrical and regular, and the agreement of the registered times of high water with those predicted in my tide-table closer than I had ever known them before. The mean error of the predicted times (found, not algebraically, by taking the balance of those + and those −, but by adding all the magnitudes together, regardless of signs) was from the 10th of April to the 24th of October only 2½ minutes; and during about six weeks, namely, from the 17th of August to the 27th of September, it was less than 1·9 minute per tide.

The diagram No. 4 shows the whole of these six months' times and heights of high water, both as predicted and registered; and is interesting chiefly because it so clearly exhibits the two Diurnal Inequalities. No one who looks at it can fail to detect, in an

instant, the existence of them both; and the one is as manifest as the other. The agreement everywhere seen between the computed and observed Diurnal Inequalities, the laws of which it has cost me so much labour to attain, has, I confess, afforded me no small gratification.

Accompanying this diagram, I have enclosed a sheet taken from the Cylinder of the Tide-Gauge, containing the original markings of the pencil, on which an ink line has been very carefully drawn. It registered some of the tides of the remarkably tranquil period which has been already referred to, and is sent as a specimen of the great regularity which the curves sometimes exhibit*.

## Postscript.

Received October 27, 1866.

### *Barometric Inequality.*

In a letter of mine inserted in the Report of the British Association in 1841, I stated that from a comparison of three or four years' computed and observed Heights of High Water at Bristol, I had found that a fall of 1 inch in the mercurial column was accompanied by an average rise of about 13½ inches of tide. I have since obtained for fourteen additional years the following proportions of tide and mercury:—

| | Tide. in. | |
|---|---|---|
| 1841. . . . | 13·0 | |
| 1842. . . . | 11·4 | |
| 1843. . . . | 13·2 | |
| 1844. . . . | 11·4 | |
| 1845. . . . | 10·6 | |
| 1846. . . . | 14·7 | |
| 1847. . . . | 16·0 | to 1 inch of mercury. |
| 1848. . . . | 13·7 | |
| 1849. . . . | 10·0 | |
| 1850. . . . | 9·5 | |
| 1851. . . . | 11·0 | |
| 1852. . . . | 11·7 | |
| 1853. . . . | 12·0 | |
| 1854. . . . | 12·0 | |
| | 14)170·2 | |
| Mean . . | 12·157 | inches of tide to 1 inch of mercury. |

The mean of all the twenty-one years I have thus examined, viz. 1834 to 1854, is 12·772 inches of tide to 1 inch of mercury.

* It has not been deemed necessary to give a Plate of this Tide-Gauge Sheet, or of Diagram No. 4. The latter is, however, preserved in the Archives of the Society, accompanied by a detailed explanation.

## Postscript.

On comparing, *inter se*, the curves of the diurnal inequality of time, a very close resemblance is found between those which differ 6 months in respect of date, and 12 hours in respect of transit. For example, the curve for April 28th, $0^h$, $6^h$, $12^h$, $18^h$, $0^h$, is almost identical with that of October 28th, $12^h$, $18^h$, $0^h$, $6^h$, $12^h$.

The curves of the diurnal inequality of height present a good deal of the same resemblance.—*April* 1867.

## II. *A Supplementary Memoir on Caustics.* *By* A. Cayley, *F.R.S.*

Received November 15,—Read November 22, 1866.

It is near the conclusion of my "Memoir on Caustics," Philosophical Transactions, vol. cxlvii. (1857), pp. 273–312, remarked that for the case of parallel rays refracted at a circle, the ordinary construction for the secondary caustic cannot be made use of (the entire curve would in fact pass off to an infinite distance), and that the simplest course is to measure off the distance GQ from a line through the centre of the refracting circle perpendicular to the direction of the incident rays. The particular secondary caustic, or orthogonal trajectory of the refracted rays, obtained on the above supposition was shown to be a curve of the order 8; and it was further shown (by consideration of the case wherein the distance GQ is measured off from an arbitrary line perpendicular to the incident rays) that the general secondary caustic or orthogonal trajectory of the refracted rays was a curve of the same order 8. The last-mentioned curve in the case of reflexion, or for $\mu=-1$, degenerates into a curve of the order 6; and I propose in the present supplementary memoir to discuss this sextic curve, viz. the sextic curve which is the general secondary caustic or orthogonal trajectory of parallel rays reflected at a circle.

1. For parallel rays refracted at a circle, taking the equation of the circle to be $x^2+y^2=1$, and the incident rays to be parallel to the axis of $x$, then if $x=m$ be an arbitrary line perpendicular to the direction of the incident rays, the secondary caustic is the envelope of the circle,

$$\mu^2\{(x-\alpha)^2+(y-\beta)^2\}-(x-m)^2=0,$$

where $(\alpha, \beta)$ are the coordinates of a variable point on the refracting circle, and as such satisfy the equation $\alpha^2+\beta^2=1$. Or, what is the same thing, writing $\alpha=\cos\theta$, $\beta=\sin\theta$, the secondary caustic is the envelope of the circle

$$\mu^2\{(x-\cos\theta)^2+(y-\sin\theta)^2\}-(x-m)^2=0,$$

where $\theta$ is a variable parameter.

2. The equation may be written

$$A\cos 2\theta+B\sin 2\theta+C\cos\theta+D\sin\theta+E=0,$$

where

$$A=1,$$

$$B=0,$$

$$C=4\mu^2x-4m,$$

$$D=4\mu^2y,$$

$$E=-2\mu^2(x^2+y^2)-2\mu^2+1+2m^2,$$

and which in the case of reflexion, or for $\mu=-1$, become

$$\begin{aligned} A&=1,\\ B&=0,\\ C&=4x-4m,\\ D&=4y,\\ E&=-2(x^2+y^2)-1+2m^2, \end{aligned}$$

viz. the equation of the variable circle is in this case

$$\cos 2\theta+4(x-m)\cos\theta+4y\sin\theta+2m^2-1-2(x^2+y^2)=0.$$

3. Now in general for the equation

$$A\cos 2\theta+B\sin 2\theta+C\cos\theta+D\sin\theta+E=0,$$

where the coefficients are any functions whatever of the coordinates $(x, y)$, the equation of the envelope is

$$S^3-T^2=0,$$

where

$$S=12(A^2+B^2)-3(C^2+D^2)+4E^2,$$

$$-T=27\,A(C^2-D^2)+54BCD-\big(72(A^2+B^2)+9(C^2+D^2)\big)E+8E^3.$$

4. Hence, substituting for A, B, C, D, E the above reflexion values, we find

$$\begin{aligned} S&=12-48\big((x-m)^2+y^2\big)+4(2m^2-1-2x^2-2y^2)^2,\\ -T&=432\big((x-m)^2-y^2\big)\\ &\quad-72\Big(12+144\big((x-m)^2+y^2\big)\Big)(2m^2-1-2x^2-2y^2)\\ &\quad+\ 8(2m^2-1-2x^2-2y^2)^3. \end{aligned}$$

Writing in these equations

$$\begin{aligned} (x-m)^2+y^2&=x^2+y^2-2mx+m^2,\\ (x-m)^2-y^2&=2x^2-2mx+m^2-(x^2+y^2), \end{aligned}$$

then after some simple reductions, we find

$$\begin{aligned} S&=16\{(x^2+y^2-m^2-1)^2+6m(x-m)\},\\ T&=32\{2(x^2+y^2-m^2-1)^3+18m(x-m)(x^2+y^2-m^2-1)-27(x-m)^2\}, \end{aligned}$$

and thence

$$S^3-T^2=1024(x-m)^2U,$$

where

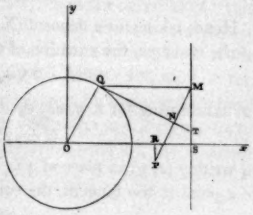
$$\begin{aligned} U=&\quad 4(x^2+y^2-m^2-1)^3\\ &+\ 4m^2(x^2+y^2-m^2-1)^2\\ &+36m\,(x^2+y^2-m^2-1)(x-m)\\ &-27\quad(x-m)^2\\ &+32m^2(x-m), \end{aligned}$$

or, what is the same thing,

$$\begin{aligned} U= \quad & 4(x^2+y^2)^3 \\ & -(8m^2+12)\ (x^2+y^2)^2 \\ & +(36mx+4m^4-20m^2+12)\ (x^2+y^2) \\ & -27x^2+(-4m^3+18)mx+m^2-4; \end{aligned}$$

so that the equation of the secondary caustic is $U=0$, or the secondary caustic is, as stated above, a sextic curve.

5. It is easy to see that the foregoing envelope may be geometrically constructed as follows: viz. if from the point Q (coordinates $\cos\theta$, $\sin\theta$) on the reflecting circle we draw QM perpendicular to the line $x-m=0$, and then from the point M draw MN perpendicular to QT, the tangent at T, and produce MN to a point P such that PN=NM, then P is a point of the envelope; and we thence obtain for the coordinates $(x, y)$ of a point P of the envelope the values




$$\begin{aligned} x= \quad & m-2(m-\cos\theta)\cos^2\theta, \\ y= \quad & \sin\theta-2(m-\cos\theta)\cos\theta\sin\theta, \end{aligned}$$

or, what is the same thing,

$$\begin{aligned} x= \quad & 2\cos^3\theta-m(2\cos^2\theta-1), \\ y= \quad & \sin\theta(2\cos^2\theta+1)-2m\sin\theta\cos\theta, \end{aligned}$$

or, as these equations may also be written,

$$\begin{aligned} x= \quad & \tfrac{3}{2}\cos\theta-m\cos 2\theta+\tfrac{1}{2}\cos 3\theta, \\ y= \quad & \tfrac{3}{2}\sin\theta-m\sin 2\theta+\tfrac{1}{2}\sin 3\theta. \end{aligned}$$

6. This result may be verified by showing that these values satisfy the equation

$$\cos 2\theta+4(x-m)\cos\theta+4y\sin\theta+2m^2-1-2(x^2+y^2)=0,$$

and also the derived equation

$$\sin 2\theta+2(x-m)\sin\theta-2y\cos\theta=0.$$

We in fact have

$$\begin{aligned} x\sin\theta-y\cos\theta= \quad & m\sin\theta-\tfrac{1}{2}\sin 2\theta, \\ x\cos\theta+y\sin\theta= \quad & \tfrac{3}{2}-m\cos\theta+\tfrac{1}{2}\cos 2\theta, \end{aligned}$$

and thence

$$(x-m)\sin\theta-y\cos\theta= \quad -\tfrac{1}{2}\sin 2\theta,$$

which is one of the equations to be verified; and also

$$(x-m)\cos\theta+y\sin\theta=\tfrac{3}{2}-2m\cos\theta+\tfrac{1}{2}\cos 2\theta.$$

We have moreover

$$x^2+y^2= \quad \tfrac{5}{2}+m^2-4m\cos\theta+\tfrac{3}{2}\cos 2\theta;$$

and, by means of these last equations, the other equation

$$\cos 2\theta + 4(x-m)\cos\theta + 4y\sin\theta + 2m^2 - 1 - 2(x^2+y^2) = 0,$$

is also verified.

7. The foregoing values of $(x, y)$ give

$$dx = (-\tfrac{1}{2}\sin\theta + 2m\sin 2\theta - \tfrac{3}{2}\sin 3\theta)d\theta = -\sin 2\theta(3\cos\theta - 2m)d\theta,$$
$$dy = (\ \tfrac{1}{2}\cos\theta - 2m\cos 2\theta + \tfrac{3}{2}\cos 3\theta)d\theta = \ \ \cos 2\theta(3\cos\theta - 2m)d\theta,$$

or, what is the same thing,

$$dx : dy = -\sin 2\theta : \cos 2\theta.$$

Hence taking for a moment (X, Y) as the current coordinates of a point in the tangent of the envelope, the equation of the tangent of the envelope is

$$X\,dy - Y\,dx = x\,dy - y\,dx,$$

or, substituting for $x$, $y$, $dx$, $dy$ their values, this equation takes the very simple form

$$X\cos 2\theta - Y\sin 2\theta - 2\cos\theta + m = 0,$$

or writing $(x, y)$ in place of (X, Y), that is taking now $(x, y)$ as the current coordinates of a point in the tangent, the equation of the tangent is

$$x\cos 2\theta - y\sin 2\theta - 2\cos\theta + m = 0;$$

whence observing that this equation may be expressed as a rational equation of the fourth order in terms of the parameter $\tan\frac{1}{2}\theta$ (or $\cos\theta + \sqrt{-1}\sin\theta$), it appears that the class of the secondary caustic is $=4$.

8. The secondary caustic may be considered as the envelope of the tangent, and the equation be obtained in this manner. Comparing with the general equation

$$A\cos 2\theta + D\sin 2\theta + C\cos\theta + D\sin\theta + E = 0,$$

we have

$$A = x,$$
$$B = -y,$$
$$C = -2,$$
$$D = 0,$$
$$E = m,$$

and thence

$$S = 4\{3(x^2+y^2) + m^2 - 3\},$$
$$T = 4\{18m(x^2+y^2) - 27x - 2m^3 + 9m\},$$

giving

$$S^3 - T^2 = 16V,$$

if for a moment

$$V = 4\{3(x^2+y^2) + m^2 - 3\}^3 - \{18m(x^2+y^2) - 27x - 2m^3 + 9m\}^2.$$

The equation of the curve is thus obtained in the form $V=0$; this should of course be equivalent to the before-mentioned equation $U=0$; and by developing V, and com-

paring with the second of the two expressions of U, it appears that we in fact have V=27U.

9. Taking as parameter $\tan\frac{1}{2}\theta$, or if we please $\cos\theta+\sqrt{-1}\sin\theta$, the foregoing values of $(x, y)$ in terms of $\theta$ give $(x, y, 1)$ proportional to rational and integral functions of the parameter of the degree 6; so that not only the curve is a sextic curve, but it is a unicursal sextic, or curve of the order 6 with the maximum number, =10, of nodes and cusps; that is, if $\delta$ be the number of nodes and $\varkappa$ the number of cusps, we have $\delta+\varkappa=10$. Moreover, introducing the same parameter into the equation of the tangent, this equation is seen to be of the degree 4 in the parameter; that is, the class of the curve is =4: this implies $2\delta+3\varkappa=26$, and we have therefore $\delta=4$, $\varkappa=6$. To verify these numbers, it is to be remarked that it appears by the equation of the curve that there is at each of the circular points at infinity a triple point in the nature of the point $x=0$, $y=0$ on the curve $y^3=x^4$; such a point is in fact equivalent to two cusps and a node, and we have thus the two circular points at infinity counting together as 2 nodes and 4 cusps; there should therefore besides be 2 nodes and 2 cusps, and I proceed to establish the existence of these by means of the expressions for $(x, y)$ in terms of $\theta$.

10. To find the cusps, we have

$$\frac{dx}{d\theta}=-\sin 2\theta(3\cos\theta-2m)=0,$$

$$\frac{dy}{d\theta}=\quad\cos 2\theta(3\cos\theta-2m)=0,$$

which are each of them satisfied if only $3\cos\theta-2m=0$, or $\cos\theta=\frac{2m}{3}$; the corresponding values of $(x, y)$ are found to be

$$x=m-\frac{8m^3}{27},\quad y=\pm\left(1-\frac{4m^2}{9}\right)^{\frac{3}{2}},$$

and we have thus two cusps situate symmetrically in regard to the axis of $x$; the cusps are real if $m<\frac{3}{2}$, imaginary if $m>\frac{3}{2}$; for $m=\frac{3}{2}$, the two cusps unite together at the point $x=\frac{1}{2}$ on the axis of $x$, giving rise to a higher singularity, which will be further examined, *post*, No. 12.

11. The curve is symmetrical in regard to the axis of $x$, and hence any intersection with the axis of $x$, not being a point where the curve cuts the axis at right angles, will be a node. Hence, in order to find the nodes, writing $y=0$, this is

$$\sin\theta(1-2m\cos\theta+2\cos^2\theta)=0,$$

giving $\sin\theta=0$, that is,

$$\theta=0,\quad x=\quad 2-m;$$

or

$$\theta=\pi,\quad x=-2-m;$$

but these are each of them ordinary points on the axis of $x$; or else giving

$$1-2m\cos\theta+2\cos^2\theta=0,$$

that is,

$$\cos\theta=\tfrac{1}{2}(m\pm\sqrt{m^2-2}).$$

The corresponding values of $x$ are

$$x=\cos\theta(2\cos^2\theta-2m\cos\theta)+m,\ =m-\cos\theta,\ =\tfrac{1}{2}(m\mp\sqrt{m^2-2});$$

each of the points in question, viz. the points

$$x=\tfrac{1}{2}(m\mp\sqrt{m^2-2}),\quad y=0,$$

is a node on the axis of $x$.

12. It is to be observed that for $m<\sqrt{2}$ the nodes are both imaginary; for $m=\sqrt{2}$ they coincide together at the point $x=\frac{1}{\sqrt{2}}$; for $m>\sqrt{2}$ they are both real: it is to be further noticed that

$$\text{node } x=\tfrac{1}{2}(m+\sqrt{m^2-2}) \text{ corresponds to } \cos\theta=\tfrac{1}{2}(m-\sqrt{m^2-2}),$$

where ($m$ being $>\sqrt{2}$) the point ($\cos\theta$, $\sin\theta$) is a real point on the circle $x^2+y^2=1$; in fact for $m<\frac{3}{2}$ (that is, $m=\sqrt{2}$ to $m=\frac{3}{2}$) we have $\frac{1}{2}(m-\sqrt{m^2-2})<\frac{1}{2}m$, that is, $\cos\theta<\frac{3}{4}$; but $m=$ or $>\frac{3}{2}$, then $\cos\theta=\frac{1}{2}(m-\sqrt{m^2-2})=\frac{1}{m+\sqrt{m^2-2}}$ is $=$ or $<\frac{1}{2}$, and

$$\text{node } x=\tfrac{1}{2}(m-\sqrt{m^2-2}) \text{ corresponds to } \cos\theta=\tfrac{1}{2}(m+\sqrt{m^2-2}),$$

where ($m$ being $>\sqrt{2}$) the point ($\cos\theta$, $\sin\theta$) is a real point on the circle $x^2+y^2=1$ so long as $m$ is not $>\frac{3}{2}$, that is, from $m=\sqrt{2}$ to $m=\frac{3}{2}$; but if $m>\frac{3}{2}$, then the point in question is an imaginary point on the circle—whence also the node $x=\frac{1}{2}(m-\sqrt{m^2-2})$ is an acnode or isolated point.

In the case $m=\frac{3}{2}$ we have

$$\text{node } x=1 \text{ corresponding to } \cos\theta=\tfrac{1}{2} \text{ or } \theta=60^\circ,$$
$$\text{,, } x=\tfrac{1}{2} \text{ ,, } \cos\theta=1 \text{ or } \theta=0^\circ,$$

the last-mentioned point $x=\frac{1}{2}$ being in fact the point of union of two cusps in the case $m=\frac{3}{2}$ now in question. Hence in this case we have at $(x=\frac{1}{2}, y=0)$ a triple point equivalent to two cusps and a node; visibly, there is only a single branch cutting the axis of $x$ at right angles.

In the case $m=\sqrt{2}$, the nodes coincide as above mentioned at the point $x=\frac{1}{\sqrt{2}}$ on the axis; for this value of $m$ the coordinates of the cusps are

$$x=\tfrac{11}{12}\sqrt{2}\left(=\tfrac{33}{34}\frac{1}{\sqrt{2}}, \text{ which is} <\frac{1}{\sqrt{2}}\right);\quad y=\pm\tfrac{1}{17}.$$

13. Starting from the equation $1024\,(x-m)^3U=S^3-T^2=0$, it is clear that the cusps are included among the intersections of the curves $S=0$, $T=0$; these two curves intersect in 24 points which lie $9+9$ at the circular points at infinity, $2+2$ at the points $x=m$, $y^2-1=0$, and $1+1$ are the cusps, or points $x=m-\frac{8m^3}{27}$, $y^2=\left(1-\frac{4m^2}{9}\right)^3$. To verify this, writing for a moment

$$S'=\ (x^2+y^2-m^2-1)^2+\ 6m(x-m),$$
$$T'=2(x^2+y^2-m^2-1)^3+18m(x-m)(x^2+y^2-m^2-1)-27(x-m)^2,$$

then we have

$$T'-2(x^2+y^2-m^2-1)S' = 6m(x-m)(x^2+y^2-m^2-1)-27(x-m)^2$$
$$= 3(x-m)\{2m(x^2+y^2-m^2-1)-9(x-m)\};$$

so that the equations $S=0$, $T=0$, or, what is the same thing, $S'=0$, $T'=0$ give

$$(x-m)\{2m(x^2+y^2-m^2-1)-9(x-m)\}=0,$$

that is, $x-m=0$, or else $x^2+y^2-m^2-1=\frac{9}{2m}(x-m)$. And combining herewith the equation $S'=(x^2+y^2-m^2-1)^2+6m(x-m)=0$, we have $x-m=0$, $(y^2-1)^2=0$, or else

$$(x^2+y^2-m^2-1)^2=\frac{81}{4m^2}(x-m)^2=6m(x-m),$$

and therefore

$$(x-m)\frac{3}{4m^2}\{27(x-m)-8m^2\}=0,$$

the second factor of which gives $x=m-\frac{8}{27}m^2$, and thence $x^2+y^2-m^2-1=-\frac{4}{3}m^2$, that is, $x^2+y^2=1-\frac{1}{3}m^2$, and therefore $y^2=(1-\frac{1}{3}m^2)-(m-\frac{8}{27}m^2)^2$, $=(1-\frac{4}{3}m^2)^2$, that is, we have

$$x=m-\tfrac{8}{27}m^2, \quad y^2=(1-\tfrac{2}{3}m^2)^2,$$

which, as appears above, gives the two cusps.

14. Similarly, in the equation $16V=S^3-T^2=0$, the intersections of the curves $S=0$, $T=0$ must include the cusps; the curves in question are the two circles

$$3\ (x^2+y^2)+m^2-3=0,$$
$$18m(x^2+y^2)-27x-2m^3+9m=0,$$

meeting in the circular points at infinity, and in the two cusps. It is to be added that the tangent at the cusp coincides with the tangent of the last-mentioned circle,

$$18m(x^2+y^2)-27x-2m^3+9m=0,$$

or, as this may also be written,

$$\left(x-\frac{3}{4m}\right)^2+y^2=\left(\frac{4m^2-9}{12m}\right)^2.$$

15. The axis of $x$ meets the secondary caustic in the two nodes counting as 4 intersections, and besides in 2 points, viz. the points $x=2-m$, $x=-2-m$; these correspond to the values $\theta=0$ and $\theta=\pi$ respectively. But to verify them by means of the equation

$$16V=S^3-T^2=0$$

of the curve, it may be remarked that for $y=0$ we have

$$S=4(3x^2+m^2-3),\ T=4(18mx^2-27x-2m^3+9m);$$

and writing herein $x=\pm 2-m$, we find

$$S=4(2m\mp 3)^2, \quad T=8(2m\mp 3)^3,$$

values which satisfy the equation $S^3-T^2=0$.

16. In the equation U=0 of the curve, writing $x-m=0$, the equation becomes

$$4(y^2-1)^3+4m^2(y^2-1)^2=0,$$

that is,

$$4(y^2-1)^2(y^2-1+m^2)=0,$$

and the line $(x-m)=0$ is thus a double tangent to the curve touching it at the points $x=m$, $y=\pm 1$, and besides meeting it at the points $x=m$, $y=\pm\sqrt{1-m^2}$, that is, at the intersections of the line $x-m=0$, with the circle $x^2+y^2=1$.

17. The maximum or minimum values of $y$ correspond to the values $\theta=\frac{\pi}{4}$, $\theta=\frac{3\pi}{4}$, $\theta=\frac{5\pi}{4}$, $\theta=\frac{7\pi}{4}$ of $\theta$; and we have for

$$\theta=\tfrac{\pi}{4}, \quad x=\tfrac{1}{2}\sqrt{2}, \quad y=\sqrt{2}-m,$$

$$\theta=\tfrac{3\pi}{4}, \quad x=-\tfrac{1}{2}\sqrt{2}, \quad y=\sqrt{2}+m,$$

$$\theta=\tfrac{5\pi}{4}, \quad x=-\tfrac{1}{2}\sqrt{2}, \quad y=-\sqrt{2}-m,$$

$$\theta=\tfrac{7\pi}{4}, \quad x=\tfrac{1}{2}\sqrt{2}, \quad y=-\sqrt{2}+m.$$

18. It is now easy to trace the secondary caustic; we may without loss of generality assume that $m$ is positive, and the values to be considered are

$$m=0, \quad m=1, \quad m=\sqrt{2}, \quad m=\tfrac{3}{2},$$

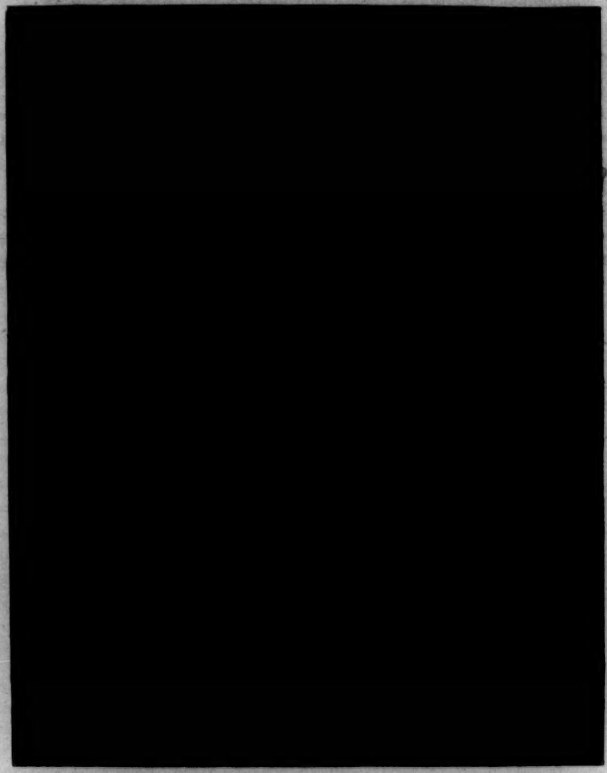
with the intermediate values $m>0<1$, &c. ..., and $m>\frac{3}{2}$. I have for convenience delineated in the figure only a portion of each curve, viz. the figure is terminated at the negative value $x=-\frac{1}{2}\sqrt{2}$, which corresponds to the maximum value $y=\sqrt{2}+m$; as $x$ increases negatively, the value of the ordinate $y$ diminishes continuously from this maximum value, becoming $=0$ for the value $x=-2-m$, and the curve at this point cutting the axis of $x$ at right angles; this is a sufficient explanation of the form of the curves beyond the limits of the figure. Moreover the curve is symmetrical in regard to the axis of $x$, and I have within the limits of the figure delineated only one of the two halves of the curve.

19. For $m>\frac{3}{2}$ the cusps are both imaginary, the nodes both real, but one of them is an isolated point or acnode (shown in the figure by a small cross). The curve has an interior loop, as shown in the figure, and there is also the acnode lying within the loop.

For $m=\frac{3}{2}$, there is still an interior loop, but the acnode has united itself to the loop, the point of union, although presenting no visible singularity, being really a triple point equivalent to a node and two cusps. And in all the cases which follow there are two real cusps.

For $m=<\frac{3}{4}>\sqrt{2}$, the loop has altered its form in such wise as to exhibit the node and two cusps, the curve has therefore two real nodes.

For $m=\sqrt{2}$, the two nodes unite together into a tacnode, so that the loop is on the point of disappearing; and for $m<\sqrt{2}>1$ the nodes are imaginary, and there is thus no longer any loop.



In all the above forms the double tangent $x=m$ touches the curve at the points $y=\pm 1$, but the other two intersections of the double tangent with the curve are imaginary.

For $m=1$, the double tangent has the two coincident real intersections $y=0$, or it is in fact a triple tangent.

For $m<1>0$, the double tangent has with the curve two real intersections, viz. they are the points where the double tangent meets the circle $x^2+y^2=1$.

And finally, for $m=0$, the points in question unite themselves with the points of contact, the double tangent $x=0$ being in this case the common tangent at the two cusps $x=0$, $y=\pm 1$.

Added May 13, 1867.—A. C.

20. As remarked in the original memoir, p. 312, the secondary caustic, in the last-mentioned case $m=0$, is a curve similar to and double the magnitude of the caustic itself (viz. the caustic for parallel rays reflected at a circle), the position of the two curves differing by a right angle.

The secondary caustics corresponding to the different values of $m$ form, it is clear, a system of parallel curves; and, by the remark just referred to, it appears that this system is similar to the system of curves parallel to the caustic for parallel rays reflected at a circle.

## III. *On the Muscular Arrangements of the Bladder and Prostate, and the manner in which the Ureters and Urethra are closed.* By James Bell Pettigrew, *M.D. Edin., First Assistant in the Museum of the Royal College of Surgeons of England. Communicated by* Dr. Sharpey, *Sec. R.S.*

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The arrangement of the muscular fibres of the bladder is exceedingly intricate, and notwithstanding the large share of attention devoted to it, remains to a considerable extent unexplained. A cursory examination of the literature of the subject will serve to show that many of the statements advanced in elucidation of this difficult point of minute anatomy are more or less conflicting. Lieutaud* and Sabatier† described the fibres as pursuing no definite course, but as crossing in all directions to form a complete network of unequal meshes; while Galen‡ enumerated three distinct orders of fibres, viz. a longitudinal, an oblique, and a circular, "Vesicarum tunicæ rectos rotundos et obliquos habent villos."

Duverney§, Lauth‖, Huschke¶, and Cruveilhier** were of opinion that the fibres of the bladder might be separated into an external layer consisting of straight or longitudinal fibres, "musculor detrusor urinæ" (Lauth), and an internal layer composed of oblique circular fibres, the oblique fibres, according to them, being developed most fully towards the cervix, where they form the "sphincter vesicæ."

Guthrie†† gives a very similar account. He recognizes an external longitudinal layer and an internal one, the fibres of which run in a spiral, oval, and transverse direction.

Barkow‡‡ and Ellis§§ indicate three layers, an external longitudinal, a middle circular, and an internal reticular or submucous.

Winslow‖‖ carried the separation further, and maintained that the fleshy fibres of

* Hist. de l'Acad. des Sciences, 1753, p. 99.
† Traité Compl. d'Anat. Paris, 1777.
‡ De usu partium, lib. v. cap. xi.
§ Œuvres Anat. 1761.
‖ Nouveau Manuel de l'Anatomiste. Paris, 1835.
¶ Encyclop. Anat. trad. Jourdan. Paris, 1845.
** Anat. Descript. 3e édit. 1852.
†† On the Anatomy and Diseases of the Urinary Organs and Sexual Organs. By G. J. Guthrie, F.R.S. Lond. 1843.
‡‡ Anatomy of the Muscular Fibres of the Human Bladder. Breslau, 1858.
§§ "An Account of the Urinary and certain of the Generative Organs of the Human Body. By George Viner Ellis, Prof. Anat. Univ. Coll. Lond.," Med. Chir. Trans. vol. xxxix.
‖‖ The Anatomy of the Human Body. Lond. 1766, sect. viii. § 452.

the bladder may be divided into several layers, the fibres of the first or most external layer having a longitudinal direction, those which come next being inclined on either side of the longitudinal one, the succeeding or deepest fibres, which are the most oblique, gradually becoming transverse. This author pointed out the important fact that the fibres mutually cross each other.

A. Sabatier* in a recent memoir divides the fibres into a superficial layer of longitudinal fibres—a deeper layer of oval fibres, a still deeper one of circular or elliptical fibres, and a layer of internal or deep longitudinal fibres. The external longitudinal fibres, in his opinion, surround the summit of the bladder, and form with those of the opposite side a sort of cravat (*cravate*) behind the base of the urachus, other and similar fibres passing beneath the neck of the bladder to form an inferior cravat. The superficial fibres, as will be observed from this description, are continuous towards the apex and base. The oval fibres, he remarks, form loops (*anses*), which, being transverse on the anterior surface and oblique on the sides, converge and diverge posteriorly to form the italic letter *x*. The circular or elliptical fibres, according to Sabatier, extend from the summit of the bladder to the cervix vesicæ and form complete circles; the deep longitudinal or internal ones, which he regards as the internal prolongations of the urachus, intersecting the circular fibres at right angles, and being continued into the prostatic portion of the urethra.

Other authors might be cited, as Douglas, Rutty, Pearson, Thompson, Sappey, Mercier, &c., but the opinions already quoted embrace, as far as I am aware, everything at present known of the general arrangement. The sphincter, trigone, and muscles of Bell have been described separately and variously. Guthrie†, assisted by Messrs. Taylor, Bedford, and Hancock, altogether failed to detect a sphincter for the bladder, and came to the conclusion that no circular fibres surround the neck of the organ; whereas Rutty ‡, as early as 1750, speaks of a complete sphincter in the shape of a small muscle of circular fibres which invests the neck of the bladder and prevents the involuntary emission of the urine. The sphincter vesicæ was regarded by Huschke† as being less regular and distinct than the internal sphincter of the anus, and Sir Charles Bell § describes it as consisting of a semicircular band of fibres about half an inch in breadth, and particularly strong on the under surface of the cervix; the fibres of the upper portion of the band, which are fewer in number and weaker, dispersing themselves in the substance of the bladder. Sabatier† and Bell† gave it as their belief that the trigone and luette are the most sensible parts of the bladder, and Mor-

* Recherches Anatomiques et Physiologiques sur les Appareils Musculaires Correspondants à la vessie et à la prostate dans les deux sexes. Paris, 1864.

† *Op. cit.*

‡ A Treatise on the Urinary Passages. By William Rutty, M.D. Lond. 1750.

§ A Treatise on the Diseases of the Urethra, Vesica Urinaria, Prostate, and Rectum. By Charles Bell. 3rd edit. With Notes by John Shaw. Lond. 1822.

GAGNI* and SANTORINI† state that at the points where the ureters terminate in the bladder there arises from each of them a thick, round, compact, fleshy body, which takes a downward direction, and having proceeded a little way, unites with its fellow and terminates in the caput gallinaceum. The bodies referred to were described by LIEUTAUD‡ under the term "La trigone de la vessie," and Sir CHARLES BELL regarded them as distinct muscles. This anatomist believed that they terminated in the third lobe of the prostate, and not in the caput gallinaceum, as averred by MORGAGNI and SANTORINI. In this, however, as I shall have occasion to show, he was mistaken.

The accounts given of the distribution of the fibres in the prostate are few in number, from the fact that the authors who have turned their attention to the anatomy of the gland have for the most part described either its microscopical structure or pathological conditions. ELLIS‡ regards the prostate as being essentially a muscular body, consisting of circular or orbicular involuntary fibres, which are directly continuous behind with the circular fibres of the bladder, and in front with the circular fibres of the membranous portion of the urethra. It is, in his estimation, "only a largely developed portion of the circular muscular layer that invests all the urethra behind the bulb, or the spongy portion." The few longitudinal fibres which, according to ELLIS, occur on the upper surface of the prostate, are, he says, derived from the external layer of the bladder, and can scarcely be said to form part of the gland. According to HODGSON§, the prostate is made up of two structures, a soft yellowish series of vesicles and their ducts, and a more or less firm tissue laid between the glandular matter and connecting it together. The firm tissue in some instances forms the greater part of the gland, and in his opinion proceeds from the internal aspect of the capsule. It consists of unstriped muscular fibres, with white fibrous and yellow elastic tissues intervening. This author agrees with ELLIS in believing that the circular muscular fibres of the prostatic portion of the urethra are not separable from the muscular structure of the prostate itself, the division being, as he states, the result of dissection and artificial. His description runs as follows:—"The muscular structure, from the mucous membrane of the urethra to the capsule of the prostate, may be considered as the general muscular coat of the urethra interspersed with glandular tissue, and somewhat altered in arrangement and form to adapt it to this condition." A. SABATIER‡ thus tabulates the several layers of the urethra and prostate, as seen in section and enumerated from within. 1. Mucous membrane. 2. Longitudinal fibres. 3. Layer of deep circular fibres concentric with the canal of the urethra, and continuous with the circular layer of the bladder. 4. Longitudinal fibres placed outside that layer, and only existing behind the canal. The longitudinal fibres are the terminations of the oval fibres which enter the prostate directly. 5. A very thick layer of circular superficial fibres which compose almost the whole mass of the prostate.

* MORGAGNI, Adversaria, i. n. 9, Adversaria iii. Animadver. xlii.

† Observationes Anatomicæ, cap. x. sec. xxi. ‡ *Op cit.*

§ The Prostate Gland and its Enlargement in Old Age. By DECIMUS HODGSON, M.D. Edin., M.R.C.S. Eng. Lond. 1856.

These fibres are eccentric to the deep circular fibres surrounding the canal of the urethra, and contain the glandular substance in a kind of network. 6, and lastly. The capsule, composed of fibrous tissue, which serves at once for a covering for the gland and a tendon of insertion for some of the fibres of the bladder. The prostate, according to SABATIER, consists almost exclusively of circular fibres, but the gland, in his opinion, is not a continuation of the bladder, although some of the fibres of the bladder terminate in it. He differs from ELLIS and HODGSON in describing two sets of circular fibres; the one belonging to the urethra, the other to the prostate proper. In this he is followed by HENLE *, who figures the two sets of circular fibres referred to, and indicates (towards the apex of the gland) the presence of striated or voluntary muscular fibres.

LEUCKHART † describes a true rudimentary prostate in the female, which consists principally of mucous follicles, and is situated between the beginning of the urethra and the reflexion of the vagina ‡.

From what has been stated, it will be evident that the arrangement of the fibres in the prostate, and their precise relations to those of the urethra and bladder, is by no means well ascertained. It will further appear that those who have described the fibres of the bladder as running in all directions have not attempted to trace their courses or localize them; while those who have done the latter have given a very imperfect account of the direction of the fibres forming the layers they have enumerated. In conclusion, no attempt has been made to reconcile the statements of SABATIER and HENLE with those of ELLIS and HODGSON, or the descriptions of LIEUTAUD and SABATIER with those of DUVERNEY, LAUTH, HUSCHKE, and CRUVEILHIER—the former describing structures remarkable for their complexity, the latter for their extreme simplicity.

The difficulties experienced in unravelling and tracing the fibres of the bladder and prostate sufficiently account for the discrepancies. With a view to simplifying the arrangement, I have in the present instance distended the bladder and urethra with liquid plaster of Paris, this substance preserving the soft contour of the viscus, and, when set, enabling me to dissect even individual fibres with a surprising degree of precision §. In many instances I have hardened the bladders so prepared in alcohol, bisected them in various directions, and removed the plaster, so as to render them

* Handbuch der Systematischen Anatomie des Menschen, von Dr. J. HENLE, 1866.

† *Vide* article on WEBER's Organ, in Illustr. Med. Zeitung, 1, 2.

‡ "VIRCHOW also admits the existence of the rudimentary prostate in females, and says that he has often found at the neck of the bladder, especially in old women when the internal orifice is thickened, round greyish-yellow enlargements, in which there are gradually formed firm dark-coloured bodies lying imbedded in the mucous membrane. These bodies he considers identical with, or analogous to, the concretions found in the prostatic portion of the urethra" (VIRCHOW's Archives, vol. iii. 1853).

§ In order to obtain a dark background, against which to contrast the direction of the fibres, I added ultramarine blue to the plaster before mixing it with water. The deep colour shines through the semi-transparent walls, and causes the fibres to stand out in relief. The plaster sets in a very few minutes and keeps perfectly in spirit.

transparent. In this way I have obtained illustrative views of all parts of the parietes, and particularly of the apex and fundus. By rendering the bladder transparent, the various sets of fibres can be accurately traced, their direction contrasted, and their relative position in the vesical parietes determined both from without and from within. The varying degrees of thickness in the muscular wall can likewise readily be made out. The prostate has been examined by slicing it in all directions, and by macerating it and tracing out the fibres by the aid of needles. My dissections, some sixty in number, have in all instances supplied me with my description, and have for the most part been carefully photographed in illustration. They are preserved and catalogued in the Museum of the Royal College of Surgeons of England; and I take this opportunity of acknowledging my obligations to the Council of the College for the facilities afforded me in their preparation. By adopting the methods explained, I have obtained remarkably uniform results, and am fully persuaded that the fibres of the bladder and prostate, contrary to the received opinion, are curvilinear, and in fact spiral, or, more properly, lemniscate. The fibres, with few exceptions, form figure-of-eight loops, and the loops are variously shaped, according as they are superficial or deep. The most external and the most internal loops are attenuated or drawn out, and in this respect resemble longitudinal or vertical fibres, the deeper or more central ones being flattened from above downwards, and resembling circular fibres. The loops from this circumstance are divisible into *two orders*, viz. an external and an internal, and these orders may be subdivided, the former into *three sets of external* loops, the latter into *three sets of internal* ones. An intermediate or central set of loops occurs between. The seven sets of loops (external, internal, and central) represent so many layers or strata more or less perfect, and are indicated as well by their direction as by their position in the vesical parietes, the first and seventh sets (the most external and the most internal) being feebly developed, and having a more or less longitudinal or vertical direction, the second and sixth sets consisting of stronger fibres, which cross at acute angles after the manner of an attenuated figure of eight, the third and fifth sets, which occur in strong flattened fasciculi, crossing at obtuse angles, or more obliquely as in a figure of eight flattened from above or expanded laterally. The fourth, or central layer, consists of figure-of-eight loops, so crushed down or flattened that they appear to form complete circles. The crossing, however, is readily made out when sought for, and occasions that reticulated structure so conspicuous in the central layer. The terminal expansions of the loops forming the other layers contribute to the formation of the fourth or central layer, particularly towards the apex and base, and this accounts for its greater thickness. The various sets of external and internal loops are directed towards the apex and base, and are distributed on the anterior, posterior, and lateral aspects respectively. The anterior and posterior fibres are most strongly pronounced, the lateral ones being rudimentary and less fully developed. As a result of this distribution, and because of the puckering occasioned by the constrictions which originally separate the bladder from the intestinal tube, the walls of the viscus are of

unequal thickness, the thickest portion corresponding to the cervix in the neighbourhood of the sphincter, the second thickest portion to the apex in the vicinity of the urachus, the third to the trigone, the fourth and fifth to the anterior and posterior walls, and the sixth and seventh to the right and left lateral walls *. The most external and most internal loops are confined principally to one or other of the aspects indicated; but the deeper or more central loops radiate and expand towards the apex and base, so that they come to embrace the entire circumference of the bladder in these directions. The expansion referred to is greatest towards the apex, and the aggregation of the terminal loops of the anterior and posterior fibres at the cervix (assisted by the lateral ones) form a well-marked sphincter in this situation. The sphincter is bilaterally symmetrical and oval in shape, the long axis being directed transversely, or from side to side, as represented at *m m'*, *y y'* of fig. 11, Plate III., and *l y* of diagrams 10 & 20, Plate V. The two sets of lateral fibres (Plate V. diagram 10, *z r*, *v s*) which assist in the formation of the sphincter, intersect the angles formed by the crossing of the anterior and posterior fibres, and render its aperture somewhat circular in appearance. This circumstance, taken in connexion with the fact that the fibres pursue a very oblique direction, has given rise to the belief that the fibres of the sphincter and neck of the bladder generally are circular fibres, which is not the case. The fibres of the sphincter are best seen by inverting the bladder and dissecting from within, or by making transverse sections of the prostatic portion of the urethra in the direction of the fundus (Plate IV. figs. 19 & 20, *m*). They are most strongly pronounced at the cervix, but are continued forward on the urethra, and backwards into the bladder. In the female they extend even to the meatus urinarius. The apex and base of the bladder are similarly constructed, and resemble in their general configuration the other portions of the vesical walls; *i. e.* they are composed of longitudinal or vertical, slightly oblique, oblique, and very oblique spiral fibres, which cross in all directions externally and internally (Plate IV. figs. 13, 14 & 15). The longitudinal or vertical, slightly oblique, oblique, and very oblique external and internal fibres at the base are continued forwards within the prostate to the membranous portion of the urethra and the external and internal surfaces of the corpus spongiosum (*vide* Cervices of figs. 1, 4 & 7, Plate III.). The coats of the urethra are therefore to be regarded as the proper continuation of the walls of the bladder in an anterior direction. The external or longitudinal, slightly oblique, oblique, and very oblique spiral fibres which form the three outer and the central or circular tunic of the bladder and urethra, are curiously enough repeated in the prostate of the male and the analogous structure in the female, so that this gland would seem to be composed chiefly of fibrous offsets from the longitudinal, slightly oblique, oblique, and very oblique or circular fibres in question. To understand the relations existing between the bladder, prostate, and urethra, it will be most convenient to regard the four outer layers of the bladder as splitting up at the neck of the viscus,

* The right and left lateral walls are strengthened by the plexuses of blood-vessels and nerves, and by the cellular and fibrous tissues which abound in these situations.

one-half of the longitudinal, slightly oblique, oblique, and very oblique or circular fibres going to the prostate (Plate V. diagram 8. 1′, 2′, 3′, 4′), the other to the external or outer part of the prostatic portion of the urethra (1, 2, 3, 4). That this is the true arrangement may be readily ascertained by making vertical, horizontal, and antero-posterior or transverse sections of the parts concerned. In such sections (proceeding from without inwards) we find, first, the longitudinal (Plate V. diagram 8. 1′), slightly oblique (2′), oblique (3′) and very oblique or circular fibres (4′) of the prostate; second, the longitudinal (1), slightly oblique (2), oblique (3), and very oblique fibres (4) of the external or outer half of the urethra; and third, the very oblique (4), oblique (5), slightly oblique (6) and longitudinal or submucous fibres (7) of the internal or inner half of the urethra. The longitudinal, slightly oblique, oblique, and very oblique fibres belonging to the outer half of the urethra occupy an intermediate or central position with regard to those of the prostate and inner half of the urethra, and form a partition or septum similar in many respects to the septum ventriculorum of the heart. It is just possible that the septum in either case is formed by a simple reduplication, the folding in the bladder occurring at the cervix, and taking the form of an intussusception from below upwards *. Mr. Hancock, in his work on Stricture (p. 14), states his belief that the outer coat of the bladder passes forwards on the outside of the prostate gland, and laterally and inferiorly joins the fibres derived from the inner coat in front of the gland to assist in forming the organic muscular covering of the membranous portion of the urethra; but my dissections show that the external fibres of the prostate (Plate IV. figs. 30 & 32, *w*; Plate V. diagram 7, *p*) and the internal or submucous fibres of the urethra (Plate IV. figs. 30 & 32, *r*; Plate V. diagram 7, $a''$) are altogether independent of each other, and are separated by a wide interval—the interval being occupied by the slightly oblique, oblique, and very oblique fibres peculiar to the prostate and urethra. The very oblique or circular fibres of the prostate (Plate IV. figs. 24 & 27, *o*) and urethra (*m*), as will be seen from this account, are likewise separated by a considerable interval. The interval is widest at the base of the gland (Plate IV. fig. 24, *m*, *o*), where the sphincter is most fully developed, and at the apex (fig. 27, *m*, *o*). Towards the centre of the prostate the circular fibres (Plate IV. figs. 25 & 26, *o*) of the gland curve in an upward direction into the veromontanum or caput gallinaginis (*r*), where they blend with the circular fibres of the urethra (*m*). I draw attention to these facts because, as has been stated, Hodgson†, Ellis‡, and other investigators are of opinion that the circular fibres of the prostate are identical with those of the bladder and urethra, which is contrary to my experience. The circular fibres of the prostate are for the most part not only widely removed from those of

* For an explanation of the manner in which the septum is formed in the heart see paper by the author "On the Arrangement of the Muscular Fibres in the Ventricles of the Vertebrate Heart," Phil. Trans. Part III. 1864, p. 464.

† The Prostate Gland and its Enlargement in Old Age. By Decimus Hodgson, M.D. Edin. Lond. 1856.

‡ *Op. cit.* p. 4.

the urethra, but they have, as was shown by SABATIER, separate axes—the circular fibres of the urethra being concentric to the canal of the urethra, and eccentric to the circular fibres of the prostate (Plate IV. fig. 24, *m*, *o*). The only fibres which can with accuracy be regarded as continuous with the circular fibres of the bladder, are the corresponding fibres of the urethra (Plate III. fig. 9, *m*, *n*). The various sets of fibres in the vesical parietes are elaborately interlaced, the most external or superficial ones being connected directly and indirectly with the slightly oblique external, the slightly oblique external with the oblique external, and the oblique external with the very oblique or central fibres. The very oblique internal fibres, on the other hand, are connected with the oblique internal, these in their turn being connected with the slightly oblique internal, and the slightly oblique internal with the longitudinal or vertical internal. In some instances the longitudinal external are connected directly with the longitudinal internal, and so of the slightly oblique, oblique, and very oblique external and internal fibres. The bladder, urethra, and prostate are bilaterally symmetrical, and the fibres composing them pursue something like seven directions, the fibres crossing with remarkable precision at wider and wider vertical angles as the centre of either is reached, as in the stomach and heart *. In fact the fibres of the bladder, stomach, and heart have a strictly analogous arrangement, and I am inclined to believe that functionally also they possess points of resemblance. Very similar remarks may be made regarding the structure and functions of the uterus.

From a careful examination of a large number of mammalian bladders, I am of opinion that fundamentally the fibres are arranged in two principal directions, viz. vertically or longitudinally (Plate IV. fig. 34 & 36, *s*), and transversely or circularly (*m*). In the primary or typical bladder the vertical fibres, particularly in the undistended condition, are grouped together, and form ridges which are raised considerably above the level of the transverse fibres. The ridges, two in number, run from the urethra anteriorly to the urachus and urethra posteriorly, and from side to side. The former bisects the bladder in an antero-posterior direction, the other laterally. As the urachus naturally disconnects the ridges at the apex of the bladder, they may be conveniently described as the anterior, posterior, and right and left lateral ridges. They are seen to advantage in the bladders of the Ox (Plate IV. fig. 36, *a b*, *o p*, *s*), Cat (Plate IV. fig. 38, *a b'*, *c d*, *o p*), Sheep, Roebuck, and Wombat, and map out the circular fibres into four distinct regions. The fibres constituting them are united to the circular ones by short oblique fibres (Plate IV. fig. 36, *m*), and it may be stated that the so-called longitudinal and circular fibres are, with few exceptions, the most strongly marked. This is precisely what we should expect if, as BAER and RATHKE affirm, the bladder is formed originally from the intestinal tube. The ridges are not always persistent, and the disappearance of a ridge necessitates a higher degree of differentiation in the fibres themselves, *i. e.* it demands an increase in the number of oblique fibres. In the Ox, Sheep, and Pig the lateral ridges are but feebly developed, and in the Horse the posterior one has all but disappeared.

* See paper by the author "On the Arrangement of the Muscular Fibres in the Ventricles of the Vertebrate Heart," Phil. Trans. Part III. 1864.

In the Koala no posterior ridge can be discerned, but the remaining three, viz. the anterior and two lateral (Plate IV. fig. 34, *o p*, *s t*), are strongly pronounced. In the human and other bilateral bladders (to which my subsequent remarks more particularly apply) two ridges alone persist, viz. the anterior (Plate III. fig. 1, *a b*) and posterior (Plate III. fig. 6, *o*). They correspond with what has been described as the great anterior and posterior longitudinal bands of fibres, and during the distended state of the viscus have their height so greatly reduced by lateral stretching as scarcely any longer to deserve the name of ridges. They, however, have their homologues in bladders of a lower type, and occasion a thickening of the wall of the bladder anteriorly and posteriorly which cannot otherwise be easily accounted for. It is a curious fact, and not without interest as far as the comparative anatomy of the bladder is concerned, that traces of the fourfold arrangement alluded to above may not unfrequently be detected in the human bladder, particularly at the apex and base. In one specimen which I dissected and preserved, it is especially distinct (Plate IV. fig. 16, *a o*, *k l*; fig. 18, *b p*, *m n*).

Although it is usual to speak of the fibres constituting the muscular coats of the bladder as longitudinal and circular, it may be as well to state that, strictly speaking, those terms are inapplicable, the so-called longitudinal fibres for the most part curving and diverging either in an antero-posterior or lateral direction, and the circular ones representing the aggregate of terminal loops, or very oblique fibres crossing at very obtuse angles. The terms therefore have been retained in the text rather with a view to assimilating my description with that of previous writers, and because it is convenient to have an ideal standard of comparison than because of their correctness. The best standard by which to compare the direction of the fibres is a line drawn from the cervix of the bladder anteriorly to the urachus and cervix posteriorly, another being made to extend from the right side of the cervix to the urachus and left side. A third line may be drawn transversely, or at right angles to both.

It has likewise been customary to regard the fibres constituting the vesical parietes as consisting of layers, but here, again, it is necessary to explain that the strata of the anatomist find no exact counterpart in the bladder itself, the fibres rarely, if ever, occupying precisely the same plane and running exactly parallel. They moreover split up and become fused with each other, with corresponding or homologous fibres, and with fibres which are either superimposed or underlie them. The term layer, which has also been retained, is consequently used in a restricted sense.

In the subjoined account I make no distinction between the male and female bladder, the difference which exists between the two referring rather to shape and to the greater length of the urethra in the former than to the general arrangement of the fibres themselves. I have, moreover, taken my description from young and adult normal bladders, from a feeling that Bell, Sabatier, and other investigators have given undue prominence to certain fibres or sets of fibres from having dissected abnormally thickened or hypertrophied bladders. As mere verbal descriptions, however voluminous and precise, would fail to convey an adequate conception of the various combinations formed by the fibres

of the bladder, I have confined my remarks to elucidating the plan or general arrangement, and have trusted for minor details to the figures and diagrams which will, I trust, be found sufficiently numerous for this purpose.

### EXTERNAL FIBRES.

*First or superficial layer, consisting principally of longitudinal fibres.*—When the bladder of a young adult has been distended and carefully divested of its blood-vessels, nerves, and cellular tissue, it is found to be bilaterally symmetrical, the fibres comprising its parietes pursuing complicated but definite directions.

On the anterior aspect a certain number on either side of the mesial line proceed in a nearly vertical direction from the cervix to the urachus, and have from this circumstance been termed longitudinal (Plate III. figs. 1 & 3, *a b*; Plate V. diag. 1, *a b*). Of these, some arise from the posterior surface of the pubes by means of the anterior true ligaments, and some from the upper or dorsal surface of the fibrous capsule investing the prostate (Plate IV. fig. 29, *a b*), a certain number proceeding from the dorsal surface of the urethra (*a*). Others (Plate IV. fig. 28, *a*) pass in a more or less horizontal direction through the gland, and in this manner reach the under or ventral surface (*c*) of the prostatic capsule to which they adhere, the greater number losing themselves in the glandular stroma (*f*). In the opposite direction, or towards the urachus, some of the fibres are continuous with the urachus itself, while others are inserted into the peritoneal fold investing the top of the bladder. The greater proportion curve over and become continuous with the vertical or longitudinal fibres posteriorly (Plate III. figs. 4, 5, 6 & 8, *o p*). The posterior longitudinal fibres in their turn are connected with the fibres of the compressor urethræ, the under or ventral surface of the fibrous capsule of the prostate (Plate IV. fig. 29, *o p*), and the ventral surface of the urethra (*w*). Of these, a certain number curve in an upward direction (Plate IV. fig. 28, *h i*) and pass through the gland (*f*) to reach the dorsal surface of the prostate. The anterior and posterior longitudinal fibres, as they appear on the dorsal and ventral surfaces of the prostatic portion of the urethra, are seen at *b* and *p* of figs. 1 & 4, Plate III. On the right and left lateral aspects of the bladder longitudinal or vertical fibres, similar to those occurring on the anterior and posterior aspects, can be detected (Plate IV. fig. 9, *a b*); and I regard their occurrence in these situations as favouring the idea that originally the bladder is formed from the intestine. The fourfold arrangement of the fibres, especially at the apex and base, is well seen at Plate IV. fig. 16, *a o*, *k l*; fig. 18, *b p*, *m n*. The lateral longitudinal fibres are attached to the fibrous capsule of the prostate on its right and left aspects, some continuing their course on the sides of the urethra (Plate III. figs. 7 & 8, *b*), while others pass through the gland transversely (Plate IV. figs. 33 & 35, *a c*), or become lost in its substance. SABATIER*, when speaking of the anterior longitudinal fibres, says that in some instances this layer is extended laterally, and that in such cases the internal border of the aponeurosis of the levator ani also serves for insertion.

* *Op. cit.*

The fibres which can with any degree of propriety be regarded as pursuing a longitudinal or vertical direction are comparatively few, and are confined to the anterior, posterior, or lateral surfaces. They are best seen in the bladders of the lower animals.

2nd. *External layer, consisting of fibres arranged in attenuated figure-of-eight loops.*—Returning to the anterior aspect, we observe that at a point about half an inch above the cervix the fibres diverge from the mesial line gradually and with great regularity in an upward and downward direction; they in fact cross each other, and form figure-of-eight loops (Plate III. figs. 1 & 2, *c d e f*), the more attenuated of which are nearest to the line in question. These loops are placed beneath the longitudinal fibres, and are so arranged that they include the urachus (*x*) and urethra (*b*) in their terminal portions. SABATIER*, as was explained, observed a similar looped arrangement, and described the fibres which embrace the urachus and urethra, and are continued on the anterior surface as *oval fibres*. He was not aware that the fibres crossed each other and formed figure-of-eight loops. If a single fibre were taken, it might be regarded as setting out from the left side of the urachus and running in a spiral direction from left to right downwards, until it reached the cervix, round which it curves to proceed in an opposite direction, or from right to left upwards. The course of the fibre in question is indicated at Plate V. diagram 1, *c d e f*. The looped fibres form a figure-of-eight patch or layer which extends from the apex to the base for a little distance on either side of the mesial line. On the posterior aspect the fibres likewise diverge from the mesial line (Plate III. figs. 4 & 5, *q t*). In this instance, however, the divergence and crossing is less obvious, owing to the greatly attenuated condition of the loops occasioned by the greater vertical measurement of the bladder posteriorly. In many cases those loops, or modifications of them, extend forwards upon the urethra and give rise to a series of slightly oblique fibres not formerly described. The oblique fibres referred to can also be traced in the prostate. A similar figure-of-eight looped arrangement of the fibres is discovered on the sides of the bladder. The four sets of fibres described extend in the direction of each other, and invest a considerable proportion of the surface of the bladder. They form the second layer.

3rd. *External layer, consisting of fibres arranged in figure-of-eight loops flattened from above or extended laterally.*—If the longitudinal and slightly oblique fibres forming the first and second external layers be removed from the anterior aspect, a third set of looped fibres, diverging still more decidedly from the mesial line and forming a more perfect figure-of-eight, is discovered (Plate III. figs. 1 & 2, *g h i j*). Those fibres which have a deeper position diverge from the mesial line and cross each other obliquely at a point about three-fourths of an inch above the cervix. They proceed from the left of the mesial line posteriorly, curve round the left side of the bladder in a spiral oblique direction, and cross the mesial line anteriorly at the point indicated. Continuing their downward course, they curve round behind the cervix, where they alter their direction and are lost to

† *Op. cit.*

view. They reappear on the anterior surface, recross the anterior mesial line, and wind round in an upward spiral direction until they reach the right of the mesial line posteriorly, where we began to trace them. Their course is given in Plate V. diagram 1, *g h i j*.

The fibres under consideration form a broad expansion on the upper third of the bladder posteriorly (Plate III. fig. 7, *g*). On the sides and anterior aspect (*g d*) they converge until they reach the anterior mesial line, where they cross each other, after which they again slightly expand in a backward direction to assist in forming the posterior half of the sphincter vesicæ (Plate III. fig. 4, *v w*). They constitute a patch or layer of considerable thickness. Their direction at the upper third of the bladder posteriorly, and at the posterior aspect of the cervix, is nearly transverse, owing to the large curves made by the terminal portions of the loops. In those situations they form, with incidental or accessory fibres which are developed between them, part of the so-called transverse or circular layer of the older anatomists. Turning to the posterior surface, a similar disposition of the fibres can be made out. Here, however, owing, as has been already explained, to the greater posterior measurement of the bladder, the looped arrangement is not so obvious (Plate III. figs. 4 & 6, *u v w x*). That the fibres diverge from the mesial line in an upward direction is at once apparent, but that they cross and diverge in a downward direction is not so evident. Sabatier* was of opinion that they did not cross, and has represented them as coming together and separating as in the italic letter *x*. The crossing, however, as I have abundantly satisfied myself, does occur. Its site corresponds to a point in the mesial line about two inches above the base of the prostate.

The external fibres forming the third layer or patch posteriorly may be briefly described. They proceed from the left of the mesial line anteriorly, and curve in a downward and backward spiral direction until they reach the point indicated. Continuing their downward course, they cross the median line posteriorly, and curve round on the cervix anteriorly, where they assist in forming the anterior half of the sphincter vesicæ (Plate III. fig. 1, *h i*). They reappear on the posterior surface, recross the posterior median line, and curve in an upward and forward spiral direction until they reach the right of the median line anteriorly whence they set out. Their course is traced in Plate V. diagram 3, *u v w x*.

The posterior fibres of the third layer in this manner form figure-of-eight loops like the anterior ones; the loops of either set being so arranged that the terminal portions of each appear either on the anterior or posterior surface. They are continued forwards upon the dorsal and ventral surfaces of the urethra (Plate V. diagram 8. *s*), and likewise upon the dorsal and ventral surfaces of the prostate (*s*). This disposition of the fibres imparts a highly symmetrical appearance to the bladder, the terminal loops being arranged in two sets at the apex (Plate IV. fig. 15) and cervix (fig. 17), rendering it strikingly bilateral in these directions. I am axious to be explicit on this point, as it

* *Op. cit.*

bears upon the true structure of the sphincter vesicæ, which has up till the present been regarded as consisting of circular fibres instead of two sets of very oblique spiral fibres intersecting each other at two fixed points, those points corresponding to the right and left aspects of the cervix.

I have chosen to speak of anterior and posterior looped fibres, but this description, as will readily be perceived, is purely arbitrary, the fibres which begin on the posterior surface curving round and crossing each other anteriorly and the reverse. The method followed has been suggested by the direction and the crossing of the fibres, as well as by the position of the fibres in the parietes; it so happening that those portions of the fibres which form the terminal loops occupy a deeper situation than that occupied by the same fibres where they cross. By considering the fibres as anterior which cross anteriorly, and *vice versâ*, we are enabled to compare and contrast the direction of the fibres on the anterior, posterior, and lateral aspects of the bladder without in any way obscuring or interfering with their continuity. As it is to the direction pursued by the fibres that we must look for an explanation of the manner in which the bladder contracts, the plan adopted has many advantages,

The anterior and posterior fibres of the third layer, whose upper terminal loops spread out on the upper third of the bladder anteriorly and posteriorly, and whose lower terminal loops converge to assist in the formation of the sphincter, form two twisted slings (Plate III. fig. 8, *h g*, *o q*), the influence exercised by which in dragging the summit of the bladder towards the cervix or fixed point must be very considerable. The anterior and posterior sets of fibres which spread out on the summit, in addition to being continuous upon themselves to form loops, are continuous with each other on the upper third of the lateral aspects of the bladder. In these situations they unite with considerable regularity, after the manner of a groined arch (Plate III. fig. 6, *q l*), so that they form a kind of hood or capsule, which envelopes the apex. The fibres of the anterior and posterior sets interchange slightly with the fibres which are above and beneath them.

Two sets of fibres corresponding in their essential features to the anterior and posterior sets of fibres just described, may by a little careful examination be detected on the sides of the viscus. The fibres in question are feebly developed when compared with the anterior and posterior ones; but their distribution and general configuration is the same. Thus, if we trace the fibres from the median line intersecting the left half of the bladder, we shall find that they wind round anteriorly in a downward spiral direction, and cross that which intersects the right half at a point nearly midway between the apex and fundus (Plate III. fig. 9, *q r h g*). Pursuing their downward spiral course, they curve round the left side of the fundus, and reverse their direction to reappear on the right side. They then proceed in an upward spiral direction, recross the mesial line of the right side, and curve round until they regain that on the left, thus completing the figure of eight. These two sets of lateral fibres spread out on the anterior and posterior aspects of the upper third of the bladder, their terminal loops confining them-

selves to the right and left sides respectively. They converge at the cervix to assist in forming the sphincter, as explained.

*Fourth, or central layer, consisting of fibres arranged in figure-of-eight loops, crushed together to produce the maximum of lateral distention.*—Returning, for a fourth time, to the anterior aspect, we discover a still deeper and more oblique set of fibres. These, like their predecessors, form figure-of-eight loops; the loops in this instance being flattened from above downwards, and from below upwards. Proceeding with the description as before, we trace the fibres from the left of the posterior median line round upon the left side (Plate III. fig. 1, *k*), then on the anterior aspect, then on the right side (*l*), and onward to the posterior of the cervix, where they reverse their course to proceed in an upward and very oblique spiral direction (*m n*). They ultimately reach the point from which they started, and in this way become continuous at the apex and base. Their course is indicated at Plate V. diag. 1, *k l m n*. These fibres spread out on the lower two thirds of the bladder posteriorly, and form a layer of great thickness. They thus complete, by their flattened terminal loops, the lower portion of the so-called circular layer on this aspect (Plate III. fig. 5, *y z*; fig. 12, *k l*). They converge on the sides, and on reaching the anterior mesial line at a point half an inch or so above the cervix, are plainly seen to cross each other. On the posterior aspect of the cervix they are crowded together, and are principally concerned in forming THE POSTERIOR LIP OF THE SPHINCTER VESICÆ (Plate III. fig. 4, *y y′*, *z z′*; Plate V. diagram 4, *z z z*, *y′ y′ y′*). Many of them are prolonged on the ventral aspect of the urethra (Plate III. fig. 9, *m*), especially its prostatic portion, while others proceed to the prostate itself (Plate V. diag. 8, *g g′*). Turning to the posterior aspect, a corresponding set of fibres is met with (Plate V. fig. 4, *y y′*, *z z′*). These fibres come from the mesial line anteriorly, and curve round on the left side in a downward and very oblique spiral direction until they reach the mesial line posteriorly. They then proceed to the right, and curve round the cervix anteriorly, so as to change their course, and return in an opposite and upward direction to where we began to trace them. These fibres are greatly developed, and in their turn spread out on the lower two thirds of the anterior aspect, to complete what was wanting of the circular layer in this direction (Plate III. fig. 3, *k l*). They converge at the apex anteriorly, and are largely concerned in the formation of THE ANTERIOR LIP OF THE SPHINCTER (Plate III. fig. 1, *k l m n*; Plate V. diagram 2, *m m′ m″*, *l l′ l″*). Many of them are continued forwards on the dorsum of the urethra, and form, with corresponding fibres found on the ventral aspect, the so-called circular layer of the prostatic and membranous portions of the urethra. The lower portions of the bladder, the sphincter, and the prostatic and membranous portions of the urethra are thus plainly bilateral.

Two sets of similar, though less fully developed fibres occur on the sides of the bladder (Plate III. fig. 9, *m n*), and contribute to the formation of the lower portion of the central layer, sphincter, and urethral canal in these directions. It will be evident from this description that the so-called central or circular layer is not composed of circular fibres having a uniform direction, as figured by SABATIER, ELLIS, and others, but of very oblique

spiral fibres crossing at very obtuse angles, mixed up with the greatly expanded terminal portions of the loops forming the several layers. It ought, however, to be mentioned that a certain number of incidental or accessory fibres is developed between the crossings formed by the very oblique fibres which confer upon the layer a certain degree of uniformity. The layer in question is seen anteriorly at Plate III. figs. 3 & 11, *k l*, posteriorly at figs. 5 & 12, *y z' k l*, and laterally at fig 9, *m n*. It is traced in outline at Plate V. diagram 2, *k n*, *m l*; diagram 4, *y z*; diagram 6, *k n*, *m l*, and diagram 10, *y l*. The fibres forming the great central layer are united to each other where they intersect; they also interchange fibres with the layers to the outside and inside of them.

INTERNAL FIBRES.

*Comprising layers* 5, 6, *and* 7 (*imperfect*).—If the central layer be removed, which, owing to its great thickness and the tenuity of the internal fibres, is a matter of difficulty, it will be seen that the fibres to the inside of it, or the internal fibres proper, are arranged very much in the same way as the external fibres already described. The internal fibres are best seen when the bladder is inverted and the dissection conducted from within. They are, as compared with the external fibres, rudimentary and scantily developed. They do not, however, pursue the simply longitudinal course usually attributed to them. On the contrary, their direction is varied as in the external fibres. Thus those occurring in the mesial line anteriorly (Plate III. fig. 10, *a b*) and posteriorly (fig. 12, *o o*, *p p*) are more or less vertical, while those to the outside of them or nearer the central layer, diverge from the line in question, and show a decided tendency to the looped arrangement (Plate III. fig. 10, *c d e f*); those still deeper making unmistakeable spiral curves (*g h i j*) analogous to those made by corresponding external fibres. The same remarks, if allowance be made for their more fragmentary condition, may be made regarding the course and distribution of the internal fibres occurring on the sides of the bladder. The various sets of internal fibres cross each other like the external ones, the more longitudinal fibres at acute angles, the more circular or deeper ones at obtuse angles. They are continued on the ureters and urethra, and in the latter situation, owing to the smaller space occupied by them, are more plentiful (Plate III. fig. 10, *b*). Their general distribution in the interior of the urethra is given in the section headed *trigone* further on. The internal fibres of the bladder in some instances are so straggling that they form a network of large meshes. They are united by accessory slips to corresponding external ones, and the external in some instances originate the internal as in the heart. The internal fibres are so delicate in comparison with the fibres of the great central layer (Plate III. fig. 12, *k l*), that in the hypertrophied bladder the circular ones tear them asunder so that they partially or altogether disappear.

*Distribution of the external and internal fibres at the apex and fundus.*—The various directions pursued by the external and internal fibres on the walls of the bladder generally are readily made out at the apex and fundus, both from without and from

within. In fact if a cylinder composed of external and internal longitudinal slightly oblique, oblique, and very oblique fibres be constricted at either end, the general scheme of the arrangement of the fibres at the summit and base is at once apparent. The longitudinal or vertical fibres, particularly the external ones, bend over and unite, to form a crucial arrangement, the anterior joining with the posterior and the lateral with each other (Plate IV. fig. 16, *a o*, *k l*; fig. 18, *b p*, *m n*). The slightly oblique and oblique fibres in like manner bend over and unite, those because of their obliquity occasioning a stellate arrangement, which causes at the apex a corresponding thickening of the wall (Plate IV. figs. 13 & 15, *x*), and at the cervix a thickening and funnel-shaped puckering, which largely contributes to the closure of the urethral orifice (Plate IV. figs. 14 & 17, *z*, *y*). The very oblique fibres, because of the greater size of the loops formed by their junction, furnish what are known as the circular fibres, and are continued on the body of the viscus generally (Plate IV. figs. 13, 15, & 16, *k l*, and figs. 14, 17 & 18, *m n*). The circular fibres, which are plentifully developed at the cervix, where they contribute largely to the formation of the sphincter, occur to the outside of the internal stellate fibres, and, with the elasticity natural to the parts, are principally concerned in the closure of the urethral orifice. This orifice, it may be remarked, is as impervious as the urachus itself when the urine is not actually passing through it, a considerable degree of pressure being necessary, even in the dead bladder, to force a passage. As might be supposed from this explanation, the apex and base, if allowance be made for the apertures of the ureters and urethra and the expanded condition of the viscus in the latter direction, are structurally identical. This is particularly evident when the apex and base are rendered transparent, and compared by being held against the light. When viewed from within, the various sets of vertical, slightly oblique, oblique, and very oblique fibres can be made out without difficulty. The crossing of the oblique and very oblique fibres concerned in the formation of the central layer is especially evident. These fibres, in conjunction with the accessory fibres developed between them, form tolerably perfect circles or rings, which invest the summit and base, particularly that portion of the base which corresponds with the cervix (Plate IV. fig. 17, *z y*). I draw attention to this arrangement because of the symmetry it everywhere secures, and because in no work with which I am acquainted has the arrangement of the fibres at the apex and base been either described or figured. The fibres in question are shown at Plate IV. figs. 13, 14, 15, 16, 17 & 18.

### TRIGONE.

*Uvula, verumontanum; ureters and urethra—closure of, &c.*—The only points requiring further elucidation pertain to the ureters and urethra, and that triangular space which occurs between them familiarly known as the trigone (Plate V. diagram 7, *z v s*). The space referred to has received a large share of attention, partly on account of its surgical importance, and partly because many authors suppose that the fibres and tissues composing it are specially constructed, and have particular functions assigned to them.

Sabatier and Bell, it will be remembered, were of opinion that the trigone and luette are the most sensible parts of the bladder; and Bell* and Guthrie† agreed in assigning to those parts a separate and increased nervous supply. Morgagni‡, Santorini§, and Lieutaud, as was explained, described two fleshy bodies which run from the uretral orifices to the verumontanum, those bodies being subsquently, though wrongly, described by Bell‖ as separate muscles.

Without entering into too great detail, it may be stated in a general way that the fibres of the trigone are very similarly arranged to those of corresponding internal fibres anywhere else at the fundus (Plate IV. figs. 14 & 17, *a*, *r*); the only difference being that they are more mixed up with fibrous tissue, the fibrous tissue causing a matting and thickening of the vesical parietes in this situation. The following account is intended to show how the ureters enter the bladder, and how the fibres of the trigone are continued into the uvula and median ridge of the female, and the caput gallinaginis or verumontanum of the male (Plate V. diagram 7, *s*); those ridges playing an important part in the closure of the urethra.

The ureters, as is well known, enter the walls of the bladder obliquely on its posterior aspect (Plate III. fig. 6, *a*). They enter midway between the posterior longitudinal and lateral longitudinal fibres at a point an inch and a half or so from the mesial line, and about the same distance from the base of the prostate. They are crossed externally and internally by the longitudinal slightly oblique, oblique, and very oblique fibres of the bladder, from all of which they receive accessions (Plate IV. fig. 14). The very oblique fibres, which are by much the strongest, run nearly at right angles to the longitudinal fibres of the ureters. Those portions of the ureters (*a*, *a*) which lay within the vesical parietes are therefore invested with fibres from the bladder, analogous to those surrounding the prostatic portion of the urethra. The ureters, in virtue of this arrangement, are more or less under the influence of the fibres of the walls of the bladder in their immediate vicinity, the longitudinal and slightly oblique ones tending to obliterate the uretral canals during contraction by the thickening they undergo, the oblique fibres plaiting above, beneath, and around, and closing them from without inwards or centripetally. The closure is aided by the elasticity of the parts and the great obliquity of the uretral canals, those portions of the parietes which correspond with the track of the canals, particularly where the ureters open, being exceedingly thin, and acting in those situations as a moveable partition or valve. The valve in question responds to the

* "Account of the Muscles of the Ureters, and their effects on the irritable states of the Bladder." By Charles Bell, Esq.," Med. Chir. Trans. vol. iii. 1812.

† On the Anatomy and Diseases of the Urinary Organs and Sexual Organs. By G. J. Guthrie, Esq., F.R.S. 3rd. edit. London, 1843.

‡ Morgagni, Adversaria, i. n. 9, Adversaria iii. Animadver xlii.

§ Observationes Anatomicæ, cap. x. sec. xxi.

‖ A Treatise on the Diseases of the Urethra, Vesica urinaria, Prostate, and Rectum. By Charles Bell, Esq., 3rd edit. with notes by John Shaw, Esq. London, 1822.

slightest pressure, and resembles in some respects that found at the mouths of sinuses and in the smaller veins*.

The ureters enter the bladder obliquely in a twofold sense, viz., from above downwards, and from without inwards with regard to the posterior mesial line, and from above downwards and from without inwards with regard to the walls of the bladder itself. The degree of obliquity varies according to the degree of distention. In the flaccid or undistended bladder the ureters are directed towards each other at a considerable angle (Plate IV. fig. 14, *x x*), and the extent of wall traversed by them from the time they enter the bladder until they reach its interior, equals, as nearly as may be, half an inch. In the distended state, on the other hand, the ureters come together in a nearly straight line (Plate IV. fig. 18, *x x*), and the extent of wall traversed is increased to three-fourths of an inch, or thereabouts. The openings of the ureters in the flaccid bladder are separated by an interval, which varies in different bladders, and in the same bladders according to the degree of distension. Usually it is about an inch and a quarter in the flaccid bladder, and from an inch and a half to two inches in the distended one. Although the openings of the ureters are thus widely apart, it does not follow that the ureters, or rather the muscular fibres composing them, terminate where the openings occur. On the contrary, the muscular fibres of the ureters are continued between the uretral openings, and are as strongly pronounced in the interior of the bladder as they are on the exterior and within the walls. There is in fact no breach of continuity in the muscular fibres of the ureters within the bladder, and the two ureters unite in the mesial line to form a strong loop or girder in which the viscus may be said to be suspended or slung. This arrangement prevents puckering of the coats of the bladder at the points where the ureters open, and secures to those portions of the ureters imbedded in the walls of the bladder a relative and definite position. The continuity referred to is best seen when the preparation is rendered transparent and held against the light. When so viewed the ureters (continued into each other) appear as a strong dark band, which is as distinctly pronounced between the orifices of the ureters as within the walls of the bladder itself (Plate IV. figs. 14, 17, & 18, *x x*; Plate V. diagram 7, *v z*). In addition to being continuous with each other, the fibres of the ureters, as was satisfactorily shown by Hodgson† and Ellis‡, are continuous with those of the neck of the bladder and the urethra. The fibres which connect the ureters with those investing the urethral canal, converge in a downward direction, those which are nearest the median line (Plate V. diagram 7, *a a'*) coming together and crossing at very acute angles (*c c*), those which are deeper coming together and crossing at wider angles (*g g*), the deepest having a nearly transverse direction, as in the so-called central layer

* *Vide* paper by the Author "On the Relations, Structure, and Functions of the Valves of the Vascular System in Vertebrata," Trans. Roy. Soc. Edin. vol. xxiii. part 3. p. 763.

† The Prostate Gland and its Enlargement in Old Age. By Decimus Hodgson, M.D. Edin. London, 1856.

‡ "An Account of the arrangement of the Muscular Substance in the Urinary and certain of the Generative Organs of the Human Body. By George Viner Ellis," Med.-Chir. Trans. vol. xxxix.

(*k k*). The superficial and median fibres are continued into, and are principally concerned in the structure of the uvula and verumontanum (*s*). The deeper oblique fibres, which also assist in forming those structures, are mixed up with the very oblique or circular fibres of the prostate (Plate IV. figs. 25 & 26, *n*). They converge towards and cross at the verumontanum as in the letter X, so that the sinus pocularis is surrounded by fibres which radiate in every direction. This arrangement is useful in maintaining the relative position of the glandular ducts which open at this point. The verumontanum, as will be seen from this description, is essentially a muscular structure. It however contains, as KOBELT pointed out, a small quantity of erectile tissue, and in this many of the fibres terminate. The uvula and median ridge in the female and the caput gallinaginis in the male are analogous in structure, and both are connected with the ureters at their junction in the median line. While, therefore, the ureters are continuous with and drag upon each other directly (Plate IV. diagram 7, *v z*), they are continuous with and drag upon the median ridge (*s'*) and verumontanum (*s*) indirectly. This is important, as the ureters act against each other, and the two together tend to elevate or raise the median ridge and verumontanum during contraction. The shape of the verumontanum, on which its uses to a certain extent depend, is that of an inverted pyramid, the base of the pyramid being turned towards the base of the prostate. Its narrow end is consequently directed downwards and forwards. It is attached by one side of the pyramid to the posterior wall of the prostatic portion of the urethra, the two sides which are free terminating in a well pronounced crest. As the prostatic portion of the urethral canal is triangular in form (Plate IV. diagrams 12, 13, 14, 15, 16, & 17) and fitted upon or to the verumontanum so closely that water cannot be passed even in the dead bladder without exercising a considerable degree of pressure, it is not difficult to perceive that in the living organism, when the parts are injected with blood, the obliteration must be very complete. The urine moreover by its own weight will tend to force the wedge formed by the verumontanum in a downward direction, the circular fibres of the sphincter and its own structure and connexions confining the wedging within certain limits. When the bladder contracts, the longitudinal fibres, which connect the verumontanum with the ureters where they meet in the mesial line, have the effect of elevating or withdrawing the wedge and thus assist in rendering the orifice of the urethra patent*. SABATIER speaks of the verumontanum as the gate-keeper of the prostatic portion of the urethra.

In giving this explanation of the action of the verumontanum, I am aware that the office hitherto assigned to it is that of checking the reflux of the seminal fluid into the bladder. The semen, however, is passed so seldom when compared with the urine, that this must be regarded as a secondary rather than a primary function. In the female, moreover, where no corresponding action can be performed, a median ridge or modified verumontanum can be detected. The uneasy feeling experienced by the patient when

* The longitudinal fibres referred to are fully an inch and a half in length, so that their elevating power must be very considerable.

the catheter is being passed through the prostatic portion of the urethra, and which is not felt by the female, is owing probably not so much to the supposed heightened sensibility of the parts as to mechanical obstruction and entanglement. That the pain in question is referable to the region of the verumontanum and not to the trigone, as is commonly believed, seems certain from a fact first stated by GUTHRIE*, that in the healthy bladder even when moderately distended the triangular space descends so as to be beyond the reach of the catheter. The extreme sensitiveness of the trigone in disease, or when irritated by the presence of a stone, is moreover no proof of its increased sensibility in the normal condition of the parts. If the trigone, as SABATIER, BELL, and others affirm, be so delicately sensitive, it is difficult to understand how the urine, which naturally collects in the neck of the bladder, is not expelled almost as soon as it is received. BELL and GUTHRIE lay considerable emphasis upon the unusually large supply of nerves to this part, but repeated and very careful dissections of the nerves of the bladder, not only in man but in the ox, sheep, monkey, and other animals, induce me to dissent from their views. The supply to the trigone in no way exceeds that to the parts surrounding the neck of the bladder generally. The nerves and likewise the blood-vessels are more numerous at the neck and fundus than they are towards the apex and higher up, but the distribution is uniform, and consists of a complete network, which extends itself over the external and internal surfaces and within the walls. The network, I may observe, is remarkable for the immense number of ganglia it everywhere displays, these in some instances, particularly on the sides of the bladder, being exceedingly large. In the ox I have found them of the size of a small millet seed, and in man and in the monkey they are correspondingly developed.

*Muscles of* BELL.—The muscular fibres which run between the orifices of the ureters and urethra, and which form the lateral boundaries of the trigone, had undue prominence assigned them by BELL†, who described them as separate structures under the title of muscles of the ureters. These muscles, he says, are inserted by tendons into the middle lobe of the prostate, and their function is to preserve the due obliquity of the orifices of the ureters. They have, however, neither the insertion nor function indicated. They are continued into the verumontanum and urethra, and apart from the other fibres of the trigone have no existence. The obliquity of the ureters, moreover, as has been shown, is primarily secured by the fibres of the ureters being continued into each other within the bladder. Sir CHARLES seems to have been misled by dissecting hypertrophied bladders from without, and by supposing that the coat of the bladder which contracts is on the outside of the oblique passages of the ureters, an arrangement necessitating, as he thought, some counteracting power on the inside to draw down the

* On the Anatomy and Diseases of the Urinary Organs and Sexual Organs. By G. J. GUTHRIE, F.R.S. 3rd edit. Lond. 1843, p. 6.

† "Account of the Muscles of the Ureters and their effects on the irritable states of the Bladder. By CHARLES BELL, Esq." Med. Chir. Trans. vol. iii. 1812.

uretral orifices. The ureters, however, as the reader is aware, are in reality situated within the walls of the bladder, and are grasped alike by the external and internal fibres (Plate V. figs. 14 & 17, *z*, *z*), an arrangement which dispenses with the necessity for special muscles. What therefore was delegated to separate structures is more conveniently and effectively performed by the fibres of the bladder itself.

The longitudinal and oblique fibres occurring between the muscles of Bell, and which enter principally into the formatian of the uvula and median ridge of the female and the verumontanum of the male, are continued forwards on the urethra of the latter. They are variously disposed. At the apex of the prostate they diverge and bifurcate to a greater or less degree. On the membranous portion of the canal some are oblique and some straight. Further forwards they diverge and then converge to form an oval patch corresponding to the posterior third of the spongy portion; they subsequently diverge in the direction of the glans penis, where they spread out to embrace the fossa navicularis. Here they apparently terminate in loops.

### RECAPITULATION.

The points which I have sought more especially to establish in the present memoir are the following:—

I have endeavoured to show that the fibres of the bladder are spiral continuous fibres arranged for the most part in the form of figure-of-eight loops, the loops being directed towards and embracing the urachus and urethra respectively.

The fibres distribute themselves on the anterior, posterior, and lateral surfaces, and are divisible into seven layers or strata, which are more or less perfect, viz. three external, a fourth or central, and three internal. The fibres of the first and seventh layers (the most external and most internal) pursue a nearly vertical direction and are feebly developed; the fibres of the second and sixth layers, which are stronger and occupy a deeper situation, running in a slightly oblique spiral direction and crossing at acute angles; those of the third and fifth layers, which are still stronger and deeper than any of the others, running in a spiral oblique direction and crossing at obtuse angles. The fibres of the fourth or central layer pursue a very oblique spiral course, and cross at such obtuse angles as to have been up till the present regarded as circular fibres. The fibres in this manner increase in strength and in obliquity, both from without and from within, and form by their interlacings a structure remarkable alike for its complexity and its beauty.

The apex and base are structurally identical, and consist of longitudinal, slightly oblique, oblique, and very oblique or circular external and internal fibres, crossing and interlacing as in the other portions of the vesical parietes.

The longitudinal or vertical fibres have a crucial arrangement at the apex and base, and the slightly oblique ones are drawn together at the urachus and cervix by the constrictions which in the embryo separate the bladder from the allantois and urethra. This stellate arrangement occasions a thickening of the walls of the bladder at the points indicated, and renders the bladder impervious in both directions; the urethra, unless

when the urine is actually passing through it, being perfectly closed. This is interesting, as it shows how an orifice patent only at long intervals assimilates itself to one, viz. the urachus, closed at birth. The closure of the urethra is favoured by the contraction of the very oblique or circular fibres forming the sphincter, and by the prominence of the uvula vesicæ (luette vésicale) and median ridge in the female and the caput gallinaginis or verumontanum in the male. The longitudinal, slightly oblique, oblique and very oblique external and internal fibres are continued into the prostatic portion of the urethra, so that the urethra is to be regarded as the proper continuation of the bladder in an antero-posterior direction. It ought, however, to be mentioned that the four outer layers of the bladder split up or bifurcate at the cervix, the one half going to the external or outer half of the urethra, the other to the prostate. There is consequently no portion of the bladder, urethra, or prostate in which longitudinal, slightly oblique, oblique, and very oblique or circular fibres may not be found.

The longitudinal fibres of the prostate and urethra are separated by a considerable interval, and the very oblique or circular fibres, which are widely distinct and have separate axes at the cervix where the sphincter is most fully developed, curve into and are blended with each other in the region of the verumontanum. This is important, as it shows how the sphincter may act independently of the prostate, and the reverse.

The very oblique or circular fibres have been specially described from the fact of their entering largely into the formation of the bladder, urethra, and prostate, and because they are principally concerned in the formation of the sphincter.

The sphincter vesicæ, the existence of which has been doubted, is composed of an anterior and posterior set of oblique and very oblique or circular fibres which is largely developed, and by a right and left lateral set which is accessory and less fully developed.

The fibres of the sphincter are continuous with the oblique and very oblique spiral fibres of the urethra and bladder generally, and this circumstance, more than any other, has induced anatomists to deny its presence. As well, however, might we argue against the existence of a sphincter in the stomach or rectum, for in both of those cases, as is well known, there is continuity of structure. The fibres of the trigone are similarly arranged to the other internal fibres at the fundus, the very oblique or circular ones passing across between the uretral orifices to blend with the fibres of the ureters themselves, while the olique, slightly oblique, and vertical pass in a downward direction and converge prior to reaching the verumontanum, where they cross, and for the most part terminate. The verumontanum is thus directly connected with the fibres of the trigone, and indirectly with the internal fibres of the cervix generally. During the distended or passive condition of the bladder it acts in a downward direction as a mechanical wedge, and with the aid of the sphincter completely occludes the passage of the urethra. In the active state, or when the urine is being expelled, the verumontanum is elevated or withdrawn by the contraction of the more vertical fibres of the trigone, the other vertical fibres of the fundus acting in harmony and elevating, and in this manner opening up, dilating, or expanding the funnel-shaped cavity within the

sphincter, the sphincter relaxing simultaneously and affording a clear channel for the escape of the fluid contents of the bladder*.

The ureters enter the vesical parietes at a very obtuse angle, and the angle increases according to the degree of distention of the bladder. They receive accessions of fibres from the longitudinal, slightly oblique, oblique, and very oblique external and internal fibres in their vicinity, and are continued upon each other within the bladder in the form of a strong transverse band. The transverse band which connects the ureters together within the bladder, or between the uretral orifices, is equal in volume to the ureters themselves within the vesical parietes. It is best seen when the base of the bladder is detached and held against the light, and seems to be formed by a partial obliteration of the uretral tubes.

The uretral canals seek the internal surface of the bladder even more obliquely than the ureters, and the inner surfaces of the ureters become so thin, particularly towards the uretral orifices, that they act mechanically as moveable partitions or valves, as in the smaller veins. The canals of the ureters are consequently closed, partly by the contractions of the muscular walls, and partly by the mechanical pressure exercised by the urine about to be expelled.

From the foregoing *résumé* it will be evident that the various sets of external and internal fibres forming the bladder, urethra, and prostate are arranged so as to coordinate each other, the loops formed by the anterior fibres crossing each other at more or less acute angles according to their depths, the anterior fibres, as a whole, crossing the posterior or homologous fibres as a whole. While, therefore, the fibres, in virtue of their twisted looped arrangement, coordinate each other individually, the aggregation of the fibres in any one region coordinate a similar aggregation of fibres at an opposite point, the anterior fibres, *e. g.*, acting on the posterior, and the right lateral upon the left lateral†. This arrangement, which is productive of great strength, ensures that the external and internal fibres shall act in unison or together, and fully explains the views of the older anatomists, who described the bladder as consisting of fibres crossing in every direction, and forming an intricate network. It likewise accords with the more modern opinion, that the fibres of the bladder may be divided into strata or layers.

It is difficult to estimate the precise effect which the twisted looped arrangement of the fibres may have on the contraction of the bladder; but the fibres are disposed so

* The structures which take part in the expulsion of the urine have been tabulated by Sir Charles Bell as follows:—"The proper internal sphincter of the bladder, the compressor prostatæ, the levator ani, the levator or compressor urethræ of Mr. Wilson, the ejaculator seminis, the internal and oblique perinal muscles. These, he says, are of the class of sphincter muscles, their opponents being the detrusor urinæ or muscular coat of the bladder (and in consent with it), the abdominal muscles and diaphragm." This author, it will be observed, makes no mention of the verumontanum.

† The principle here foreshadowed seems to attain its full development in the voluntary system of muscles where the extensors coordinate the flexors, the abductors the adductors, the pronators the supinators, &c.

symmetrically, and so nicely balanced as regards length, strength, and direction, that I am of opinion the order of contraction is very precise and well defined. I am further inclined to believe that the fibres of the several layers contract towards certain points, the more longitudinal anterior, posterior, and lateral fibres contracting from above downwards in the direction of the urethra so as to approximate the apex and fundus, the slightly oblique, oblique, and very oblique spiral fibres contracting towards the points where they cross each other, viz. towards the anterior, posterior, and right and left lateral raphe. That this centripetal and downward action of the oblique fibres towards the points of intersection takes place is rendered probable by the fact, that in the contracted bladder the anterior, posterior, and lateral ridges (where they exist) are thrown into bold relief*, the sides of the viscus in some instances coming together so completely that their mucous surfaces adhere. In such cases, if any urine be present, it is, as a rule, confined to the immediate vicinity of the uretral orifices, and not diffused throughout the cavity of the bladder generally, as it would most likely be if there were not a strong persistent lateral action.

## Explanation of Plates.

### PLATE III.

Fig. 1. Anterior view of young adult male bladder, showing longitudinal (*a b*), slightly oblique (*c d e f*), oblique (*g h i j*), and very oblique (*k l m n*) spiral figure-of-eight fibres, as seen in layers 1, 2, 3, & 4.
*x*. Urachus.

Fig. 2. Anterior view of adult female bladder. Shows slightly oblique (*c d e f*) and oblique (*g h i j*) spiral figure-of-eight fibres, with a few oval fibres, as observed in layers 2 & 3.
*x*. Urachus.

Fig. 3. Anterior view of adult female bladder (walls rendered transparent). Shows longitudinal and slightly oblique spiral figure-of-eight fibres mixed up (*a b*), as seen in layers 1 & 2. Likewise the very oblique spiral fibres commonly regarded as circular (*k l*), forming the fourth or central layer.

Fig. 4. Posterior view of young adult male bladder, showing longitudinal (*o p*), slightly oblique (*q t*), oblique (*u v w x*), and very oblique (*y y′, z z′*) spiral figure-of-eight fibres, as seen in layers 1, 2, 3, & 4.
*z, z*. Portions of ureters.

Fig. 5. Posterior view of adult female bladder (transparent). Shows longitudinal (*o p*), slightly oblique (*q t*), and very oblique or circular fibres (*y z*), as seen in layers 1, 2, & 4.

Fig. 6. Posterior view of adult male bladder. Shows longitudinal (*o*), slightly oblique

* *Vide* Plate IV. figs. 34, 36, & 38, *a b*, *o p*, *s t*.

($q$ $t$), oblique ($u$ $v$ $w$ $x$), and very oblique or circular fibres ($y$), as seen in layers 1, 2, 3, & 4.

$z$ $z$ Portions of ureters.

Fig. 7. Left lateral view of young adult male bladder, showing longitudinal ($a$ $b$, $o$ $p$), slightly oblique ($c$ $d$), oblique ($g$ $i$ $h$), and very oblique ($k$ $l$ $m$ $n$) spiral figure-of-eight fibres, as seen in layers 1, 2, 3, & 4.

$x$. Urachus.

Fig. 8. Right lateral view of adult male bladder, showing longitudinal ($a$ $b$, $o$ $p$), slightly oblique ($q$), oblique ($g$ $g$, $h$ $h$), and very oblique spiral fibres ($n$), as seen in layers 1, 2, 3, & 4.

$z$ Portion of ureter.

$x$. Urachus.

Fig. 9. Left lateral view of adult male bladder (transparent), showing longitudinal ($a$ $b$), oblique ($q$ $q$, $r$ $r$, $h$ $h$, $g$ $g$), and very oblique or circular fibres ($m$ $n$), as seen in layers 1, 3, & 4.

$z$ Portion of ureter.

$x$. Urachus.

Fig. 10. Anterior view of adult male bladder inverted, showing longitudinal ($a$ $b$), slightly oblique ($c$ $d$ $e$ $f$), oblique ($g$ $h$ $i$ $j$), and very oblique ($k$) spiral figure-of-eight fibres, as seen in layers 7, 6, 5, & 4. The internal fibres are fewer in number and more rudimentary than the external ones, but their directions, as a little careful examination will show, are the same.

$x$. Urachus inverted.

Fig. 11. Anterior half of adult male bladder, seen from within (transparent). Shows longitudinal ($a$) and very oblique or circular fibres, forming the fourth or central layer ($k$ $l$); also the continuations of those fibres in a downward direction towards the cervix, where they are arranged in two sets ($m$ $m'$, $y$ $y'$), and are principally concerned in the formation of the sphincter vesicæ.

$x$. Urachus, from within.

Fig. 12. Posterior view of adult male bladder inverted. Shows longitudinal ($o$ $o$, $p$ $p$) and very oblique or circular fibres ($k$ $k$, $l$ $l$), as seen in layers 7 & 4.

## PLATE IV.

Fig. 13. Apex of adult male bladder placed upon its posterior surface (transparent). Shows longitudinal ($a$ $o$), slightly oblique ($e$ $f$ $q$ $t$), oblique ($g$ $j$ $u$ $x$), and very oblique or circular fibres ($k$ $l$), similar to those occurring on the walls of the bladder generally. The very oblique fibres are arranged in two sets ($k$ $l$), as at the fundus. Compare with fibres marked $m$ $m'$, $y$ $y'$ in fig. 11, Plate III.

Fig. 14. Fundus and cervix of adult female bladder placed on its posterior surface (transparent). Shows longitudinal ($b$ $p$), slightly oblique ($d$ $e$, $r$ $s$), oblique

(*h i v w*), and very oblique (*m n*) fibres; the latter curving round the urethra (*s*) in two sets (*m* and *n*), and contributing largely to the formation of the sphincter, which is bilaterally symmetrical. The sphincter receives accessions of fibres from the ureters (*a, r*).

*a, a.* Ureters penetrating the walls of the bladder and appearing continuous within it as a strong transverse band.

Fig. 15. Apex of adult male bladder placed on its posterior surface, seen from within, the converse of fig. 13 (transparent). Shows longitudinal (*a o*), slightly oblique (*e f q t*), oblique (*g j u x*), and very oblique fibres (*k l*).

*x*. Thickened stellate patch of fibres corresponding to attachment of urachus, and occasioned probably by the constriction which originally separates the bladder from the allantois.

Fig. 16. Apex of adult male bladder placed on its posterior surface (transparent). Shows anterior (*a*), posterior (*o*), and right (*k*) and left (*l*) longitudinal fibres arranged in a crucial form, with the urachus (*x*) as the central point, and in a minor degree the slightly oblique (*c f q t*), oblique (*g j u x*), and very oblique (*k l*) fibres seen in fig. 13. Compare with similar arrangement in fig. 18, which represents the fundus and cervix of the same bladder.

Fig. 17. Fundus and cervix of adult female bladder placed on its posterior surface, seen from within (tranparent). Shows longitudinal (*b p*), slightly oblique (*d e r s*), oblique (*h i v w*), and very oblique or circular (*m n*) fibres, but principally the longitudinal and circular; also the continuity of the ureters with each other in the mesial line, and with the fibres of the uvula, &c. It likewise shows the trigone (*z, z*) and the funnel-shaped thickening occurring at the cervix, occasioned probably by the constriction which separates the bladder from the urethra.

Fig. 18. Fundus and cervix of adult male bladder placed on its posterior surface (transparent). Shows anterior (*b*), posterior (*p*), and right (*m*) and left (*n*) longitudinal fibres arranged in a crucial form with the prostate (*z*) as a centre; and in a minor degree, the slightly oblique (*d e r s*), oblique (*h i v w*), and very oblique or circular (*m n*) fibres seen in fig. 14. Compare with similar arrangement in fig. 16, which represents the apex of the same bladder.

*z z*. Ureters penetrating the walls of the bladder and appearing continuous within it as a dark transverse band.

Fig. 19. Transverse section of prostate and urethra at cervix (male).

*m*. Very oblique or circular fibres of urethra forming the sphincter. Compare with *m m′ m″*, *l l′ l″* of diagram 2, and *z z z*, *y y′ y″* of diagram 4, Plate V.

Fig. 20. Transverse section of prostate and urethra ½ of an inch from the cervix (male).

*m*. Very oblique or circular fibres of the prostatic portion of the urethra. Compare with *m m′ m″*, *l l′ l″* of diagram 2, and *z z z*, *y y′ y″* of diagram 4, Plate V.

*x*. Oval band of fibres surrounding the ducts of the vesiculæ seminales.

Fig. 21. Transverse section of prostate and urethra ½ an inch from the cervix (male).

*m*. Very oblique or circular fibres of the urethra blending with similar fibres belonging to the prostate (*o*).

*n*. Fibres belonging partly to the urethra and partly to the prostate, radiating in the substance of the gland from the verumontanum (*r*) as a central point.

*x*. Circular band of fibres embracing the ducts of the vesiculæ seminales.

Fig. 22. Transverse section of female urethra near the cervix, showing the very oblique or circular fibres (*m*) constituting the sphincter vesicæ.

*y*. Opening for vessels.

*f f'*. Upper surface of vagina.

Fig. 23. The same, nearer the meatus urinarius.

*m*. Very oblique or circular fibres of the urethra.

*n*. Ditto, in subjacent tissue.

*z f*. Upper and under surface of urethra.

*y*. Opening for vessels.

Fig. 24. Transverse section of prostate and prostatic portion of urethra at base of gland (male).

*m*. Very oblique or circular fibres of urethra where sphincter is most fully developed.

*o*. Corresponding fibres of the prostate. Those fibres are distinct from each other at this point, and are separated by a considerable interval.

*x*. Oval band of fibres surrounding ducts of vesiculæ seminales.

Fig. 25. Transverse section of prostate and prostatic portion of urethra rather more than ½ of an inch from the base (male).

*m*. Very oblique or circular fibres of urethra.

*o*. Very oblique or circular fibres of prostate curving into the verumontanum (*r*), where they blend with the circular and other fibres of the urethra (*m*). The relation existing between the urethra and prostate in this and the succeeding section is of the most intimate description.

Fig. 26. A similar section, rather more than ½ an inch from the base (male).

*m*. Very oblique or circular fibres of the urethra blending with corresponding fibres belonging to the prostate (*o*).

*n*. Fibres which belong partly to the prostate and partly to the urethra, and which radiate from the verumontanum (*r*) as a centre.

Fig. 27. Another and similar section at the apex (male).

*m*. Very oblique or circular fibres of the urethra.

*o*. Corresponding fibres of the prostate. In this section, as in that represented at fig 24, a considerable interval occurs between the two sets of circular fibres.

*g g*. Under surface of prostate.

Fig. 28. Vertical section of fundus of bladder and prostate (male). Shows intimate relation existing between bladder and prostate, and how some of the longitudinal fibres from the anterior wall (*a*) proceed to the dorsal surface of the gland (*i*), some passing through it (*f*) and reaching its ventral surface (*e*). It also shows how some of the longitudinal fibres from the posterior wall (*h*) pass on to the ventral surface (*x*), while others curve in an upward direction to reach the dorsal surface (*i y*).

Fig. 29. A similar vertical section in the vicinity of the urethra, exhibiting a still closer relation of the parts (male).

*a*. Longitudinal fibres from anterior wall of bladder proceeding to dorsum of prostate (*b*) and dorsal surface of urethra (*w*).

*o*. Longitudinal fibres from posterior wall of bladder proceeding to ventral surface of prostate (*p*) and ventral surface of urethra (*w*).

Fig. 30. A third and similar vertical section through centre of prostate and urethra (male). Shows anterior longitudinal fibres diverging at cervix (*r*) and going to dorsum of prostate (*w*) and dorsal surface of urethra.

*s*. Internal longitudinal submucous fibres continued into the urethra. These fibres are independent of those marked (*r*).

*t*. Posterior longitudinal fibres going to ventral surface of urethra.

*v*. Substance of prostate in which is to be found slightly oblique, oblique, and very oblique fibres.

Fig. 31. Vertical section of the parts at the neck of the bladder in the adult female.

*a*. Longitudinal fibres from anterior wall, bifurcating, some proceeding to dorsum of urethra (*b*), others in a downward direction (*l*).

*o*. Longitudinal fibres from posterior wall proceeding to the ventral surface of urethra (*p*), and in an upward direction (*l*).

*m*. Very oblique or circular fibres surrounding urethra.

Fig. 32. Vertical mesial section through cervix of bladder and urethra in adult female. The lettering and description corresponds to that given under fig. 30.

Fig. 33. Horizontal section of cervix and prostate in adult male.

*a*. Longitudinal fibres from right side of bladder, bifurcating, some passing to right side of prostate (*b*), others passing in a lateral direction to left side of bladder (*c*).

*c*. Longitudinal fibres from left side of bladder proceeding to left side of prostate (*d*), and to right side of bladder (*a*). These fibres represent certain of the terminal loops.

*s*. Oblique passage of urethra.

Fig. 34. Right lateral view of bladder of Koala, showing how in some of the lower animals the longitudinal (*a b, o p, s t*) and very oblique or circular fibres (*m*) predominate, and how the longitudinal fibres are thrown into ridges during contraction.

Fig. 35. Horizontal section of cervix and prostate in adult male.

*a.* Longitudinal fibres from right lateral wall of bladder passing across to left lateral aspect of prostate (*d*).

*c.* Longitudinal fibres from left wall of lateral bladder passing to right lateral aspect of prostate (*b*).

*m.* Some of the terminal loops of the posterior figure-of-eight fibres.

*o.* Oblique passage of urethra.

Fig. 36. Right lateral view of heifer's bladder in a state of contraction. Shows anterior (*a b*) and posterior (*o p*) longitudinal fibres raised in the form of ridges. The lateral ridge (*s*) is less strongly marked than those marked *a b*, *o p*.

*m.* Oblique and very oblique or circular fibres mixed up with the longitudinal.

Fig. 37. Horizontal section of cervix and prostate nearer centre of gland (male).

*a.* Longitudinal fibres from right lateral wall of bladder proceeding to right side of prostate (*b*) and across to left side of bladder (*c*).

*c.* Longitudinal fibres from left lateral wall of bladder proceeding to left side of prostate (*d*) and across to right side of bladder (*a*).

*n.* Peculiar stellate arrangement of fibres.

Fig. 38. Posterior view of cat's bladder in the contracted state, showing longitudinal fibres (*a b*, *c d*, *o p*) elevated into ridges; the central posterior ridge (*o p*) having oblique fibres (*g t*) proceeding from beneath it.

## PLATE V.

Diagram 1 represents in outline the various sets of fibres occurring on the anterior aspect of the bladder, as seen in layers 1, 2, 3, & 4.

*a b.* Longitudinal or vertical fibres forming layer 1.

*c d e f.* Slightly oblique spiral figure-of-eight fibres embracing urachus (*x*) and urethra (*b*) posteriorly and forming layer 2.

*g h i j.* Oblique spiral figure-of-eight fibres embracing upper third of bladder and lower portion of cervix posteriorly and forming layer 3.

*k l m n.* Very oblique spiral figure-of-eight fibres embracing lower two-thirds of bladder and upper portion of cervix posteriorly, and forming the fourth or central layer. The fibres of this layer enter principally into the formation of the sphincter, and, contrary to the received opinion, cross each other at very obtuse vertical angles.

Diagram 2 shows the same as diagram 1, and, in addition, the manner in which the sphincter and fourth or circular layer is formed posteriorly.

*g j.* Terminal expansion or loop representing the spiral oblique fibres which spread out on the upper third of the bladder posteriorly, and assist in forming the central, transverse, or circular layer in this direction. Its

concomitant or companion loop embraces the cervix posteriorly, and assists in forming the posterior lip of the sphincter vesicæ.

*k n*, *k' n'*, *k'' n''*. Terminal expansions or loops representing very oblique spiral figure-of-eight fibres, forming the lower two-thirds of the fourth or central layer posteriorly. Their companion loops occur on the posterior aspect of the cervix, and are principally concerned in the formation of the posterior lip of the sphincter (*m l*, *m' l'*, *m'' l''*). The sphincter and the other portions of the fourth or central layer, as will be seen from this explanation, are not composed of circular fibres as is generally believed.

Diagram 3 represents the various sets of fibres occurring on the posterior aspect of the bladder, as seen in layers 1, 2, 3, & 4.

*o p*. Longitudinal or vertical fibres forming layer 1.

*q r s t*. Slightly oblique spiral figure-of-eight fibres embracing urachus and urethra anteriorly, and forming layer 2.

*u v w x*. Oblique spiral figure-of-eight fibres embracing upper third of bladder and lower portion of cervix anteriorly, and forming layer 3.

*y y' z z'*. Very oblique spiral figure-of-eight fibres embracing lower two-thirds of bladder and upper portion of cervix anteriorly, and forming the fourth or central layer.

Diagram 4 shows the same as diagram 3, and, in addition, the manner in which the sphincter and fourth or central layer is formed anteriorly.

*u x*. Terminal expansion or loop representing the spiral oblique fibres which spread out on the upper third of the bladder anteriorly, and assist in forming the central, transverse, or circular layer in this direction. Its companion loop embraces the cervix anteriorly, and assists in forming the anterior lip of the sphincter.

*y' z*, *y' z'*, *y'' z''*. Terminal expansions or loops representing the very oblique spiral figure-of-eight fibres, forming the lower two-thirds of the fourth or central layer anteriorly. Their companion loops occur on the anterior aspect of the cervix, and are principally concerned in the formation of the sphincter, *z y'*, *z y'*, *z y'*.

Diagram 5 represents the various sets of fibres occurring on the left lateral aspect of the bladder, as seen in layers 1, 2, 3, & 4.

*a b*, *o p*. Anterior and posterior longitudinal fibres, as seen in layer 1.

*c d e*, *s t*. Slightly oblique spiral figure-of-eight fibres embracing urachus (*x*) and urethra, as seen in layer 2.

*g h h'*, *i i' j*. Oblique spiral figure-of-eight fibres spreading out on upper third of right side of bladder and right side of sphincter, as seen in layer 3.

*k l m n*. Very oblique spiral figure-of-eight fibres embracing lower two-thirds of right side of bladder and right side of sphincter.

Diagram 6 represents the manner in which the sphincter and the fourth or central layer is formed on the right side of the bladder.

*k n*, *k n*, *k n*. Terminal expansions or loops formed by the very oblique spiral figure-of-eight fibres which spread out on the right side of the bladder and form the fourth or circular layer in this situation. Their companion loops occur on the left side of the cervix, and contribute to the formation of the sphincter in this situation (*m m m*, *l l l*).

Diagram 7. Horizontal section of fundus, cervix, prostate, and urethra of adult male. Shows relation of ureters to each other, to fibres of trigone, verumontanum, &c.

*v*, *v*. Ureters continued into each other at *a*, and connected more or less directly with the longitudinal (*a'*), slightly oblique (*c c*), oblique (*g g*), and very oblique (*k k*) fibres of the trigone. Those fibres converge towards, and in many instances cross at the verumontanum (*s*), where they terminate. A considerable number, however, are continued downwards on the membranous portion of the urethra (*a''*).

*m*, *m'*. External half of right and left wall of bladder splitting up (*n*, *n'*) a certain proportion of the fibres (*p p'*) going to the prostate gland (*q q'*); the greater number to the urethra (*o o'*). See diagram 8.

Diagram 8. Vertical section of neck of bladder, urethra, and prostate (enlarged). Shows how the urethra is to be regarded as the proper continuation of the bladder in an antero-posterior direction, and how the prostate is formed by the splitting up of the four outer tunics of the bladder.

*v*. Longitudinal fibres of first layer splitting up at cervix—a certain number (*a a'*) investing the prostate (*q*) on its dorsal aspect; some proceeding to the dorsal surface of the urethra (*b b'*), and some (*c'*) to the under or ventral surface of the prostate (*q'*).

*v*. Slightly oblique spiral fibres of second layer splitting up (*d*), and passing into the substance of the prostate (*d' d' d'*) and into the second layer of the urethra (*e e*).

*v*. Oblique spiral fibres of third layer splitting up (*f*); some passing through the prostate (*f' f'*), others proceeding to the third layer of the urethra.

*v*. Cut ends of a portion of the very oblique spiral fibres of fourth layer, forming the so-called circular fibres of the prostate (*g g'*), the remainder occupying the centre of the walls of the urethra.

1, 2 & 3. The three external layers of the bladder and urethra.

4. The central or circular layer. A portion of each of these layers, as has been explained, go to form the prostate.

5, 6, & 7. The three internal layers of the bladder and urethra. These layers are peculiar to the urethra, and are quite distinct from the three external layers forming the prostate and the outer half of the urethra, unless in the region of the verumontanum, where they are more or less blended with them.

Diagram 9 illustrates the structure of the prostatic portion of the urethra, and the relation of the ureters to each other and to the urethra, as seen in the inverted bladder.

*a*. Longitudinal submucous fibres continued on the urethra.

*b*. Ditto, posterior longitudinal fibres.

*c*. Slightly oblique spiral fibres curving forward on the urethra.

*g*. Ditto, oblique spiral fibres.

*z*. Similar fibres from the right ureter.

*v*. Very oblique spiral fibres from the left ureter curving round the urethra (*l*).

*s*. The fibres of the right ureter passing across to join with those of the left in the mesial line (*b*).

Diagram 10 shows the conformation of the fundus and the bilateral nature of the sphincter in the inverted bladder, the prostatic portion of urethra being removed. In this diagram the anterior and posterior sets of fibres only are shown.

*a b*. Anterior and posterior longitudinal submucous fibres.

*l l l*, *l l l*. Terminal expansions or loops of the very oblique spiral fibres forming the fourth layer at the fundus posteriorly, and especially concerned in the formation of the anterior lip of the sphincter (*l'*).

*y y y*, *y y y*. Terminal expansions or loops of the very oblique spiral fibres forming the fourth layer at the fundus anteriorly, and especially concerned in the formation of the posterior lip of the sphincter (*y'*).

*r* and *s*. Spiral oblique fibres from the right and left ureters (*z v*), which give off filaments to assist in the formation of the sphincter (*l' y'*).

Diagrams 11, 12, 13, 14, 15, 16, 17, 18, & 19. Accurate outline sketches of transverse sections of male prostate and urethra, showing the precise shape and degree of obliquity in the urethral canal at different points, and the part which the verumontanum plays in the closure of it. (Dissections preserved.)

*a*. Urethra at base of prostate, oval in shape, and quite open.

*b*. Urethra a little further forward, oval, floor (*j*) slightly elevated.

*c*. Urethra somewhat triangular in shape; the verumontanum (*k*) beginning to project from the floor or base of the triangle.

*d* and *e*. Urethra more decidedly triangular, the verumontanum (*l* and *m*) projecting to such an extent as almost to obliterate the urethral canal.

*f* and *g*. Urethra bayonet-shaped and entirely closed by the projection of the verumontanum (*n*, *o*), which acts at this point as a mechanical wedge.

*h i p q*. Urethra circular in form and again becoming patent.

Diagram 20. Sketch of preparation showing the anterior (*l*) and posterior (*y*) lips of the sphincter at the cervix.

## IV. *On the Dynamical Theory of Gases.* By J. Clerk Maxwell, *F.R.S. L. & E*

Received May 16,—Read May 31, 1866.

Theories of the constitution of bodies suppose them either to be continuous and homogeneous, or to be composed of a finite number of distinct particles or molecules.

In certain applications of mathematics to physical questions, it is convenient to suppose bodies homogeneous in order to make the quantity of matter in each differential element a function of the coordinates, but I am not aware that any theory of this kind has been proposed to account for the different properties of bodies. Indeed the properties of a body supposed to be a uniform *plenum* may be affirmed dogmatically, but cannot be explained mathematically.

Molecular theories suppose that all bodies, even when they appear to our senses homogeneous, consist of a multitude of particles, or small parts the mechanical relations of which constitute the properties of the bodies. Those theories which suppose that the molecules are at rest relative to the body may be called statical theories, and those which suppose the molecules to be in motion, even while the body is apparently at rest, may be called dynamical theories.

If we adopt a statical theory, and suppose the molecules of a body kept at rest in their positions of equilibrium by the action of forces in the directions of the lines joining their centres, we may determine the mechanical properties of a body so constructed, if distorted so that the displacement of each molecule is a function of its coordinates when in equilibrium. It appears from the mathematical theory of bodies of this kind, that the forces called into play by a small change of form must always bear a fixed proportion to those excited by a small change of volume.

Now we know that in fluids the elasticity of form is evanescent, while that of volume is considerable. Hence such theories will not apply to fluids. In solid bodies the elasticity of form appears in many cases to be smaller in proportion to that of volume than the theory gives*, so that we are forced to give up the theory of molecules whose displacements are functions of their coordinates when at rest, even in the case of solid bodies.

The theory of moving molecules, on the other hand, is not open to these objections. The mathematical difficulties in applying the theory are considerable, and till they are surmounted we cannot fully decide on the applicability of the theory. We are able, however, to explain a great variety of phenomena by the dynamical theory which have not been hitherto explained otherwise.

The dynamical theory supposes that the molecules of solid bodies oscillate about their

* [In glass, according to Dr. Everett's second series of experiments (1866), the ratio of the elasticity of form to that of volume is greater than that given by the theory. In brass and steel it is less.—March 7, 1867.]

positions of equilibrium, but do not travel from one position to another in the body. In fluids the molecules are supposed to be constantly moving into new relative positions, so that the same molecule may travel from one part of the fluid to any other part. In liquids the molecules are supposed to be always under the action of the forces due to neighbouring molecules throughout their course, but in gases the greater part of the path of each molecule is supposed to be sensibly rectilinear and beyond the sphere of sensible action of the neighbouring molecules.

I propose in this paper to apply this theory to the explanation of various properties of gases, and to show that, besides accounting for the relations of pressure, density, and temperature in a single gas, it affords a mechanical explanation of the known chemical relation between the density of a gas and its equivalent weight, commonly called the Law of Equivalent Volumes. It also explains the diffusion of one gas through another, the internal friction of a gas, and the conduction of heat through gases.

The opinion that the observed properties of visible bodies apparently at rest are due to the action of invisible molecules in rapid motion is to be found in LUCRETIUS. In the exposition which he gives of the theories of DEMOCRITUS as modified by EPICURUS, he describes the invisible atoms as all moving downwards with equal velocities, which, at quite uncertain times and places, suffer an imperceptible change, just enough to allow of occasional collisions taking place between the atoms. These atoms he supposes to set small bodies in motion by an action of which we may form some conception by looking at the motes in a sunbeam. The language of LUCRETIUS must of course be interpreted according to the physical ideas of his age, but we need not wonder that it suggested to LE SAGE the fundamental conception of his theory of gases, as well as his doctrine of ultramundane corpuscles.

Professor CLAUSIUS, to whom we owe the most extensive developments of the dynamical theory of gases, has given* a list of authors who have adopted or given countenance to any theory of invisible particles in motion. Of these, DANIEL BERNOULLI, in the tenth section of his 'Hydrodynamics,' distinctly explains the pressure of air by the impact of its particles on the sides of the vessel containing it.

CLAUSIUS also mentions a book entitled "Deux Traités de Physique Mécanique, publiés par PIERRE PREVOST, comme simple Éditeur du premier et comme Auteur du second," Genève et Paris, 1818. The first memoir is by G. LE SAGE, who explains gravity by the impact of "ultramundane corpuscles" on bodies. These corpuscles also set in motion the particles of light and various æthereal media, which in their turn act on the molecules of gases and keep up their motions. His theory of impact is faulty, but his explanation of the expansive force of gases is essentially the same as in the dynamical theory as it now stands. The second memoir, by PREVOST, contains new applications of the principles of LE SAGE to gases and to light. A more extensive application of the theory of moving molecules was made by HERAPATH†. His theory of the collisions of

* POGGENDORFF'S 'Annalen,' Jan. 1862. Translated by G. C. FOSTER, B.A., Phil. Mag. June 1862.

† Mathematical Physics, &c., by JOHN HERAPATH, Esq. 2 vols. London: Whittaker & Co., and Herapath's Railway Journal Office, 1847.

perfectly hard bodies, such as he supposes the molecules to be, is faulty, inasmuch as it makes the result of impact depend on the absolute motion of the bodies, so that by experiments on such hard bodies (if we could get them) we might determine the absolute direction and velocity of the motion of the earth*. This author, however, has applied his theory to the numerical results of experiment in many cases, and his speculations are always ingenious, and often throw much real light on the questions treated. In particular, the theory of temperature and pressure in gases and the theory of diffusion are clearly pointed out.

Dr. JOULE† has also explained the pressure of gases by the impact of their molecules, and has calculated the velocity which they must have in order to produce the pressure observed in particular gases.

It is to Professor CLAUSIUS, of Zurich, that we owe the most complete dynamical theory of gases. His other researches on the general dynamical theory of heat are well known, and his memoirs "On the kind of Motion which we call Heat," are a complete exposition of the molecular theory adopted in this paper. After reading his investigation‡ of the distance described by each molecule between successive collisions, I published some propositions§ on the motions and collisions of perfectly elastic spheres, and deduced several properties of gases, especially the law of equivalent volumes, and the nature of gaseous friction. I also gave a theory of diffusion of gases, which I now know to be erroneous, and there were several errors in my theory of the conduction of heat in gases which M. CLAUSIUS has pointed out in an elaborate memoir on that subject‖.

M. O. E. MEYER¶ has also investigated the theory of internal friction on the hypothesis of hard elastic molecules.

In the present paper I propose to consider the molecules of a gas, not as elastic spheres of definite radius, but as small bodies or groups of smaller molecules repelling one another with a force whose direction always passes very nearly through the centres of gravity of the molecules, and whose magnitude is represented very nearly by some function of the distance of the centres of gravity. I have made this modification of the theory in consequence of the results of my experiments on the viscosity of air at different temperatures, and I have deduced from these experiments that the repulsion is inversely as the *fifth* power of the distance.

If we suppose an imaginary plane drawn through a vessel containing a great number of such molecules in motion, then a great many molecules will cross the plane in either direction. The excess of the mass of those which traverse the plane in the positive

* Mathematical Physics, &c., p. 134.

† Some Remarks on Heat and the Constitution of Elastic Fluids, Oct. 3, 1848.

‡ Phil. Mag. Feb. 1859.

§ Illustrations of the Dynamical Theory of Gases, Phil. Mag. 1860, January and July.

‖ POGGENDORFF, Jan. 1862; Phil. Mag. June 1862.

¶ Ueber die innere Reibung der Gase (POGGENDORFF, vol. CXXV. 1865).

direction over that of those which traverse it in the negative direction, gives a measure of the flow of gas through the plane in the positive direction.

If the plane be made to move with such a velocity that there is no excess of flow of molecules in one direction through it, then the velocity of the plane is the mean velocity of the gas resolved normal to the plane.

There will still be molecules moving in both directions through the plane, and carrying with them a certain amount of momentum into the portion of gas which lies on the other side of the plane.

The quantity of momentum thus communicated to the gas on the other side of the plane during a unit of time is a measure of the force exerted on this gas by the rest. This force is called the pressure of the gas.

If the velocities of the molecules moving in different directions were independent of one another, then the pressure at any point of the gas need not be the same in all directions, and the pressure between two portions of gas separated by a plane need not be perpendicular to that plane. Hence, to account for the observed equality of pressure in all directions, we must suppose some cause equalizing the motion in all directions. This we find in the deflection of the path of one particle by another when they come near one another. Since, however, this equalization of motion is not instantaneous, the pressures in all directions are perfectly equalized only in the case of a gas at rest, but when the gas is in a state of motion, the want of perfect equality in the pressures gives rise to the phenomena of viscosity or internal friction. The phenomena of viscosity in all bodies may be described, independently of hypothesis, as follows:—

A distortion or strain of some kind, which we may call S, is produced in the body by displacement. A state of stress or elastic force which we may call F is thus excited. The relation between the stress and the strain may be written $F=ES$, where E is the coefficient of elasticity for that particular kind of strain. In a solid body free from viscosity, F will remain $=ES$, and

$$\frac{dF}{dt}=E\frac{dS}{dt}.$$

If, however, the body is viscous, F will not remain constant, but will tend to disappear at a rate depending on the value of F, and on the nature of the body. If we suppose this rate proportional to F, the equation may be written

$$\frac{dF}{dt}=E\frac{dS}{dt}-\frac{F}{T},$$

which will indicate the actual phenomena in an empirical manner. For if S be constant,

$$F=ESe^{-\frac{t}{T}},$$

showing that F gradually disappears, so that if the body is left to itself it gradually loses any internal stress, and the pressures are finally distributed as in a fluid at rest.

If $\frac{dS}{dt}$ is constant, that is, if there is a steady motion of the body which continually

increases the displacement,

$$F=ET\frac{dS}{dt}+Ce^{-\frac{t}{T}},$$

showing that F tends to a constant value depending on the rate of displacement. The quantity ET, by which the rate of displacement must be multiplied to get the force, may be called the coefficient of viscosity. It is the product of a coefficient of elasticity, E, and a time T, which may be called the "time of relaxation" of the elastic force. In mobile fluids T is a very small fraction of a second, and E is not easily determined experimentally. In viscous solids T may be several hours or days, and then E is easily measured. It is possible that in some bodies T may be a function of F, and this would account for the gradual untwisting of wires after being twisted beyond the limit of perfect elasticity. For if T diminishes as F increases, the parts of the wire furthest from the axis will yield more rapidly than the parts near the axis during the twisting process, and when the twisting force is removed, the wire will at first untwist till there is equilibrium between the stresses in the inner and outer portions. These stresses will then undergo a gradual relaxation; but since the actual value of the stress is greater in the outer layers, it will have a more rapid rate of relaxation, so that the wire will go on gradually untwisting for some hours or days, owing to the stress on the interior portions maintaining itself longer than that of the outer parts. This phenomenon was observed by Weber in silk fibres, by Kohlrausch in glass fibres, and by myself in steel wires.

In the case of a collection of moving molecules such as we suppose a gas to be, there is also a resistance to change of form, constituting what may be called the linear elasticity, or "rigidity" of the gas, but this resistance gives way and diminishes at a rate depending on the amount of the force and on the nature of the gas.

Suppose the molecules to be confined in a rectangular vessel with perfectly elastic sides, and that they have no action on one another, so that they never strike one another, or cause each other to deviate from their rectilinear paths. Then it can easily be shown that the pressures on the sides of the vessel due to the impacts of the molecules are perfectly independent of each other, so that the mass of moving molecules will behave, not like a fluid, but like an elastic solid. Now suppose the pressures at first equal in the three directions perpendicular to the sides, and let the dimensions $a$, $b$, $c$ of the vessel be altered by small quantities, $\delta a$, $\delta b$, $\delta c$.

Then if the original pressure in the direction of $a$ was $p$, it will become

$$p\left(1-3\frac{\delta a}{a}-\frac{\delta b}{b}-\frac{\delta c}{c}\right);$$

or if there is no change of volume,

$$\frac{\delta p}{p}=-2\frac{\delta a}{a},$$

showing that in this case there is a "longitudinal" elasticity of form of which the coefficient is $2p$. The coefficient of "Rigidity" is therefore $=p$.

This rigidity, however, cannot be directly observed, because the molecules continually deflect each other from their rectilinear courses, and so equalize the pressure in all directions. The rate at which this equalization takes place is great, but not infinite; and therefore there remains a certain inequality of pressure which constitutes the phenomenon of viscosity.

I have found by experiment that the coefficient of viscosity in a given gas is independent of the density, and proportional to the absolute temperature, so that if ET be the viscosity, $ET \propto \frac{p}{\rho}$.

But $E=p$, therefore T, the time of relaxation, varies inversely as the density and is independent of the temperature. Hence the number of collisions producing a given deflection which take place in unit of time is independent of the temperature, that is, of the velocity of the molecules, and is proportional to the number of molecules in unit of volume. If we suppose the molecules hard elastic bodies, the number of collisions of a given kind will be proportional to the velocity, but if we suppose them centres of force, the angle of deflection will be smaller when the velocity is greater; and if the force is inversely as the fifth power of the distance, the number of deflections of a given kind will be independent of the velocity. Hence I have adopted this law in making my calculations.

The effect of the mutual action of the molecules is not only to equalize the pressure in all directions, but, when molecules of different kinds are present, to communicate motion from the one kind to the other. I formerly showed that the final result in the case of hard elastic bodies is to cause the average *vis viva* of a molecule to be the same for all the different kinds of molecules. Now the pressure due to each molecule is proportional to its *vis viva*, hence the whole pressure due to a given number of molecules in a given volume will be the same whatever the mass of the molecules, provided the molecules of different kinds are permitted freely to communicate motion to each other.

When the flow of *vis viva* from the one kind of molecules to the other is zero, the temperature is said to be the same. Hence equal volumes of different gases at equal pressures and temperatures contain equal numbers of molecules.

This result of the dynamical theory affords the explanation of the "law of equivalent volumes" in gases.

We shall see that this result is true in the case of molecules acting as centres of force. A law of the same general character is probably to be found connecting the temperatures of liquid and solid bodies with the energy possessed by their molecules, although our ignorance of the nature of the connexions between the molecules renders it difficult to enunciate the precise form of the law.

The molecules of a gas in this theory are those portions of it which move about as a single body. These molecules may be mere points, or pure centres of force endowed with inertia, or the capacity of performing work while losing velocity. They may be systems of several such centres of force, bound together by their mutual actions, and in

this case the different centres may either be separated, so as to form a group of points, or they may be actually coincident, so as to form one point.

Finally, if necessary, we may suppose them to be small solid bodies of a determinate form; but in this case we must assume a new set of forces binding the parts of these small bodies together, and so introduce a molecular theory of the second order. The doctrines that all matter is extended, and that no two portions of matter can coincide in the same place, being deductions from our experiments with bodies sensible to us, have no application to the theory of molecules.

The actual energy of a moving body consists of two parts, one due to the motion of its centre of gravity, and the other due to the motions of its parts relative to the centre of gravity. If the body is of invariable form, the motions of its parts relative to the centre of gravity consist entirely of rotation, but if the parts of the body are not rigidly connected, their motions may consist of oscillations of various kinds, as well as rotation of the whole body.

The mutual interference of the molecules in their courses will cause their energy of motion to be distributed in a certain ratio between that due to the motion of the centre of gravity and that due to the rotation, or other internal motion. If the molecules are pure centres of force, there can be no energy of rotation, and the whole energy is reduced to that of translation; but in all other cases the whole energy of the molecule may be represented by $\frac{1}{2}Mv^2\beta$, where $\beta$ is the ratio of the total energy to the energy of translation. The ratio $\beta$ will be different for every molecule, and will be different for the same molecule after every encounter with another molecule, but it will have an average value depending on the nature of the molecules, as has been shown by Clausius. The value of $\beta$ can be determined if we know either of the specific heats of the gas, or the ratio between them.

The method of investigation which I shall adopt in the following paper, is to determine the mean values of the following functions of the velocity of all the molecules of a given kind within an element of volume:—

($\alpha$) the mean velocity resolved parallel to each of the coordinate axes;

($\beta$) the mean values of functions of two dimensions of these component velocities;

($\gamma$) the mean values of functions of three dimensions of these velocities.

The rate of translation of the gas, whether by itself, or by diffusion through another gas, is given by ($\alpha$), the pressure of the gas on any plane, whether normal or tangential to the plane, is given by ($\beta$), and the rate of conduction of heat through the gas is given by ($\gamma$).

I propose to determine the variations of these quantities, due, 1st, to the encounters of the molecules with others of the same system or of a different system; 2nd, to the action of external forces such as gravity; and 3rd, to the passage of molecules through the boundary of the element of volume.

I shall then apply these calculations to the determination of the statical cases of the final distribution of two gases under the action of gravity, the equilibrium of tempe-

rature between two gases, and the distribution of temperature in a vertical column. These results are independent of the law of force between the molecules. I shall also consider the dynamical cases of diffusion, viscosity, and conduction of heat, which involve the law of force between the molecules.

*On the Mutual Action of Two Molecules.*

Let the masses of these molecules be $M_1$, $M_2$, and let their velocities resolved in three directions at right angles to each other be $\xi_1$, $\eta_1$, $\zeta_1$ and $\xi_2$, $\eta_2$, $\zeta_2$. The components of the velocity of the centre of gravity of the two molecules will be

$$\frac{\xi_1 M_1 + \xi_2 M_2}{M_1 + M_2}, \quad \frac{\eta_1 M_1 + \eta_2 M_2}{M_1 + M_2}, \quad \frac{\zeta_1 M_1 + \zeta_2 M_2}{M_1 + M_2}.$$

The motion of the centre of gravity will not be altered by the mutual action of the molecules, of whatever nature that action may be. We may therefore take the centre of gravity as the origin of a system of coordinates moving parallel to itself with uniform velocity, and consider the alteration of the motion of each particle with reference to this point as origin.

If we regard the molecules as simple centres of force, then each molecule will describe a plane curve about this centre of gravity, and the two curves will be similar to each other and symmetrical with respect to the line of apses. If the molecules move with sufficient velocity to carry them out of the sphere of their mutual action, their orbits will each have a pair of asymptotes inclined at an angle $\frac{\pi}{2} - \theta$ to the line of apses. The asymptotes of the orbit of $M_1$ will be at a distance $b_1$ from the centre of gravity, and those of $M_2$ at a distance $b_2$, where

$$M_1 b_1 = M_2 b_2.$$

The distance between two parallel asymptotes, one in each orbit, will be

$$b = b_1 + b_2.$$

If, while the two molecules are still beyond each other's action, we draw a straight line through $M_1$ in the direction of the relative velocity of $M_1$ to $M_2$, and draw from $M_2$ a perpendicular to this line, the length of this perpendicular will be $b$, and the plane including $b$ and the direction of relative motion will be the plane of the orbits about the centre of gravity.

When, after their mutual action and deflection, the molecules have again reached a distance such that there is no sensible action between them, each will be moving with the same velocity relative to the centre of gravity that it had before the mutual action, but the direction of this relative velocity will be turned through an angle $2\theta$ in the plane of the orbit.

The angle $\theta$ is a function of the relative velocity of the molecules and of $b$, the form of the function depending on the nature of the action between the molecules.

If we suppose the molecules to be bodies, or systems of bodies, capable of rotation,

internal vibration, or any form of energy other than simple motion of translation, these results will be modified. The value of $\theta$ and the final velocities of the molecules will depend on the amount of internal energy in each molecule before the encounter, and on the particular form of that energy at every instant during the mutual action. We have no means of determining such intricate actions in the present state of our knowledge of molecules, so that we must content ourselves with the assumption that the value of $\theta$ is, on an average, the same as for pure centres of force, and that the final velocities differ from the initial velocities only by quantities which may in each collision be neglected, although in a great many encounters the energy of translation and the internal energy of the molecules arrive, by repeated small exchanges, at a final ratio, which we shall suppose to be that of 1 to $\beta-1$.

We may now determine the final velocity of $M_1$ after it has passed beyond the sphere of mutual action between itself and $M_2$.

Let V be the velocity of $M_1$ relative to $M_2$, then the components of V are

$$\xi_1-\xi_2, \quad \eta_1-\eta_2, \quad \zeta_1-\zeta_2.$$

The plane of the orbit is that containing V and $b$. Let this plane be inclined $\varphi$ to a plane containing V and parallel to the axis of $x$; then, since the direction of V is turned round an angle $2\theta$ in the plane of the orbit, while its magnitude remains the same, we may find the value of $\xi_1$ after the encounter. Calling it $\xi_1'$,

$$\xi_1'=\xi_1+\frac{M_2}{M_1+M_2}\{(\xi_2-\xi_1)2\sin^2\theta+\sqrt{(\eta_2-\eta_1)^2+(\zeta_2-\zeta_1)^2}\sin 2\theta\cos\varphi\}. \quad . \quad . \quad (1)$$

There will be similar expressions for the components of the final velocity of $M_1$ in the other coordinate directions.

If we know the initial positions and velocities of $M_1$ and $M_2$ we can determine V, the velocity of $M_1$ relative to $M_2$; $b$ the shortest distance between $M_1$ and $M_2$ if they had continued to move with uniform velocity in straight lines; and $\varphi$ the angle which determines the plane in which V and $b$ lie. From V and $b$ we can determine $\theta$, if we know the law of force, so that the problem is solved in the case of two molecules.

When we pass from this case to that of two systems of moving molecules, we shall suppose that the time during which a molecule is beyond the action of other molecules is so great compared with the time during which it is deflected by that action, that we may neglect both the time and the distance described by the molecules during the encounter, as compared with the time and the distance described while the molecules are free from disturbing force. We may also neglect those cases in which three or more molecules are within each other's spheres of action at the same instant.

### *On the Mutual Action of Two Systems of Moving Molecules.*

Let the number of molecules of the first kind in unit of volume be $N_1$, the mass of each being $M_1$. The velocities of these molecules will in general be different both in magnitude and direction. Let us select those molecules the components of whose velocities

lie between

$$\xi_1 \text{ and } \xi_1+d\xi_1, \quad \eta_1 \text{ and } \eta_1+d\eta_1, \quad \zeta_1 \text{ and } \zeta_1+d\zeta_1,$$

and let the number of these molecules be $dN_1$. The velocities of these molecules will be very nearly equal and parallel.

On account of the mutual actions of the molecules, the number of molecules which at a given instant have velocities within given limits will be definite, so that

$$dN_1 = f_1(\xi_1,\eta_1,\zeta_1)d\xi_1 d\eta_1 d\zeta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We shall consider the form of this function afterwards.

Let the number of molecules of the second kind in unit of volume be $N_2$, and let $dN_2$ of these have velocities between $\xi_2$ and $\xi_2+d\xi_2$, $\eta_2$ and $d\eta_2+\eta_2$, $\zeta_2$ and $\zeta_2+d\zeta_2$, where

$$dN_2 = f_2(\xi_2,\eta_2,\zeta_2)d\xi_2 d\eta_2 d\zeta_2.$$

The velocity of any of the $dN_1$ molecules of the first system relative to the $dN_2$ molecules of the second system is V, and each molecule $M_1$ will in the time $\delta t$ describe a relative path $V\delta t$ among the molecules of the second system. Conceive a space bounded by the following surfaces. Let two cylindrical surfaces have the common axis $V\delta t$ and radii $b$ and $b+db$. Let two planes be drawn through the extremities of the line $V\delta t$ perpendicular to it. Finally, let two planes be drawn through $V\delta t$ making angles $\varphi$ and $\varphi+d\varphi$ with a plane through V parallel to the axis of $x$. Then the volume included between the four planes and the two cylindric surfaces will be $Vbdbd\varphi\delta t$.

If this volume includes one of the molecules $M_2$, then during the time $\delta t$ there will be an encounter between $M_1$ and $M_2$, in which $b$ is between $b$ and $b+db$, and $\varphi$ between $\varphi$ and $\varphi+d\varphi$.

Since there are $dN_1$ molecules similar to $M_1$ and $dN_2$ similar to $M_2$ in unit of volume, the whole number of encounters of the given kind between the two systems will be

$$Vbdbd\varphi\delta t dN_1 dN_2.$$

Now let Q be any property of the molecule $M_1$, such as its velocity in a given direction, the square or cube of that velocity or any other property of the molecule which is altered in a known manner by an encounter of the given kind, so that Q becomes Q′ after the encounter, then during the time $\delta t$ a certain number of the molecules of the first kind have Q changed to Q′, while the remainder retain the original value of Q, so that

$$\delta Q dN_1 = (Q'-Q)Vbdbd\varphi\delta t dN_1 dN_2,$$

or

$$\frac{\delta Q dN_1}{\delta t} = (Q'-Q)Vbdbd\varphi dN_1 dN_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here $\frac{\delta Q dN_1}{\delta t}$ refers to the alteration in the sum of the values of Q for the $dN_1$ molecules, due to their encounters of the given kind with the $dN_2$ molecules of the second sort. In order to determine the value of $\frac{\delta Q N_1}{\delta t}$, the rate of alteration of Q among all the molecules of the first kind, we must perform the following integrations:—

1st, with respect to $\varphi$ from $\varphi=0$ to $\varphi=2\pi$.

2nd, with respect to $b$ from $b=0$ to $b=\infty$. These operations will give the results of the encounters of every kind between the $dN_1$ and $dN_2$ molecules.

3rd, with respect to $dN_2$, or $f_2(\xi_2,\eta_2,\zeta_2)d\xi_2 d\eta_2 d\zeta_2$.

4th, with respect to $dN_1$, or $f_1(\xi_1,\eta_1,\zeta_1)d\xi_1 d\eta_1 d\zeta_1$.

These operations require in general a knowledge of the forms of $f_1$ and $f_2$.

1st. *Integration with respect to* $\varphi$.

Since the action between the molecules is the same in whatever plane it takes place, we shall first determine the value of $\int_0^{2\pi}(Q'-Q)d\varphi$ in several cases, making Q some function of $\xi$, $\eta$, and $\zeta$.

($\alpha$) Let $Q=\xi_1$ and $Q'=\xi_1'$, then

$$\int_0^{2\pi}(\xi_1'-\xi_1)d\varphi=\frac{M_2}{M_1+M_2}(\xi_2-\xi_1)4\pi\sin^2\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

($\beta$) Let $Q=\xi_1^2$ and $Q'=\xi_1'^2$,

$$\int_0^{2\pi}(\xi_1'^2-\xi_1^2)d\varphi=\frac{M_2}{(M_1+M_2)^2}\left\{(\xi_2-\xi_1)(M_1\xi_1+M_2\xi_2)8\pi\sin^2\theta+M_2\left((\eta_2-\eta_1)^2+(\zeta_2-\zeta_1)^2-2(\xi_2-\xi_1)^2\right)\pi\sin^2 2\theta\right\}. \quad (5)$$

By transformation of coordinates we may derive from this

$$\int_0^{2\pi}(\xi_1'\eta_1'-\xi_1\eta_1)d\varphi=\frac{M_2}{(M_1+M_2)^2}\left\{\left(M_2\xi_2\eta_2-M_1\xi_1\eta_1+\tfrac{1}{2}(M_1-M_2)(\xi_1\eta_2+\xi_2\eta_1)\right)8\pi\sin^2\theta-3M_2(\xi_2-\xi_1)(\eta_2-\eta_1)\right\}, \quad (6)$$

with similar expressions for the other quadratic functions of $\xi$, $\eta$, $\zeta$.

($\gamma$) Let $Q=\xi_1(\xi_1^2+\eta_1^2+\zeta_1^2)$, and $Q'=\xi_1'(\xi_1'^2+\eta_1'^2+\zeta_1'^2)$; then putting $\xi_1^2+\eta_1^2+\zeta_1^2=V_1^2$, $\xi_1\xi_2+\eta_1\eta_2+\zeta_1\zeta_2=U$, $\xi_2^2+\eta_2^2+\zeta_2^2=V_2^2$, and $(\xi_2-\xi_1)^2+(\eta_2-\eta_1)^2+(\zeta_2-\zeta_1)^2=V^2$, we find

$$\left.\begin{aligned}\int_0^{2\pi}(\xi_1'V_1'^2-\xi_1V_1^2)d\varphi=&\frac{M_2}{M_1+M_2}4\pi\sin^2\theta\{(\xi_2-\xi_1)V_1^2+2\xi_1(U-V_1^2)\}\\&+\left(\frac{M_2}{M_1+M_2}\right)^2(8\pi\sin^2\theta-3\pi\sin^2 2\theta)2(\xi_2-\xi_1)(U-V_1^2)\\&+\left(\frac{M_2}{M_1+M_2}\right)^2(8\pi\sin^2\theta+2\pi\sin^2 2\theta)\xi_1V^2\\&+\left(\frac{M_2}{M_1+M_2}\right)^2(8\pi\sin^2\theta-2\pi\sin^2 2\theta)2(\xi_2-\xi_1)V^2.\end{aligned}\right\} \quad . \quad . \quad (7)$$

These are the principal functions of $\xi$, $\eta$, $\zeta$ whose changes we shall have to consider; we shall indicate them by the symbols $\alpha$, $\beta$, or $\gamma$, according as the function of the velocity is of one, two, or three dimensions.

2nd. *Integration with respect to* b.

We have next to multiply these expressions by $bdb$, and to integrate with respect to $b$ from $b=0$ to $b=\infty$. We must bear in mind that $\theta$ is a function of $b$ and V, and can only be determined when the law of force is known. In the expressions which we have

to deal with, $\theta$ occurs under two forms only, namely, $\sin^2\theta$ and $\sin^2 2\theta$. If, therefore, we can find the values of

$$B_1=\int_0^\infty 4\pi b db \sin^2\theta, \text{ and } B_2=\int_0^\infty \pi b db \sin^2 2\theta, \quad \ldots\ldots\ldots (8)$$

we can integrate all the expressions with respect to $b$.

$B_1$ and $B_2$ will be functions of V only, the form of which we can determine only in particular cases, after we have found $\theta$ as a function of $b$ and V.

*Determination of $\theta$ for certain laws of Force.*

Let us assume that the force between the molecules $M_1$ and $M_2$ is repulsive and varies inversely as the $n$th power of the distance between them, the value of the moving force at distance unity being K, then we find by the equation of central orbits,

$$\frac{\pi}{2}-\theta=\int_0^{x'}\frac{dx}{\sqrt{1-x^2-\frac{2}{n-1}\left(\frac{x}{\alpha}\right)^{n-1}}}, \quad \ldots\ldots\ldots (9)$$

where $x=\frac{b}{r}$, or the ratio of $b$ to the distance of the molecules at a given time: $x$ is therefore a numerical quantity; $\alpha$ is also a numerical quantity and is given by the equation

$$\alpha=b\left(\frac{V^2 M_1 M_2}{K(M_1+M_2)}\right)^{\frac{1}{n-1}} \quad \ldots\ldots\ldots (10)$$

The limits of integration are $x=0$ and $x=x'$, where $x'$ is the least positive root of the equation

$$1-x^2-\frac{2}{n-1}\left(\frac{x}{\alpha}\right)^{n-1}=0. \quad \ldots\ldots\ldots (11)$$

It is evident that $\theta$ is a function of $\alpha$ and $n$, and when $n$ is known $\theta$ may be expressed as a function of $\alpha$ only.

Also

$$bdb=\left(\frac{K(M_1+M_2)}{V^2 M_1 M_2}\right)^{\frac{2}{n-1}} \alpha d\alpha; \quad \ldots\ldots\ldots (12)$$

so that if we put

$$A_1=\int_0^\infty 4\pi\alpha d\alpha \sin^2\theta, \quad A_2=\int_0^\infty \pi\alpha d\alpha \sin^2 2\theta, \quad \ldots\ldots\ldots (13)$$

$A_1$ and $A_2$ will be definite numerical quantities which may be ascertained when $n$ is given, and $B_1$ and $B_2$ may be found by multiplying $A_1$ and $A_2$ by $\left(\frac{K(M_1+M_2)}{M_1 M_2}\right)^{\frac{2}{n-1}} V^{\frac{-4}{n-1}}$.

Before integrating further we have to multiply by V, so that the form in which V will enter into the expressions which have to be integrated with respect to $dN_1$ and $dN_2$ will be

$$V^{\frac{n-5}{n-1}}.$$

It will be shown that we have reason from experiments on the viscosity of gases to believe that $n=5$. In this case V will disappear from the expressions of the form (3), and they will be capable of immediate integration with respect to $dN_1$ and $dN_2$.

If we assume $n=5$ and put $\alpha'=2\cot^2 2\phi$ and $x=\sqrt{1-\tan^2\phi}\cos\psi$,

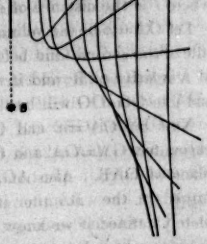
$$\left.\begin{aligned}\frac{\pi}{2}-\theta&=\sqrt{\cos 2\phi}\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-\sin^2\phi\sin^2\psi}}\\&=\sqrt{\cos 2\phi}\ F_{\sin\phi},\end{aligned}\right\}\quad\ldots\ldots\quad(14)$$

where $F_{\sin\phi}$ is the complete elliptic function of the first kind and is given in LEGENDRE'S Tables. I have computed the following Table of the distance of the asymptotes, the distance of the apse, the value of $\theta$, and of the quantities whose summation leads to $A_1$ and $A_2$.

| $\phi$. | $\delta$. | Distance of apse. | $\theta$. | $\sin^2\theta$ / $\sin^2 2\phi$ | $\sin^4 2\theta$ / $\sin^2 2\phi$ |
|---|---|---|---|---|---|
| ° ′ | ° ′ | | ° ′ | | |
| 0 0 | infinite | infinite | 0 0 | 0 | 0 |
| 5 0 | 2381 | 2391 | 0 31 | ·00270 | ·01079 |
| 10 0 | 1658 | 1684 | 1 53 | ·01464 | ·03689 |
| 15 0 | 1316 | 1366 | 4 47 | ·02781 | ·11048 |
| 20 0 | 1092 | 1172 | 8 45 | ·05601 | ·21885 |
| 25 0 | 916 | 1036 | 14 15 | ·10325 | ·38799 |
| 30 0 | 760 | 931 | 21 42 | ·18228 | ·62942 |
| 35 0 | 603 | 845 | 31 59 | ·31772 | ·71433 |
| 40 0 | 420 | 772 | 47 20 | ·55749 | 1·02427 |
| 41 0 | 374 | 758 | 51 32 | ·62515 | ·96763 |
| 42 0 | 324 | 745 | 56 26 | ·70197 | ·85838 |
| 43 0 | 264 | 732 | 62 22 | ·78872 | ·67868 |
| 44 0 | 187 | 719 | 70 18 | ·88745 | ·40338 |
| 44 30 | 132 | 713 | 76 1 | ·94190 | ·21999 |
| 45 0 | 0 | 707 | 90 0 | 1·00000 | ·00000 |

$$A_1=\int 4\pi a da\sin^2\theta=2\cdot6595.\quad\ldots\ldots\quad(15)$$

$$A_2=\int\pi a da\sin^2 2\theta=1\cdot3682.\quad\ldots\ldots\quad(16)$$




The paths described by molecules about a centre of force S, repelling inversely as the fifth power of the distance, are given in the figure.

The molecules are supposed to be originally moving with equal velocities in parallel paths, and the way in which their deflections depend on the distance of the path from S is shown by the different curves in the figure.

3rd. *Integration with respect to* $dN_1$.

We have now to integrate expressions involving various functions of $\xi$, $\eta$, $\zeta$, and V with respect to all the molecules of the second sort. We may write the expression to

be integrated

$$\iiint Q V^{\frac{n-1}{n-1}} f_s(\xi_s \eta_s \zeta_s) d\xi_s d\eta_s d\zeta_s,$$

where Q is some function of $\xi$, $\eta$, $\zeta$, &c., already determined, and $f_s$ is the function which indicates the distribution of velocity among the molecules of the second kind.

In the case in which $n=5$, V disappears, and we may write the result of integration

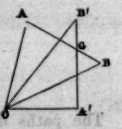
$$\overline{Q} N_s,$$

where $\overline{Q}$ is the mean value of Q for all the molecules of the second kind, and $N_s$ is the number of those molecules.

If, however, $n$ is not equal to 5, so that V does not disappear, we should require to know the form of the function $f_s$ before we could proceed further with the integration.

The only case in which I have determined the form of this function is that of one or more kinds of molecules which have by their continual encounters brought about a distribution of velocity such that the number of molecules whose velocity lies within given limits remains constant. In the Philosophical Magazine for January 1860, I have given an investigation of this case, founded on the assumption that the probability of a molecule having a velocity resolved parallel to $x$ lying between given limits is not in any way affected by the knowledge that the molecule has a given velocity resolved parallel to $y$. As this assumption may appear precarious, I shall now determine the form of the function in a different manner.

*On the Final Distribution of Velocity among the Molecules of Two Systems acting on one another according to any Law of Force.*




From a given point O let lines be drawn representing in direction and magnitude the velocities of every molecule of either kind in unit of volume. The extremities of these lines will be distributed over space in such a way that if an element of volume $dV$ be taken anywhere, the number of such lines which will terminate within $dV$will be $f(r)dV$, where $r$ is the distance of $dV$ from O.

Let $OA=a$ be the velocity of a molecule of the first kind, and $OB=b$ that of a molecule of the second kind before they encounter one another, then BA will be the velocity of A relative to B; and if we divide AB in G inversely as the masses of the molecules, and join OG, OG will be the velocity of the centre of gravity of the two molecules.

Now let $OA'=a'$ and $OB'=b'$ be the velocities of the two molecules after the encounter, $GA=GA'$ and $GB=GB'$, and $A'GB'$ is a straight line not necessarily in the plane of OAB. Also $AGA'=2\theta$ is the angle through which the relative velocity is turned in the encounter in question. The relative motion of the molecules is completely defined if we know BA the relative velocity before the encounter, $2\theta$ the angle through which BA is turned during the encounter, and $\phi$ the angle which defines the direction of the plane in which BA and B'A' lie. All encounters in which the magnitude and direction of BA, and also $\theta$ and $\phi$, lie within certain almost contiguous limits,

we shall class as encounters of the given kind. The number of such encounters in unit of time will be

$$n_1 n_2 F de, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where $n_1$ and $n_2$ are the numbers of molecules of each kind under consideration, and F is a function of the relative velocity and of the angle $\theta$, and $de$ depends on the limits of variation within which we class encounters as of the same kind.

Now let A describe the boundary of an element of volume $dV$ while AB and A′B′ move parallel to themselves, then B, A′, and B′ will also describe equal and similar elements of volume.

The number of molecules of the first kind, the lines representing the velocities of which terminate in the element $dV$ at A, will be

$$n_1 = f_1(a) dV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The number of molecules of the second kind which have velocities corresponding to OB will be

$$n_2 = f_2(b) dV; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and the number of encounters of the given kind between these two sets of molecules will be

$$f_1(a) f_2(b) dV^2 F de. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The lines representing the velocities of these molecules after encounters of the given kind will terminate within elements of volume at A′ and B′, each equal to $dV$.

In like manner we should find for the number of encounters between molecules whose original velocities corresponded to elements equal to $dV$ described about A′ and B′, and whose subsequent velocities correspond to elements equal to $dV$ described about A and B,

$$f_1(a') f_2(b') dV^2 F' de, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where F′ is the same function of B′A′ and A′GA that F is of BA and AGA′. F is therefore equal to F′.

When the number of pairs of molecules which change their velocities from OA, OB to OA′ OB′ is equal to the number which change from OA′, OB′ to OA, OB, then the final distribution of velocity will be obtained, which will not be altered by subsequent exchanges. This will be the case when

$$f_1(a) f_2(b) = f_1(a') f_2(b'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Now the only relation between $a$, $b$ and $a'$, $b'$ is

$$M_1 a^2 + M_2 b^2 = M_1 a'^2 + M_2 b'^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

whence we obtain

$$f_1(a) = C_1 e^{-\frac{a^2}{\alpha^2}}, \quad f_2(b) = C_2 e^{-\frac{b^2}{\beta^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where

$$M_1 \alpha^2 = M_2 \beta^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

By integrating $\iiint C_1 e^{-\frac{\xi^2+\eta^2+\zeta^2}{\alpha^2}} d\xi\, d\eta\, d\zeta$, and equating the result to $N_1$, we obtain the value of $C_1$. If, therefore, the distribution of velocities among $N_1$ molecules is such that

the number of molecules whose component velocities are between $\xi$ and $\xi+d\xi$, $\eta$ and $\eta+d\eta$, and $\zeta$ and $\zeta+d\zeta$ is

$$dN_1=\frac{N_1}{\alpha^3\pi^{\frac{3}{2}}}e^{-\frac{\xi^2+\eta^2+\zeta^2}{\alpha^2}}d\xi\,d\eta\,d\zeta, \quad \ldots\ldots\ldots \quad (26)$$

then this distribution of velocities will not be altered by the exchange of velocities among the molecules by their mutual action.

This is therefore a possible form of the final distribution of velocities. It is also the only form; for if there were any other, the exchange between velocities represented by OA and OA′ would not be equal. Suppose that the number of molecules having velocity OA′ increases at the expense of OA. Then since the total number of molecules corresponding to OA′ remains constant, OA′ must communicate as many to OA″, and so on till they return to OA.

Hence if OA, OA′, OA″, &c. be a series of velocities, there will be a tendency of each molecule to assume the velocities OA, OA′, OA″, &c. in order, returning to OA. Now it is impossible to assign a reason why the successive velocities of a molecule should be arranged in this cycle, rather than in the reverse order. If, therefore, the direct exchange between OA and OA′ is not equal, the equality cannot be preserved by exchange in a cycle. Hence the direct exchange between OA and OA′ is equal, and the distribution we have determined is the only one possible.

This final distribution of velocity is attained only when the molecules have had a great number of encounters, but the great rapidity with which the encounters succeed each other is such that in all motions and changes of the gaseous system except the most violent, the form of the distribution of velocity is only slightly changed.

When the gas moves in mass, the velocities now determined are compounded with the motion of translation of the gas.

When the differential elements of the gas are changing their figure, being compressed or extended along certain axes, the values of the mean square of the velocity will be different in different directions. It is probable that the form of the function will then be

$$f_1(\xi,\eta,\zeta)=\frac{N_1}{\alpha\beta\gamma\pi^{\frac{3}{2}}}e^{-\left(\frac{\xi^2}{\alpha^2}+\frac{\eta^2}{\beta^2}+\frac{\zeta^2}{\gamma^2}\right)}, \quad \ldots\ldots\ldots \quad (27)$$

where $\alpha$, $\beta$, $\gamma$ are slightly different. I have not, however, attempted to investigate the exact distribution of velocities in this case, as the theory of motion of gases does not require it.

When one gas is diffusing through another, or when heat is being conducted through a gas, the distribution of velocities will be different in the positive and negative directions, instead of being symmetrical, as in the case we have considered. The want of symmetry, however, may be treated as very small in most actual cases.

The principal conclusions which we may draw from this investigation are as follows. Calling $\alpha$ the modulus of velocity,

1st. The mean velocity is

$$\bar{v}=\frac{2}{\sqrt{\pi}}\alpha. \quad \ldots\ldots\ldots \quad (28)$$

2nd. The mean square of the velocity is $\overline{v^2}=\frac{3}{2}\alpha^2$ . . . . . (29)

3rd. The mean value of $\xi^2$ is $\overline{\xi^2}=\frac{1}{2}\alpha^2$. . . . . (30)

4th. The mean value of $\xi^4$ is $\overline{\xi^4}=\frac{3}{4}\alpha^4$. . . . . (31)

5th. The mean value of $\xi^2\eta^2$ is $\overline{\xi^2\eta^2}=\frac{1}{4}\alpha^4$. . . . . (32)

6th. When there are two systems of molecules

$$M_1\alpha^2=M_2\beta^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

whence

$$M_1\overline{v_1^2}=M_2\overline{v_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

or the mean *vis viva* of a molecule will be the same in each system. This is a very important result in the theory of gases, and it is independent of the nature of the action between the molecules, as are all the other results relating to the final distribution of velocities. We shall find that it leads to the law of gases known as that of Equivalent Volumes.

*Variation of Functions of the Velocity due to encounters between the Molecules.*

We may now proceed to write down the values of $\frac{\delta\overline{Q}}{\delta t}$ in the different cases. We shall indicate the mean value of any quantity for all the molecules of one kind by placing a bar over the symbol which represents that quantity for any particular molecule, but in expressions where all such quantities are to be taken at their mean values, we shall, for convenience, omit the bar. We shall use the symbols $\delta_1$ and $\delta_2$ to indicate the effect produced by molecules of the first kind and second kind respectively, and $\delta_3$ to indicate the effect of external forces. We shall also confine ourselves to the case in which $n=5$, since it is not only free from mathematical difficulty, but is the only case which is consistent with the laws of viscosity of gases.

In this case V disappears, and we have for the effect of the second system or the first,

$$\frac{\delta Q}{\delta t}=N_2\left(\frac{K(M_1+M_2)}{M_1M_2}\right)^{\frac{1}{2}}A\int_0^{2\pi}(Q'-Q)d\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where the functions of $\xi$, $\eta$, $\zeta$ in $\int(Q'-Q)d\phi$ must be put equal to their mean values for all the molecules, and $A_1$ or $A_2$ must be put for A according as $\sin^2\theta$ or $\sin^2 2\theta$ occurs in the expressions in equations (4), (5), (6), (7). We thus obtain

$$(\alpha)\quad \frac{\delta_3\xi_1}{\delta t}=\left(\frac{K}{M_1M_2(M_1+M_2)}\right)^{\frac{1}{2}}N_2M_2A_1(\xi_2-\xi_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$(\beta)\quad \left.\begin{aligned}\frac{\delta_3\xi_1^2}{\delta t}=&\left(\frac{K}{M_1M_2(M_1+M_2)}\right)^{\frac{1}{2}}\frac{N_2M_2}{M_1+M_2}\\ &\{2A_1(\xi_2-\xi_1)(M_1\xi_1+M_2\xi_2)+A_2M_2(\overline{\eta_2-\eta_1}^2+\overline{\zeta_2-\zeta_1}^2-2\overline{\xi_2-\xi_1}^2)\};\end{aligned}\right\} \quad . \quad . \quad (37)$$

$$\left.\begin{aligned}\frac{\delta_3\xi_1\eta_1}{\delta t}=&\left(\frac{K}{M_1M_2(M_1+M_2)}\right)^{\frac{1}{2}}\frac{N_2M_2}{M_1+M_2}\\ &\left\{A_1\left(2M_2\xi_2\eta_2-2M_1\xi_1\eta_1+(M_1-M_2)(\xi_1\eta_2+\xi_2\eta_1)\right)-3A_2M_2(\xi_2-\xi_1)(\eta_2-\eta_1)\right\};\end{aligned}\right\} \quad (38)$$

$$(\gamma)\quad \frac{\delta_2 \overline{\xi_1 V_1}}{\delta t} = \left(\frac{K}{M_1 M_2 (M_1+M_2)}\right)^{\frac{1}{2}} N_2 M_2 \Big\{ A_1 (\overline{\xi_2 - \xi_1} \overline{V_1^2} + 2\overline{\xi_1}(U - V_1^2)) + \frac{M_2}{M_1+M_2}(2A_1 - 3A_2) 2(\overline{\xi_2 - \xi_1})(U - V_1^2) + \frac{M_2}{M_1+M_2}(2A_1 + 2A_2)\overline{\xi_1 V^2} + \left(\frac{M_2}{M_1+M_2}\right)^2 (2A_1 - 2A_2) 2(\overline{\xi_2 - \xi_1}) V^2 \Big\}; \quad (39)$$

using the symbol $\delta_2$ to indicate variations arising from the action of molecules of the second system.

These are the values of the rate of variation of the mean values of $\xi_1$, $\xi_1^2$, $\xi_1\eta_1$, and $\xi_1 V_1^2$, for the molecules of the first kind due to their encounters with molecules of the second kind. In all of them we must multiply up all functions of $\xi$, $\eta$, $\zeta$, and take the mean values of the products so found. As this has to be done for all such functions, I have omitted the bar over each function in these expressions.

To find the rate of variation due to the encounters among the particles of the same system, we have only to alter the suffix (2) into (1) throughout, and to change K, the coefficient of the force between $M_1$ and $M_2$ into $K_1$, that of the force between two molecules of the first system. We thus find

$$(\alpha)\quad \frac{\delta_1 \overline{\xi_1}}{\delta t} = 0; \quad (40)$$

$$(\beta)\quad \frac{1\overline{\xi_1^2}}{dt} = \left(\frac{K_1}{2M_1^2}\right)^{\frac{1}{2}} M_1 N_1 A_2 \{\overline{\eta_1^2} + \overline{\zeta_1^2} - 2\overline{\xi_1^2} - (\overline{\eta_1}.\overline{\eta_1} + \overline{\zeta_1}.\overline{\zeta_1} - 2\overline{\xi_1}.\overline{\xi_1})\}; \quad (41)$$

$$\frac{\delta_1 \overline{\xi_1 \eta_1}}{\delta t} = \left(\frac{K_1}{2M_1^2}\right)^{\frac{1}{2}} M_1 N_1 A_2 3\{\overline{\xi_1}.\overline{\eta_1} - \overline{\xi_1 \eta_1}\}; \quad (42)$$

$$(\gamma)\quad \frac{\delta_1 \overline{\xi_1 V_1^2}}{\delta t} = \left(\frac{K_1}{2M_1^2}\right)^{\frac{1}{2}} M_1 N_1 A_2 3(\overline{\xi_1}.\overline{V_1^2} - \overline{\xi_1 V_1^2}). \quad (43)$$

These quantities must be added to those in equations (36) to (39) in order to get the rate of variation in the molecules of the first kind due to their encounters with molecules of both systems. When there is only one kind of molecules, the latter equations give the rates of variation at once.

*On the Action of External Forces on a System of Moving Molecules.*

We shall suppose the external force to be like the force of gravity, producing equal acceleration on all the molecules. Let the components of the force in the three coordinate directions be X, Y, Z. Then we have by dynamics for the variations of $\xi$, $\xi^2$, and $\xi V^2$ due to this cause,

$$(\alpha)\quad \frac{\delta_3 \overline{\xi}}{\delta t} = X; \quad (44)$$

$$(\beta)\quad \frac{\delta_2 \cdot \xi^2}{\delta t} = 2\xi X; \qquad (45)$$

$$\frac{\delta_2 \cdot \xi\eta}{\delta t} = \eta X + \xi Y; \qquad (46)$$

$$(\gamma)\quad \frac{\delta_2 \cdot \xi V^2}{\delta t} = 2\xi(\xi X + \eta Y + \zeta Z) + X V^2; \qquad (47)$$

where $\delta_2$ refers to variations due to the action of external forces.

*On the Total rate of change of the different functions of the velocity of the molecules of the first system arising from their encounters with molecules of both systems and from the action of external forces.*

To find the total rate of change arising from these causes, we must add

$$\frac{\delta_1 Q}{\delta t},\ \frac{\delta_2 Q}{\delta t},\ \text{and}\ \frac{\delta_3 Q}{\delta t},$$

the quantities already found. We shall find it, however, most convenient in the remainder of this investigation to introduce a change in the notation, and to substitute for

$$\xi,\ \eta,\ \text{and}\ \zeta,\quad u+\xi,\quad v+\eta,\ \text{and}\ w+\zeta, \qquad (48)$$

where $u$, $v$, and $w$ are so chosen that they are the mean values of the components of the velocity of all molecules of the same system in the immediate neighbourhood of a given point. We shall also write

$$M_1 N_1 = \varrho_1,\ M_2 N_2 = \varrho_2, \qquad (49)$$

where $\varrho_1$ and $\varrho_2$ are the densities of the two systems of molecules, that is, the mass in unit of volume. We shall also write

$$\left(\frac{K_1}{2M_1^2}\right)^{\frac{1}{2}} = k_1,\ \left(\frac{K}{M_1 M_2 (M_1+M_2)}\right)^{\frac{1}{2}} = k,\ \text{and}\ \left(\frac{K_2}{2M_2^2}\right)^{\frac{1}{2}} = k_2; \qquad (50)$$

$\varrho_1$, $\varrho_2$, $k_1$, $k_2$, and $k$ are quantities the absolute values of which can be deduced from experiment. We have not as yet experimental data for determining M, N, or K.

We thus find for the rate of change of the various functions of the velocity,

$$(\alpha)\quad \frac{\delta u_1}{\delta t} = k A_1 \varrho_2 (u_2 - u_1) + X; \qquad (51)$$

$$\left.\begin{aligned}(\beta)\quad \frac{\delta \cdot \xi_1^2}{\delta t} &= k_1 A_2 \varrho_1 \{\eta_1^2 + \zeta_1^2 - 2\xi_1^2\} \\ &+ k\varrho_2 \frac{M_2}{M_1+M_2}\left\{2A_1(u_2-u_1)^2 + A_2\left(\overline{v_2-v_1}^2 + \overline{w_2-w_1}^2 - 2\overline{u_2-u_1}^2\right)\right\} \\ &+ \frac{k\varrho_2}{M_1+M_2}\left\{2A_1(M_2\xi_2^2 - M_1\xi_1^2) + A_2 M_2(\eta_1^2+\zeta_1^2-2\xi_1^2+\eta_2^2+\zeta_2^2-2\xi_2^2)\right\};\end{aligned}\right\} \qquad (52)$$

also

$$\left.\begin{aligned}\frac{\delta \cdot \xi\eta}{\delta t} &= -3k_1 A_2 \varrho_1 \xi_1 \eta_1 + k\varrho_2 \frac{M_2}{M_1+M_2}(2A_1 - 3A_2)(u_2-u_1)(v_2-v_1) \\ &+ \frac{k\varrho_2}{M_1+M_2}\left\{2A_1(M_2\xi_2\eta_2 - M_1\xi_1\eta_1) - 3A_2 M_2(\xi_1\eta_1 + \xi_2\eta_2)\right\}.\end{aligned}\right\} \qquad (53)$$

(γ) As the expressions for the variation of functions of three dimensions in mixed media are complicated, and as we shall not have occasion to use them, I shall give the case of a single medium,

$$\frac{\partial}{\partial t}(\xi_1^2+\xi_1\eta_1^2+\xi_1\zeta_1^2)=-3k_1\varepsilon_1 A_2(\xi_1^2+\xi_1\eta_1^2+\xi_1\zeta_1^2)+X(3\xi_1^2+\eta_1^2+\zeta_1^2)+2Y\xi_1\eta_1+2Z\xi_1\zeta_1. \quad (54)$$

*Theory of a Medium composed of Moving Molecules.*

We shall suppose the position of every moving molecule referred to three rectangular axes, and that the component velocities of any one of them, resolved in the directions of $x$, $y$, $z$, are

$$u+\xi, \quad v+\eta, \quad w+\zeta,$$

where $u$, $v$, $w$ are the components of the mean velocity of all the molecules which are at a given instant in a given element of volume, and $\xi$, $\eta$, $\zeta$ are the components of the relative velocity of one of these molecules with respect to the mean velocity.

The quantities $u$, $v$, $w$ may be treated as functions of $x$, $y$, $z$, and $t$, in which case differentiation will be expressed by the symbol $d$. The quantities $\xi$, $\eta$, $\zeta$, being different for every molecule, must be regarded as functions of $t$ for each molecule. Their variation with respect to $t$ will be indicated by the symbol $\delta$.

The mean values of $\xi^a$ and other functions of $\xi$, $\eta$, $\zeta$ for all the molecules in the element of volume may, however, be treated as functions of $x$, $y$, $z$, and $t$.

If we consider an element of volume which always moves with the velocities $u$, $v$, $w$, we shall find that it does not always consist of the same molecules, because molecules are continually passing through its boundary. We cannot therefore treat it as a mass moving with the velocity $u$, $v$, $w$, as is done in hydrodynamics, but we must consider separately the motion of each molecule. When we have occasion to consider the variation of the properties of this element during its motion as a function of the time we shall use the symbol $\partial$.

We shall call the velocities $u$, $v$, $w$ the velocities of translation of the medium, and $\xi$, $\eta$, $\zeta$ the velocities of agitation of the molecules.

Let the number of molecules in the element $dx\,dy\,dz$ be $N\,dx\,dy\,dz$, then we may call N the number of molecules in unit of volume. If M is the mass of each molecule, and $\rho$ the density of the element, then

$$MN=\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

*Transference of Quantities across a Plane Area.*

We must next consider the molecules which pass through a given plane of unit area in unit of time, and determine the quantity of matter, of momentum, of heat, &c. which is transferred from the negative to the positive side of this plane in unit of time.

We shall first divide the N molecules in unit of volume into classes according to the value of $\xi$, $\eta$, and $\zeta$ for each, and we shall suppose that the number of molecules in unit of volume whose velocity in the direction of $x$ lies between $\xi$ and $\xi+d\xi$, $\eta$ and $\eta+d\eta$, $\zeta$ and $\zeta+d\zeta$ is $dN$, $dN$ will then be a function of the component velocities, the sum of

which being taken for all the molecules will give N the total number of molecules. The most probable form of this function for a medium in its state of equilibrium is

$$dN = \frac{N}{\alpha^3\sqrt{\pi^3}} e^{-\frac{\xi^2+\eta^2+\zeta^2}{\alpha^2}} d\xi\, d\eta\, d\zeta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

In the present investigation we do not require to know the form of this function.

Now let us consider a plane of unit area perpendicular to $x$ moving with a velocity of which the part resolved parallel to $x$ is $u'$. The velocity of the plane relative to the molecules we have been considering is $u'-(u+\xi)$, and since there are $dN$ of these molecules in unit of volume it will overtake

$$\big(u'-(u+\xi)\big)dN$$

such molecules in unit of time, and the number of such molecules passing from the negative to the positive side of the plane, will be

$$(u+\xi-u')dN.$$

Now let Q be any property belonging to the molecule, such as its mass, momentum, *vis viva*, &c., which it carries with it across the plane, Q being supposed a function of $\xi$ or of $\xi$, $\eta$, and $\zeta$, or to vary in any way from one molecule to another, provided it be the same for the selected molecules whose number is $dN$, then the quantity of Q transferred across the plane in the positive direction in unit of time is

$$\int(u-u'+\xi)Q dN,$$

or

$$(u-u')\int Q dN + \int \xi Q dN. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

If we put $\overline{Q}N$ for $\int Q dN$, and $\overline{\xi Q}N$ for $\int \xi Q dN$, then we may call $\overline{Q}$ the mean value of Q, and $\xi Q$ the mean value of $\xi Q$, for all the particles in the element of volume, and we may write the expression for the quantity of Q which crosses the plane in unit of time

$$(u-u')\overline{Q}N + \overline{\xi Q}N. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

(α) *Transference of Matter across a Plane—Velocity of the Fluid.*

To determine the quantity of matter which crosses the plane, make Q equal to M the mass of each molecule; then, since M is the same for all molecules of the same kind, $\overline{M}=M$; and since the mean value of $\xi$ is zero, the expression is reduced to

$$(u-u')MN = (u-u')\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

If $u=u'$, or if the plane moves with velocity $u$, the whole excess of matter transferred across the plane is zero; the velocity of the fluid may therefore be defined as the velocity whose components are $u$, $v$, $w$.

(β) *Transference of Momentum across a Plane—System of Pressures at any point of the Fluid.*

The momentum of any one molecule in the direction of $x$ is $M(u+\xi)$. Substituting this for Q, we get for the quantity of momentum transferred across the plane in the

positive direction

$$(u-u')u\varrho+\overline{\xi^2}\varrho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

If the plane moves with the velocity $u$, this expression is reduced to $\overline{\xi^2}\varrho$, where $\overline{\xi^2}$ represents the mean value of $\xi^2$.

This is the whole momentum in the direction of $x$ of the molecules projected from the negative to the positive side of the plane in unit of time. The mechanical action between the parts of the medium on opposite sides of the plane consists partly of the momentum thus transferred, and partly of the direct attractions or repulsions between molecules on opposite sides of the plane. The latter part of the action must be very small in gases, so that we may consider the pressure between the parts of the medium on opposite sides of the plane as entirely due to the constant bombardment kept up between them. There will also be a transference of momentum in the directions of $y$ and $z$ across the same plane,

$$(u-u')v\varrho+\overline{\xi\eta}\varrho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

and

$$(u-u')w\varrho+\overline{\xi\zeta}\varrho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

where $\overline{\xi\eta}$ and $\overline{\xi\zeta}$ represent the mean values of these products.

If the plane moves with the mean velocity $u$ of the fluid, the total force exerted on the medium on the positive side by the projection of molecules into it from the negative side will be

| | | |
|---|---|---|
| a normal pressure | $\overline{\xi^2}\varrho$ | in the direction of $x$, |
| a tangential pressure | $\overline{\xi\eta}\varrho$ | in the direction of $y$, |
| and a tangential pressure | $\overline{\xi\zeta}\varrho$ | in the direction of $z$. |

If X, Y, Z are the components of the pressure on unit of area of a plane whose direction cosines are $l$, $m$, $n$,

$$\left.\begin{aligned} X&=l\overline{\xi^2}\varrho+m\overline{\xi\eta}\varrho+n\overline{\xi\zeta}\varrho,\\ Y&=l\overline{\xi\eta}\varrho+m\overline{\eta^2}\varrho+n\overline{\eta\zeta}\varrho,\\ Z&=l\overline{\xi\zeta}\varrho+m\overline{\eta\zeta}\varrho+n\overline{\zeta^2}\varrho. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

When a gas is not in a state of violent motion the pressures in all directions are nearly equal, in which case, if we put

$$\overline{\xi^2}\varrho+\overline{\eta^2}\varrho+\overline{\zeta^2}\varrho=3p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

the quantity $p$ will represent the mean pressure at a given point, and $\overline{\xi^2}\varrho$, $\overline{\eta^2}\varrho$, and $\overline{\zeta^2}\varrho$ will differ from $p$ only by small quantities; $\overline{\eta\zeta}\varrho$, $\overline{\zeta\xi}\varrho$, and $\overline{\xi\eta}\varrho$ will then be also small quantities with respect to $p$.

*Energy in the Medium—Actual Heat.*

The actual energy of any molecule depends partly on the velocity of its centre of gravity, and partly on its rotation or other internal motion with respect to the centre of gravity. It may be written

$$\tfrac{1}{2}M\{(u+\xi)^2+(v+\eta)^2+(w+\zeta)^2\}+\tfrac{1}{2}EM, \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

where $\frac{1}{2}EM$ is the internal part of the energy of the molecule, the form of which is at present unknown. Summing for all the molecules in unit of volume, the energy is

$$\tfrac{1}{2}(u^2+v^2+w^2)\rho+\tfrac{1}{2}(\xi^2+\eta^2+\zeta^2)\rho+\tfrac{1}{2}\overline{E}\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

The first term gives the energy due to the motion of translation of the medium in mass, the second that due to the agitation of the centres of gravity of the molecules, and the third that due to the internal motion of the parts of each molecule.

If we assume with CLAUSIUS that the ratio of the mean energy of internal motion to that of agitation tends continually towards a definite value $(\beta-1)$, we may conclude that, except in very violent disturbances, this ratio is always preserved, so that

$$\overline{E}=(\beta-1)(\xi^2+\eta^2+\zeta^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

The total energy of the invisible agitation in unit of volume will then be

$$\tfrac{1}{2}\beta(\xi^2+\eta^2+\zeta^2)\rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

or

$$\tfrac{3}{2}\beta p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

This energy being in the form of invisible agitation, may be called the total heat in the unit of volume of the medium.

### ($\gamma$) *Transference of Energy across a Plane—Conduction of Heat.*

Putting

$$Q=\tfrac{1}{2}\beta(\xi^2+\eta^2+\zeta^2)M, \quad \text{and } u=u', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

we find for the quantity of heat carried over the unit of area by conduction in unit of time

$$\tfrac{1}{2}\beta(\overline{\xi^2}+\overline{\xi\eta^2}+\overline{\xi\zeta^2})\rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

where $\overline{\xi^2}$, &c. indicate the mean values of $\xi^2$, &c. They are always small quantities.

### *On the Rate of Variation of* Q *in an Element of Volume,* Q *being any property of the Molecules in that Element.*

Let Q be the value of the quantity for any particular molecule, and $\overline{Q}$ the mean value of Q for all the molecules of the same kind within the element.

The quantity $\overline{Q}$ may vary from two causes. The molecules within the element may by their mutual action or by the action of external forces produce an alteration of $\overline{Q}$, or molecules may pass into the element and out of it, and so cause an increase or diminution of the value of $\overline{Q}$ within it. If we employ the symbol $\delta$ to denote the variation of Q due to actions of the first kind on the individual molecules, and the symbol $\partial$ to denote the actual variation of Q in an element moving with the mean velocity of the system of molecules under consideration, then by the ordinary investigation of the increase or diminution of matter in an element of volume as contained in treatises on Hydrodynamics,

$$\left.\begin{aligned}\frac{\partial\overline{Q}N}{\partial t}=\frac{\delta\overline{Q}}{\delta t}N-\frac{d}{dx}\left\{(u-u')\overline{Q}N+\overline{\xi Q}N\right\}\\ -\frac{d}{dy}\left\{(v-v')\overline{Q}N+\overline{\eta Q}N\right\}-\frac{d}{dz}\left\{(w-w')\overline{Q}N+\overline{\zeta Q}N\right\},\end{aligned}\right\} \quad . \quad . \quad . \quad (72)$$

where the last three terms are derived from equation (59) and two similar equations, and denote the quantity of Q which flows out of an element of volume, that element moving with the velocities $u'$, $v'$, $w'$. If we perform the differentiations and then make $u'=u$, $v'=v$, and $w'=w$, then the variation will be that in an element which moves with the actual mean velocity of the system of molecules, and the equation becomes

$$\frac{\partial \overline{Q}N}{\partial t}+\overline{Q}N\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)+\frac{d}{dx}(\overline{\xi Q}N)+\frac{d}{dy}(\overline{\eta Q}N)+\frac{d}{dz}(\overline{\zeta Q}N)=\frac{\delta Q}{\delta t}N. \quad . \quad . \quad (73)$$

*Equation of Continuity.*

Put Q=M the mass of a molecule; M is unalterable, and we have, putting $MN=\varrho$,

$$\frac{\partial \varrho}{\partial t}+\varrho\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

which is the ordinary equation of continuity in hydrodynamics, the element being supposed to move with the velocity of the fluid. Combining this equation with that from which it was obtained, we find

$$N\frac{\partial \overline{Q}}{\partial t}+\frac{d}{dx}(\overline{\xi Q}N)+\frac{d}{dy}(\overline{\eta Q}N)+\frac{d}{dz}(\overline{\zeta Q}N)=N\frac{\delta Q}{\delta t}, \quad . \quad . \quad . \quad . \quad . \quad (75)$$

a more convenient form of the general equation.

*Equations of Motion* ($\alpha$).

To obtain the Equation of Motion in the direction of $x$, put $Q=M_1(u_1+\xi_1)$, the momentum of a molecule in the direction of $x$.

We obtain the value of $\frac{\delta Q}{\delta t}$ from equation (51), and the equation may be written

$$\varrho_1\frac{\partial u_1}{\partial t}+\frac{d}{dx}(\varrho_1\overline{\xi_1^2})+\frac{d}{dy}(\varrho_1\overline{\xi_1\eta_1})+\frac{d}{dz}(\varrho_1\overline{\xi_1\zeta_1})=kA_{12}\varrho_1\varrho_2(u_2-u_1)+X\varrho_1. \quad . \quad . \quad . \quad (76)$$

In this equation the first term denotes the efficient force per unit of volume, the second the variation of normal pressure, the third and fourth the variations of tangential pressure, the fifth the resistance due to the molecules of a different system, and the sixth the external force acting on the system.

The investigation of the values of the second, third, and fourth terms must be deferred till we consider the variations of the second degree.

*Condition of Equilibrium of a Mixture of Gases.*

In a state of equilibrium $u_1$ and $u_2$ vanish, $\varrho_1\overline{\xi_1^2}$ becomes $p_1$, and the tangential pressures vanish, so that the equation becomes

$$\frac{dp_1}{dx}=X\varrho_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

which is the equation of equilibrium in ordinary hydrostatics.

This equation, being true of the system of molecules forming the first medium inde-

pendently of the presence of the molecules of the second system, shows that if several kinds of molecules are mixed together, placed in a vessel and acted on by gravity, the final distribution of the molecules of each kind will be the same as if none of the other kinds had been present. This is the same mode of distribution as that which DALTON considered to exist in a mixed atmosphere in equilibrium, the law of diminution of density of each constituent gas being the same as if no other gases were present.

This result, however, can only take place after the gases have been left for a considerable time perfectly undisturbed. If currents arise so as to mix the strata, the composition of the gas will be made more uniform throughout.

The result at which we have arrived as to the final distribution of gases, when left to themselves, is independent of the law of force between the molecules.

### *Diffusion of Gases.*

If the motion of the gases is slow, we may still neglect the tangential pressures. The equation then becomes for the first system of molecules

$$\rho_1 \frac{\partial u_1}{\partial t} + \frac{dp_1}{dx} = kA_{12}\rho_1\rho_2(u_2 - u_1) + X\rho_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

and for the second,

$$\rho_2 \frac{\partial u_2}{\partial t} + \frac{dp_2}{dx} = kA_{12}\rho_1\rho_2(u_1 - u_2) + X\rho_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

In all cases of quiet diffusion we may neglect the first term of each equation. If we then put $p_1 + p_2 = p$, and $\rho_1 + \rho_2 = \rho$, we find by adding,

$$\frac{dp}{dx} = X\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

If we also put $p_1u_1 + p_2u_2 = pu$, then the volumes transferred in opposite directions across a plane moving with velocity $u$ will be equal, so that

$$p_1(u_1 - u) = p_2(u - u_2) = \frac{p_1p_2}{p\rho_1\rho_2 kA_1} \cdot \left(X\rho_1 \frac{dp_1}{dx}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Here $p_1(u_1 - u)$ is the volume of the first gas transferred in unit of time across unit of area of the plane reduced to pressure unity, and at the actual temperature; and $p_2(u - u_2)$ is the equal volume of the second gas transferred across the same area in the opposite direction.

The external force X has very little effect on the quiet diffusion of gases in vessels of moderate size. We may therefore leave it out in our definition of the coefficient of diffusion of two gases.

When two gases not acted on by gravity are placed in different parts of a vessel at equal pressures and temperatures, there will be mechanical equilibrium from the first, and $u$ will always be zero. This will also be approximately true of heavy gases, provided the denser gas is placed below the lighter. Mr. GRAHAM has described in his paper on the Mobility of Gases*, experiments which were made under these conditions. A vertical

* Philosophical Transactions, 1863.

tube had its lower tenth part filled with a heavy gas, and the remaining nine-tenths with a lighter gas. After the lapse of a known time the upper tenth part of the tube was shut off, and the gas in it analyzed, so as to determine the quantity of the heavier gas which had ascended into the upper tenth of the tube during the given time.

In this case we have $u=0$, . . . . . (82)

$$p_1 u_1 = -\frac{p_1 p_2}{\rho_1 \rho_2 k A_1} \frac{1}{p} \frac{dp_1}{dx}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

and by the equation of continuity,

$$\frac{dp_1}{dt} + \frac{d}{dx}(p_1 u_1) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

whence

$$\frac{dp_1}{dt} = \frac{p_1 p_2}{\rho_1 \rho_2 k A_1} \frac{1}{p} \frac{d^2 p_1}{dx^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

or if we put $D = \frac{p_1 p_2}{\rho_1 \rho_2 k A_1} \frac{1}{p}$,

$$\frac{dp_1}{dt} = D \frac{d^2 p_1}{dx^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

The solution of this equation is

$$p_1 = C_1 + C_2 e^{-n^2 D t} \cos(nx + \alpha) + \&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

If the length of the tube is $a$, and if it is closed at both ends,

$$p_1 = C_1 + C_2 e^{-\frac{\pi^2 D}{a^2} t} \cos\frac{\pi x}{a} + C_3 e^{-4\frac{\pi^2 D}{a^2} t} \cos 2\frac{\pi x}{a} + \&c., \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

where $C_1$, $C_2$, $C_3$ are to be determined by the condition that when $t=0$, $p_1=p$, from $x=0$ to $x=\frac{1}{10}a$, and $p_1=0$ from $x=\frac{1}{10}a$ to $x=a$. The general expression for the case in which the first gas originally extends from $x=0$ to $x=b$, and in which after a time $t$ the gas from $x=0$ to $x=c$ is collected, is

$$\frac{p_1}{p} = \frac{b}{a} + \frac{2a}{\pi^2 c}\left\{e^{-\frac{\pi^2 D}{a^2} t} \sin\frac{\pi b}{a} \sin\frac{\pi c}{a} + \frac{1}{2^2} e^{-4\frac{\pi^2 D}{a^2} t} \sin\frac{2\pi b}{a} \sin\frac{2\pi c}{a} + \&c.\right\}, \quad . \quad . \quad . \quad (89)$$

where $\frac{p_1}{p}$ is the proportion of the first gas to the whole in the portion from $x=0$ to $x=c$.

In Mr. GRAHAM's experiments, in which one-tenth of the tube was filled with the first gas, and the proportion of the first gas in the tenth of the tube at the other end ascertained after a time $t$, this proportion will be

$$\frac{p_1}{p} = \frac{1}{10} - \frac{20}{\pi^2}\left\{e^{-\frac{\pi^2 D}{a^2} t} \sin^2\frac{\pi}{10} - e^{-9\frac{\pi^2 D}{a^2} t} \sin^2 2\frac{\pi}{10} + e^{-9^2\frac{\pi^2 D}{a^2} t} \sin^2 3\frac{\pi}{10} - \&c.\right\}. \quad . \quad . \quad . \quad (90)$$

We find for a series of values of $\frac{p_1}{p}$ taken at equal intervals of time T, where

$$T = \frac{\log_e 10}{10\pi^2} \frac{a^2}{D}.$$

| Time. | $\frac{p_1}{p}$ |
|---|---|
| 0 | 0 |
| T | ·01193 |
| 2T | ·02305 |
| 3T | ·03376 |
| 4T | ·04366 |
| 5T | ·05267 |
| 6T | ·06072 |
| 8T | ·07321 |
| 10T | ·08227 |
| 12T | ·08845 |
| ∞ | ·10000 |

Mr. GRAHAM'S experiments on carbonic acid and air, when compared with this Table, give T=500 seconds nearly for a tube 0·57 metre long. Now

$$D=\frac{\log_e 10}{10\pi^2}\frac{a^2}{T}, \quad \ldots \ldots \ldots \ldots \quad (91)$$

whence

$$D=\cdot 0235$$

for carbonic acid and air, in inch-grain-second measure.

*Definition of the Coefficient of Diffusion.*

D is the volume of gas reduced to unit of pressure which passes in unit of time through unit of area when the total pressure is uniform and equal to $p$, and the pressure of either gas increases or diminishes by unity in unit of distance. D may be called the coefficient of diffusion. It varies directly as the square of the absolute temperature, and inversely as the total pressure $p$.

The dimensions of D are evidently $L^2T^{-1}$, where L and T are the standards of length and time.

In considering this experiment of the interdiffusion of carbonic acid and air, we have assumed that air is a simple gas. Now it is well known that the constituents of air can be separated by mechanical means, such as passing them through a porous diaphragm, as in Mr. GRAHAM'S experiments on Atmolysis. The discussion of the interdiffusion of three or more gases leads to a much more complicated equation than that which we have found for two gases, and it is not easy to deduce the coefficients of interdiffusion of the separate gases. It is therefore to be desired that experiments should be made on the interdiffusion of every pair of the more important pure gases which do not act chemically on each other, the temperature and pressure of the mixture being noted at the time of experiment.

Mr. GRAHAM has also published in BRANDE'S Journal for 1829, pt. 2, p. 74, the results

of experiments on the diffusion of various gases out of a vessel through a tube into air. The coefficients of diffusion deduced from these experiments are—

| | |
|---|---|
| Air and Hydrogen . . . . . | ·026216 |
| Air and Marsh-gas . . . . . | ·010240 |
| Air and Ammonia . . . . . | ·00962 |
| Air and Olefiant gas . . . . . | ·00771 |
| Air and Carbonic acid . . . . | ·00682 |
| Air and Sulphurous acid . . . | ·00582 |
| Air and Chlorine . . . . . | ·00486 |

The value for carbonic acid is only one third of that deduced from the experiment with the vertical column. The inequality of composition of the mixed gas in different parts of the vessel is, however, neglected; and the diameter of the tube at the middle part, where it was bent, was probably less than that given.

Those experiments on diffusion which lasted ten hours, all give smaller values of D than those which lasted four hours, and this would also result from the mixture of the gases in the vessel being imperfect.

*Interdiffusion through a small hole.*

When two vessels containing different gases are connected by a small hole, the mixture of gases in each vessel will be nearly uniform except near the hole; and the inequality of the pressure of each gas will extend to a distance from the hole depending on the diameter of the hole, and nearly proportional to that diameter.

Hence in the equation

$$\rho_1 \frac{\partial u_1}{dt} + \frac{dp_1}{dx} = k A \rho_1 \rho_2 (u_2 - u_1) + X \rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

the term $\frac{dp_1}{dx}$ will vary inversely as the diameter of the hole, while $u_1$ and $u_2$ will not vary considerably with the diameter.

Hence when the hole is very small the right-hand side of the equation may be neglected, and the flow of either gas through the hole will be independent of the flow of the other gas, as the term $kA\rho_1\rho_2(u_2-u_1)$ becomes comparatively insignificant.

One gas therefore will escape through a very fine hole into another nearly as fast as into a vacuum; and if the pressures are equal on both sides, the volumes diffused will be as the square roots of the specific gravities inversely, which is the law of diffusion of gases established by GRAHAM*.

*Variation of the invisible agitation* ($\beta$).

By putting for Q in equation (75)

$$Q = \frac{M}{2}\left((u_1+\xi_1)^2+(v_1+\eta_1)^2+(w_1+\zeta_1)^2+(\beta-1)(\xi_1^2+\eta_1^2+\zeta_1^2)\right), \quad . \quad . \quad (93)$$

* Trans. Royal Society of Edinburgh, vol. xii. p. 222.

and eliminating by means of equations (76) and (52), we find

$$\left.\begin{aligned}
&\tfrac{1}{2}\rho_1\frac{\partial}{\partial t}\beta_1(\xi_1^2+\eta_1^2+\zeta_1^2)+\rho_1\xi_1^2\frac{du_1}{dx}+\rho_1\eta_1^2\frac{dv_1}{dy}+\rho_1\zeta_1^2\frac{dw_1}{dz}\\
&\quad+\rho_1\eta_1\zeta_1\left(\frac{dv_1}{dz}+\frac{dw_1}{dy}\right)+\rho_1\zeta_1\xi_1\left(\frac{dw_1}{dx}+\frac{du_1}{dz}\right)+\rho_1\xi_1\eta_1\left(\frac{du_1}{dy}+\frac{dv_1}{dx}\right)\\
&\quad+\beta_1\left\{\frac{d}{dx}(\rho_1\xi_1^3+\rho_1\xi_1\eta_1^2+\rho_1\xi_1\zeta_1^2)+\frac{d}{dy}(\rho_1\eta_1\xi_1^2+\rho_1\eta_1^3+\rho_1\eta_1\zeta_1^2)+\frac{d}{dz}(\rho_1\zeta_1\xi_1^2+\rho_1\zeta_1\eta_1^2+\rho_1\zeta_1^3)\right\}\\
&=\frac{k\rho_1\rho_2A_1}{M_1+M_2}\left\{M_2[(u_2-u_1)^2+(v_2-v_1)^2+(w_2-w_1)^2]+M_2(\xi_2^2+\eta_2^2+\zeta_2^2)-M_1(\xi_1^2+\eta_1^2+\zeta_1^2)\right\}.
\end{aligned}\right\}\quad(94)$$

In this equation the first term represents the variation of invisible agitation or heat; the second, third, and fourth represent the cooling by expansion; the fifth, sixth, and seventh the heating effect of fluid friction or viscosity; and the last the loss of heat by conduction. The quantities on the other side of the equation represent the thermal effects of diffusion, and the communication of heat from one gas to the other.

The equation may be simplified in various cases, which we shall take in order.

1st. *Equilibrium of Temperature between two Gases.—Law of Equivalent Volumes.*

We shall suppose that there is no motion of translation, and no transfer of heat by conduction through either gas. The equation (94) is then reduced to the following form,

$$\tfrac{1}{2}\rho_1\frac{\partial}{\partial t}\beta_1(\xi_1^2+\eta_1^2+\zeta_1^2)=\frac{k\rho_1\rho_2A_1}{M_1+M_2}\left\{M_2(\xi_2^2+\eta_2^2+\zeta_2^2)-M_1(\xi_1^2+\eta_1^2+\zeta_1^2)\right\}.\quad\ldots\quad(95)$$

If we put

$$\frac{M_1}{M_1+M_2}(\xi_1^2+\eta_1^2+\zeta_1^2)=Q_1,\quad\text{and}\quad\frac{M_2}{M_1+M_2}(\xi_2^2+\eta_2^2+\zeta_2^2)=Q_2,\quad\ldots\quad(96)$$

we find

$$\frac{\partial}{\partial t}(Q_2-Q_1)=-\frac{2kA_1}{M_1+M_2}(M_2\rho_2\beta_1+M_1\rho_1\beta_2)(Q_2-Q_1),\quad\ldots\quad(97)$$

or

$$Q_2-Q_1=Ce^{-nt},\quad\text{where } n=\frac{2kA_1}{M_1+M_2}(M_2\rho_2\beta_2+M_1\rho_1\beta_1)\frac{1}{\beta_1\beta_2}.\quad\ldots\quad(98)$$

If, therefore, the gases are in contact and undisturbed, $Q_1$ and $Q_2$ will rapidly become equal. Now the state into which two bodies come by exchange of invisible agitation is called equilibrium of heat or equality of temperature. Hence when two gases are at the same temperature,

$$Q_1=Q_2,\quad\ldots\quad(99)$$

or

$$1=\frac{Q_1}{Q_2}=\frac{M_1(\xi_1^2+\eta_1^2+\zeta_1^2)}{M_2(\xi_2^2+\eta_2^2+\zeta_2^2)}$$

$$=\frac{M_1\frac{p_1}{\rho_1}}{M_2\frac{p_2}{\rho_2}}.$$

Hence if the pressures as well as the temperatures be the same in two gases,

$$\frac{M_1}{\rho_1}=\frac{M_2}{\rho_2}, \quad \ldots \quad (100)$$

or the masses of the individual molecules are proportional to the density of the gas.

This result, by which the relative masses of the molecules can be deduced from the relative densities of the gases, was first arrived at by GAY-LUSSAC from chemical considerations. It is here shown to be a necessary result of the Dynamical Theory of Gases; and it is so, whatever theory we adopt as to the nature of the action between the individual molecules, as may be seen by equation (34), which is deduced from perfectly general assumptions as to the nature of the law of force.

We may therefore henceforth put $\frac{s_1}{s_2}$ for $\frac{M_1}{M_2}$, where $s_1$, $s_2$ are the specific gravities of the gases referred to a standard gas.

If we use $\theta$ to denote the temperature reckoned from absolute zero of a gas thermometer, $M_0$ the mass of a molecule of hydrogen, $V_0^2$ its mean square of velocity at temperature unity, $s$ the specific gravity of any other gas referred to hydrogen, then the mass of a molecule of the other gas is

$$M=M_0 s. \quad \ldots \quad (101)$$

Its mean square of velocity,

$$V^2=\frac{1}{s}V_0^2\theta. \quad \ldots \quad (102)$$

Pressure of the gas,

$$p=\tfrac{1}{3}\rho\theta V_0^2. \quad \ldots \quad (103)$$

We may next determine the amount of cooling by expansion.

*Cooling by Expansion.*

Let the expansion be equal in all directions, then

$$\frac{du}{dx}=\frac{dv}{dy}=\frac{dw}{dz}=-\frac{1}{3\rho}\frac{\partial\rho}{\partial t}, \quad \ldots \quad (104)$$

and $\frac{du}{dy}$ and all terms of unsymmetrical form will be zero.

If the mass of gas is of the same temperature throughout there will be no conduction of heat, and the equation (94) will become

$$\tfrac{1}{2}\rho\beta\frac{\partial\overline{V^2}}{t}-\tfrac{1}{3}\overline{V^2}\frac{\partial\rho}{\partial t}=0, \quad \ldots \quad (105)$$

or

$$2\frac{\partial\rho}{\rho}=3\beta\frac{\partial\overline{V^2}}{\overline{V}}=3\beta\frac{\partial\theta}{\theta}, \quad \ldots \quad (106)$$

or

$$\frac{\partial\theta}{\theta}=\frac{2}{3\beta}\frac{\partial\rho}{\rho}, \quad \ldots \quad (107)$$

which gives the relation between the density and the temperature in a gas expanding

without exchange of heat with other bodies. We also find

$$\frac{\partial p}{p}=\frac{\partial \varrho}{\varrho}+\frac{\partial \theta}{\theta}$$

$$=\frac{2+3\beta}{3\beta}\frac{\partial \varrho}{\varrho}, \quad \ldots \ldots \ldots \ldots \quad (108)$$

which gives the relation between the pressure and the density.

*Specific Heat of Unit of Mass at Constant Volume.*

The total energy of agitation of unit of mass is $\beta V^2=E$, or

$$E=\frac{3\beta}{2}\frac{p}{\varrho}. \quad \ldots \ldots \ldots \ldots \quad (109)$$

If, now, additional energy in the form of heat be communicated to it without changing its density,

$$\partial E=\frac{3\beta}{2}\frac{\partial p}{\varrho}=\frac{3\beta}{2}\frac{p}{\varrho}\frac{\partial \theta}{\theta}. \quad \ldots \ldots \ldots \quad (110)$$

Hence the specific heat of unit of mass of constant volume is in dynamical measure

$$\frac{\partial E}{\partial \theta}=\frac{3\beta}{2}\frac{p}{\varrho\theta}. \quad \ldots \ldots \ldots \ldots \quad (111)$$

*Specific Heat of Unit of Mass at Constant Pressure.*

By the addition of the heat $\partial E$ the temperature was raised $\partial\theta$ and the pressure $\partial p$. Now, let the gas expand without communication of heat till the pressure sinks to its former value, and let the final temperature be $\theta+\partial'\theta$. The temperature will thus sink by a quantity $\partial\theta-\partial'\theta$, such that

$$\frac{\partial\theta-\partial'\theta}{\theta}=\frac{2}{2+3\beta}\frac{\partial p}{p}=\frac{2}{2+3\beta}\frac{\partial\theta}{\theta},$$

whence

$$\frac{\partial'\theta}{\theta}=\frac{3\beta}{2+3\beta}\frac{\partial\theta}{\theta}; \quad \ldots \ldots \ldots \quad (112)$$

and the specific heat of unit of mass at constant pressure is

$$\frac{\partial E}{\partial'\theta}=\frac{2+3\beta}{2}\frac{p}{\varrho\theta}. \quad \ldots \ldots \ldots \quad (113)$$

The ratio of the specific heat at constant pressure to that of constant volume is known in several cases from experiment. We shall denote this ratio by

$$\gamma=\frac{2+3\beta}{3\beta}, \quad \ldots \ldots \ldots \quad (114)$$

whence

$$\beta=\frac{2}{3}\frac{1}{\gamma-1}. \quad \ldots \ldots \ldots \quad (115)$$

The specific heat of unit of volume in ordinary measure is at constant volume

$$\frac{1}{\gamma-1}\frac{p}{J\theta}, \quad \ldots \ldots \ldots \quad (116)$$

and at constant pressure

$$\frac{\gamma}{\gamma-1}\frac{p}{\mathrm{J}\delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (117)$$

where J is the mechanical equivalent of unit of heat.

From these expressions Dr. RANKINE* has calculated the specific heat of air, and has found the result to agree with the value afterwards determined experimentally by M. REGNAULT†.

*Thermal Effects of Diffusion.*

If two gases are diffusing into one another, then, omitting the terms relating to heat generated by friction and to conduction of heat, the equation (94) gives

$$\left.\begin{aligned}&\tfrac{1}{2}\varrho_1\frac{\partial}{\partial t}\beta_1(\xi_1^2+\eta_1^2+\zeta_1^2)+\tfrac{1}{2}\varrho_2\frac{\partial}{\partial t}\beta_2(\xi_2^2+\eta_2^2+\zeta_2^2)+p_1\left(\frac{du_1}{dx}+\frac{dv_1}{dy}+\frac{dw_1}{dz}\right)+p_2\left(\frac{du_2}{dx}+\frac{dv_2}{dy}+\frac{dw_2}{dz}\right)\\&=k\varrho_1\varrho_2\mathrm{A}_1\{(u_1-u_2)^2+(v_1-v_2)^2+(w_1-w_2)^2\}.\end{aligned}\right\} \quad 118)$$

By comparison with equations (78), (79), the right-hand side of this equation becomes

$$\begin{aligned}&\mathrm{X}(\varrho_1u_1+\varrho_2u_2)+\mathrm{Y}(\varrho_1v_1+\varrho_2v_2)+\mathrm{Z}(\varrho_1w_1+\varrho_2w_2)\\&\quad-\left(\frac{dp_1}{dx}u_1+\frac{dp_1}{dy}v_1+\frac{dp_1}{dz}w_1\right)-\left(\frac{dp_2}{dx}u_2+\frac{dp_2}{dy}v_2+\frac{dp_2}{dz}w_2\right)\\&\quad-\tfrac{1}{2}\varrho_1\frac{\partial}{\partial t}(u_1^2+v_1^2+w_1^2)-\tfrac{1}{2}\varrho_2\frac{\partial}{\partial t}(u_2^2+v_2^2+w_2^2).\end{aligned}$$

The equation (118) may now be written

$$\left.\begin{aligned}&\tfrac{1}{2}\varrho_1\frac{\partial}{\partial t}\left(u_1^2+v_1^2+w_1^2+\beta_1(\xi_1^2+\eta_1^2+\zeta_1^2)\right)+\tfrac{1}{2}\varrho_2\frac{\partial}{\partial t}\left(u_2^2+v_2^2+w_2^2+\beta_2(\xi_2^2+\eta_2^2+\zeta_2^2)\right)\\&=\mathrm{X}(\varrho_1u_1+\varrho_2u_2)+\mathrm{Y}(\varrho_1v_1+\varrho_2v_2)+\mathrm{Z}(\varrho_1w_1+\varrho_2w_2)-\left(\frac{d.pu}{dx}+\frac{d.pv}{dy}+\frac{d.pw}{dz}\right).\end{aligned}\right\} \quad (119)$$

The whole increase of energy is therefore that due to the action of the external forces *minus* the cooling due to the expansion of the mixed gases. If the diffusion takes place without alteration of the volume of the mixture, the heat due to the mutual action of the gases in diffusion will be exactly neutralized by the cooling of each gas as it expands in passing from places where it is dense to places where it is rare.

*Determination of the Inequality of Pressure in different directions due to the Motion of the Medium.*

Let us put

$$\varrho_1\overline{\xi_1^2}=p_1+q_1 \text{ and } \varrho_2\overline{\xi_2^2}=p_2+q_2. \quad \ldots\ldots\ldots \quad (120)$$

Then by equation (52),

$$\left.\begin{aligned}\frac{\delta q_1}{\delta t}=&-3k_1\mathrm{A}_2\varrho_1q_1-\frac{k}{\mathrm{M}_1+\mathrm{M}_2}(2\mathrm{M}_1\mathrm{A}_1+3\mathrm{M}_2\mathrm{A}_2)\varrho_1q_1-k(3\mathrm{A}_2-2\mathrm{A}_1)\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}\varrho_1q_2\\&-k\varrho_1\varrho_2\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}\mathrm{A}_2-\frac{2}{3}\mathrm{A}_1)(2\overline{u_1-u_2}-\overline{v_1-v_2}-\overline{w_1-w_2}),\end{aligned}\right\} \quad (121)$$

* Transactions of the Royal Society of Edinburgh, vol. xx. (1850). † Comptes Rendus, 1853.

the last term depending on diffusion; and if we omit in equation (75) terms of three dimensions in $\xi$, $\eta$, $\zeta$, which relate to conduction of heat, and neglect quantities of the form $\xi\eta\varrho$ and $\varrho\xi^2-p$, when not multiplied by the large coefficients $k$, $k_1$, and $k_2$, we get

$$\frac{\partial q}{\partial t}+2p\frac{du}{dx}-\frac{2}{3}p\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)=\frac{\delta q}{\delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

If the motion is not subject to any very rapid changes, as in all cases except that of the propagation of sound, we may neglect $\frac{\partial q}{\partial t}$. In a single system of molecules

$$\frac{\overline{\delta q}}{\delta t}=-3kA_2\varrho q, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

whence

$$q=-\frac{2p}{3kA_2\varrho}\left\{\frac{du}{dx}-\frac{1}{3}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

If we make

$$\frac{1}{3}\frac{1}{kA_2}\frac{p}{\varrho}=\mu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125)$$

$\mu$ will be the coefficient of viscosity, and we shall have by equation (120),

$$\left.\begin{aligned}\varrho\xi^2&=p-2\mu\left\{\frac{du}{dx}-\frac{1}{3}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right\},\\ \varrho\eta^2&=p-2\mu\left\{\frac{dv}{dy}-\frac{1}{3}\left(\frac{du}{dx}+\frac{dv}{d}+\frac{dw}{dz}\right)\right\},\\ \varrho\zeta^2&=p-2\mu\left\{\frac{dw}{dz}-\frac{1}{3}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right\};\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

and by transformation of coordinates we obtain

$$\left.\begin{aligned}\varrho\eta\zeta&=-\mu\left(\frac{dv}{dz}+\frac{dw}{dy}\right),\\ \varrho\zeta\xi&=-\mu\left(\frac{dw}{dx}+\frac{du}{dz}\right),\\ \varrho\xi\eta&=-\mu\left(\frac{du}{dy}+\frac{dv}{dx}\right).\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

These are the values of the normal and tangential stresses in a simple gas when the variation of motion is not very rapid, and when $\mu$, the coefficient of viscosity, is so small that its square may be neglected.

*Equations of Motion corrected for Viscosity.*

Substituting these values in the equation of motion (76), we find

$$\varrho\frac{\partial u}{\partial t}+\frac{dp}{dx}-\mu\left\{\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}\right\}-\frac{1}{3}\mu\frac{d}{dx}\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)=X\varrho, \quad . \quad . \quad . \quad . \quad (128)$$

with two other equations which may be written down from symmetry. The form of these equations is identical with that of those deduced by POISSON* from the theory of

* Journal de l'École Polytechnique, 1829, tom. xiii. cah. xx. p. 139.

elasticity, by supposing the strain to be continually relaxed at a rate proportional to its amount. The ratio of the third and fourth terms agrees with that given by Professor STOKES*.

If we suppose the inequality of pressure which we have denoted by $q$ to exist in the medium at any instant, and not to be maintained by the motion of the medium, we find, from equation (123),

$$q_1 = Ce^{-2k\mathrm{A}_{s^2}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

$$= Ce^{-\frac{t}{\mathrm{T}}} \text{ if } \mathrm{T} = \frac{1}{3k\mathrm{A}_{s^2}} = \frac{\mu}{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (130)$$

the stress $q$ is therefore relaxed at a rate proportional to itself, so that

$$\frac{\delta q}{q} = \frac{\delta t}{\mathrm{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

We may call T the modulus of the time of relaxation.

If we next make $k=0$, so that the stress $q$ does not become relaxed, the medium will be an elastic solid, and the equation

$$\frac{\partial(\rho\xi^2 - p)}{\partial t} + 2p\frac{du}{dx} - \tfrac{2}{3}p\left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

may be written

$$\frac{\partial}{\partial t}\left\{(p_{xx} - p) + 2p\frac{d\alpha}{dx} - \tfrac{2}{3}p\left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz}\right)\right\} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

where $\alpha$, $\beta$, $\gamma$ are the displacements of an element of the medium, and $p_{xx}$ is the normal pressure in the direction of $x$. If we suppose the initial value of this quantity zero, and $p_{xx}$ originally equal to $p$, then, after a small displacement,

$$p_{xx} = p - p\left(\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz}\right) - 2p\frac{d\alpha}{dx}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

and by transformation of coordinates the tangential pressure

$$p_{xy} = -p\left(\frac{d\alpha}{dy} + \frac{d\beta}{dx}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

The medium has now the mechanical properties of an elastic solid, the rigidity of which is $p$, while the cubical elasticity is $\frac{5}{3}p$†.

The same result and the same ratio of the elasticities would be obtained if we supposed the molecules to be at rest, and to act on one another with forces depending on the distance, as in the statical molecular theory of elasticity. The coincidence of the properties of a medium in which the molecules are held in equilibrium by attractions and repulsions, and those of a medium in which the molecules move in straight lines without acting on each other at all, deserves notice from those who speculate on theories of physics.

The fluidity of our medium is therefore due to the mutual action of the molecules, causing them to be deflected from their paths.

* "On the Friction of Fluids in Motion and the Equilibrium and Motion of Elastic Solids," Cambridge Phil. Trans. vol. viii. (1845), p. 297, equation (12).

† Ibid. p. 311, equation (29).

The coefficient of instantaneous rigidity of a gas is therefore $p$.
The modulus of the time of relaxation is T.
The coefficient of viscosity is $\mu = p\mathrm{T}$. . . . (136)

Now $p$ varies as the density and temperature conjointly, while T varies inversely as the density.

Hence $\mu$ varies as the absolute temperature, and is independent of the density.

This result is confirmed by the experiments of Mr. GRAHAM on the Transpiration of Gases*, and by my own experiments on the Viscosity or Internal Friction of Air and other Gases†.

The result, that the viscosity is independent of the density, follows from the Dynamical Theory of Gases, whatever be the law of force between the molecules. It was deduced by myself‡ from the hypothesis of hard elastic molecules, and M. O. E. MEYER§ has given a more complete investigation on the same hypothesis.

The experimental result, that the viscosity is proportional to the absolute temperature, requires us to abandon this hypothesis, which would make it vary as the square root of the absolute temperature, and to adopt the hypothesis of a repulsive force inversely as the fifth power of the distance between the molecules, which is the only law of force which gives the observed result.

Using the foot, the grain, and the second as units, my experiments give for the temperature of 62° FAHRENHEIT, and in dry air,

$$\mu = 0{\cdot}0936.$$

If the pressure is 30 inches of mercury, we find, using the same units,

$$p = 477360000.$$

Since $p\mathrm{T} = \mu$, we find that the modulus of the time of relaxation of rigidity in air of this pressure and temperature is

$$\frac{1}{5099100000} \text{ of a second.}$$

This time is exceedingly small, even when compared with the period of vibration of the most acute audible sounds; so that even in the theory of sound we may consider the motion as steady during this very short time, and use the equations we have already found, as has been done by Professor STOKES‖.

### *Viscosity of a Mixture of Gases.*

In a complete mixture of gases, in which there is no diffusion going on, the velocity at any point is the same for all the gases.

* Philosophical Transactions, 1846 and 1849.

† Proceedings of the Royal Society, February 8, 1866; Philosophical Transactions, 1866, p. 249.

‡ Philosophical Magazine, January 1860. § POGGENDORFF's 'Annalen,' 1865.

‖ "On the effect of the Internal Friction of Fluids on the motion of Pendulums," Cambridge Transactions, vol. ix. (1850), art. 79.

Putting

$$\frac{2}{3}\left(2\frac{du}{dx}-\frac{dv}{dy}-\frac{dw}{dz}\right)=\mathrm{U}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

equation (122) becomes

$$p_1\mathrm{U}=-3k_1\mathrm{A}_2\varrho_1q_1-\frac{k}{\mathrm{M}_1+\mathrm{M}_2}(2\mathrm{M}_1\mathrm{A}_1+3\mathrm{M}_2\mathrm{A}_2)\varrho_2q_1-k(3\mathrm{A}_2-2\mathrm{A}_1)\frac{\mathrm{M}_2}{\mathrm{M}_1+\mathrm{M}_2}\varrho_1q_2. \quad . \quad (138)$$

Similarly,

$$p_2\mathrm{U}=-3k_2\mathrm{A}_2\varrho_2q_2-\frac{k}{\mathrm{M}_1+\mathrm{M}_2}(2\mathrm{M}_2\mathrm{A}_1+3\mathrm{M}_1\mathrm{A}_2)\varrho_1q_2-k(3\mathrm{A}_2-2\mathrm{A}_1)\frac{\mathrm{M}_1}{\mathrm{M}_1+\mathrm{M}_2}\varrho_2q_1. \quad . \quad (139)$$

Since $p=p_1+p_2$ and $q=q_1+q_2$, where $p$ and $q$ refer to the mixture, we shall have

$$\mu\mathrm{U}=-q=-(q_1+q_2),$$

where $\mu$ is the coefficient of viscosity of the mixture.

If we put $s_1$ and $s_2$ for the specific gravities of the two gases, referred to a standard gas, in which the values of $p$ and $\varrho$ at temperature $\theta_0$ are $p_0$ and $\varrho_0$,

$$\mu=\frac{p_0\theta}{\varrho_0\theta_0}\cdot\frac{\mathrm{E}p_1^2+\mathrm{F}p_1p_2+\mathrm{G}p_2^2}{3\mathrm{A}_2k_1s_1\mathrm{E}p_1^2+\mathrm{H}p_1p_2+3\mathrm{A}_1k_2s_2\mathrm{G}p_2^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

where $\mu$ is the coefficient of viscosity of the mixture, and

$$\left.\begin{aligned}
\mathrm{E}&=\frac{ks_1}{s_1+s_2}(2s_2\mathrm{A}_1+3s_1\mathrm{A}_2),\\
\mathrm{F}&=3\mathrm{A}_2(k_1s_1+k_2s_2)-(3\mathrm{A}_2-2\mathrm{A}_1)k\frac{2s_1s_2}{s_1+s_2},\\
\mathrm{G}&=\frac{ks_2}{s_1+s_2}(2s_1\mathrm{A}_1+3s_2\mathrm{A}_2),\\
\mathrm{H}&=3\mathrm{A}_2s_1s_2\{3k_1k_2\mathrm{A}_2+2k^2\mathrm{A}_1\}.
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

This expression is reduced to $\mu_1$ when $p_2=0$, and to $\mu_2$ when $p_1=0$. For other values of $p_1$ and $p_2$ we require to know the value of $k$, the coefficient of mutual interference of the molecules of the two gases. This might be deduced from the observed values of $\mu$ for mixtures, but a better method is by making experiments on the interdiffusion of the two gases. The experiments of GRAHAM on the transpiration of gases, combined with my experiments on the viscosity of air, give as values of $k_1$ for air, hydrogen, and carbonic acid,

| | | |
|---|---|---|
| Air . . . . . | $k_1=$ | $4{\cdot}81\times10^{10}$, |
| Hydrogen . . . | $k_1=$ | $142{\cdot}8\times10^{10}$, |
| Carbonic acid . . | $k_1=$ | $3{\cdot}9\times10^{10}$. |

The experiments of GRAHAM in 1863, referred to at page 73, on the interdiffusion of air and carbonic acid, give the coefficient of mutual interference of these gases,

$$\text{Air and carbonic acid} \quad . \quad k=5{\cdot}2\times10^{10};$$

and by taking this as the absolute value of $k$, and assuming that the ratios of the coefficients of interdiffusion given at page 76 are correct, we find

$$\text{Air and hydrogen} \quad . \quad . \quad k=29{\cdot}8\times10^{10}.$$

These numbers are to be regarded as doubtful, as we have supposed air to be a simple gas in our calculations, and we do not know the value of $k$ between oxygen and nitrogen. It is also doubtful whether our method of calculation applies to experiments such as the earlier observations of Mr. GRAHAM.

I have also examined the transpiration-times determined by GRAHAM for mixtures of hydrogen and carbonic acid, and hydrogen and air, assuming a value of $k$ roughly, to satisfy the experimental results about the middle of the scale. It will be seen that the calculated numbers for hydrogen and carbonic acid exhibit the peculiarity observed in the experiments, that a small addition of hydrogen *increases* the transpiration-time of carbonic acid, and that in both series the times of mixtures depend more on the slower than on the quicker gas.

The assumed values of $k$ in these calculations were—

For hydrogen and carbonic acid $k=12{\cdot}5\times10^{10}$,

For hydrogen and air . . . . $k=18{\cdot}8\times10^{10}$;

and the results of observation and calculation are, for the times of transpiration of mixtures of—

| Hydrogen and Carbonic acid. | | Observed. | Calculated. | Hydrogen and Air. | | Observed. | Calculated. |
|---|---|---|---|---|---|---|---|
| 100 | 0 | ·4321 | ·4375 | 100 | 0 | ·4434 | ·4375 |
| 97·5 | 2·5 | ·4714 | ·4750 | 95 | 5 | ·5282 | ·5300 |
| 95 | 5 | ·5157 | ·5089 | 90 | 10 | ·5880 | ·6028 |
| 90 | 10 | ·5722 | ·5678 | 75 | 25 | ·7488 | ·7438 |
| 75 | 25 | ·6786 | ·6822 | 50 | 50 | ·8179 | ·8488 |
| 50 | 50 | ·7339 | ·7652 | 25 | 75 | ·8790 | ·8946 |
| 25 | 75 | ·7535 | ·7468 | 10 | 90 | ·8880 | ·8983 |
| 10 | 90 | ·7521 | ·7361 | 5 | 95 | ·8960 | ·8996 |
| 0 | 100 | ·7470 | ·7272 | 0 | 100 | ·9000 | ·9010 |

The numbers given are the ratios of the transpiration-times of mixtures to that of oxygen as determined by Mr. GRAHAM, compared with those given by the equation (140) deduced from our theory.

*Conduction of Heat in a Single Medium* ($\gamma$).

The rate of conduction depends on the value of the quantity

$$\tfrac{1}{3}\beta\rho(\overline{\xi^2}+\overline{\eta^2}+\overline{\zeta^2}),$$

where $\overline{\xi^2}$, $\overline{\eta^2}$, and $\overline{\zeta^2}$ denote the mean values of those functions of $\xi$, $\eta$, $\zeta$ for all the molecules in a given element of volume.

As the expressions for the variations of this quantity are somewhat complicated in a mixture of media, and as the experimental investigation of the conduction of heat in gases is attended with great difficulty, I shall confine myself here to the discussion of a single medium.

Putting

$$Q=M(u+\xi)\{u^2+v^2+w^2+2u\zeta+2v\eta+2w\zeta+\beta(\xi^2+\eta^2+\zeta^2)\}, \quad . \quad . \quad . \quad (142)$$

and neglecting terms of the forms $\xi\eta$ and $\xi^3$ and $\xi\eta^2$ when not multiplied by the large coefficient $k_1$, we find by equations (75), (77), and (54),

$$\left.\begin{aligned}\rho\frac{\partial}{\partial t}\beta(\xi^2+\xi\eta^2+\xi\zeta^2)+\beta\frac{d}{dx}\cdot\rho(\xi^4+\xi^2\eta^2+\xi^2\zeta^2)-\beta(\xi^2+\eta^2+\zeta^2)\frac{dp}{dx}-2\beta\xi^2\frac{dp}{dx}\\ =-3k_1\rho^2A_2\beta\{\xi^3+\xi\eta^2+\xi\zeta^2\}.\end{aligned}\right\} \quad . \quad (143)$$

The first term of this equation may be neglected, as the rate of conduction will rapidly establish itself. The second term contains quantities of four dimensions in $\xi$, $\eta$, $\zeta$, whose values will depend on the distribution of velocity among the molecules. If the distribution of velocity is that which we have proved to exist when the system has no external force acting on it and has arrived at its final state, we shall have by equations (29), (31), (32),

$$\overline{\xi^4}=\overline{3\xi^2}.\overline{\xi^2}=3\frac{p^2}{\rho^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

$$\overline{\xi^2\eta^2}=\overline{\xi^2}.\overline{\eta^2}=\frac{p^2}{\rho^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

$$\overline{\xi^2\zeta^2}=\overline{\xi^2}.\overline{\zeta^2}=\frac{p^2}{\rho^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

and the equation of conduction may be written

$$5\beta\frac{p^2}{\rho^2}\frac{d\theta}{dx}=-3k_1\rho^2A_2\beta\{\xi^3+\xi\eta^2+\xi\zeta^2\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

[Addition made December 17, 1866.]

[*Final Equilibrium of Temperature.*]

[The left-hand side of equation (147), as sent to the Royal Society, contained a term $2(\beta-1)\frac{p}{\rho}\frac{dp}{dx}$, the result of which was to indicate that a column of air, when left to itself, would assume a temperature varying with the height, and greater above than below. The mistake arose from an error* in equation (143). Equation (147), as now corrected, shows that the flow of heat depends on the variation of temperature only, and not on the direction of the variation of pressure. A vertical column would therefore, when in thermal equilibrium, have the same temperature throughout.

When I first attempted this investigation I overlooked the fact that $\overline{\xi^4}$ is not the same as $\overline{\xi^2}.\overline{\xi^2}$, and so obtained as a result that the temperature diminishes as the height increases at a greater rate than it does by expansion when air is carried up in mass. This leads at once to a condition of instability, which is inconsistent with the second law of thermodynamics. I wrote to Professor Sir W. THOMSON about this result, and the difficulty I had met with, but presently discovered *one* of my mistakes, and arrived at the conclusion that the temperature would increase with the height. This does not lead to mecha-

* The last term on the left-hand side was not multiplied by $\beta$.

nical instability, or to any self-acting currents of air, and I was in some degree satisfied with it. But it is equally inconsistent with the second law of thermodynamics. In fact, if the temperature of any substance, when in thermic equilibrium, is a function of the height, that of any other substance must be the same function of the height. For if not, let equal columns of the two substances be enclosed in cylinders impermeable to heat, and put in thermal communication at the bottom. If, when in thermal equilibrium, the tops of the two columns are at different temperatures, an engine might be worked by taking heat from the hotter and giving it up to the cooler, and the refuse heat would circulate round the system till it was all converted into mechanical energy, which is in contradiction to the second law of thermodynamics.

The result as now given is, that temperature in gases, when in thermal equilibrium, is independent of height, and it follows from what has been said that temperature is independent of height in all other substances.

If we accept this law of temperature as the actual one, and examine our assumptions, we shall find that unless $\overline{\xi^2}=3\overline{\xi^2}.\overline{\xi^2}$, we should have obtained a different result. Now this equation is derived from the law of distribution of velocities to which we were led by independent considerations. We may therefore regard this law of temperature, if true, as in some measure a confirmation of the law of distribution of velocities.]

### *Coefficient of Conductivity.*

If C is the coefficient of conductivity of the gas for heat, then the quantity of heat which passes through unit of area in unit of time measured as mechanical energy, is

$$C\frac{d\vartheta}{dx}=\frac{5}{6}\frac{\beta}{k_1 A_2}\frac{p^2}{\rho^2\vartheta}\frac{d\vartheta}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

by equation (147).

Substituting for $\beta$ its value in terms of $\gamma$ by equation (115), and for $k_1$ its value in terms of $\mu$ by equation (125), and calling $p_0$, $\rho_0$, and $\vartheta_0$ the simultaneous pressure, density, and temperature of the standard gas, and $s$ the specific gravity of the gas in question, we find

$$C=\frac{5}{3(\gamma-1)}\frac{p_0}{\rho_0\vartheta_0}\frac{\mu}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

For air we have $\gamma=1{\cdot}409$, and at the temperature of melting ice, or 274°·6 C. above absolute zero, $\sqrt{\frac{p}{\rho}}=918{\cdot}6$ feet per second, and at 16°·6 C., $\mu=0{\cdot}0936$ in foot-grain-second measure. Hence for air at 16°·6 C the conductivity for heat is

$$C=1172. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

That is to say, a horizontal stratum of air one foot thick, of which the upper surface is kept at 17° C., and the lower at 16° C., would in one second transmit through every square foot of horizontal surface a quantity of heat the mechanical energy of which is equal to that of 2344 grains moving at the rate of one foot per second.

Principal FORBES* has deduced from his experiments on the conduction of heat in bars, that a plate of wrought iron one foot thick, with its opposite surfaces kept 1° C. different in temperature, would, when the mean temperature is 25° C., transmit in one minute through every square foot of surface as much heat as would raise one cubic foot of water 0°·0127 C.

Now the dynamical equivalent in foot-grain-second measure of the heat required to raise a cubic foot of water 1° C. is $1{\cdot}9157 \times 10^{10}$.

It appears from this that iron at 25° C. conducts heat 3525 times better than air at 16°·6 C.

M. CLAUSIUS, from a different form of the theory, and from a different value of $\mu$, found that lead should conduct heat 1400 times better than air. Now iron is twice as good a conductor of heat as lead, so that this estimate is not far different from that of M. CLAUSIUS in actual value.

In reducing the value of the conductivity from one kind of measure to another, we must remember that its dimensions are $MLT^{-3}$, when expressed in absolute dynamical measure.

Since all the quantities which enter into the expression for C are constant except $\mu$, the conductivity is subject to the same laws as the viscosity, that is, it is independent of the pressure, and varies directly as the absolute temperature. The conductivity of iron diminishes as the temperature increases.

Also, since $\gamma$ is nearly the same for air, oxygen, hydrogen, and carbonic oxide, the conductivity of these gases will vary as the ratio of the viscosity to the specific gravity. Oxygen, nitrogen, carbonic oxide, and air will have equal conductivity, while that of hydrogen will be about seven times as great.

The value of $\gamma$ for carbonic acid is 1·27, its specific gravity is $1\frac{1}{2}$ of oxygen, and its viscosity $\frac{4}{5}$ of that of oxygen. The conductivity of carbonic acid for heat is therefore about $\frac{1}{2}$ of that of oxygen or of air.

* "Experimental Inquiry into the Laws of the Conduction of Heat in Bars," Edinburgh Transactions, 1861–62.

# V. *Experimental Researches in Magnetism and Electricity.* By H. Wilde, *Esq.* *Communicated by Dr.* Faraday.

Received March 26,—Read April 26, 1866.

§ 1. *On some new and paradoxical Phenomena in Electro-magnetic Induction, and their relation to the Principle of the Conservation of Physical Force.*

§ 2. *On a new and powerful Generator of Dynamic Electricity.*

1. The principle of the conservation of force, as I apprehend it, is the definite quantitative relation existing between all the phenomena of the universe whatsoever, both in direction and amount, whether such phenomena be considered in the relation of cause and effect, or as antecedent and consequent events.

2. In the particular application of this principle to the advancement of physical science, and also to the invention of new engines and machinery to meet the progressive requirements of society, problems not unfrequently present themselves which involve the consideration of static and dynamic force, from several different aspects; and the solution of these problems often brings out results which are as surprising as they are paradoxical. Of such cases, in which the idea of paradox alluded to is involved, may be mentioned the one contained in the 36th Proposition of Newton's 'Principia' (Book 2, Cor. 2)*, and in D. Bernoulli's 'Hydrodynamica,' p. 279; in which the repulsive force of a jet of water issuing from a hole in the bottom or side of a vessel with a velocity which a body would acquire in falling freely from the surface, is equal to the weight of a column of water of which the base is equal to the section of the contracted vein and about twice the height of the column which produces the flowing pressure; the *static force* of reaction being thus double that which, without experiment, had been predicted†. An instance in which the quantity of *dynamic force* is increased paradoxically may be seen in that curious and useful piece of apparatus the injector, by means of which a boiler containing steam of high pressure is able to feed itself with water through a hole in its shell, though this hole is open to the atmosphere; or the steam from a low-pressure boiler is enabled to drive the feed-water through a hole (also open to the atmosphere) into a high-pressure boiler. Although, when rightly interpreted, these examples of paradox, as well as many others of a similar character, are in strict accordance with the principle of conservation, yet they are at the same time contrary to the inferences which are generally drawn from analogical reasonings, and to some of those maxims of science which are framed for the instruction of the unlearned. As the examples cited are only

* Principia, 2nd Edition.

† Ibid. 1st Edition, Book 2. Prop. 37.

adduced for the purpose of illustrating some analogous phenomena observed in connexion with certain combinations of static and dynamic force in molecular mechanics which form the subject of the present research, it is not my intention to enter into the rationale of either of them, but to direct attention to some new and paradoxical phenomena arising out of FARADAY'S important discovery of magneto-electric induction, the close consideration of which has resulted in the discovery of a means of producing dynamic electricity in quantities unattainable by any apparatus hitherto constructed.

3. If round a piece of iron forming the armature of a permanent magnet a quantity of insulated wire be wound at right angles to the line which joins the poles of the magnet, and if the free ends of the wire be connected together directly, or indirectly by the interposition of some conductor, a momentary wave of electricity, as is well known, is generated in the wire every time the armature is suddenly removed from the magnet, or suddenly approaches it; and the wave of electricity generated by the removal of the armature moves in the opposite direction to that generated by the approach of the armature. With a description of this simple experiment, FARADAY announced (in 1831) the discovery of magneto-electricity*, which was found to possess all the distinguishing characteristics of electricity derived from any other source.

4. The force of a permanent magnet is usually estimated by the weight which is required to separate the armature or submagnet from its poles; and if the question were asked, for the first time, what relation existed between the sustaining-power of an electro-magnet excited by means of a magneto-electric machine, and the sustaining-power of the permanent magnet from which the electricity was derived, it would probably be answered, that since the permanent magnet was the primary cause of the phenomena, the electro-magnet would possess, at the most, no greater sustaining-power than the permanent magnet. This, however, is not the case; for I have found that an indefinitely small amount of magnetism, or of dynamic electricity, is capable of inducing an indefinitely large amount of magnetism. And again, that an indefinitely small amount of dynamic electricity, or of magnetism, is capable of evolving an indefinitely large amount of dynamic electricity.

5. That FARADAY himself stood on the threshhold of this discovery, will be obvious from the following observations made by him in a paper "On the Physical Character of the Lines of Magnetic Force"†, in which, when speaking of the magnet as a source of electricity, he says, "Its analogy with the helix is wonderful, nevertheless there is, as yet, a striking experimental distinction between them; for whereas an unchangeable magnet can never raise up a piece of soft iron to a state more than equal to its own, as measured by the moving wire (3219), a helix carrying a current can develope in an iron core magnetic force, of a hundred or more times as much power as that possessed by itself, when measured by the same means. In every point of view, therefore, the magnet deserves the utmost exertions of the philosopher for the development of its nature, both as a magnet and also as a source of electricity, that we may become acquainted with the

* Philosophical Transactions, 1832, vol. cxxii. † Philosophical Magazine, June 1852, par. 3273.

great law under which the apparent anomaly may disappear, and by which all these various phenomena presented to us shall become *one*."

6. As the investigations which led to the paradoxical conclusions enunciated above (4) were not originally intended for publication, but were undertaken for my own instruction, I find that it will be much more convenient to describe the experimental results in a more methodical manner than that in which they were obtained.

7. The numerical determinations, derived from the experiments to be described, will be given with sufficient exactitude to allow of a comparison being made between them and those of other experimentalists. Other quantitative determinations will, for the present purpose, be sufficiently expressed by the terms "greater" and "less," as attention will be chiefly confined to a description of well-defined phenomenal effects.

8. Though I make use of certain conventional terms in connexion with the subject of these researches, it is not thence to be inferred that I hold to the opinion that specific entities distinct from ordinary matter are concerned in the production of phenomena of any kind whatever.

9. The apparatus with which the experiments were made is shown in Plate VI. figs. 1–10. Two blocks of cast iron, A, A, of the form shown in figs. 1, 2, 3, and two pieces of brass, B, B, of the same length as the cast-iron blocks, were bolted together at the top and bottom with small brass bolts, in such a manner as to form a compound hollow cylinder of brass and iron, hereafter called the magnet-cylinder. A smooth and parallel hole C, $1\frac{5}{8}$ inch in diameter, was bored through the magnet-cylinder; and the thickness of the brass packings, B, B, separating the iron sides of the cylinder from one another, was three-quarters of an inch, or nearly half the diameter of the hole. Two pillars of wrought iron, D, D (fig. 3), were screwed into the cast-iron projections E, E (figs. 1, 2, 3) at each end of the magnet-cylinder, for the purpose of holding the cross-heads F, F, fig. 3. These cross-heads were made of brass, and were bored out concentrically with the hole through the magnet-cylinder, so as to form suitable bearings in which the journals of an armature might revolve.

10. The armature, which was of the same form as that used by SIEMENS (figs. 4, 5, 6, 7), was made of cast iron, and was turned parallel throughout its entire length, and about one-twentieth of an inch less in diameter than the hole in the magnet-cylinder, for the purpose of allowing it to revolve inside the cylinder in close proximity to it, but without touching it. The thickness of the rib G, uniting the segmental sides of the armature (figs. 4, 5, 7), was one-quarter of an inch. Two brass disks or caps, H, H', having concentric prolongations for holding the steel journals I, I, were fitted by means of screws, one at each end of the armature. A pulley, K, for driving the armature was fixed upon the cylindrical axis of the cap H', and upon the axis of the cap H at the other end of the armature, a commutator, L, L', of hardened steel was fixed.

11. About 163 feet of copper wire 0·03 of an inch in diameter, insulated with silk, was wound upon the armature (fig. 6) in the direction of its length. The inner extremity of the wire was placed in good metallic contact with the armature, and its outer

extremity was connected with the insulated half L′ of the commutator by means of a clip and binding screw. Bands, encircling the armature at intervals, and sunk below the surface of the iron in grooves turned out for their reception, prevented the convolutions of insulated wire from flying out of position by the centrifugal force attending their rapid revolution. The armature is represented complete in fig. 6, and in section in fig. 7.

12. A number of exactly similar permanent magnets (of the form shown in fig. 1), 8 inches long, were made from bars of steel 1 inch wide and a quarter of an inch thick, and the distance between the inner edges of the polar extremities of the magnets was a little less than 2 inches. The magnets weighed about one pound each, and they received very nearly equal degrees of magnetism, which enabled them to support a weight of about ten pounds each.

13. On each side of the magnet-cylinder was a flat raised surface, M, M, figs. 2, 3, extending the whole length of the cylinder between the projections E, E. These surfaces were planed parallel with each other and with the axis of the magnet-cylinder. When the magnets, the legs of which were somewhat less than 2 inches apart, were sprung upon the cylinder in the position shown in fig. 1, they were held in sufficiently good contact for the magnetism to diffuse itself equally throughout the entire mass of the cylinder; the two cast-iron sides of which, consequently, formed the poles of the magnets. On the lower part of the magnet-cylinder four projections or feet, N, N, N, N, figs. 2, 3, were cast, by means of which it was bolted firmly to a wooden base.

14. When the armature was made to revolve rapidly in the interior of the magnet-cylinder, waves of magneto-electricity were generated in the coils by the reversals of the magnetism in the rib G; and from the peculiar construction of the cylinder and armature, two waves of electricity, moving in alternate directions, were generated for each revolution of the armature.

15. The rapid succession of alternating waves thus generated could be taken from the machine as an intermittent current moving in one direction, by means of two steel springs (shown in the perspective drawing, fig. 10), when they were made to rub against the opposite sides of the commutator L.

16. The waves of electricity could also be taken in alternate directions from the machine when required, by adjusting the springs so as to rub against the unbroken cylindrical part of each half of the commutator.

17. The springs were placed in metallic connexion with the respective polar terminals of the machine, and to these terminals wires were attached for making the necessary connexions with those of a galvanometer or of an electro-magnet.

18. The first series of experiments with the magneto-electric machine thus described, was made for the purpose of ascertaining what influence the number of magnets on the cylinder had upon the quantity of electricity generated by the machine, as indicated by the galvanometer.

19. During these investigations, the armature of this machine was driven by steam-power at a constant velocity of three thousand revolutions (equivalent to six thousand

waves of electricity) (14) per minute. The direct current from the machine (15) was transmitted through one of POUILLET's galvanometers of tangents, constructed by RUHMKORFF, which was placed beyond the influence of the magnetism of the machine. The resistance of the galvanometer coils was so small in proportion to the resistances of the other circuits employed in these researches, as to render it unnecessary to take it into account.

20. Four permanent magnets (12) were placed successively upon the magnet-cylinder at a distance of half an inch from one another, and as each additional magnet was placed upon the cylinder, the deviation of the galvanometer-needle was read off after it had taken up a steady position. The results of these experiments are shown in the following Table.

TABLE I.

| No. of magnets on cylinder. | Deviations of galvanometer. | Tangents of deviations. |
|---|---|---|
| 1 | 29°·25 | 0·56 |
| 2 | 52·00 | 1·28 |
| 3 | 62·75 | 1·94 |
| 4 | 67·75 | 2·44 |

21. In making these experiments, which have often been repeated at different times, it was invariably found that, when only one magnet was on the cylinder, the quantity of electricity generated by the machine was proportionately less than when two or more magnets were placed on the cylinder. This deficiency appears to me to be due to the small amount of magnetism of a single magnet having to diffuse its influence through the comparatively large masses of iron of which the cylinder and armature were composed. After making allowance for this discrepancy, together with errors of observation, it will be seen from an inspection of the above Table, that, within the limits of these experiments, the quantity of electricity generated in the wire surrounding the armature of the magneto-electric machine is in direct proportion to the number of magnets on the magnet-cylinder, or to the quantity of magnetism induced in it.

22. A second series of experiments was made with the view of ascertaining the relation existing between the lifting-power of the permanent magnets on the magnet-cylinder, and that of an electro-magnet excited by the electricity derived from the magneto-electric machine. In these investigations the apparatus shown in fig. 8 was used, which consisted of a small electro-magnet, made by bolting to an iron block, forming the base of the electro-magnet, two plates of iron 6 inches long, 3 inches wide, and $\frac{3}{8}$ths of an inch thick. The inside distance between the two plates was about 2 inches; and the polar surfaces of the magnet were truly planed, as was also the under surface of the keeper or submagnet used in connexion with it. This submagnet was made of a small block of iron about 3 inches square and 1 inch in thickness, and was connected, by means of a link and swivel, to one end of a scale-beam supported at its centre by an upright pillar. From the other end of the beam depended a scale-pan, which was weighted so as to

exactly counterbalance the weight of the submagnet. The stand supporting the scale-beam was firmly bolted to an iron lathe-bed, as was also the electro-magnet, which was placed in a vertical position beneath the submagnet.

23. Around each side or plate of the electro-magnet, a length of 100 feet of insulated copper wire 0·05 of an inch in diameter was coiled, and the ends of the wires were so arranged that they could, at pleasure, be coupled up so as to form a single circuit of 200 feet, or a double circuit of 100 feet in length. One foot of the wire on the armature of the magneto-electric machine had a resistance equal to 3 feet of the single wire on the electro-magnet.

24. Experiments were made, in the first instance, for the purpose of ascertaining what influence the number of magnets on the magnet-cylinder had upon the attractive force mutually exerted by the electro-magnet and the submagnet. As the scale-beam was of too delicate a construction to allow of the submagnet being placed in direct metallic contact with the electro-magnet, a piece of thin cardboard was fastened upon the poles, by means of gum. The wires of the electro-magnet were coupled up so as to form a double circuit 100 feet in length, the resistance of which was about one-tenth of the resistance of the armature circuit. The electro-magnet was excited by the direct current from the magneto-electric machine. The submagnet was then placed upon the covered poles of the electro-magnet, and small weights were introduced into the scale-pan of the balance until the submagnet was separated from the electro-magnet.

25. In order that a more rigid comparison might be made between the quantities of electricity derived from the magneto-electric machine and the amount of the attractive force mutually exerted by the electro-magnet and the submagnet, the particular experiments, the results of which are given in Tables I. and II., were made simultaneously, the tangent galvanometer at the same time forming part of the same metallic circuit as the electro-helices and the wire surrounding the armature.

26. Coordinate results, such as are shown in Tables I. and II., were, however, obtained, whether the first and second series of experiments were made either together or separately.

TABLE II.

| No. of magnets on cylinder. | Weight, in ounces, required to separate submagnet from electro-magnet. | Squares of magnetic force of the magnet-cylinder. |
|---|---|---|
| 1 | 2·50 | 2·50 |
| 2 | 11·25 | 10·00 |
| 3 | 24·00 | 22·50 |
| 4 | 38·00 | 40·00 |

27. The results arrived at, as shown in the above Table, are somewhat remarkable, and have amongst themselves a well-defined ratio, such as would hardly have been expected from a bare consideration of the nature of the magnetism of the permanent magnet; for when one magnet was placed on the cylinder, the weight required to separate the submagnet from the electro-magnet was 2·5 ozs. It might therefore have been

expected that two magnets placed on the cylinder would have induced a double amount of magnetic force in the electro-magnet, whereas the force required to detach the submagnet was equal to a weight of 11·25 ozs., or was a little more than quadrupled. From a further comparison of the numbers contained in the Table, it will be seen that (within the limits and conditions of these experiments, and after making allowance for a certain degree of imperfection in them) the amount of magnetism induced in the electro-magnet, as measured by the weight required to separate the submagnet, is as the square of the inducing magnetism of the permanent magnets of the electro-magnetic machine.

28. Experiments were then made for the purpose of ascertaining to what extent an alteration in the length and section of the wires surrounding the electro-magnet would influence the quantity of magnetism induced in it. The electro-helices were therefore coupled up so as to form a single circuit 200 feet in length (23), and its resistance was about four-tenths of that of the wire surrounding the armature. The experiments were conducted in the same order as those in the preceding series, the thin cardboard being still interposed between the submagnet and the electro-magnet (24), and the following Table contains the results obtained.

TABLE III.

| No. of magnets on cylinder. | Weight, in ounces, required to separate submagnet from electro-magnet. | Squares of magnetic force of magnet-cylinder. |
|---|---|---|
| 1 | 5·00 | 5 |
| 2 | 28·00 | 20 |
| 3 | 76·00 | 45 |
| 4 | 144·00 | 80 |

29. From a comparison of the numbers in this Table with those in Table II., it will be seen that the ratio of increase, as well as the amount of the magnetism induced in the electro-magnet, has been considerably augmented by an increase in the length of the electro-magnetic circuit.

30. Experiments were also made with the submagnet in direct contact with the electro-magnet without the interposition of the cardboard, the arrangement of the electro-helices remaining the same as in the preceding experiments (28). The small scale-beam and stand were removed from the lathe-bed, and were replaced by a stronger apparatus of a similar construction. The results of these experiments are shown in the following Table.

TABLE IV.

| No. of magnets on cylinder. | Weight, in pounds, required to separate submagnet from electro-magnet. | Squares of magnetic force of magnet-cylinder. |
|---|---|---|
| 1 | 31·50 | 31·50 |
| 2 | 98·00 | 126·00 |
| 3 | 150·50 | 283·50 |
| 4 | 178·50 | 504·00 |

31. From an examination of the results of these experiments, it will be seen that when the submagnet was in direct contact with the electro-magnet, the force required to separate them was very greatly increased; but the ratio of this increase, as measured by the same means as in the former experiments (22), is very considerably diminished; for when one magnet was placed on the cylinder, the addition of a second magnet increased the sustaining-power of the electro-magnet by 66·5 lbs., whereas when three magnets were placed on the cylinder, the addition of a fourth magnet was only attended by an increase of 28 lbs. in its sustaining-power.

32. But the most extraordinary fact brought out in connexion with the latter series of experiments, is the development of a much greater amount of magnetism in the electro-magnet than that which existed in the permanent magnets employed in exciting it; for while the four permanent magnets on the cylinder were only capable, collectively, of sustaining a weight of about 40 lbs., the electro-magnet, as will be seen from the Table, would sustain a weight of 178·5 lbs.

33. In order that this remarkable property might be exhibited in a more striking manner, a large electro-magnet was constructed by screwing into a heavy iron block, 6 inches in thickness, two cylinders of wrought iron 24 inches in length and 3½ inches in diameter. Round each of these cylinders an insulated strand of copper wires, each 950 feet in length and 0·15 of an inch in diameter, was wound from end to end of the cylinders in several concentric layers, and the two electro-helices were coupled up so as to form one continuous helix 1900 feet in length. The cylindrical poles of the electro-magnet were 8½ inches distant from centre to centre, and were furnished with a suitable submagnet, which was connected by means of a link with a strong lever, for the purpose of measuring the amount of force necessary to separate the submagnet from the electro-magnet.

34. When the four permanent magnets (20) were placed on the cylinder of the magneto-electric machine, and the electricity from it was transmitted through the electro-magnetic helices, a weight of not less than 1088 lbs. was required to overcome the attractive force of the electro-magnet, or twenty-seven times the weight which the four permanent magnets used in exciting it were collectively able to sustain. It will, however, be shown hereafter (77) that this difference between the sustaining-power of a permanent magnet and that of an electro-magnet excited through its agency, great as it is, is very far from reaching the limits to which it can be carried.

35. The question now arose, how the results obtained from these experiments were to be reconciled with the principle of the conservation of force, since it is now generally held by physicists that the calorific, magnetic, and other properties of the electric circuit are correlated, both in direction and amount; and to admit the coexistence of any one of these properties along with the others in a greater or less degree, under like conditions, would involve the idea of the miraculous or the paradoxical.

36. In experimenting with the magneto-electric machine, it was found that the dead point of the armature, or that position during its revolution in which no electricity is

evolved, varied with the length or the resistance of the wires which joined the poles of the machine. It therefore became necessary to change the position of the commutator on the armature axis, to suit the different circuits through which the electricity was transmitted, so that the burning effects of the spark at the junction of the two halves of the commutator might be avoided when the direction of the current was changed.

37. When the commutator was properly adjusted to the resistance of the wires surrounding the electro-magnet (33), I observed that so long as the magneto-electric machine was allowed to run without its poles being connected, either with the wires of the electro-magnet or any other conductor, a brilliant star of light appeared at the points where the springs were in contact with the commutator (15); but when the poles of the machine were connected by means of a short piece of wire, the bright light at the commutator instantly disappeared. It was also observed that when the poles of the machine were connected with the long helices of the large electro-magnet, a perceptible interval of time elapsed before the light at the commutator disappeared. Besides this, it was also observed that, at the moment of breaking contact between the wires of the electro-magnet and the poles of the machine, a much more brilliant spark appeared at the points of separation, and a much more severe shock was felt when the body formed part of the circuit, than could be produced by the direct action of the machine alone. The latter effects could not, however, be obtained until a certain interval of time had elapsed after contact had been made between the electro-helices and the wires of the machine.

38. Moreover, I found that a spark could be obtained from the wires surrounding the electro-magnet even after they had been entirely disconnected from the machine. This curious result was obtained by first holding the free extremities of the wires surrounding the electro-magnet, one in each hand, in contact for a few seconds with the respective polar terminals of the machine, and while contact was still maintained, bringing the ends of the electro-helices into metallic contact with each other, so that they formed a continuous metallic loop or closed circuit. The loop so formed was then suddenly removed from the polar terminals, and while thus entirely disconnected from them, the ends of the loop were suddenly separated, and a bright spark appeared at the point of disjunction. With a larger and more powerful electro-magnet (67, 68), a bright spark was in like manner obtained, twenty-five seconds after all connexion with the magneto-electric machine had been broken*.

39. None of the effects described, such as the great sustaining-power of the electro-magnet above that of the permanent magnets (34), the increased brilliancy of the spark at the point of disjunction of the wires (37), or the spark from the electro-helices after all connexion with the magneto-electric machine had been broken (38), were observed when the alternating current from the machine (16) was transmitted through the electro-helices, instead of the direct current from the commutator (15). Under these conditions

* Since this paper was read I have discovered that Dr. Page, in 1851, succeeded in obtaining a spark from an electro-magnet, coiled with a very long wire, half a second after all connexion with the battery had been broken.—Silliman's American Journal of Science, vol. xi. p. 88.

it was found that neither the small electro-magnet (22, 23) nor the large electro-magnet (33) would sustain even a weight of 1 lb.

40. It was at first thought that the great difference observed between the sustaining-power of the electro-magnet and that of the permanent magnets which excited it, might be due to the helices surrounding the electro-magnet absorbing or retaining the electricity transmitted through them in a static condition, in the manner observed in insulated submarine and subterranean telegraph wires; an investigation of which phenomenon, as it was manifested in gutta-percha-covered wires submerged in a canal, and in similar wires laid underground between London and Manchester, was made by FARADAY in 1853*.

41. For the purpose of ascertaining whether this view of the subject was correct, a very small and delicate electro-magnetic balance was constructed, similar in principle to the one shown in fig. 8. The small electro-magnet, fixed beneath one end of the balance, was coiled with a strand of insulated copper wires 6 feet in length and 0·15 of an inch in diameter. The submagnet was prevented from coming into contact with the poles of the electro-magnet by means of regulating-screws. Other regulating-screws limited the movements of the balance in the opposite direction; and the distance of the submagnet from the electro-magnet could also be adjusted, by means of the same regulating-screws, to suit the different quantities of electricity transmitted through the electro-helices.

42. This electro-magnetic balance was placed in the middle of the circuit of the electro-magnet (33), *i. e.* at the point where the two electro-helices were joined together. The poles of the magneto-electric machine were then connected with the free extremities of the electro-helices for a few seconds; and after the spark from the commutator had disappeared, the submagnet was so counterpoised, by means of small weights, as to respond immediately to the attractive force of the small electro-magnet placed beneath it, so long as the electricity from the machine was transmitted through the helices of the large electro-magnet; but when contact with the machine and the electro-helices was broken, it was observed that the submagnet was instantly withdrawn from the poles of its electro-magnet by the weights placed in the scale-pan at the opposite end of the balance.

43. The apparatus being thus arranged, it would follow that if the charge in the wire surrounding the electro-magnet were identical with that which is observed in insulated submarine-telegraph cables, a certain interval of time would elapse, after contact with the magneto-electric machine had been made, before the balance in the middle of the circuit would respond to the attractive force of the small electro-magnet placed beneath it. On making the experiment it was found that when contact was made with the machine, 1·5 second elapsed before the balance responded to the attractive force.

44. When placed in the middle of the circuit of a larger and more powerful electro-magnet (67, 68), excited by the same magneto-electric machine (18), the electro-magnetic balance did not respond to the attractive force until an interval of 15 seconds had elapsed.

* Proceedings of the Royal Institution, Jan. 20th, 1854.

45. But if the retardation of the current, as indicated by the balance when placed in the middle of the circuit, had been the effect of an accumulation of static electricity in the electro-helices, it would also have been instantly attended by a rush of the full current of electricity into the helices at the beginning of the circuit, such as was observed in the before-mentioned experiments made by FARADAY (40). On making the experiment this was not, however, found to be the case; for when the balance was removed from the middle and placed at the beginning of the electro-magnetic circuit, the wires being again joined up so as to form a continuous helix as before (33), it was still found that 1·5 second, and with the large electro-magnet (67, 68) 15 seconds, elapsed before the electricity acquired sufficient power to bring down the submagnet of the balance.

46. When the large electro-magnet (67, 68) was excited by the electricity from a larger and more powerful machine (63), driven at a velocity of 2000 revolutions (equivalent to 4000 waves) per minute, instead of that from the small magneto-electric machine, which produced 6000 waves per minute, an interval of only four seconds elapsed before the balance responded to the attractive force of its electro-magnet.

47. Moreover, the direction of the current in the electro-helices, as shown by the galvanometer, was the same *after* as it was *before* connexion with the electromotor was broken; whereas had the current, as shown by the spark obtained (38), been the result of a static charge of the kind observed in insulated telegraph wires, the electricity would have discharged itself, when the place of disjunction was at the electromotor, in the opposite direction to that in which it entered the electro-helices.

48. The conclusions drawn from a consideration of these experiments are therefore opposed to the supposition that the effects described are the consequence of a static charge of the kind observed to be retained by insulated submarine and subterranean telegraph wires; but some of the phenomena described,—such as the retardation of the current when contact was made with the magneto-electric machine (43, 45), and the exalted electrical condition of the wire surrounding the electro-magnet, as shown by the voluminous spark seen and the severe shock felt when contact with the machine was broken (37),—are identical with the phenomena of electric induction observed by Dr. HENRY* and investigated by FARADAY with the aid of the voltaic battery, and which form the subject of his Ninth Series of Researches in Electricity†.

49. That an electro-magnet possesses the power of retaining a charge of electricity in a manner analogous to that in which it is retained in insulated submarine cables and in the Leyden jar, but not identical with it, is evident from the appearance of a spark at the point of disjunction of the wires a considerable time after all connexion with the electromotor has been cut off. The production of this spark appears to me to arise from the comparatively slow manner in which large masses of iron return to their normal condition after having attained a highly exalted degree of magnetism; the rate of decrease, however, being sufficiently rapid to allow the induction-current to manifest

* SILLIMAN's American Journal of Science, 1832, vol. xxii. p. 408.

† Philosophical Transactions, 1835, vol. CXXV.

itself in the electro-helices, with a decreasing intensity, simultaneously with the decreasing flux of magnetism in the iron itself.

50. It is this important retentive property of the electro-magnet which maintains its attractive force unimpaired, notwithstanding the intermittent character of the electricity transmitted through the electro-helices; for, as is well known, no current whatever is produced from the armature of the magneto-electric machine when in certain positions during its revolution. These positions correspond, in some measure, with the dead points of the crank of a steam-engine, the fly-wheel of which performs the same function *dynamically*, as that which the electro-magnet performs *statically*, in the case of the magneto-electric machine.

51. That the charge retained by the electro-magnet is, as has already been observed, much more powerful than that which the magneto-electric machine is of itself capable of producing, is evident from the severe shock which is felt when the body forms part of the circuit, and also from the more voluminous spark which appears at the point of disjunction of the wires when contact with the machine is broken.

52. That this increase of electric force in the electro-magnet is the consequence or effect of a certain number of electrical waves transmitted through the electro-helices, and succeeding each other with sufficient rapidity to sustain the increasing flux of magnetism in the iron, is manifest from the time which elapses before the electricity transmitted through the helices attains a permanent degree of intensity, and before the electro-magnet acquires its greatest amount of magnetism (45)*.

53. That the length of time which was observed to elapse, and the number of waves which required to be transmitted through the electro-helices before the current from the magneto-electric machine attained a permanent degree of intensity, and the electro-magnet acquired its greatest amount of magnetism, are dependent upon the magnitude of the waves of electricity transmitted through the electro-helices, is evident from the fact that the same degree of intensity of the current (as measured by the balance), and the same amount of magnetism in the electro-magnet, were obtained with a much smaller number of waves, and in a shorter time, from a large electromotor, than could be obtained with a much greater number of waves from a small electromotor (46). These observations will be further confirmed by experiments to be hereafter adduced.

54. The cause of the great difference between the attractive force of a permanent magnet and that of an electro-magnet excited through its agency, and also the agreement of the phenomena with the principle of the conservation of force, now become sufficiently manifest to render it unnecessary, at the present time, to institute a more rigorous comparison between the quantities of magnetism and electricity of the magneto-electric machine, and the quantities of the same forces developed in the electro-magnets (23, 33). The general conclusion which may, however, be drawn from a consideration of the preceding experiments is, that when an electro-magnet is excited through the agency of a permanent magnet, the large amount of magnetism manifested in the electro-magnet,

* Philosophical Transactions, 1846, p. 6.

simultaneously with the small amount manifested in the permanent magnet, is the constant accompaniment of, at least, a correlative amount of electricity evolved from the magneto-electric machine, either all at once, in a large quantity, or by a continuous succession of small quantities (45, 46),—the power which the metals (but more particularly iron) possess of accumulating and retaining a temporary charge of electricity, or of magnetism, or of both together (according to the mode in which these forces are viewed by physicists), giving rise to the paradoxical phenomena which form the subject of this research*.

### § 2. *On a new and powerful Generator of Dynamic Electricity.*

55. The fact that a large amount of magnetism can be developed in an electro-magnet by means of a permanent magnet of much smaller power having been established, and as from the first series of experiments (Table I.) it was shown that definite quantities of magnetism are accompanied by the evolution of proportionate quantities of dynamic electricity, and since an electro-magnet, when excited by means of a voltaic battery, possesses all the properties of a permanent magnet, it appeared reasonable to suppose that a large electro-magnet excited by means of a small magneto-electric machine could, by suitable arrangements, be made instrumental in evolving a proportionately large quantity of dynamic electricity, notwithstanding the pulsatory character of the electricity transmitted through the wires surrounding the electro-magnet.

56. Two magnet-cylinders, of similar construction to the one already described (9)

* Since the publication of the abstract of this paper in the Proceedings of the Royal Society, my attention has been directed to several accounts of experiments in which electro-magnets, excited by means of magneto-electric machines, have been made to sustain considerable weights. The most important of these accounts which have come under my notice, is one contained in Silliman's Journal of Science for 1845, vol. xlviii. p. 393, in which it is stated that Dr. Page, by means of a magneto-electric machine, made an electro-magnet sustain a weight of 1000 lbs.

Another account to which I have been referred, is contained in a Treatise on the Electric Telegraph, by M. l'Abbé Moigno, Paris, 1849, in which it is stated (page 15, p. 72 in the second edition) that the Abbés Moigno and Raillard, by means of a small machine, made an electro-magnet sustain a weight of 600 kilogrammes.

In neither of these accounts, however, does any direct comparison appear to have been made between the sustaining-power of the permanent and the electro-magnets, as no mention is therein made of the sustaining-power of the permanent magnets, nor are they (the permanent magnets) specifically mentioned.

In a brief notice of my experiments which appeared in 'Les Mondes' of September 6th, 1866, of which Journal M. l'Abbé Moigno is the editor, he gives what professes to be a quotation from his 'Traité de Télégraphie Électrique,' in which he has introduced a statement specifying the sustaining-power of the permanent magnets used in his experiments, although no such statement is to be found in the treatise from which the quotation is taken.

Another discrepancy with reference to the account of Moigno's experiments also occurs in an article on "Wilde's Magneto-electric Machine," in the Quarterly Journal of Science for October 1866, in which the writer would seem to have mistaken a small electro-magnet (used only as an adjunct to a magneto-electric machine, and which Moigno states would only support a few grammes) for the permanent magnets which excited the electro-magnet; and from this error it is made to appear that the permanent magnets used by Moigno would only sustain a few grammes.

(figs. 1, 2, 3, 9), were therefore made, having a bore of $2\frac{1}{2}$ inches and a length of $12\frac{1}{2}$ inches, or five times the diameter of the bore.

57. As frequent mention will have to be made of the different-sized machines employed in these investigations, they will in future be distinguished by their calibre, or the diameter of the bore of the magnet-cylinder.

58. Each cylinder was fitted with pillars, cross-heads, and an armature similar to those already described (9, 10) (figs. 3, 6). Around each armature was coiled an insulated strand of copper wires 67 feet in length and 0·15 of an inch in diameter, the extremities of which were respectively connected with the two halves of a commutator fixed on the axis of each armature (10). Upon one of the magnet-cylinders (fig. 1) sixteen permanent magnets, of the form shown in the figure, 12 inches in length, were fixed. Each of the magnets weighed 3 lbs., and would sustain a weight of 20 lbs.

59. To the sides of the other magnet-cylinder, an end view of which is shown in fig. 9, two rectangular pieces of boiler plate, O, O, $12\frac{1}{2}$ inches long, 9 inches wide, and $\frac{3}{4}$ of an inch thick, were bolted parallel with each other and between the iron packings P, P, P', P'. The upper extremities of these plates were united by means of a hollow bridge, Q, to which they were bolted, along with iron packings similar to those on their lower extremities. The bridge was made of two thicknesses of the same boiler-plate iron as that of which the sides were made; and for the purpose of ensuring good contact, its edges, as well as those parts of the sides of the rectangular plates in contact with them and with the magnet-cylinder, were planed to a true surface. An insulated strand of copper wires, 350 feet in length, and of the same diameter as that on the armature (58), was coiled round each of the rectangular iron plates in a direction parallel with the axis of the magnet-cylinder. The two coils were united so as to form a single circuit 700 feet in length, the free ends of which were furnished with suitable terminal screws, for the purpose of connecting them with the wires from the $2\frac{1}{2}$-inch magneto-electric machine. A perspective view of this machine is shown in fig. 10, but on a much larger scale than the magneto-electric machine which is placed on the top of it. The $2\frac{1}{2}$-inch magneto-electric and electro-magnetic machines were placed side by side upon a strong wooden base, and their armatures were driven simultaneously from the same driving shaft, at an equal velocity of about 2500 revolutions per minute.

60. Experiments were then made for the purpose of comparing the quantities of electricity evolved from the magneto-electric and electro-magnetic machines, as measured, approximately, by their heating effects. When the alternating waves from the magneto-electric machine were transmitted through a piece of No. 20 iron wire, 0·04 of an inch in diameter, a length of 3 inches of this wire was raised to a red heat.

61. The direct current (15) from the magneto-electric machine was then transmitted through the coils surrounding the iron plates O, O, which being united by the bridge Q, formed a powerful electro-magnet similar in construction to that invented by JOULE*, and having for its poles the two sides of the magnet-cylinder. When the alter-

* Philosophical Magazine, S. 4. vol. iii. p. 32.

nating waves from this electro-magnetic machine were transmitted through the same-sized iron wire as was used in the preceding experiment, 8 inches of it were melted, and a length of 24 inches was raised to a red heat.

62. A comparison of the heating effects of the two machines, as found by these experiments, brings out the important fact, that a much greater amount of electricity is evolved from the electro-magnetic machine than is evolved simultaneously from the magneto-electric machine. Moreover, considering the smallness of the quantity of iron of which the armature was made (only five pounds), and of the copper wire surrounding it, the weight of which was only $3\frac{1}{2}$ pounds, the heating effects of the electro-magnetic machine are very remarkable. One of the most energetic generators of dynamic electricity is that invented by GROVE, and it was found from experiments made with four new cells of this battery, the platina plates of which were $6 \times 3\frac{1}{2}$ inches, with double zinc plates well amalgamated, and charged with concentrated nitric acid and a strong solution of sulphuric acid, that ten cells of this powerful arrangement would be required in order to produce the same heating effects as those produced by the electro-magnetic machine.

63. For the purpose of ascertaining in what ratio the power of the electro-magnetic machine would be increased by an enlargement of its dimensions, a machine was constructed double the size of the one described (59), but of precisely the same proportions. The bore of the cylinder was 5 inches in diameter, and its length 25 inches. The copper wire strand surrounding the electro-magnet was 1170 feet in length, and weighed about 390 lbs. The armature of this machine was coiled with an insulated copper strand 84 feet in length, the weight of which was about 28 lbs.

64. When the electro-magnet of the 5-inch machine was excited by the $2\frac{1}{2}$-inch magneto-electric machine, the armature of the 5-inch machine being driven at about 2000 revolutions per minute, the electricity from it melted 15 inches of No. 15 iron wire 0·075 of an inch in diameter. Now it was found that a piece of No. 15 iron wire 15 inches in length, was about seven times the weight of 8 inches of the wire melted by the $2\frac{1}{2}$-inch electro-magnetic machine (61); and as the 5-inch machine was about eight times the weight of the $2\frac{1}{2}$-inch electro-magnetic machine, the increase of power of the 5-inch machine, as measured by its heating effects, appears to me to be nearly in direct proportion to the increase of its cubical dimensions, after allowance has been made for the diminished speed at which the armature was driven, together with the small increase of power which might have been obtained had the electro-magnet been excited by a more powerful magneto-electric machine.

65. For the purpose of learning to what extent the power of the electro-magnetic machine might be increased by an exaltation of the magnetism of the electro-magnet, without changing the speed at which the armature was driven, the electro-magnet of the 5-inch machine was excited by the direct current from the $2\frac{1}{2}$-inch electro-magnetic machine (59), the latter being in its turn excited by the $2\frac{1}{2}$-inch magneto-electric machine (58). On making the experiment, it was found that although the magnetism of the

electro-magnet (63) was considerably increased, yet this increase was only accompanied by a very small additional quantity of electricity from the armature; thus showing that the full power of the 5-inch machine had been very nearly attained, when its electro-magnet was excited by the 2½-inch magneto-electric machine alone.

66. Having found that an increase in the dimensions of the electro-magnetic machine was accompanied by a proportionate and satisfactory increase of the magnetic and electric forces, a 10-inch electro-magnetic machine was constructed; and as its calorific and illuminating powers are very remarkable, some particulars respecting the machine, together with a few experiments made with it, may perhaps be found to possess some interest, especially for those physicists who are engaged in determining the quantitative relations existing between the various forces as manifested to the senses.

67. In describing the different parts of the machine, reference will still be made to figures 1–9, which have been drawn to a proportionate scale. A perspective view of the machine complete is shown in fig. 10. Each side of the electro-magnet O, O, fig. 9, is made of a plate of rolled iron 48 inches in length, 39 inches wide, and1½ inch in thickness. The wrought-iron bars P, P, P', P', bolted to the upper and lower extremities of the plates, are 6 inches wide and 2 inches thick. These bars are somewhat longer than the width of the plates, and are secured to the sides of the magnet-cylinder, with the plates between them, by means of iron bolts 1 inch in diameter. The bridge Q extends the whole length of the bars P', P', and is made of two plates of rolled iron 43 inches long, 16 inches wide, and 1½ inch thick, separated from each other by an iron packing 3 inches in thickness, which makes the entire depth of the bridge equal to the width of the bars P', P'. The bridge is fixed between the side plates by means of long iron bolts 1 inch in diameter, extending from one side of the magnet to the other, as shown in the figure. All the component parts of the electro-magnet which required to be fitted together were planed to a true surface, for the purpose of ensuring intimate ferruginous contact throughout the entire mass. The total weight of the iron of the electro-magnet, exclusive of the magnet-cylinder, is a little more than 1·5 ton.

68. Each side of the electro-magnet was coiled with an insulated conductor, consisting of a bundle of thirteen No. 11 copper wires, each 0·125 of an inch in diameter, laid parallel with each other, and bound together with a double covering of linen tape, after the manner adopted by JOULE in the construction of his electro-magnets*. The length of conductor coiled round each side of the electro-magnet is 2400 feet, and the outer extremities of the coils are coupled up so as to form a continuous circuit 4800 feet in length. The inner extremities of the coils terminate in two insulated metal studs fixed upon the wooden top of the machine (fig. 10). The total weight of the two coils is 1·3 ton.

69. The magnet-cylinder consists of two masses of cast iron 50 inches in length, separated from each other by an interval of 5 inches, by means of blocks of brass placed at intervals along the top and bottom of the cylinder. All the flat surfaces of the latter,

* Annals of Electricity, vol. v. p. 472.

as well as those of the brass blocks in contact with them, are truly planed, and the several parts of the cylinder are bolted together at the top and bottom by means of twelve copper bolts 1 inch in diameter. The bore of the magnet-cylinder is 10 inches, and its weight, when fitted up with iron pillars and brass cross-heads, is 1·1 ton.

70. The machine is furnished with two armatures, one for the production of "intensity," and the other for the production of "quantity" effects. These armatures are made of cast iron, and are precisely alike in dimensions, as they were cast from the same pattern. The thickness of the rib G, fig. 5, uniting the segmental sides of the armatures is 1·75 inch, and the latter are turned one-eighth of an inch less in diameter than the bore of the magnet-cylinder. A pulley, 10 inches in diameter, is keyed upon one end of each armature, and upon the other end is fixed a commutator, by means of which the waves of electricity from the armature can be taken, either in the same or in alternate directions as required (15, 16).

71. The intensity armature is coiled with an insulated conductor consisting of a bundle of thirteen No. 11 copper wires, each 0·125 of an inch in diameter, the same as that coiled round the sides of the electro-magnet (68). The conductor is 376 feet in length and weighs 232 lbs., and is covered with a casing of wood extending the entire length of the armature, for the purpose of protecting it from external injury. Strong bands of sheet brass, 1 inch in width, encircle the armature at intervals of 6 inches, for the purpose of keeping the casing and the convolutions of the conductor in position during their rapid revolution (11). The total weight of this armature with all its fittings is 0·3 of a ton.

72. The quantity armature is enveloped with the folds of an insulated conductor, consisting of four plates of copper, each 67 feet in length, 6 inches in width, and one-sixteenth of an inch in thickness. These plates are superposed in metallic contact with each other, so as to form a single copper plate one-quarter of an inch in thickness, 67 feet in length, and nearly wide enough to occupy the entire width between the segmental sides of the armature. This division of the conductor into four plates was made for the greater convenience of bending it round the armature. The inner extremity of the conductor is held in intimate contact with the body of the armature by means of flat-headed screws; and the convolutions are insulated from one another by placing between them a band of thick cotton and india-rubber fabric, of the same length and width as the laminated conductor; and the edges of the latter are insulated from the sides of the armature by means of thin pieces of wood. The outer extremity of the conductor is terminated by a thick copper stud which connects it with the insulated half of the commutator fixed on the armature axis; and the convolutions are retained in position, by means of bands, in the same manner as those of the intensity armature (11, 71). The weight of the laminated copper conductor is 344 lbs., and the total weight of the armature is 0·35 of a ton.

73. The armatures were accurately balanced before being placed in the magnet-cylinder, for the purpose of avoiding the excessive vibrations which were produced when they revolved at high velocities without being balanced. By means of a small carriage, the

quantity and intensity armatures could easily be withdrawn from the magnet-cylinder, and interchanged, when required, in the course of a few minutes, the cross-head at the driving end of the machine being readily removeable for that purpose.

74. Though the total weight of the 10-inch electro-magnetic machine complete is very considerable, being nearly 4·5 tons, yet its dimensions are comparatively small, since the entire length of the machine, including the brass cross-heads, is only 80 inches, its width 24 inches, and its height 60 inches.

75. Experiments were first made with the machine, for the purpose of testing its power when the large electro-magnet was excited by magneto-electric and electro-magnetic machines of various sizes. The 10-inch machine, as well as the machines used for exciting it, were all driven from the same countershaft by means of pulleys of suitable dimensions. The intensity and quantity armatures (71, 72) were driven at a uniform velocity of about 1500 revolutions per minute, by means of a broad leather belt of the strongest description. The springs for taking the electricity from the 10-inch machine were connected, by means of large copper conductors, with two insulated plates of copper let into the under side of an experimental table. On the upper side of this table were two moveable brass studs, sliding in good contact with the copper plates, and forming the polar terminals of the machine.

76. The quantity armature was first placed in the cylinder, and the springs were so arranged as to take the alternating currents of electricity from the polar terminals (16). The direct current from the small magneto-electric machine, having on its cylinder six permanent magnets, such as were used in the first series of experiments (12), was then transmitted through the coils of the electro-magnet of the 5-inch electro-magnetic machine (63); and the direct current from the latter was simultaneously, and in like manner, transmitted through the coils of the electro-magnet of the 10-inch machine.

77. This combination of the machines, when in full action, was attended by the development of an amount of magnetic force in the large electro-magnet far exceeding anything which has hitherto been produced, together with the evolution of a quantity of dynamic electricity from the armature so enormous as to melt pieces of cylindrical iron rod 15 inches in length, and fully one-quarter of an inch in diameter. With the same arrangement, the electricity from the quantity armature also melted 15 inches of No. 11 copper wire 0·125 of an inch in diameter.

78. When the intensity armature (71) was placed in the cylinder, the combination of the machines remaining the same as in the preceding experiments (76), the alternating current from the armature melted 7 feet of No. 16 iron wire 0·065 of an inch in diameter, and made a length of 21 feet of the same wire red-hot.

79. The illuminating power of the electricity from the intensity armature is, as might be expected, of the most splendid description. Two rods of gas-carbon, half an inch square, were placed in the carbon-holders of the beautiful apparatus for regulating the electric light, recently invented by M. Foucault, behind which was fixed a parabolic reflector 20 inches in diameter, so adjusted as to cause the rays of light to diverge from

it at a considerable angle. When the electric lamp was placed at the top of a lofty building, the light evolved from it was sufficient to cast the shadows from the flames of the street-lamps a quarter of a mile distant upon the neighbouring walls. When viewed from that distance, the light was a very magnificent object to behold, the rays proceeding from the reflector having all the rich effulgence of sunshine.

80. A piece of the ordinary sensitized paper, such as is used for photographic printing, when exposed to the action of the light for twenty seconds, at a distance of 2 feet from the reflector, was darkened to the same degree as was a piece of the same sheet of paper when exposed for a period of one minute to the direct rays of the sun, at noon, on a very clear day in the month of March.

81. The extraordinary calorific and illuminating powers of the 10-inch machine are all the more remarkable from the fact that they have their origin in six small permanent magnets, weighing only 1 lb. each (12), and only capable, at most, of sustaining collectively a weight of 60 lbs.; while the electricity from the magneto-electric machine which was employed in exciting the electro-magnet was, of itself, incapable of heating to redness the shortest length of iron wire of the smallest size manufactured.

82. The production of so large an amount of electricity was only obtained (as might have been anticipated by the physicist) by a correspondingly large expenditure of mechanical force, as the machine when in full action absorbed, as nearly as could be estimated, from eight to ten horse-power. When the $2\frac{1}{2}$-inch magneto-electric machine (58) was substituted for the $1\frac{5}{8}$-inch machine, in the combination before described (76), the magnetism developed in the electro-magnet of the 10-inch machine was exalted to such a degree that, although the strong leather belt from the main shaft, used for driving the countershaft, was 6 inches in width and acted upon a pulley 10 inches in diameter, it was scarcely able to drive the machine.

83. It was, however, found, as in the case of the 5-inch electro-magnetic machine, that beyond certain limits a great augmentation of the magnetism of the electro-magnet was only attended by a small increase of electricity from the armature (65). The results of a number of experiments, in which various quantities of electricity were transmitted through the coils of the electro-magnet of the 10-inch machine, proved that when it was excited through the agency of the six permanent magnets, combined with the 5-inch machine, (76), the maximum amount of electricity from the machine, when working at a speed of 1500 revolutions per minute, had been nearly obtained.

84. It was also found that the maximum amount of power, as measured by the quantity of wire melted, was very nearly obtained from the 10-inch machine when its electro-magnet was excited by means of a 5-inch magneto-electric machine alone, instead of the combination of magneto-electric and electro-magnetic machines used for that purpose (76).

85. When the electro-magnet of the 10-inch machine was excited by means of the $2\frac{1}{2}$-inch magneto-electric machine alone (58) (as shown on the wooden top of the machine in fig. 10), about two-thirds of the maximum amount of power from the 10-inch machine was obtained.

## VI. *On the Anatomy of the Fovea Centralis of the Human Retina.* *By* J. W. Hulke, *F.R.C.S., Assistant Surgeon to the Middlesex and Royal London Ophthalmic Hospitals. Communicated by* William Bowman, *F.R.S.*

Received May 26,—Read June 14, 1866.

The following account of the fovea centralis is chiefly drawn from a study of this part in a perfectly healthy eye excised during life (in order to allow the extirpation of a tumour) and *instantly* put into a solution of chromic acid. By *instantly* hardening the retina, the formation of the *Plica* and the resulting disturbance of the fovea were prevented, and it was possible to make extremely regular sections.

The fovea centralis is a minute circular pit in that central spot of the retina called, from its yellow colour, the macula lutea—yellow spot. This colour proceeds from a diffuse stain of the retinal tissues, and not from the presence of granulated pigment as in the choroid. It is most intense at the centre of the spot, and fades towards the circumference of the spot, where it ceases gradually without a distinct margin, so that neither in the fresh retina nor in that which has been artificially hardened in chromic acid (which destroys the colour) can we use this as a measure of the extent of the spot. The size of the spot is, however, more certainly fixed by the extent of certain anatomical peculiarities which do not occur in any other part of the retina.

The ophthalmoscopic appearance of the spot in the living eye is that of an ellipse, the major axis of which is horizontal. It is rather larger than the disk of the optic nerve, and comes into view when the eye under observation looks directly at the sight-hole in the mirror. It is distinguished from the parts around by its deeper colour and by a certain dullness, difficult to describe but readily appreciated when seen, which proceeds from the faintness or absence of the reflection occurring at the inner surface of the retina elsewhere. In children, particularly in those of dark complexion, the retinal reflection ceases at the periphery of the macula with such a definite edge that the macula appears as a dull, dark-coloured spot encircled by a bright ring. In the middle of the macula lutea a bright dot marks the position of the fovea centralis.

According to Helmholtz the *visual line* (the straight line which joins the most sensitive part of the retina and the object directly seen) does not coincide with the axis of the eyeball, as is commonly thought, but includes an angle with it; and the most sensitive point in the retina (the fovea) lies slightly outwards, and usually a little below the posterior end of the axis of the globe*.

In an artificially hardened normal eye, in which the relations of the parts were undis-

* Cornelius, 'Die Theorie des Sehens und räumliches Vorstellens.' Halle, 1861, Abth. 1. S. 250.

turbed, I found the distance of the fovea from the centre of the optic nerve exactly equalled $1\frac{5}{8}'''$.

In vertical sections through the artificially hardened macula the fovea centralis is seen to be produced by the radial divergence of the cone-fibres from a central point, and by the thinning and curving of the inner retinal layers towards the outer surface of the retina as they approach this point. The thickness of the retina decreases in a rapid uniform curve from the edge to the centre of the fovea, and very slowly from the foveal edge towards the ora retinæ. Since the maximum thickness coincides with the edge, and the minimum with the centre of the fovea, the former is the most elevated, the latter the most depressed part in the macula.

At the centre of the fovea, where the retina is thinnest, passing from the outer to the inner surface, we meet with the following structures:—the bacillary layer, the outer limiting membrane, a thin band of areolated connective tissue which transmits a few cone-fibres, the inner granule and ganglionic layers represented by scattered cells and not separated by a distinct granular layer, optic nerve-fibres, and lastly, the inner limiting membrane.

*Minute Structure.*

1. The bacillary layer (Plate VII. fig. 1, 1) in the fovea contains cones only; and rods first appear midway between the centre of the fovea and the outer border of the macula*.

The foveal cones (figs. 2 & 3) are longer and more slender than those distant from the macula, and there is a similar difference in the length and stoutness of the rods. In both rods and cones an outer and an inner segment are observable; the outer in profile is a slender rectangle; the inner is flask-shaped, and is in the cones much stouter than the outer segment, while in the rods it only slightly exceeds the outer segment.

The inner segment is always associated in the usual manner, by inclusion or by a communicating fibre, with an "outer granule" (figs. 2 & 3). Connexion by a fibre obtains (1) with the central cones whose associate outer granules are peripherally displaced, (2) with all cones associated with a distant outer granule, and (3) always with the rods whose smaller diameter does not allow them to include their granule.

There is a definite outer limiting membrane which has the same relations to the bacillary and outer granule-layers as in other situations.

*Outer Granule-layer* (fig. 1, 2).—Owing to the peripheral situation of the outer granules belonging to the central cones, this layer is absent from the centre of the fovea. It begins at a short distance from this latter, attains its maximum thickness near the foveal edge, then decreases gradually, and again increases towards the border of the macula. This variation is due to the presence, at the margin of the fovea, of the granules belonging to the central cones in addition to those connected with the cones of this part, and to

* Kölliker says there are no rods in the macula (Handbuch der Gewebelehre, Aufl. iv. S. 684). This difference perhaps proceeds from the difficulty in fixing the exact extent of the macula in chromic-acid preparations, in which the colour of the spot is destroyed.

the increase of outer granules connected with the occurrence of rods towards the border of the macula.

The relation of the outer granules to the cones and rods (figs. 2 & 3) (mentioned in the description of the bacillary layer) indicates them to be nuclei of the inner segments of these. In fresh specimens they are roundly oval bodies. I have not been able to distinguish any constant difference between those associated with cones and those with rods. The lozenge-shape of some granules in chromic-acid preparations results, I suspect, from their compression by the sheathing membrane of the rod or cone which shrinks under the action of the acid, or from traction exerted upon them by the cone- or rod-fibre.

The fibres produced from the inner segments of the cones and rods—primitive bacillary fibres (some of which connect these with the outer granules, fig. 1, 3)—traverse the layer obliquely from its outer to its inner surface, and radially from the centre of the fovea towards the ora retinæ. At the inner surface of the layer they combine in a plexus (fig. 1, 4, *Cone-fibre plexus*), which at the centre of the fovea (where the outer granule-layer is absent) lies between the bacillary and inner granule-layers, but at the margin of the fovea between the outer and inner granule-layers. The thickness of this layer (which in the chameleon I termed the cone-fibre plexus, H. Müller's intergranule-layer) at the margin of the fovea equals or slightly exceeds that of the combined outer and inner granule-layers. The general direction of the bundles of the plexus coincides with that of the primitive bacillary fibres in the outer granule-layer; it becomes, however, less oblique in the inner part of the plexus, where the bundles run nearly parallel to the surface of the inner granule-layer.

These inner bundles midway between the centre of the fovea and the edge of the macula form a stratum parallel to the surface of the inner granule-layer. Beyond this point, with increasing distance from the centre of the fovea, the obliquity of the bundles increases until at the margin of the macula their direction is vertical.

At its inner surface the bundles of the plexus (fig. 4) resolve themselves into primitive fibres which enter the inner granule-layer (fig. 1, 4) through a granular stratum of finely areolated connective tissue. At the edge of the fovea and at the border of the macula, where the bundles of the plexus are very oblique or nearly vertical, the primitive fibres pursue the same direction for a short distance in the inner granule-layer, but where the bundles are parallel to the surface of the inner granule-layer, the resultant fibres pass off nearly vertically into this layer*.

* An oblique fibrillation has long been known in the intergranule-layer of the human macula lutea. Bergman seems first to have described it as a natural appearance[1].

H. Müller and Kölliker originally regarded it as a post-mortem or an accidental change; but subsequently Müller, having discovered the oblique fibres in the chameleon's macula, saw their correspondence to the oblique fibres in the intergranule-layer of the human macula, and acknowledged the oblique direction of these to be natural[2].

Kölliker a year later described two forms of fibrillation in the human macula, and left it undecided which

[1] Bergman, Ztsch. f. rat. Med. N.F. V. S. 245.

[2] H. Müller, Würzb. Naturwiss. Ztsch. Bd. iii. S. 31.

*Inner Granular Layer* (fig. 1, 5).—At the centre of the fovea this layer is very thin, and its innermost granules are not clearly separated from the outermost cells of the ganglionic layer (fig. 6); but at the margin of the fovea it has already a considerable thickness, and the granular layer, now a distinct band, separates it from the ganglionic layer. The granules are roundly oval nuclei of ·000465″ diameter, and larger cells of ·000697″ diameter, in some of which a nucleus is discernible. The layer also contains obliquely and vertically radial fibres. The latter are connected on the one side with the membrana limitans interna, and on the other with the thin granular band lying between the cone-fibre plexus and the inner granule-layer; they are manifestly a modified connective tissue, and form conspicuous objects when examined with an $\frac{1}{4}$″ objective.

The oblique fibres are much more delicate, and require the highest magnifying powers for their demonstration (fig. 5). With $\frac{1}{9\,5}$″ object-glass I traced their identity with the fibres which enter the layer from the cone-fibre plexus (fig. 4), and I have also seen the smaller inner granules or nuclei intercalated in the oblique fibres, and observed these connect themselves with the larger granules or cells.

*The Granular Layer* (fig. 1, 6).—This does not exist as a distinct layer at the centre of the fovea, but appears at a short distance from it in the angle between the inner granule and ganglionic layers. At the margin of the fovea it has a thickness of ·001627″, which it keeps with little variation throughout the macula. It transmits the connective tissue and nerve-fibres which pass between the inner granules and the ganglion-cells.

*The Ganglionic Layer* (fig. 1, 7).—At the centre of the fovea the ganglion-cells do not lie in a continuous band, but are scattered in a double or treble series through a finely areolated matrix of connective tissue. At the margin of the fovea they lie closer together, four or five deep, in the spaces between the vertically radial connective-tissue fibres. Throughout the fovea and macula the cells are separated from the membrana limitans interna by a narrow granular band. This latter is structurally identical with the granular layer; throughout the macula and fovea it contains optic nerve-fibres.

*Connective-Tissue Structures.*—The origin and distribution of the vertically radial

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was natural: one, in which he says "the fibres in the intergranule-layer had an oblique and horizontal curve (as Bergman saw them);" another, in which "they had a generally vertical direction broken by a double almost rectangular band"[1].

It does not, however, appear in their writings that Bergman, Müller, or Kölliker actually demonstrated the connexion of the oblique fibres of the intergranule-layer of the human macula with the elementary tissues of the other layers. Kölliker says that he saw processes from the cones (which he calls Müller's fibres) in the intergranule-layer throughout the macula, even in the fovea itself, adding "they can be easily followed to the inner granule-layer;" yet as he subsequently disputes Schultze's[2] statement—that the oblique fibrillation in the human macula occurs in the inner part of the outer granule-layer (a layer which lies between the bacillary and the intergranule-layer)—he leaves the subject, even so far as his own observation goes, very ambiguous.

[1] Kölliker, Handb. der Gewebelehre, iv. Aufl. S. 674.

[2] Schultze, Sitzungber. der niederrhein. Ges. in Bonn, 1861. I regret that this paper has not been within my reach. The reference I take from Kölliker's 'Handbuch.'

fibres agree with that which obtains in other parts of the retina, only their branches are with difficulty traceable through the granular band lying between the cone-fibre plexus and the inner granule-layer, in which most of them appear to end. They are also structurally connected with the finely areolated tissue composing the granular layer, and, further, with the interstitial tissue which pervades all the layers.

*Blood-Vessels.*—In none of my sections have I found blood-vessels at the centre of the fovea; at the margin, however, capillaries occur, and small arteries are not uncommon within the limits of the macula. The vessels nowhere penetrate beyond the outer surface of the inner granule-layer.

*Deductions.*

1. Since the total of the effects of light upon living tissue will be greater as the extent of tissue traversed by it is greater, and since the relative common sensitiveness of a surface varies with the number of distinct sentient elements it contains, it follows that the greater length of the cones and rods, and their greater slenderness, which allows a larger number of them to the superficial unit, are in harmony with the greater sensitiveness of the retina at the macula lutea. Inasmuch, however, as the foveal cones are stouter than the rods, a superficial unit at the centre of the fovea contains fewer sentient (*i. e.* percipient) elements than the same unit near the periphery of the macula lutea; and on this ground the sensitiveness of the retina at the fovea should be less than that of the retina near the periphery of the macula. On the other hand, the extreme thinness of the inner layers of the retina at the centre of the fovea, places the bacillary layer here most favourably for receiving incident light.

2. The division of the rods and cones into an outer and an inner segment is natural. The facts in support of this are, the presence of the division in perfectly fresh specimens; its sharpness and constant occurrence at a definite place; the constantly rectilinear figure of the outer, and the curvilinear figure of the inner segment; the different refractive powers of the segments; and their different behaviour towards staining and chemical solutions.

3. From these structural differences it is a fair inference that the segments have different physiological meanings.

The higher refractive power, straight sides, and slender cylindrical or prismatic figure of the outer segment may be adaptations for confining within the segment light incident upon its end, and for preventing the lateral escape of light through the sides of the segment into neighbouring cones and rods. These considerations incline me to adopt the opinion that this segment has an optical function, an opinion which derives further support from the fact that, in those animals in which the segment is so wide a cylinder that a ray might be incident upon the inner surface of its sides at a small enough angle not to be reflected but to pass out, the segment is insulated by a sheath of black pigment.

The inner segments of the cones and rods are the specially modified peripheral termi-

nations of the optic nerve-fibres; and at their junction with the outer segment the conversion of light into nerve-force may take place.

4. The outer granules being the nuclei of the inner cone- and rod-segments, probably maintain the integrity of these as living tissues, and are not directly concerned in their specific functions as organs of perception.

5. The primitive bacillary fibres are the link by which the cones and rods communicate through the inner granules and ganglion-cells with the optic nerve-fibres.

6. The smaller inner granules are nuclei of the oblique bacillary fibres in the inner granule-layer; or they may be small bipolar ganglion-cells, and act specifically on the forces transmitted through the oblique fibres from the cones and rods. The larger inner granules not being distinguishable by any definite structural characters from the smaller cells of the ganglionic layer, may agree with these latter cells in function.

7. Since the ganglion-cells (of the ganglionic layer) are fewer than the inner granules, and much fewer than the cones and rods, and since it is probable that these latter communicate with the optic nerve-fibres only through the ganglion-cells, it follows that one ganglion-cell probably is in correspondence with more than one inner granule and with several cones and rods. From this it is not an improbable conjecture that the cones and rods are disposed in groups*, each of which is represented by one or more ganglion-cells, the function of which is to connect or coordinate the individual action of the separate bacillary elements in their groups in a manner analogous to that attributed to the ganglion-cells of the spinal cord by V. der Kölk.

8. There is a close general resemblance between the human fovea and that of the chameleon†.

## Description of the Plate.

### PLATE VII.

Fig. 1. A vertical section through the centre of the fovea centralis in the vertical meridian, extending about halfway towards the periphery of the macula lutea, ×240.

1. Bacillary layer.
2. Outer granule-layer.
3. Cone-fibre plexus.
4. A granular band between the latter and the inner granule-layer.
5. Inner granule-layer.
6. Granular layer.
7. Ganglionic layer.

*a*, Centre of fovea; *b*, membrana limitans externa; *c*, membrana limitans interna; *d*, section of a blood-vessel.

* I have an impression that I have seen this in a German author, but have not been able to find the passage again.

† H. Müller, "Ueber das Auge des Chamäleons," Wurzb. Naturw. Zchr. Bd. iii. S. 36.

Fig. 2. Three cones from the centre of the fovea, with outer granules lying in their diverging primitive fibres, ×1300.

Fig. 3. Cones and outer granules from near the margin of the fovea. The inner segments of the cones are coarsely granulated; three include outer granules, and all produce a fibre which runs obliquely inwards through the outer granule-layer, ×1300.

Fig. 4. A vertical section showing the passage of the primitive fibres of the cone-fibre plexus into the inner granule-layer midway between the centre of the fovea and the margin of the macula lutea, ×1300: *a*. horizontal band of inner bundles of the cone-fibre plexus detaching fibres which traverse *b*, a finely areolated band lying between the plexus, and *c* the inner granule-layer; *d*, an inner granule in one of these fibres.

Fig. 5. A vertical section through the inner granule-layer near the edge of the fovea, ×1300: *a*, the granular band marked *b* in the preceding figure; *b*, the smaller granules; *c*, the larger granules; *d*, obliquely directed bacillary fibres with which the granules are connected.

Fig. 6. A vertical section through the inner layers at the centre of the fovea, ×1300: *a*, inner granules imbedded in areolated connective tissue; *b*, ganglion-cells.

*Note.*—In order to include these figures in one Plate they have been much reduced from the size of the original drawings, the fine outlines and delicacy of which can scarcely be reproduced by lithography.

VII. *On the Laws of Connexion between the Conditions of a Chemical Change and its Amount.* *By* A. Vernon Harcourt, *M.A., Student of Christ Church, and Demonstrator of Chemistry in the University of Oxford, and* William Esson, *M.A., Fellow of Merton College, Oxford. Communicated by Sir* B. C. Brodie, *Bart., F.R.S., Professor of Chemistry in the University of Oxford.*

Received July 13,—Read November 22, 1866.

## II. *On the Reaction of Hydric Peroxide and Hydric Iodide.*

The reaction of hydric permanganate upon hydric oxalate, which formed the subject of the first part of this inquiry*, having proved to be of a complex character, consisting in fact of several distinct reactions, it became necessary to seek for investigation a simpler case of chemical change. The reaction selected must at the same time combine all the other qualifications before enumerated, that it might be possible successively to vary its conditions and to measure its conditions and its amount.

After making trial of several reactions which appeared suitable, and being as often foiled by some practical difficulty in the proposed methods of investigation, we at last succeeded in devising for a very simple case of chemical change a method of investigation at once easy and exact. The reaction is that of hydric peroxide and hydric iodide,

$$H_2O_2+2HI=2H_2O+I_2.$$

When solutions of potassic iodide and sodic peroxide are brought together in presence either of an acid or an alkaline bicarbonate, a gradual development of iodine takes place. If sodic hyposulphite be added to the solution it reconverts the iodine, as soon as it is formed, into iodide, but appears in no other way to affect the course of the reaction. Consequently, if the peroxide be in excess over the hyposulphite, the whole of the latter is changed by the action of nascent iodine into tetrathionate, while the amount of iodide remains always constant; and after this conversion of the hyposulphite is complete, free iodine makes its appearance in the solution†. The moment at which

* Philosophical Transactions, 1866, p. 193.

† A solution of sodic hyposulphite may be mixed with a large volume of a dilute solution of potassic iodide and hydric sulphate or chloride without undergoing any decomposition. It is not oxidized to sulphate, nor acted upon in any way in this solution by hydric peroxide; for its decomposition is accompanied by a formation of sulphur, which even in very minute quantity would produce a perceptible opalescence in the liquid under observation. When hydric chloride has been employed to acidulate the solution, the addition of barium chloride after or during the set of experiments produces no precipitate. The quantity of sodic hyposulphite in the solution varies in each experiment from the maximum quantity to zero; the progress of the reaction is unaffected by this variation.

this liberation of iodine begins may be most accurately observed by the help of a little starch previously added to the liquid.

In all the experiments whose results are here recorded the same apparatus and methods were employed. The apparatus consisted of a glass cylinder about 12 inches high and 3 broad, round which, within 2½ inches of the top, a fine line was etched: into the cylinder, through a bung closing its mouth, passed a thermometer and an inverted funnel-tube; the latter, which occupied the axis of the cylinder and reached nearly to the bottom, was connected with an apparatus for generating carbonic acid; a third hole in the bung, which served to give access to the contents of the cylinder, was ordinarily closed with a small cork. The method of performing an experiment was as follows. A quantity of water, purified from organic matter by redistillation off potassic permanganate, was boiled for some time to expel dissolved oxygen, and then allowed to cool in an atmosphere of carbonic acid. When cold it was poured into the cylinder which had previously been filled with carbonic acid, and a current of this gas, ascending in large bubbles from the inverted funnel, was kept passing through the liquid until the close of the experiment. These bubbles of gas, whose diameter is nearly half that of the cylinder, serve the purpose of stirring the fluid constantly and to any required degree, without causing loss or exposure to the air, and without danger to the thermometer. Measured quantities of the standard solutions were then introduced according to the particular experiment which was to be made; for example, 50 cub. centims. of hydric sulphate and 10 cub. centims. of potassic iodide, together with in all cases a few cub. centims. of starch. Next, the liquid having been brought to the proper temperature, the cylinder was placed on a level stand, and so much more water added as would make the upper surface of the fluid exactly coincide with the line etched upon the vessel. In every experiment the same quantity of the remaining ingredient was taken, namely, 10 cub. centims. of a dilute solution of hydric peroxide*; thus the total volume was in every experiment the same. Two operations, however, had still to be performed before starting the reaction by the addition of peroxide. First, it was necessary to make sure in each case that the fluid contained no trace of any oxidizing or reducing substance. To this end the colour of the fluid was brought to the faintest possible blue by the addition,

* The solution employed in most of these sets of experiments was prepared by dissolving a weighed quantity of pure sodic peroxide in water, and adding twice the quantity of hydric sulphate required to neutralize it. The alkaline solution of sodic peroxide, and the solution obtained by neutralizing this with hydric sulphate, decompose slowly but perceptibly from day to day; the addition of a second proportion of acid renders the solution almost absolutely stable. In some sets of experiments a pure solution of hydric peroxide was employed, which was obtained by distilling the acidified solution of sodic peroxide. The first portions of the distillate consist of water containing but little peroxide; as the acid liquid becomes more concentrated and the temperature rises, hydric peroxide comes over in considerable quantities, but finally decomposition sets in, and the liquid in the retort effervesces with escaping oxygen. About ⅓ of the peroxide may thus be collected in a simple distillation. The proportion is not much increased by distilling under a diminished atmospheric pressure. It is probable that by supplying water continuously so as to keep the fluid in the retort at that degree of concentration at which the peroxide begins to come over in quantity, nearly the whole might be distilled.

according to circumstances, of a trace of sodic hyposulphite or hydric hypochlorite. If at the end of half an hour the blue tint had neither deepened nor disappeared, it was considered that the experiment might be proceeded with. Secondly, it was necessary to add a little measure of sodic hyposulphite, the first of a series of similar measures which were to play the part already indicated in the observation of the course of the reaction. These measures needed to fulfil two somewhat incompatible conditions; they must be exactly equal, or at least must stand to one another in a known ratio; and they must be of very small volume in order that their addition might not materially augment the total volume of the fluid. In the earlier experiments a pipette of about half a cubic centimetre capacity, with a capillary tube at either end, was filled with the hyposulphite solution by means of a siphon-tube provided with india-rubber nozzle and clamp. The lower end of the pipette having been wiped and pressed gently against a pad of blotting-paper, was inserted into the cylinder through the hole in the bung so as to dip beneath the surface of the fluid. By applying the mouth to a piece of india-rubber tubing slipt over the upper end of the pipette, and alternately blowing and sucking, the charge of hyposulphite was driven and completely washed into the great body of the fluid within the cylinder. This mode of measurement proved to be susceptible of great accuracy, but it only satisfied imperfectly the second condition, that of producing no material increase in the total volume of fluid used in the experiment. It will be seen that in some of the experiments hereafter recorded as many as twenty observations of the return of the blue colour were made; the total quantity of hyposulphite that had been added at the close of such an experiment was therefore 10 cub. centims., causing an increase of 1 per cent. in the volume of the fluid.

Subsequently a method of measurement was devised by which this source of error was avoided. It consisted in collecting single drops of a strong solution of hyposulphite under circumstances favourable to their perfect uniformity, and introducing these in succession into the cylinder. The drops were formed at the end of a siphon of which the shorter limb passed into a bottle containing the standard solution, while the longer limb, clamped firmly to a solid stand, was protected at its extremity by an outer tube which extended slightly below it and served to shield the growing drop from currents of air. The siphon was at one point so contracted as to deliver not more than five drops in a minute. The drops were received in small tubes about 6 inches long, open at both ends; in the side of each tube near one extremity a round hole had been blown such as would be made for the purpose of joining on another tube at right angles. Two forks were so placed on either side of the long limb of the siphon as that when one of these tubes had been passed through and rested upon them it lay horizontally just under the dropping-point, and could easily be so adjusted as to receive a drop through its lateral opening. When a drop had fallen, the tube containing it was withdrawn and replaced by another tube until a sufficient number of drops had been collected. An india-rubber cap was then slipped over the dropping-point to stop the flow of the liquid. The whole apparatus remained always in readiness, needing only the removal of its cap whenever it

was desired to collect a fresh series of drops. The width of the bottle containing the standard solution is so great, in comparison with the quantity of solution taken for any one set of experiments, that the available length of the siphon and the rate of flow, upon whose constancy that of the drops depends, varies in no appreciable degree. Numerous determinations were made with standard iodine solution of the values of drops thus collected, and they proved to be perfectly equal. To introduce a drop into the fluid in the cylinder, the end of one of the tubes thus charged was dipped into it and moved up and down, while an active stirring was carried on by means of the bubbles of carbonic acid.

When, then, the preparations already described had been completed and a sufficient number of tubes, each loaded with its drop, were lying in readiness, it remained to add to the contents of the cylinder a measure of hydric peroxide, and to mix it as thoroughly and as rapidly as possible with the rest of the fluid. Since, however, the addition and mixing are far from being instantaneous, an experiment was not made to date from this point, but from the moment of the first appearance of the blue colour. In order that the second at which this change occurred might be accurately noted, the cylinder was placed on a sheet of white paper in a good light, and opposite to it was stationed a clock beating seconds. The paper lay on an iron plate, one end of which was heated more or less gently by a lamp according to the temperature at which the set of experiments was to be performed and that of the surrounding air. By moving the cylinder nearer to or further from the heated end of the plate, the temperature of the fluid could be conveniently regulated. The observations were made by looking down upon the column of fluid and watching the appearance of the disk forming its upper surface, listening at the same time to the beat of the clock and counting the seconds. So suddenly does the blue shade pass over the clear and brightly illuminated disk, that a practised observer can generally feel sure as to the second in which the change begins. And where the reaction is proceeding very rapidly it would often be possible to subdivide the second. As soon as the observation had been made, a drop of hyposulphite was introduced, which speedily restores the liquid to its normal colourless condition. The time that elapses between two successive appearances of the blue colour becomes continually greater as the amount of peroxide in the solution diminishes, and finally the last measure of hyposulphite requires for its conversion more iodine than the residual peroxide can furnish, and the blue colour never returns. The values of the measure of peroxide and of the drops are readily compared by means of a standard solution of potassic permanganate. To apply this reagent to the estimation of sodic hyposulphite, it is necessary to add to the solution potassic iodide and hydric sulphate, together with a little starch. The determination may thus be performed directly with the same result as though an acidified solution of potassic iodide were first decomposed by permanganate, and the liberated iodine were then used to measure the hyposulphite. The relation between the two reactions which occur simultaneously in this determination is strictly parallel to that between the reaction of hydric peroxide and hydric iodide, which forms the subject, and the reaction of iodine

and hyposulphite, which furnishes the method of our investigation. But whereas hydric peroxide acts on hydric iodide much more slowly than iodine acts on hyposulphite, hydric permanganate appears under the same circumstances to form iodine even more rapidly than it is reduced. So that in this case it is possible actually to see the double reaction, for each drop of permanganate as it enters the fluid developes for an instant the colour of iodine. But the fact of the alternate action is proved independently of this observation; for while, as has been stated, the result obtained by the addition of permanganate to the mixture of hydric iodide and hyposulphite is the same as that obtained when the two reactions are caused to occur successively, the result of the direct action of hydric permanganate on hydric hyposulphite is widely different.

The relation between the measure of peroxide and the drops of hyposulphite may also be determined in another manner. It is possible at the close of the actual set of experiments, having allowed the liquid in the cylinder to stand until the action has come practically to an end, to determine the excess of hyposulphite by means of a standard solution of iodine or permanganate, and then to determine by the same means the value of an entire drop subsequently added. Hence we know what fraction of a drop, in addition to the entire drops previously introduced, has been required to reduce the whole of the peroxide, and this quantity (the whole number and the fraction) expresses the value of the peroxide at the commencement of the experiment. If we represent by $r$ the amount of residual hyposulphite at the close of the experiment, and by $d$ the amount in one drop, and by $n$ the number of drops added between the first and last appearances of the blue colour, then $\frac{r}{d}$ is the fraction of a drop which remained unacted upon, and $\frac{d-r}{d}$ is the fraction of a drop acted upon by the last portion of peroxide; and putting $p$ equal to $\frac{d-r}{d}$, $n+p$ is the whole quantity acted on, and may therefore represent also the quantity of peroxide at the moment of the first observation. At the moment of the second observation the quantity of peroxide is $n-1+p$, and at the moments of subsequent observations it is successively $n-2+p$, $n-3+p$, &c., until finally at the moment of the last observation only $p$ remains. Now the decrease of the peroxide is a measure of the amount of chemical change. Each time that the operation represented by

$$H_2O_2+2HI=2H_2O+I_2$$

is performed a molecule of peroxide disappears. We may therefore regard the change by which $n+p$ parts of peroxide become $n-1+p$ parts as a definite portion of chemical change. Representing, then, the observed times by $t_0$, $t_1$, $t_2$, &c., $t_1-t_0$, $t_2-t_1$, &c. are the successive intervals in which as the experiment proceeds this portion of chemical change is accomplished. Now if all the conditions of the reaction could be kept constant, if it were possible to reconvert the water which is formed into hydric peroxide, as it is possible, by placing sodic hyposulphite in the solution, to reconvert the iodine which is formed into hydric iodide, then, the same event occurring always under the same con-

ditions, the intervals $t_1-t_0$, $t_2-t_1$, &c. would be equal. But, as it is, one condition varies, namely, the quantity of hydric peroxide in the solution; and as this quantity diminishes, the amount of chemical change in a unit of time diminishes, or the time required for the accomplishment of a unit of chemical change increases. The former of these (the amount of chemical change occurring within a given time) was the quantity which we were able to determine when investigating the reaction in which hydric permanganate is gradually reduced by an excess of hydric oxalate. The latter (the time required for a given amount of chemical change) is that which we are able to measure in the experiment which we have described. Either determination provides us with the means of calculating the relation between the amount of chemical change and the varying condition, that is, the continually diminishing amount of one of the active substances.

The following Table contains the results of one of our first sets of experiments. The standard solutions employed in it and in subsequent sets were (1) dilute hydric sulphate containing ·37 grm. in a cub. centim., (2) a solution of potassic iodide containing ·06 grm. in a cub. centim., (3) a solution of sodic peroxide containing ·00127 grm. in the same volume. Of the first of these 100 cub. centims. were taken and 10 of each of the others. The total volume of the solution was very nearly 1 litre. The measures of hyposulphite were such that 21·45 of them were equivalent when determined by permanganate to the measure of sodic peroxide. Before starting the experiment, by adding the solution of peroxide, half a measure of hyposulphite was introduced. At the moment, then, of the first appearance of the blue colour, from which moment the observed times in column II. date, the amount of peroxide in the solution measured in drops of hyposulphite, was 20·95. The numbers in column I. express the quantities of peroxide present in the solution at the observed times, those in column IV. the intervals between two successive observations, and those in column III. the amounts of chemical change that occurred in those intervals.

We shall find it convenient to speak of such a series of observations made after the addition of successive measures of hyposulphite as a set of experiments, and to apply the term experiment to each addition of hyposulphite and the two observations which determine the corresponding interval.

TABLE I.—Weights of substances taken:—sodic peroxide ·0127 grm., hydric sulphate 37·1 grms., potassic iodide ·6 grm., volume of solution 993 cub. centims. Temperature 17° C.; $y$= residue of peroxide after $t$ mins.; $t'-t$= the time of a portion of chemical change by which $y$ is diminished to $y'$.

| I. | II. | III. | IV. |
|---|---|---|---|
| $y$. | $t$. | $y-y'$. | $t'-t$. |
| 20·95 | 0·00 | | |
| 19·95 | 4·57 | 1 | 4·57 |
| 18·95 | 9·37 | 1 | 4·80 |
| 17·95 | 14·50 | 1 | 5·13 |
| 16·95 | 19·87 | 1 | 5·37 |
| 15·95 | 25·57 | 1 | 5·70 |
| 14·95 | 31·68 | 1 | 6·11 |
| 13·95 | 38·20 | 1 | 6·52 |
| 12·95 | 45·23 | 1 | 7·03 |
| 11·95 | 52·82 | 1 | 7·59 |
| 10·95 | 61·12 | 1 | 8·30 |
| 9·95 | 70·15 | 1 | 9·03 |
| 8·95 | 80·08 | 1 | 9·93 |
| 7·95 | 91·27 | 1 | 11·19 |
| 6·95 | 103·88 | 1 | 12·61 |
| 5·95 | 118·50 | 1 | 14·62 |
| 4·95 | 135·85 | 1 | 17·35 |
| 3·95 | 157·00 | 1 | 21·15 |
| 2·95 | 184·53 | 1 | 27·53 |
| 1·95 | 223·45 | 1 | 38·92 |
| 0·95 | 291·18 | 1 | 67·73 |

The relation between the series of numbers in these columns is represented by the curve, Plate VIII. This curve, which is drawn through twenty experimental points, corresponds to those which served in our former communication* to exhibit the rate at which hydric permanganate is reduced by hydric oxalate. Along the axis of $x$ is measured the time of each observation, dating from the commencement of the set of experiments, and along the axis of $y$ the amount of peroxide present in the solution at each of the times. Through each experimental point a line is drawn parallel to the axis of $x$ to meet a line drawn through the point next below it parallel to the axis of $y$. These lines represent the quantities measured in each experiment, namely, the interval between two successive observations, and the amount of chemical change.

Starting, then, from the point to which our previous investigations had led us, we inquired at once whether this curve was logarithmic, that is to say, whether the amount of action had in this case varied directly with the amount of the varying active substance. The equation expressing this hypothesis has been shown† to be

$$u=ae^{-\alpha x},$$

where $a$ is the amount of active substance, $u$ the residue after a time $x$, $\alpha$ the fraction disappearing in a unit of time, and $e$ the base of Napierian logarithms. To the quantity $a$ in this equation corresponds any of the values of $y$ in the preceding Table, to the quantity $u$ corresponds the next successive value of $y$ in the Table, i. e. $y'$, and to the time $x$

* Philosophical Transactions, 1866, Plate XVIII.

† *Loc. cit.* p. 208.

corresponds the interval $t'-t$ during which the quantity $y$ has diminished to the quantity $y'$. So that the modified form of the equation which is applicable to the preceding Table is

$$y'=ye^{-\alpha(t'-t)}.$$

Now this may be written in the form

$$\frac{y}{y'}=e^{\alpha(t'-t)};$$

or taking the logarithms of both sides of the equation,

$$\log\frac{y}{y'}=(t'-t)\alpha\log e,$$

which expresses the fact the logarithms of the ratio of any two successive residues is proportional to the corresponding interval. For calculation it is convenient to express the equation in the deduced form

$$\log\log\frac{y}{y'}-\log(t'-t)=\log\alpha+\log\log e.$$

If, then, the differences between the corresponding values of $\log\log\frac{y}{y'}$ and $\log(t'-t)$ are found to be constant within the errors of experiment, it may be presumed that the hypothesis above stated is correct.

These values and their differences are given in the following Table.

TABLE II.

| $\log\log\frac{y}{y'}$. | $\log(t'-t)$. | $\log\log\frac{y}{y'}-\log(t'-t)$. |
|---|---|---|
| $\bar{2}\cdot327$ | 0·660 | $\bar{3}\cdot667$ |
| $\bar{2}\cdot349$ | 0·681 | $\bar{3}\cdot668$ |
| $\bar{2}\cdot373$ | 0·710 | $\bar{3}\,663$ |
| $\bar{2}\cdot395$ | 0·730 | $\bar{3}\cdot665$ |
| $\bar{2}\cdot421$ | 0·756 | $\bar{3}\cdot665$ |
| $\bar{2}\cdot449$ | 0·786 | $\bar{3}\cdot663$ |
| $\bar{2}\cdot478$ | 0·814 | $\bar{3}\cdot664$ |
| $\bar{2}\cdot509$ | 0·847 | $\bar{3}\cdot662$ |
| $\bar{2}\cdot543$ | 0·880 | $\bar{3}\cdot663$ |
| $\bar{2}\cdot579$ | 0·919 | $\bar{3}\cdot660$ |
| $\bar{2}\cdot619$ | 0·956 | $\bar{3}\cdot663$ |
| $\bar{2}\cdot663$ | 0·997 | $\bar{3}\cdot666$ |
| $\bar{2}\cdot711$ | 1·049 | $\bar{3}\cdot662$ |
| $\bar{2}\cdot766$ | 1·101 | $\bar{3}\cdot665$ |
| $2\cdot829$ | 1·165 | $\bar{3}\cdot664$ |
| $\bar{2}\cdot902$ | 1·239 | $\bar{3}\cdot663$ |
| $\bar{2}\cdot991$ | 1·325 | $\bar{3}\cdot666$ |
| $\bar{1}\cdot103$ | 1·440 | $\bar{3}\cdot663$ |
| $\bar{1}\cdot255$ | 1·590 | $\bar{3}\cdot665$ |
| $\bar{1}\cdot495$ | 1·831 | $\bar{3}\cdot664$ |

The mean of the values of $\log \log \frac{y}{y'} - \log (t'-t)$ is $\bar{3}\cdot664$, and it will be seen that every one of the values obtained for this difference from the several experiments approximates very closely to the mean. Those which exhibit the greatest deviation on either side are $\bar{3}\cdot668$ and $\bar{3}\cdot660$; and it is important to ascertain whether these deviations can be accounted for by possible errors of experiment. The errors may occur (1) in the measurement of the small quantities of sodic hyposulphite, (2) in the management of the temperature of the solution, (3) in the estimation of the interval $t'-t$, which depends upon two successive observations of the moment at which the colour of the solution changes. If, then, we suppose that the whole deviation is due to an error committed in one of these operations, the rest having been correctly performed, we find that it might result either (1) from a particular measure of hyposulphite having been one per cent. smaller or larger than the rest, or (2) from the temperature having been $0°\cdot13$ too high or too low, or (3) from an error of three seconds having been made in measuring an interval of five minutes. The second of these errors we may perhaps pronounce impossible: the fluctuations of the temperature of the solution seldom exceed $0°\cdot05$, and by balancing a small oscillation on one side of the degree line by a similar oscillation on the other, the mean thermometric error during an interval may generally be reduced to a much smaller quantity. But neither of the other errors is such as might not possibly occur in one or two out of a large number of measurements and observations. It is, however, most probable that the maximum deviations from the mean result are due, not to any single experimental error, but to the simultaneous occurrence of two or more errors in the same direction. For example, it may happen (and in eleven experiments it is an even chance that the case will occur) that the measure of hyposulphite is less than the mean, the temperature of the solution too high, the first observation made too late, and the second observation too soon. All these errors conspire to make the experiment in which they occur give too high a number for $\log \log \frac{y}{y'} - \log (t'-t)$. And such a divergence as that in the experiment which gives for the value of this difference $\bar{3}\cdot668$ instead of the mean $\bar{3}\cdot664$, would occur if the measure of hyposulphite were a fifth per cent. smaller than usual, the temperature $0°\cdot025$ too high, and the observed interval one second too small. Now all these errors are probable experimental errors. Hence it appears that within the limits of experimental error the numerical results here obtained accord with the hypothesis before stated. In the case of this reaction, it appears that the amount of chemical change occurring at any moment is proportional to the amount of peroxide present in the solution.

It may serve to exhibit the degree of coincidence between the experimental results and the hypothesis, if we further compare the intervals actually observed in this set of experiments with those calculated from the equation in which the hypothesis is embodied. The general equation being $\log \log \frac{y}{y'} - \log (t'-t) = \log a + \log \log e$, and the mean

value of $\log a + \log\log e$ being in this case $\bar{3}\cdot664$, the equation for calculating $t'-t$ is

$$\log(t'-t) = \log\log\frac{y}{y'} - \bar{3}\cdot664.$$

TABLE III.

| Intervals, $t'-t$. | | Intervals, $t'-t$. | |
|---|---|---|---|
| Observed. | Calculated. | Observed. | Calculated. |
| 4·57 | 4·61 | 9·03 | 9·02 |
| 4·80 | 4·84 | 9·93 | 9·98 |
| 5·13 | 5·12 | 11·19 | 11·14 |
| 5·37 | 5·38 | 12·61 | 12·65 |
| 5·70 | 5·71 | 14·62 | 14·62 |
| 6·11 | 6·09 | 17·35 | 17·30 |
| 6·52 | 6·52 | 21·15 | 21·24 |
| 7·03 | 7·00 | 27·53 | 27·48 |
| 7·59 | 7·57 | 38·92 | 39·00 |
| 8·30 | 8·23 | 67·73 | 67·73 |

In the following Table the numbers obtained in several sets of experiments are similarly compared with those calculated from equations of the same form. The sets of experiments here given are selected out of a large number equally accordant with theory, with a view to illustrate the variety of circumstances under which this reaction conforms to the law which has been enunciated. For the comparison of different sets of experiments, it will be convenient to describe each solution by stating its total volume in cubic centimetres, and how many millionths of a gramme of the several ingredients it contained in a cubic centimetre. The conditions of each set of experiments are enumerated at the head of the columns which contain the intervals actually observed and those calculated from the theoretical equation. The value of $a$ for each set of experiments is put at the head of the column which contains the calculated intervals.

TABLE IV.

| Volume, 993 cub. centims. Temperature, 30°. Hydric sulphate, 746. Hydric iodide, 1260. Sodic peroxide, 74. Sodic hyposulphite (one measure), 13·3. | | Volume, 694 cub. centims. Temperature, 17°. Hydro-sodic carbonate, 4760. Potassic iodide, 5180. Sodic peroxide, 35·7. Sodic hyposulphite (one measure), 10·7. | | Volume, 993 cub. centims. Temperature, 0°. Hydric sulphate, 18700. Potassic iodide, 1208. Sodic peroxide, 59·5. Sodic hyposulphite (one measure), 10·9. | |
|---|---|---|---|---|---|
| Intervals, $t'-t$. | | Intervals, $t'-t$. | | Intervals, $t'-t$. | |
| Observed. | Calculated, $a=·0242$. | Observed. | Calculated, $a=·085$. | Observed. | Calculated, $a=·0043$. |
| 2·42 | 2·40 | 1·17 | 1·17 | 14·27 | 13·8 |
| 2·50 | 2·55 | 1·30 | 1·30 | 14·77 | 14·6 |
| 2·71 | 2·72 | 1·45 | 1·46 | 31·38* | 32·4 |
| 2·94 | 2·91 | 1·65 | 1·66 | 17·87 | 18·1 |
| 3·10 | 3·13 | 1·93 | 1·93 | 19·62 | 19·6 |
| 3·40 | 3·39 | 2·33 | 2·32 | 21·45 | 21·4 |
| 3·68 | 3·69 | 2·88 | 2·89 | 23·65 | 23·6 |
| 4·07 | 4·05 | 3·78 | 3·83 | 26·37 | 26·3 |
| 4·52 | 4·49 | 5·70 | 5·71 | | |
| 5·02 | 5·04 | 11·73 | 11·54 | | |
| 5·77 | 5·74 | | | | |
| 6·71 | 6·67 | | | | |
| 7·92 | 7·97 | | | | |
| 9·77 | 9·88 | | | | |
| 12·91 | 13·00 | | | | |
| 19·00 | 19·00 | | | | |
| 37·00 | 36·70 | | | | |

| Volume, 993 cub. centims. Temperature, 30°. Hydric sulphate, 112000. Potassic iodide, 604. Sodic peroxide, 34·9. Sodic hyposulphite (one measure), 21·9. | | Volume, 993 cub. centims. Temperature, 30°. Hydric chloride, 13900. Potassic iodide, 604. Sodic peroxide, 27·5. Sodic hyposulphite (one measure), 14·1. | | Volume, 993 cub. centims. Temperature, 50°. Hydric sulphate, 18700. Potassic iodide, 1208. Sodic peroxide, 37·8. Sodic hyposulphite (one measure), 13·4. | |
|---|---|---|---|---|---|
| Intervals, $t'-t$. | | Intervals, $t'-t$. | | Intervals, $t'-t$. | |
| Observed. | Calculated, $a=·0949$. | Observed. | Calculated, $a=·0268$. | Observed. | Calculated, $a=·131$. |
| 2·31 | 2·32 | 6·53 | 6·56 | 1·00 | 1·01 |
| 2·98 | 2·98 | 8·05 | 7·96 | 1·18 | 1·17 |
| 4·15 | 4·17 | 10·13 | 10·13 | 1·37 | 1·38 |
| 7·01 | 7·00 | 13·95 | 14·00 | 1·70 | 1·70 |
| 30·38 | 30·27 | 22·42 | 22·60 | 2·20 | 2·19 |
| | | 67·08 | 66·84 | 3·08 | 3·09 |
| | | | | 5·33 | 5·32 |

The discrepancy between the observed and calculated intervals in the earlier experiments of the set made at 0° C. depends upon the difficulty which was experienced in managing the temperature. If it rose at all the rate of change was of course increased, and if it fell it was increased also by the separation of some of the water from the acid solution in the form of ice. With this exception it will be seen that the calculated and observed intervals agree very closely. Hence we conclude that whether the solution contains in each cub. centim. 746 millionths of a gramme of hydric sulphate, or 150 times that

* Double interval.

quantity, 604 millionths of a gramme of potassic iodide or 9 times that quantity, or whether hydric chloride or hydro-sodic carbonate be substituted for hydric sulphate, whether the temperature be 0° or 50° C., and whether the portion of change require for its accomplishment intervals of one or two minutes, or intervals of half an hour or an hour, this reaction still conforms to the law that the amount of change is at each moment proportional to the amount of changing substance.

In these experiments the actual observation has been of the rate of production of iodine. But the production of this substance is only one part of the chemical change which occurs. The whole change is represented in its simplest form by the equation

$$H_2O_2+2HI=2H_2O+I_2,$$

and we are able, knowing the quantity of iodine that has been formed, to infer from it the quantity of water formed and the quantities of hydric peroxide and iodide that have disappeared. Now if we assign a particular weight to the molecule of iodine, the equation will represent that change by which this amount of iodine is formed, together with the proportional quantity of water, while corresponding quantities of hydric peroxide and iodide disappear. We thus obtain an expression for a particular amount of change. The unit change may be defined to be that in which 254 millionths of a gramme of iodine are formed, and when the equation written above is used to express this unit change it will be written in italics. That is to say, the expression

$$H_2O_2+2HI=2H_2O+I_2$$

represents the disappearance of 34 millionths of a gramme of hydric peroxide, and of 256 millionths of a gramme of hydric iodide, and the formation of 36 millionths of a gramme of water, and 254 millionths of a gramme of iodine. The expression

$$n(H_2O_2+2HI=2H_2O+I_2)$$

represents the occurrence of $n$ units of change. Further, since in these experiments the liquid system is homogeneous, the total change which occurs during any interval of time depends upon the quantity of change occurring in each unit of volume and the number of such units. For unit of volume we may conveniently adopt the cubic centimetre. In stating the amounts of other reagents than those which appear in the equation of the reaction, it will sometimes be convenient to express these amounts in units corresponding to those proposed above, being their molecular weights taken not as relative numbers, but as so many millionths of a gramme. For example, in enumerating the conditions of a particular experiment we shall mean by $H_2SO_4$, or a unit of hydric sulphate, 98 millionths of a gramme of that substance; by $KI$, or a unit of potassic iodide, 266 millionths of a gramme of that salt.

In each set of experiments we commence with a system which contains elements capable of undergoing a certain quantity of change. We may express this by saying that there exists at starting a certain amount of potential change. As time elapses this potential change gradually becomes actual. From this point of view the change occurring in the system is analogous to the motion of a heavy body falling freely, which

at the commencement of its motion has a certain amount of potential energy capable of being transformed into actual energy. As the body falls the potential energy gradually becomes actual. Each experiment supplies data for the determination of the following quantities:—

(1) The initial potential change.
(2) The final potential change.
(3) The actual change.
(4) The time during which the actual change has occurred.

The relation existing between these quantities has been found to be of such a nature that the ratio of the initial and final potential changes in a given system depends only upon the time of the actual change, so that if this time is constant the ratio is constant; and since the actual change is simply the difference between the initial and final potential changes, it follows that for equal intervals of time the actual change is proportional to the initial potential change. Now if we could construct a system in which the potential change remained constant, it is clear that the actual change would proceed at a uniform rate, depending upon the quality of the system and proportional to the constant potential change. In all the systems upon which our experiments have been made the potential change varies, so that we are not able directly to observe this uniform rate, but we can obtain its value indirectly in the following way.

Suppose the time of actual change to be so small that its rate may be considered uniform during that time, the actual change will be so small that the initial and final potential changes may be considered to be equal; in other words, the potential change will be constant. The ratio of the small actual change to the time of its occurrence will thus represent the uniform rate of actual change when the potential change remains constant. The equation which connects the initial and final potential changes $y$, $y'$ with the time of actual change has been found to be

$$\frac{y}{y'}=e^{\alpha(t'-t)},$$

whence we obtain

$$-\frac{dy}{dt}=\alpha y.$$

Now $-dy$ is the actual change which occurs during the time $dt$, and from what is stated above the ratio of these small quantities is the uniform rate of actual change when the potential change $y$ remains constant. It follows therefore that in a given system, in which there exists a constant quantity of potential change $y$, the uniform rate of actual change is $\alpha y$. Or since $\alpha$ is a constant for the given system, the rate of actual change is proportional to the potential change. If the unit of time is one minute, $\alpha$ represents the fraction of the potential change which is converted into actual change in one minute. We may represent what we have spoken of as potential change by writing separately the left-hand side of a chemical equation. For example, the actual change in this reaction being represented by $H_2O_2+2HI=2H_2O+I_2$, the corresponding potential

change may be represented by $H_2O_2+2HI$, and the unit of potential change by $H_2O_2+2HI$. Employing this notation, the result stated above may be expressed by saying that when there exists in a system a constant quantity,

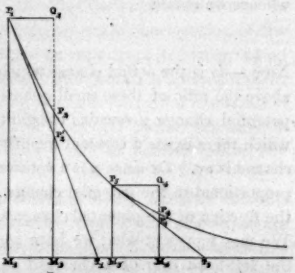
$$y(H_2O_2+2HI),$$

$$\alpha y(H_2O_2+2HI=2H_2O+I_2)$$

occurs in one minute. For example, in the sets of experiments recorded in Table IV., the following values were found for $\alpha$, ·0242, ·085, ·0043, ·0949, ·0268, ·131. These numbers represent the fractions of a unit of actual change that would occur in one minute, if there existed throughout that time in each system a unit of potential change. The difference between the values of $\alpha$ depends, it is plain, upon the different conditions of these sets of experiments which are contrasted at the heads of the several columns.

The relation between the potential change in the system and the actual change occurring at a particular time may also be represented by supposing the rate, which up to that time has continually decreased, to become thenceforward uniform. Then the whole potential change being $y$, $\alpha y$ becomes actual in each successive minute, and the whole will have become actual, *i. e.* the reaction will have come to an end, in $\frac{1}{\alpha}$ minutes. Whence it appears that if it were possible actually to make the rate uniform, as may approximately be done by continually raising the temperature of the solution, or otherwise compensating the diminution of peroxide, the time required for the completion of the reaction would be independent of the time at which the rate was thus modified, that is, of the amount of potential change existing in the system. For example, in the last set of experiments given in Table IV., if at any moment whatever while the reaction was proceeding the rate of change could have been made constant, in $(\frac{1}{\cdot131}=)$ 7·6 minutes from that time the whole change would have been accomplished. Whatever amount of peroxide the solution contained, this would equally have been the case; in 7·6 minutes the whole of the peroxide would have disappeared.

These results may be illustrated graphically by the following figure.



The curve $P_1P_2P_4$ represents the course of the reaction as determined by a set of experiments. $M_1M_2$, $M_3M_4$ represent any two equal intervals of time; $M_1P_1$, $M_2P_2$ the potential changes at the beginning and end of $M_1M_2$; $M_3P_3$, $M_4P_4$ the potential changes at the beginning and end of $M_3M_4$; $P_2Q_2$, $P_4Q_4$ the actual changes that have occurred during these intervals; $P_1T_1$, $P_3T_3$, tangents to the curve at $P_1$, $P_3$, represent the course of the reaction if it were to maintain the same rates with which it was proceeding at $P_1$, $P_3$ respectively; $P_2'Q_2$, $P_4'Q_4$ the actual changes which would occur with these uniform rates during the intervals $M_1M_2$,

$M_3M_4$, and $M_1T_1$, $M_3T_3$ the times in which under these circumstances the reaction would complete itself. The results arrived at are (1) $M_1P_1 : M_2P_2 :: M_3P_3 : M_4P_4$; and consequently (2) $P_2Q_2 : M_1P_1 :: P_4Q_4 : M_3P_3$, (3) $P_2'Q_2 : M_1P_1 :: P_4'Q_4 : M_3P_3$, (4) $M_1T_1 = M_3T_3$. If the interval $M_1M_2$ be one minute and $M_1P_1$ be the unit of potential change, then $P_2'Q_2$ represents $\alpha$ units of actual change.

We have thus investigated the relation existing between the amount of chemical change and the amount of hydric peroxide, and have shown that the former of these quantities varies directly with the latter. We now proceed to inquire how the amount of change is affected by the variation of the other conditions of the reaction.

For a particular system the amount of chemical change in a unit of time is expressed by $ay$, this expression representing the fact that if $\alpha$ is kept constant the amount of change varies directly with $y$, that is, with the amount of peroxide, and also that if $y$ is kept constant it varies directly with $\alpha$. That which has been kept constant in each set of experiments made to determine the values of $y$, and which is represented by $\alpha$, is a group of other conditions upon which the amount of chemical change depends. The systems which have been made the subjects of experiment in this investigation have all been liquid homogeneous systems. And as the quantity of water used has been always very large in comparison with that of the various reagents, they may be further characterized as aqueous systems.

Since these systems are homogeneous, they may conveniently be described by a statement of the ingredients of their unit of volume. We adopt, as before stated, the cubic centimetre for unit of volume, and shall use the units already defined (p. 128) in the measurement of the various substances. The whole amount of chemical change is a function of all the conditions of the system in which it occurs. If we call this amount $\Sigma$, the volume of the system $v$, its temperature $h$, the time during which the change proceeds $t$, and the number of units $H_2O_2$, $HI$, ... $A$, $B$, $C$, ... of the various ingredients in a unit of volume $p$, $i$, ... $a$, $b$, $c$ ....., respectively, where $A$, $B$, $C$ are units of any substance which may be introduced into the system, then

$$\Sigma = f(a, b, c, .. h, i, .. p, .. t, .. v ..).$$

The form of this function is determinate in the case of two of these conditions, viz. $t$, $v$, and has been determined experimentally for this reaction in the case of $p$, so that the equation may be written in the form

$$\Sigma = ptv \,.\, f(a, b, c, .. h, i ..).$$

Now if we keep constant all the conditions in the undeterminate part of the function except one, say $x$, the form of the equation is

$$\Sigma = ptv \,.\, \phi(x),$$

the constants in $\phi(x)$ being functions of the conditions of the system which do not vary. From this equation, knowing a series of values of $\Sigma$, $p$, $t$, $v$, $x$, it is generally possible to determine the form of the unknown function. In each set of experiments made in the manner described, $p$ and $t$ vary while $v$ and $x$ remain constant, and the law of connexion

between $p$ and $t$ is such that the amount of change within a unit of volume in a unit of time is $\alpha p$; hence $\Sigma = \alpha ptv = ptv \cdot \phi(x)$, and therefore $\alpha = \phi(x)$. If we now make a *series* of such sets of experiments, varying $x$ only, and determining the value of $\alpha$ for each set, we shall obtain a series of values of $\phi(x)$ corresponding to the several values of $x$, and may thence discover what function $\alpha$, and therefore $\Sigma$, is of $x$. And if we are able thus to determine what function $\Sigma$ is of each of the conditions $a$, $b$, $c$, ..., we shall obtain an equation expressing the laws of connexion between all the conditions of this chemical change and its amount. It is, however, to be observed, that for the determination of $\alpha$ in each set of experiments it is necessary that the introduction and variation of the condition $x$ should not alter the law of connexion between $p$ and $t$. In other words, to determine $\phi(x)$ we need to obtain a series of equations of the following form,

$$\frac{p}{p'} = e^{\alpha_1(t'-t)}, \quad \frac{p}{p'} = e^{\alpha_2(t'-t)}, \quad \frac{p}{p'} = e^{\alpha_3(t'-t)} \ldots .$$

It is plain that the investigation of a single reaction thus considered is a work requiring the performance of a very large number of experiments. In the case before us it is possible to react with the hydric peroxide either upon hydric iodide by itself, or potassic iodide and hydric sulphate, or to substitute sodic iodide for potassic, or hydric chloride for hydric sulphate, without (as appears from the numbers in Table IV.) altering the nature of the reaction. And there is little doubt that the system might be modified by the introduction or substitution of many other substances, without affecting the chemical change, except as regards its rate. Each of the conditions thus introduced furnishes a fresh subject of inquiry, namely, as to the effect of a variation in its amount upon that of the chemical change. On the effect of varying some of these conditions of the reaction we have made numerous experiments, especially on the variation of the amounts of potassic, sodic, and hydric iodide, hydric sulphate, hydric chloride (substituted for hydric sulphate), and of the temperature of the solution. We propose to include in the present communication an account of the results we have obtained by varying in successive sets of experiments the amount of iodide.

### *Variation of Iodide.*

When two salts are mixed in solution and no precipitation occurs, or change in the colour of the liquid, it is not possible to arrive at any probable conclusion as to the proportions or nature of the salts which the solution contains. In the greater part of the experiments about to be recorded, potassic, or sodic, or hydric iodide was added to a liquid containing an excess of hydric sulphate or chloride. Whether the systems thus formed contained the salts introduced into them, or hydric iodide and a metallic sulphate or chloride, or whether the metallic iodides were partially decomposed by the excess of free acid, we are unable to say. It may be hoped that the further investigation of the effect of varying these substances will throw light upon this question; but at present we must consider our results generally as depending upon the variation of iodide.

The following Table contains the numerical results of five sets of experiments, in each

of which the amount of actual change that would occur in a minute, the system containing constantly a unit of potential change, is determined by several experiments. The degree of coincidence of these experiments in each set may be observed by comparing the values of this amount calculated from the successive intervals on the hypothesis already established, that the actual change at any moment is proportional to the potential. The number of units of potential change at the commencement of a set of experiments is denoted by $p_0$, and the number of units of actual change occurring in each experiment is denoted by $a$.

The volume of the solutions employed was 993 cubic centimetres, and their temperature 30° C. The actual weights of the substances taken were, in each set,

Hydric sulphate, 37·1 grms.,
Sodic peroxide, ·0174 grm.,
Sodic hyposulphite (one measure), ·0217 grm.;

in successive sets,

Potassic iodide, ·6, ·9, 1·2, 1·5, 1·8 grm.

The weights of sodic peroxide and sodic hyposulphite are so small in comparison with that of the hydric sulphate, that the amount of acid neutralized by these salts is insignificant. The formation of a correspondingly small quantity of sodic sulphate may also be disregarded; for a separate experiment made with a large quantity of this salt proved that its influence upon the rate of change is very slight. But the potassic iodide was used in rather larger proportion; and it seems probable that some double decomposition occurs between it and the hydric sulphate.

Hence the permanent ingredients of a unit of volume of the system in each set of experiments were

$$(381{\cdot}3-1{\cdot}82\,z)H_2SO_4,\quad 1{\cdot}82\{(n-z)KI+zHI\},\quad 1{\cdot}82\,zKHSO_4,$$

$n(1{\cdot}82\,KI)$ being the weight of potassic iodide taken for every cubic centimetre of the several solutions, and $z$ being the fraction of this iodide decomposed by the hydric sulphate.

TABLE V.

| $n=2$. $p_0=·222$. $a=·044$. | | $n=3$. $p_0=·225$. $a=·044$. | | $n=4$. $p_0=·222$. $a=·044$. | | $n=5$. $p_0=·225$. $a=·044$. | | $n=6$. $p_0=·226$. $a=·044$. | |
|---|---|---|---|---|---|---|---|---|---|
| Intervals. | $\alpha$. | Intervals. | $\alpha$. | Intervals. | $\alpha$. | Intervals. | $\alpha$. | Intervals. | $\alpha$. |
| 7·33 | ·0302 | 4·78 | ·0457 | 3·65 | ·0604 | 2·87 | ·0757 | 2·35 | ·0923 |
| 9·4 | ·0302 | 6·14 | ·0455 | 4·73 | ·0600 | 3·68 | ·0755 | 3·10 | ·0895 |
| 35·47 | ·0301 | 8·61 | ·0452 | 6·59 | ·0604 | 5·12 | ·0757 | 4·25 | ·0906 |
| 103·63 | ·0303 | 14·39 | ·0449 | 11·16 | ·0601 | 8·55 | ·0748 | 7·05 | ·0900 |
| | | 53·26 | ·0451 | 52·20 | ·0601 | 29·78 | ·0762 | 23·97 | ·0902 |

The values obtained for $\alpha$ from the different experiments of each set are fairly concordant, and we may therefore assume their mean to be the true value. It is at once evident that these numbers are in arithmetical progression, and bear to one another the same ratios as the values of $n$, viz., 2 : 3 : 4 : 5 : 6. They are compared below with a series of numbers calculated on this hypothesis.

TABLE VI.

| $n$. | $a$. Observed. | $a$. Calculated. |
|---|---|---|
| 1 | ..... | ·0151 |
| 2 | ·0302 | ·0302 |
| 3 | ·0453 | ·0453 |
| 4 | ·0602 | ·0604 |
| 5 | ·0756 | ·0755 |
| 6 | ·0905 | ·0906 |

To confirm this result a second series of sets of experiments was made, differing from the former by the substitution of hydric chloride for hydric sulphate. Comparing equivalent quantities, it had been observed that hydric chloride increases the rate of change nearly twice as much as hydric sulphate. Therefore, in order to obtain a sufficient series of different rates of which the first should not be inconveniently small, nor the last inconveniently great, one molecule or half an equivalent of hydric chloride was substituted for a molecule of hydric sulphate.

Volume of the solutions 993 cub. centims., temperature 30° C.

Actual weights of substances taken in each set:

Hydric chloride, 13·8 grms.,
Sodic peroxide, ·0257 grm.,
Sodic hyposulphite (one measure), ·0135 grm.;

in successive sets,

Potassic iodide, ·3, ·6. ·9, 1·2, 1·5, 1·8, 2·1, 2·4 grms.

Hence the permanent ingredients of a unit of volume of the system in each set of experiments were

$$(381\cdot3-1\cdot82z)H\,Cl,\quad 1\cdot82((n-z)KI+zHI),\quad 1\cdot82zK\,Cl.$$

TABLE VII.

| $n=1$. $p_0=16$. $a=0274$. | | $n=2$. $p_0=17$. $a=0274$. | | $n=3$. $p_0=166$. $a=0274$. | | $n=4$. $p_0=165$. $a=0274$. | |
|---|---|---|---|---|---|---|---|
| Intervals. | $a$. | Intervals. | $a$. | Intervals. | $a$. | Intervals. | $a$. |
| 13·82 | ·0136 | 6·53 | ·0269 | 4·43 | ·0406 | 3·38 | ·0537 |
| 16·83 | ·0137 | 8·05 | ·0265 | 5·47 | ·0402 | 4·20 | ·0528 |
| 21·42 | ·0140 | 10·13 | ·0268 | 6·93 | ·0406 | 5·38 | ·0531 |
| 32·83 | ·0132 | 13·95 | ·0269 | 9·82 | ·0401 | 7·25 | ·0555 |
| | | 22·42 | ·0270 | 16·27 | ·0406 | 12·65 | ·0540 |
| | | 67·08 | ·0267 | | | | |

| $n=5$. $p_0=164$. $a=0274$. | | $n=6$. $p_0=164$. $a=0274$. | | $n=7$. $p_0=163$. $a=0274$. | | $n=8$. $p_0=161$. $a=0274$. | |
|---|---|---|---|---|---|---|---|
| Intervals. | $a$. | Intervals. | $a$. | Intervals. | $a$. | Intervals. | $a$. |
| 2·68 | ·0681 | 2·28 | ·0802 | 1·92 | ·0957 | 1·68 | ·111 |
| 3·35 | ·0666 | 2·77 | ·0809 | 2·40 | ·0940 | 2·17 | ·106 |
| ..... | ..... | 3·62 | ·0798 | 3·07 | ·0951 | 2·75 | ·108 |
| 6·05 | ·0670 | 5·05 | ·0809 | 4·40 | ·0942 | 4·02 | ·106 |
| 10·32 | ·0671 | 8·78 | ·0802 | 7·58 | ·0948 | 7·00 | ·108 |

The mean values of $\alpha$ derived from these sets of experiments are compared in the following Table with a series calculated from the equation $\alpha = \cdot 01347n$.

TABLE VIII.

| $n$. | $\alpha$. Observed. | $\alpha$. Calculated. |
|---|---|---|
| 1 | ·0136 | ·0135 |
| 2 | ·0268 | ·0269 |
| 3 | ·0404 | ·0404 |
| 4 | ·0538 | ·0539 |
| 5 | ·0672 | ·0673 |
| 6 | ·0804 | ·0808 |
| 7 | ·0948 | ·0943 |
| 8 | ·1080 | ·1078 |

Thus it appears that the amount of chemical change occurring in the solutions at any moment varies directly with the amount of iodide, if all the other conditions are the same. A few of the numbers from which the mean values of $\alpha$ are obtained differ considerably one from another. These differences were generally due to observed errors in the management of the temperature of the solution, which having through inadvertence risen or fallen a little during one interval, was made to fall or rise in a corresponding degree during the next interval, that the mean result might be correct.

In both these series the quantity of iodide was small in proportion to the quantity of acid, amounting at the most to 4 per cent. Two sets of experiments were subsequently made with systems containing in a cubic centimetre 54·5 $H_2SO_4$, and 10·42, 20·84 $KI$ respectively, at a temperature of 17° C. The values of $\alpha$ given by these two sets were ·0116 and ·0243, the latter of which is considerably more than double the former. In some other sets of experiments, in which instead of a metallic iodide different quantities of hydric iodide were added to the solutions, it was observed similarly that the increase in the rate of change was more than proportional to the increase of hydric iodide. Now, since it appears that hydric sulphate or hydric chloride, though playing no immediate part in the reaction, yet accelerates its course, it seems reasonable to suppose that a double effect may be produced by the addition of hydric iodide. For while on the one hand this addition increases the amount of substance which the hydric peroxide has to act upon, on the other hand, like the addition of hydric sulphate or hydric chloride, it increases the acidity of the solution. But further, the rate of change depends not only upon the acidity of the solution, but upon the particular acid which it contains. The two acids upon which we have experimented affect the rate of change in different degrees, hydric chloride exercising a greater influence than hydric sulphate; so that if we were to add a quantity of neutral chloride to a solution containing hydric sulphate, we should doubtless increase the rate of change; for some of the hydric sulphate would be replaced by its equivalent (in the ordinary chemical sense) of hydric chloride. In the same way it is probable that a solution into which hydric sulphate and potassic iodide have been introduced contains some hydric iodide, and that this acid also has its particular effect

upon the rate of change independently of the part which it plays in the reaction itself. Now, if equivalent quantities of hydric sulphate and iodide caused equal accelerations, the replacement in the solution of one of these acids by the other would produce no effect; but if hydric iodide, like hydric chloride, has a greater accelerating power than hydric sulphate, the effect of this replacement would be to cause an acceleration independent of and additional to that which is due to the increase of iodide. When, as in the sets of experiments recorded in Tables V. and VII., the hydric sulphate or chloride replaced by hydric iodide is but a small fraction of the whole amount in the solution, this change does not so affect the rates as to hinder us from observing the result of the simple variation of iodide; but when the experiment is pushed further, and a considerable proportion of the acid is thus changed, the effect of this second variation becomes perceptible. With the view of inquiring whether the proportional relation between the amount of iodide and the amount of chemical change still holds good when the solution does not contain an excess of acid, a number of sets of experiments were made with a constant quantity of hydric iodide and various quantities of potassic iodide. Here we encountered a fresh difficulty; the primary reaction no longer followed the law expressed by the equation $\frac{y}{y'}=e^{\alpha(t'-t)}$; and although the measurements of the solutions and the observation of the intervals were made with all possible care, we are not able to derive from the experimental data any series of values for $\alpha$, nor therefore to determine what function the total amount of change was of the amount of iodide. The following Table contains the results of these sets of experiments. To avoid the introduction of any other acid besides hydric iodide, a neutral solution of hydric peroxide was used.

The volume of the solutions was 993 cub. centims., and their temperature 30° C.

The amount of iodide in a cubic centimetre of each was 15·25 $HI$ and 7·28 $n\,KI$. The amount of peroxide at starting was about ·98 $H_2O_2$. The amount of actual change during each interval was ·13$\{H_2O_2+2HI=2H_2O+I_2\}$.

TABLE IX.

| $n=0$. | $n=1$. | $n=2$. | $n=3$. | $n=4$. | $n=5$. | $n=6$. |
|---|---|---|---|---|---|---|
| Observed intervals. | Observed intervals. | Observed intervals. | Observed intervals. | Observed intervals. | Observed intervals. | Observed intervals. |
| 6·40 | 4·05 | 2·93 | 2·28 | 1·85 | 1·55 | 1·38 |
| 7·40 | 4·90 | 3·54 | 2·75 | 2·25 | 1·92 | 1·72 |
| 9·05 | 5·92 | 4·33 | 3·50 | 2·82 | 2·40 | 2·13 |
| 11·27 | 7·58 | 5·70 | 4·65 | 3·76 | 3·26 | 2·95 |
| 15·30 | 10·57 | 8·15 | 6·80 | 5·52 | 4·74 | 4·32 |
| 24·30 | 17·41 | 13·82 | 12·20 | 9·65 | 8·06 | 7·93 |
| 59·82 | 56·07 | | | | | |

The first, and perhaps the second of these sets of experiments, gives a logarithmic curve; the remainder depart more and more widely from this relation. The cause of the departure is probably the same as that which prevented the observation of the law of

proportionality in the case of the reaction of hydric oxalate and permanganate. The measurements are no longer of the course of a single gradual action. The several series might be represented by equations expressing such a complication as it is most likely occurs, *e. g.*, the gradual oxidation of potassic iodide to iodate, and the gradual reduction of this salt by hydric iodide. But the constants of such equations cannot be determined from the experimental numbers with sufficient accuracy for much reliance to be placed upon them.

In consequence therefore of these two facts,—that the amount of change is a function both of the kind and of the amount of acid in the solution, and that the nature of the reaction is changed when a mixture of hydric and potassic iodide is used,—we have only been able to investigate the relation which exists between the amount of iodide in the solution and the amount of change in a particular case, namely, when the solution contains a sufficient quantity of free acid to render immaterial the replacement of a little of it by hydric iodide, and to determine sharply the occurrence of the single reaction, $H_2O_2+2HI=2H_2O+I_2$. With this limitation, the form of the function $\varphi(i)$ in the expression $\Sigma=ptv\,.\,\varphi(i)$ has been established by the foregoing experiments, and we may now write the general equation in the form

$$\Sigma=iptv\,.\,f(a, b, c, \ldots).$$

That is to say, the amount of change varies directly, (1) with the amount of iodide, and (2) with the amount of peroxide in a unit volume of the solution; (3) with the time during which the change proceeds; (4) with the total volume of the solution; and, finally, with some function of each of the other conditions under which the change occurs.

## VIII. *Account of Experiments on Torsion and Flexure for the Determination of Rigidities.*

*By* Joseph D. Everett, *D.C.L., Assistant to the Professor of Mathematics in the University of Glasgow. Communicated by Sir* William Thomson, *F.R.S.*

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In my Paper read February 22nd, 1866, the intention was expressed of continuing my experiments on rigidity with a modified form of apparatus. This intention was carried out during the past summer, and I have now to report the results.

In the former experiments, the rod operated on was supported at both ends, and was bent or twisted by hanging a pair of equal weights so as to act symmetrically on both ends; and the amounts of flexure and torsion were measured by the movements of two images formed by reflection upon a screen.

In the new apparatus, the rod was firmly held at one end in such a manner that this end could undergo no movement whatever, while the other end was acted on by a couple composed of the direct action of a weight and the upward pull of one arm of a balance produced by weighting the other arm. The effect produced was observed, as in Kirchhoff's experiments, by means of two telescopes looking down into two mirrors which reflected a scale of lines crossing each other at right angles placed horizontally overhead.

A B (Plate IX. fig. 1) is the rod operated on, entering a socket in the cylindrical iron bar C, in which it is firmly secured by screws (three in each set) which clamp it at two places about 2 inches asunder. The other end A passes through a brass socket (shown in cross section at fig. 2), to which it is also secured by screws in two places. This socket forms part of a piece of brass, which is shown on a larger scale in longitudinal section in fig. 3, where $n$ is a point or cone to be supported by a ring (M, fig. 1) hanging from one arm of a balance, while the lower part consists of a short cylinder $m$ (for receiving the crosspiece shown in fig. 4 and indicated by dotted lines in fig. 3) terminated by a screw which receives the nut $pp$. The circular hollow shown in the centre of the cross piece (fig. 4) fits the cylinder $m$, and the crosspiece can either be rotated about it or slipped off on loosening or removing the nut. The four arms of the crosspiece are all of equal length, and each of them has on the upper side near the end a cone or point for supporting a weight by means of a ring. F F is a cast-iron box, on the top of which the cylinder C rests in two notches one at each end, in which it turns freely when not secured by the clamp G. H is a graduated circle for turning the cylinder (and with it the rod A B) through any required angle. K, L are two mirrors clamped to the rod, and adjustable by footscrews into any position nearly parallel with the rod. One of them is shown on

a larger scale at fig. 5. By partially releasing the clamps, it was easy to rotate the mirrors about the rod without longitudinal sliding.

The point *n*, fig. 3, is supported by the flat brass ring M, which hangs by the wire N from one arm of the balance D, and a counterpoise is placed in the pan P just sufficient to keep the rod A B free from strain.

The experiments were conducted in the Natural Philosophy Lecture-room. The box F F rested on the floor, the height of mirrors above floor being 270 millims. The scale reflected by them consisted of a large sheet of paper ruled in two directions at right angles to each other with lines about a tenth of an inch apart, and was firmly fixed at the height of 4597 millims. from the floor by stretching it on a board and screwing this to two joists whose primary office was the support of a cistern. The light, which was naturally good, was improved by using a concave mirror to illuminate the scale. Two telescopes, not shown in the Plate, were clamped to a firm three-legged table, their object-glasses being about 970 millims. above the floor. They were in fixed positions, directed one towards each mirror, and were as nearly vertical as was compatible with an unobstructed view of the reflection of the scale. Their deviations from two vertical planes, one parallel and the other perpendicular to the rod, were from $\frac{1}{54}$ to $\frac{1}{30}$ in circular measure. They were inverting achromatic, of $1\frac{1}{2}$-inch aperture and 10 inches focal length, with cross wires in focus of eyepiece. A damper, consisting of a piece of thin card pressing lightly against the end A of the rod, was used on and after July 17th for the purpose of checking vibration.

The mode of observing for flexure was as follows:—The mirrors having been adjusted so as to bring the central portion of the scale into view in both telescopes, a pair of equal weights were placed, one in the scale-pan P along with the counterpoise, the other on the point S, and readings were taken in both telescopes. Then the weight at S was transferred to S′, and readings were again taken. The difference of readings in further telescope diminished by difference of readings in nearer telescope is assumed to measure the effect, on the portion of rod between the two mirrors, of a bending couple whose arm is the distance between the two points S, S′, and whose power is the force of gravity on the moveable weight.

The weight was then transferred first to T and then to T′, both telescopes being read in each case. The differences were taken in the same way as above, and the result is assumed to measure the effect, on the same portion of the rod, of a twisting couple whose power is the same as above, and whose arm is the distance between the points T, T′.

The weight was then again transferred to S′ and S, then again to T′ and T, and so on several times, both telescopes being read in each position of the weight, and no change being made in any of the adjustments. The facility of thus passing from observations of flexure to those of torsion, and *vice versâ*, gives the present form of apparatus a great superiority over that employed the previous year.

It has been observed that the arms of couple in flexure and torsion are the distances S S′, T T′ respectively, which, though nearly equal, are not absolutely identical. This

defect was easily remedied by turning the crosspiece through a right angle, so as to make S S′ change places with T T′.

Another source of error to be guarded against is want of perfect circularity in the rod operated upon. This is completely removed, if the deviation from circularity be small, by turning the rod itself through a right angle by means of the graduated circle H. This change has no effect on the torsional rigidity; and its effect on the flexural rigidity is such that the mean flexure in the two positions is the true mean for all positions, inasmuch as the flexural rigidity in any position is proportional to the moment of inertia of a section about a horizontal diameter through its centre of gravity, and by a well-known theorem the sum of the moments of inertia about two rectangular diameters is constant.

For greater security the rod was turned into six different positions, differing by 30° among themselves, so that the first and fourth positions furnished one mean, the second and fifth another, and the third and sixth another. In every one of the six positions observations of both flexure and torsion were taken; and the operation of turning the crosspiece through a right angle so as to make the arms of couple for flexure and torsion change places, occurred between the third and fourth positions.

The first rod experimented on, after much time spent in preliminary arrangements, was a flint-glass rod from the works of A. and R. Cochran, Glasgow. The weights employed for producing flexure and torsion were a pair of lead weights of 100 grms. each. One of them (distinguishable by its ring) was hung in turn on each of the four arms, and the other was always placed in the counterpoise pan.

The first complete set of observations in six positions were made July 17th and 18th, with the following results:—

| | | | |
|---|---|---|---|
| 1 (*a*). | Pointer at 135° | Torsion 539 | Flexure 435⅓ |
| 2 (*a*). | „ 165° | „ 547½ | „ 438 |
| 3 (*a*). | „ 195° | „ 546 | „ 446⅓ |
| 1 (*b*). | „ 225° | „ 548 | „ 454 |
| 2 (*b*). | „ 255° | „ 549⅓ | „ 454 |
| 3 (*b*). | „ 285° | „ 546 | „ 447⅔. |

The numbers here given as representing the amounts of torsion and flexure, are expressed in tenth parts of the scale-divisions, and are therefore approximately hundredths of an inch. Combining those positions which are mutually at right angles, we have the following means:—

| | | |
|---|---|---|
| 1 (*a*) (*b*). | Torsion 543½ | Flexure 444⅔ |
| 2 (*a*) (*b*). | „ 548⅓ | „ 446 |
| 3 (*a*) (*b*). | „ 546 | „ 447½. |

The scale-divisions were somewhat longer in one direction than in the other, being $\frac{1}{75}$ of a millimetre for torsion and $\frac{1}{75}$ millims. for flexure. In order, then, to find the true

ratio of torsion to flexure, we must divide the numbers in the first column by those in the second, and diminish the quotients by $\frac{1}{189}$ of their amounts. The quotients thus corrected are

$$1\,(a)\,(b),\ 1{\cdot}222\,;\quad 2\,(a)\,(b),\ 1{\cdot}230\,;\quad 3\,(a)\,(b),\ 1{\cdot}221,$$

whence we obtain at once for Poisson's ratio ($\sigma$) the values ·222, ·230, ·221. Some small corrections will be applied to these values hereafter, only affecting the third decimal place; but we deem it important thus early to direct attention to the strength of evidence showing that Poisson's ratio for the substance in hand is less than $\frac{1}{4}$.

An earlier set of observations, in only four positions of the rod, were taken July 13th, 14th, and 16th, the apparatus being at this time less favourably arranged, inasmuch as the rod was more distant from a vertical through the centre of the scale than in the later set. The following were the results obtained:—

| | | | |
|---|---|---|---|
| I (*a*). | Pointer at 90° | Torsion 555$\frac{1}{3}$ | Flexure 452 |
| I (*b*). | „ 0° | „ 550 | „ 430 |
| II (*a*). | „ 45° | „ 550 | „ 459$\frac{1}{3}$ |
| II (*b*). | „ 135° | „ 544$\frac{1}{4}$ | „ 437$\frac{2}{3}$ |

Giving the following means,

| | | |
|---|---|---|
| I (*a*) (*b*). | Torsion 552$\frac{2}{3}$ | Flexure 441 |
| II (*a*) (*b*). | „ 547$\frac{1}{4}$ | „ 448$\frac{1}{2}$, |

whence we obtain, after correcting for inequality of divisions, the values of Poisson's ratio ·246, ·220, the largeness of the former number being due to the large angle made by the rays of light with the vertical plane containing the rod. A correction for this defect will be applied in the sequel.

After the observations of July 17th and 18th, the rod was removed from its place, and cut at the places where the mirrors had been attached. The length of the central portion was found to be 235·6 millims., and its weights in air and water respectively 32·002 and 21·112 grms., the temperature of the water being 13·3 Reaum.

The distances S S′, T T′ were 558·2 and 557·2 millims., so that the mean arm of couple was 557·7 millims., the force being the weight of 100 grms.

The height of the scale above the mirrors was 4327 millims.; but since the deviation of a reflected ray is double of the angle turned by mirror, it will be necessary to divide the arcs traversed on the scale by twice this distance, or 8654 millims., in order to find the angles turned.

The scale-divisions for torsion were $\frac{1}{75}^{\text{th}}$ millim., but as they were subdivided by estimation to tenths, and it is in these tenths that the above torsion-numbers are expressed, the unit is to be regarded as the $\frac{1}{750}$ of a millimetre. In like manner the unit for the flexure-numbers is the $\frac{1}{750}$ of a millimetre. We shall denote the torsion-numbers and flexure-numbers, expressed in these units, by the letters T and F.

From the observations of July 17th and 18th we have the mean values T=546,

F=446·1, which, reduced to centimetres, are 13·68 and 11·24. The whole torsion and flexure in the portion of the rod between mirrors are therefore

$$\frac{13\cdot68}{865\cdot4}=\cdot0158 \text{ nearly in circular measure,}$$

and $\frac{11\cdot24}{865\cdot4}=\cdot0130$ „ „ „

We shall now investigate the corrections which must be applied to the above results.

There is, in the first place, a mechanical correction depending on the fact that the plane which contains the four points S, S', T, T', and which also happens to contain the centre of gravity of the bending apparatus (*i. e.* of the crosspiece and other pieces rigidly attached to it), does not contain the point $n$ on which the apparatus is supported. Let $a$ denote the distance of this plane below the point $n$, and W the weight of the bending apparatus. Also let A denote the horizontal distance of one of the points S or T from $n$, and $w$ the weight hung at S or T, and let $\theta$ denote the angle through which the end of the rod is bent or twisted. Then the couple which produces bending or twisting is $w(A-a\theta)$, and this is resisted by two couples, $Wa\theta$, due to the weight of the bending apparatus, and $t\theta$ or $f\theta$, due to the torsional or flexural rigidity $t$ or $f$. We have therefore, for torsion, $w(A-a\theta)=t\theta+Wa\theta$, whence $t=\frac{wA}{\theta}-(W+w)a$. The first term, $\frac{wA}{\theta}$, is the uncorrected value of $t$, and we see that it requires a subtractive correction which bears to its whole amount the ratio $\frac{(W+w)a\theta}{wA}$. Hence T, being proportional to the reciprocal of $t$, requires an additive correction bearing the above ratio to its whole amount. The correction for F is expressed by the same formula, $\theta$ having, however, a different value.

In the present case we have, in grammes and centimetres, W=373, $w$=100, A=27·9, $a$=4·3, hence $\frac{(W+w)a}{wA}=\cdot729$. Again, since the whole length of rod subjected to torsion and flexure was about 42·8, whereas the portion between the mirrors was only about 23·6, we have

For torsion, $\theta=\frac{428}{236}\times\cdot0158=\cdot0286$,

For flexure, $\theta=\frac{428}{236}\times\cdot0130=\cdot0236$,

and the products of these values of $\theta$ by ·729 are ·0208 and ·0172.

T and F therefore require the additive corrections ·0208 T and ·0172 F.

There are also two optical corrections to be considered, viz.,

1st. Correction for obliquity of ray from scale to mirror. Let $\beta$ denote this obliquity, that is to say, the angle which the projection of the ray on a vertical plane perpendicular or parallel to the rod, according as we are dealing with torsion or flexure, makes with a vertical line. Then the indicated distances on the scale are always too great in the ratio of $1:1+\beta^2$. If the angles through which the two mirrors are turned are in the ratio of $m_1:m_2$, $m_2$ being the greater, and if the corresponding values of $\beta$ are $\beta_1$ and $\beta_2$ respectively, the observed values of T and F will be too great in the ratio of $1:1+\frac{m_2\beta_2^2-m_1\beta_1^2}{m_2-m_1}$.

In the present case the ratio $m_1 : m_2$ is about 1 : 3, and the values of $\beta$ for the centre of the scale in the position occupied by the apparatus on July 17th and 18th were,

For torsion, $\beta_1 = \frac{1}{40}$, $\beta_2 = \frac{1}{40}$; for flexure, $\beta_1 = \frac{1}{34}$, $\beta_2 = \frac{1}{33}$.

Hence we find by the above formula that

T is too great by ·0025 T,
F „ „ ·0013 F.

2nd. Correction for change of distance between mirror and telescope. If the mirror is moved parallel to itself to or from the telescope by the amount $b$, and if $\phi$ denote the angle between incident and reflected ray (or rather between their projections on a vertical plane perpendicular or parallel to the rod), the change produced in the scale-reading is $b\phi$.

In the present case this correction was found to be insensible.

For the total corrections applicable to the observations of July 17th and 18th, we have therefore

$$+\cdot0208\ \mathrm{T} - \cdot0025\ \mathrm{T} = +\cdot0183\ \mathrm{T},$$
$$+\cdot0172\ \mathrm{F} - \cdot0013\ \mathrm{F} = +\cdot0159\ \mathrm{F},$$

and the resulting correction of the quotient $\frac{\mathrm{T}}{\mathrm{F}}$ is

$$(\cdot0183 - \cdot0159)\frac{\mathrm{T}}{\mathrm{F}} = \cdot0024\frac{\mathrm{T}}{\mathrm{F}}.$$

This correction reduces the values of Poisson's ratio derived from the observations of those days to ·225, ·233, ·224.

For the observations of July 13th, 14th, and 16th, the correction of F is the same as above. As regards the optical correction of T, a distinction must be made between the observations marked I $(a)(b)$ and those marked II $(a)(b)$. In the former, the central portion of the scale was on the cross wires of the telescopes, in the latter a portion of the scale nearly vertical over the mirrors. The optical correction for T applicable to the centre of the scale on the date in question was −·0089 T, and we shall apply this correction to the values I $(a)(b)$, so that the total correction of T for these values will be

$$+\cdot0208\ \mathrm{T} - \cdot0089\ \mathrm{T} = +\cdot0119\ \mathrm{T},$$

and the corresponding correction of $\frac{\mathrm{T}}{\mathrm{F}}$ will be

$$(\cdot0119 - \cdot0159)\frac{\mathrm{T}}{\mathrm{F}} = -\cdot004\frac{\mathrm{T}}{\mathrm{F}},$$

which reduces the value ·246 of Poisson's ratio to ·241. To the values II $(a)(b)$ we shall apply the same corrections as to the observations of July 17th and 18th, and the value ·220 of Poisson's ratio is thus reduced to ·223. The corrected values of Poisson's ratio ·225, ·233, ·224, ·241, ·223 give the mean value ·229; and it will be noted that every one of the five determinations (whether corrected or uncorrected) is less than one-fourth.

The five determinations of T and F uncorrected and corrected, are given below. The

correcting factor for T is, as already shown, 1·0183, except for I $(a)$ $(b)$, for which it is 1·0119. The correcting factor for F is in every case 1·0159.

| | Uncorrected. | | Corrected. | |
|---|---|---|---|---|
| | T. | F. | T. | F. |
| 1 $(a)$ $(b)$. | 543·5 | 444·7 | 553·4 | 451·8 |
| 2 $(a)$ $(b)$. | 548·5 | 446·0 | 558·5 | 453·1 |
| 3 $(a)$ $(b)$. | 546·0 | 447·5 | 556·0 | 454·6 |
| I $(a)$ $(b)$. | 552·7 | 441·0 | 559·3 | 448·0 |
| II $(a)$ $(b)$. | 547·1 | 448·5 | 557·1 | 455·6 |
| Mean of corrected values . . | | | 556·9 | 452·6 |

We now proceed to deduce, as in our former paper, the values of $t$, $f$, $n$, M, and $k$, the units being the centimetre and the weight of a gramme.

For $t$ and $f$, the torsional and flexural rigidities, we have the expressions

$$t = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{7500}{188} \div \text{T},$$
$$f = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{7500}{180} \div \text{F},$$

where twice distance $=865{\cdot}4$, force $=100$, arm $=55{\cdot}77$, length $=23{\cdot}56$. Hence we have

$$\log t = 9{\cdot}65670 - \log \text{T} = 6{\cdot}91092,$$
$$\log f = 9{\cdot}65440 - \log \text{F} = 6{\cdot}99869.$$

The volume of the rod was 10·902, being the loss of weight in water multiplied by 1·00111, which is the factor proper to the temperature 13·3 R. The length being 23·56, we find (putting $r$ for radius of rod) $\pi r^2 = {\cdot}46273$, $r = {\cdot}38378$.

For Young's modulus we have

$$\text{M} = \frac{4f}{\pi r^4} = 585{,}100{,}000;$$

for the rigidity,

$$n = \frac{2t}{\pi r^4} = 239{,}020{,}000;$$

for the resistance to compression,

$$k = \frac{\text{M}n}{3(3n - \text{M})} = 353{,}264{,}000;$$

for Poisson's ratio,

$$\sigma = \frac{\text{M}}{2n} - 1 = \frac{f}{t} - 1 = \frac{\text{T}}{\text{F}} - 1 = {\cdot}229.$$

The values found last year for another specimen of flint glass, by a different maker (see former Paper), were

$$\text{M} = 614{,}330{,}000, \quad n = 244{,}170{,}000,$$
$$k = 423{,}010{,}000, \quad \sigma = {\cdot}258,$$

the specific gravity of the present specimen being 2·935, while that of last year's was 2·942.

The differences in these determinations of M and $n$, being only about five per cent. in the former case and two in the latter, are probably real, the denser specimen being also the more rigid. The values of $k$ and $\sigma$ are liable to a larger percentage of error; but this remark is more especially applicable to last year's results, as our present apparatus affords greatly increased facilities for determining the *ratio* of flexural to torsional rigidity.

With respect to the composition of the two specimens, I am unable to give precise information, as the ingredients are mixed according to no definite rule.

The glass rod having been taken down, a rod of drawn brass was mounted in its place, the apparatus remaining in precisely the same position as in the experiments of July 17th and 18th. The following results were furnished by the first set of observations, July 27th and 28th:—

| | | | | | | |
|---|---|---|---|---|---|---|
| I (*a*). | Pointer at | 0° | Torsion | 408 | Flexure | 276 |
| II (*a*). | „ | 30° | „ | 406 | „ | 274½ |
| III (*a*). | „ | 60° | „ | 404 | „ | 275 |
| I (*b*). | „ | 90° | „ | 404 | „ | 280 |
| II (*b*). | „ | 120° | „ | 404 | „ | 276½ |
| III (*b*). | „ | 150° | „ | 407 | „ | 275¼ |

From these we obtain the following means:—

| | | | |
|---|---|---|---|
| I (*a*) (*b*). | Torsion | 406 | Flexure 278, |
| II (*a*) (*b*). | „ | 405 | „ 275·5, |
| III (*a*) (*b*). | „ | 405·5 | „ 275·1, |

whence, after correcting as before for difference of scale-divisions, we obtain for Poisson's ratio the values ·451, ·461, ·465.

The weights used in these observations were the same as for the glass rod.

A second set of observations were made July 31st, August 1st and 2nd, in which, besides the old weights, which were each 100 grms., weights of 200 grms. were also employed. These latter, however, could only be used for flexure, as when the attempt was made to employ them for torsion, it was found impossible to prevent the rod from turning in its socket. In consequence of turning which took place from this cause at the commencement of this set of observations, the following pointer-readings are not precisely comparable with the foregoing, that is to say, the zero-point may be regarded as having shifted between the two sets of observations. A slight change was also made in the position of one of the telescopes, between observations 3 (*a*) and 1 (*b*), for the purpose of obtaining better light, and at the same time a string was attached to the "damper" in such a manner that the observer could pull the damper away from the rod without removing his eye from the telescope.

The following were the results, the two sets of flexure-numbers being obtained with weights of 100 and 200 grms. respectively.

| | | | | | |
|---|---|---|---|---|---|
| 1 (*a*). | Pointer at 165° | Torsion 406¼ | Flexure 277, | 550½ |
| 2 (*a*). | „ 195° | „ 406 | „ 276, | 551 |
| 3 (*a*). | „ 225° | „ 409½ | „ 272, | 547 |
| 1 (*b*). | „ 255° | „ 410 | „ 276, | 552 |
| 2 (*b*). | „ 285° | „ 408 | „ 279, | 554 |
| 3 (*b*). | „ 315° | „ 404¼ | „ 282, | 552½ |

From these we have the following means:—

| | | | |
|---|---|---|---|
| 1 (*a*) (*b*). | Torsion 408·1 | Flexure 276·5, | 551·2 |
| 2 (*a*) (*b*). | „ 407·0 | „ 277·5, | 552·5 |
| 3 (*a*) (*b*). | „ 407·0 | „ 277·0, | 549·8 |

Correcting for difference of scale-divisions, we derive the following values of Poisson's ratio.

From torsion at 100 grms. compared with flexure at 100 grms.,

·468, ·459, ·461.

From torsion at 100 grms. compared with flexure at 200 grms.,

·473, ·465, ·473.

Collecting all the results obtained with the brass rod, we find the mean value of T to be 406·4.

The mean value of F from the six results for weights of 100 grms. is 276·6, and from the three results for weights of 200 grms. 551·2. We shall denote these two numbers by $F_1$ and $F_2$ respectively.

Reduced to centimetres, these become

$$T \times \tfrac{188}{7500} = 10{\cdot}19, \quad F_1 \times \tfrac{189}{7500} = 6{\cdot}97, \quad F_2 \times \tfrac{189}{7500} = 13{\cdot}90,$$

which, being divided by 865·4 or twice distance of scale from mirrors, give as the amounts of torsion and flexure in circular measure,

Torsion, ·01178; Flexure, ·00805 and ·0161.

The whole length of rod operated on was in the present case $\frac{477}{445}$ of the portion between mirrors; hence the values of $\theta$ for the mechanical correction are $\frac{477}{445}$ of the above angles, or

·0229, ·0157, ·0313.

The values of the factor $\frac{(W+w)a}{wA}$ are respectively

·729, ·729, ·442,

giving as the values of the mechanical correction

$+{\cdot}0167\ T$, $+{\cdot}0114\ F_1$, $+{\cdot}0138\ F_2$.

The first optical correction is the same as for July 17th and 18th, viz.

$-{\cdot}0025\ T$, $-{\cdot}0013\ F_1$, $-{\cdot}0013\ F_2$.

and the second optical correction is still inappreciable. We have therefore as the total corrections to be applied

$$+\cdot0142\ T, \qquad +\cdot0101\ F_1, \qquad +\cdot0125\ F_2,$$

from which we deduce for $\frac{T}{F_1}$ and $\frac{T}{\frac{1}{2}F_2}$ the corrections $+\cdot0041\frac{T}{F_1}$ and $+\cdot0017\frac{T}{\frac{1}{2}F_2}$.

Hence the corrected values of Poisson's ratio are—

From T and $F_1$ . . . ·457, ·467, ·471, ·474, ·465, ·467,
From T and $F_2$ . . . ·476, ·467, ·476.

The mean of these nine values is ·469, which we therefore adopt as the value of $\sigma$ for brass, being nearly double of our value for glass. The comparison of our results for these two substances with those of other experimenters is somewhat startling. It stands thus:

*Glass.* Wertheim, ·33, Maxwell, ·332, Everett, ·239,
*Brass.* Kirchhoff, ·387, „ ·469;

and our two results, ·239 and ·469, were obtained with the same apparatus in the same position, each of them being the mean of several determinations, which for glass ranged from ·223 to ·241, and for brass from ·457 to ·476.

The following are the values, uncorrected and corrected, of T and F, the latter including both $F_1$ and $\frac{1}{2}F_2$.

| | Uncorrected. | | Corrected. | |
|---|---|---|---|---|
| | T. | F. | T. | F. |
| I (*a*) (*b*). | 406·0 | 278·0 | 411·8 | 280·8 |
| II (*a*) (*b*). | 405·0 | 275·5 | 410·7 | 278·3 |
| III (*a*) (*b*). | 405·5 | 275·1 | 411·3 | 277·9 |
| 1 (*a*) (*b*). | 408·1 | 276·5 | 413·9 | 279·3 |
| 2 (*a*) (*b*). | 407·0 | 277·5 | 412·8 | 280·3 |
| 3 (*a*) (*b*). | 407·0 | 277·0 | 412·8 | 279·8 |
| 1 (*a*) (*b*). | | 275·6 | | 279·0 |
| 2 (*a*) (*b*). | $\frac{1}{2}F_2$ | 276·2 | | 279·7 $\frac{1}{2}F_2$ |
| 3 (*a*) (*b*). | | 274·9 | | 278·3 |
| Means of corrected values . . | | | 412·2 | 279·3 |

The elements for deriving $t$ from T, and $f$ from F, are the same as for the glass rod, except that the length between mirrors is 24·54 instead of 23·56.

We thus find

$$\log t = 9\cdot67439 - \log T = 7\cdot05928,$$
$$\log f = 9\cdot67209 - \log F = 7\cdot22602.$$

To determine the radius $r$ of the rod, we have weight in air = 91·361, weight in water = 80·578, the temperature of the water being 7·3 R. Hence volume in centimetres = $10\cdot783 \times 1\cdot0002 = 10\cdot785$, which, being divided by the length 24·54, gives $\pi r^2 = \cdot43949$.

Hence we find for brass,

$$M=\frac{4f}{\pi r^4}=1{,}094{,}800{,}000,$$

$$n=\frac{2t}{\pi r^4}=\ 372{,}890{,}000,$$

$$k=\frac{Mn}{3(3n-M)}=5{,}700{,}700{,}000,$$

$$\sigma=\frac{M}{2n}-1=\cdot469.$$

From comparing the above value of $k$ with its values for the two glass rods experimented on, it would appear that brass is from $13\frac{1}{2}$ to 16 times more incompressible than glass; but this result is to be received with caution, for reasons which will be stated further on.

A rod of cast steel was next operated on, with the following results, the weights used being the same as for the brass rod.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I (*a*). | Pointer at | 310° | Torsion | $204\frac{1}{3}$ | Flexure | $155\frac{1}{2}$, | 313 |
| II (*a*). | ,, | 280° | ,, | 205 | ,, | 155 , | 307 |
| III (*a*). | ,, | 250° | ,, | 207 | ,, | 154 , | 316 |
| I (*b*). | ,, | 220° | ,, | 206 | ,, | 157 , | 313 |
| II (*b*). | ,, | 190° | ,, | $206\frac{4}{3}$ | ,, | 154 , | $313\frac{1}{3}$ |
| III (*b*). | ,, | 160° | ,, | $206\frac{9}{7}$ | ,, | 156 , | $313\frac{1}{3}$ |

Hence we have the following means:—

| | | | | |
|---|---|---|---|---|
| I (*a*) (*b*). | Torsion | 205·1 | Flexure | 156·2, 313·0 |
| II (*a*) (*b*). | ,, | 205·9 | ,, | 154·5, 310·1 |
| III (*a*) (*b*). | ,, | 206·9 | ,, | 155·0, 314·7 |

Correcting for difference of scale-divisions, we obtain the following determinations of Poisson's ratio.

From torsion at 100 grms. compared with flexure at 100 grms.,

·305, ·325, ·327.

From torsion at 100 grms. compared with flexure at 200 grms.,

·304, ·321, ·308.

As the apparatus was disturbed in my absence, and the mirrors were moved from their places before any measurements had been made of their positions, it is impossible to determine with accuracy from the foregoing observations the torsional and flexural rigidities of the rod. In order to determine Poisson's ratio as accurately as the data permit, we shall assume (what is known to be near the truth) that the ratio of the whole length operated on to the length between mirrors was the same as for the brass rod, and that the optical corrections are the same. From these data, the mechanical corrections are found to be

$$+\cdot0084\ T, \qquad +\cdot0064\ F_1, \qquad +\cdot0078\ F_2,$$

which, together with the optical corrections

$$-\cdot0025\ T, \qquad -\cdot0013\ F_1, \qquad -\cdot0013\ F_2,$$

make the total corrections

$$+\cdot0059\ T, \qquad +\cdot0051\ F_1, \qquad +\cdot0065\ F_2,$$

which are so small and so nearly equal that the corrections of $\frac{T}{F}$ may be neglected. We therefore assume as the value of Poisson's ratio from these experiments, the mean of the six determinations above given, which is ·315.

All the foregoing experiments were conducted by myself in the Lecture-room during the Vacation. At the commencement of the Session, the apparatus was removed to another room, where experiments on the steel rod were continued, under my direction, by two students (Messrs. King and Walker) during the months of November and December. The room selected for this purpose being on the ground-floor, and paved with asphalt, on which the apparatus rested, was superior, as regards steadiness, to the lecture-room, which is on the first floor; and I may here remark that the inconsistencies (such as they are) which occur in the foregoing experiments, were found to be due mainly to the yielding of the floor under the feet of the observer.

On the other hand the new situation afforded less height, the scale being only 223·5 centimetres above the mirrors. It was also rather dark; but this defect was completely remedied by using a gaslight, aided by a concave reflector, to illuminate the scale. The scale used was a new one, of the same kind as the old, but with the lines nearer together, their distances, as determined by taking the means of several measurements, being such that

For torsion . . 171 scale-divisions = 23·88 centims.
For flexure . . 171 scale-divisions = 23·97 centims.

The whole length of rod subjected to torsion and flexure was 46·8, and the mirrors were attached at a greater distance apart than in any of the foregoing experiments, viz. 38·15 centims. The telescopes were at the same height above the mirrors as before, being clamped to the same table which had been previously used. The weights employed were of 100 grms., and the system of observing was the same as in the later observations above described.

The following were the values obtained for T and F in terms of their respective scale-divisions, each of these values being the mean of sixteen determinations.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 (*a*). | Pointer at | 0° | Torsion | 25·62 | Flexure | 19·68 |
| 2 (*a*). | ,, | 30° | ,, | 25·87 | ,, | 19·57 |
| 3 (*a*). | ,, | 60° | ,, | 25·87 | ,, | 19·64 |
| 1 (*b*). | ,, | 90° | ,, | 25·95 | ,, | 19·74 |
| 2 (*b*). | ,, | 120° | ,, | 25·85 | ,, | 19·82 |
| 3 (*b*). | ,, | 150° | ,, | 25·84 | ,, | 19·80 |

Hence we have the following means:—

| | | | | |
|---|---|---|---|---|
| 1 (*a*)(*b*). | Torsion | 25·78 | Flexure | 19·71 |
| 2 (*a*)(*b*). | „ | 25·86 | „ | 19·70 |
| 3 (*a*)(*b*). | „ | 25·86 | „ | 19·72 |

And applying the correction for difference of scale-divisions, which is now 1 part in 266 to be subtracted from $\frac{T}{F}$, we have as the values of Poisson's ratio, or $\frac{T}{F}-1$,

·303, ·308, ·306, giving a mean of ·306.

The mean values of T and F are respectively 25·83 and 19·71, which reduced to centimetres become 3·61 and 2·76; and as twice the height of the scale is 447, we find the amounts of torsion and flexure respectively in the portion of rod between mirrors, to be about ·00808 and ·00617. The values of $\theta$ are $\frac{448}{561}$ of these, or ·00993 and ·00758, which are to be multiplied by ·729, as before, giving for the mechanical corrections the values

$$+\cdot0072\ \mathrm{T} \text{ and } +\cdot0055\ \mathrm{F}.$$

No measurements were made to determine the optical corrections, we shall therefore assume them to be the same as in the experiments on the brass rod, viz.

$$-\cdot0025\ \mathrm{T} \text{ and } -\cdot0013\ \mathrm{F},$$

making the total corrections

$$+\cdot0047\ \mathrm{T} \text{ and } +\cdot0042\ \mathrm{F},$$

whose difference is so small that the correction for $\frac{T}{F}$ may be neglected. We therefore adopt for Poisson's ratio, as determined by these experiments, the above value ·306.

The corrected mean values of T and F are 25·95 and 19·79, and we have

$$t=447\cdot0\times100\times55\cdot77\times38\cdot15\times\tfrac{171}{23\cdot88}\div\mathrm{T},$$
$$f=447\cdot0\times100\times55\cdot77\times38\cdot15\times\tfrac{171}{23\cdot97}\div\mathrm{F},$$

whence

$$\log t=9\cdot83317-\log\mathrm{T}=7\cdot41903,$$
$$\log f=9\cdot83153-\log\mathrm{F}=7\cdot53508.$$

The weights in air and water were respectively 132·94 and 116·00 grms., the temperature of the water being 7·7 R., and the length of the portion weighed being 38·1 centims. The correction for density at this temperature may be neglected, and we have volume in centim. = loss of weight in grammes = 16·94. Hence $\pi r^2=\frac{16\cdot94}{38\cdot1}=\cdot44462$ $r=\cdot37620$.

$$\mathrm{M}=\frac{4f}{\pi r^4}=2{,}179{,}300{,}000,$$

$$n=\frac{2t}{\pi r^4}=834{,}120{,}000,$$

$$k=\frac{\mathrm{M}n}{3(3n-\mathrm{M})}=1{,}875{,}600{,}000,$$

$$\sigma=\frac{\mathrm{M}}{2n}-1=\cdot306, \text{ as above;}$$

and as the experiments in the Lecture-room gave ·315 as the value of $\sigma$, I adopt the value ·310. KIRCHHOFF's value for steel is ·294, and CLERK MAXWELL's for iron ·267.

The following are the collected results of the experiments described in both this and the former paper, the values of M, $n$, and $k$ being reduced to kilogrammes' weight per square millimetre.

| | Flint Glass, 1865. | Flint Glass, 1866. | Drawn Brass, 1866. | Cast Steel, 1866. |
|---|---|---|---|---|
| M . . . . . | 6143 | 5851 | 10948 | 21793 |
| $n$ . . . . . | 2442 | 2390 | 3729 | 8341 |
| $k$ . . . . . | 4230 | 3533 | 57007 | 18756 |
| $\sigma$ . . . . . | ·258 | ·229 | ·469 | ·310 |
| Specific gravity . | 2·942 | 2·935 | 8·471 | 7·849 |

Strictly speaking, the above values of M are the measures of resistance to longitudinal extension *parallel to the length* of the rods, and the above values of $n$ are the measures of resistance to shearing in planes *parallel or perpendicular to the length*. The values of $k$ and $\sigma$ have been deduced on the hypothesis that the materials of the rods are isotropic. If, however, as is probably the case, this hypothesis is not fulfilled, and if the deviation from isotropy be such that the resistance to shearing in planes parallel or perpendicular to the length is less than for intermediate planes, then the values of $k$ and $\sigma$ above calculated are too large; for longitudinal extension (especially if accompanied by lateral contraction) involves a certain amount of shearing in planes oblique to the length, and the resistance to this shearing is one of the constituents of M, whereas the shearing which takes place in torsion is perpendicular to the length. Such a deviation from isotropy as we are now considering will therefore increase the ratio of M to $n$, and will therefore increase $\sigma$, which is equal to $\frac{M}{2n}-1$. It will also increase $k$, since the value of $k$ may be written $\frac{M}{3(1-2\sigma)}$. This caution is specially important in the case of the brass rod, both because the operation of "drawing" appears likely to produce such a deviation from isotropy as we have been describing, and also because the value of $\sigma$ for this rod comes out so nearly equal to $\frac{1}{2}$ that the factor $1-2\sigma$ in the denominator of $k$ will be greatly affected by small errors in the value of $\sigma$. For these reasons we are not disposed to attach much weight to the very large value of $k$ which we have found for brass.

We append for comparison some of the principal results obtained by previous experimenters.

The values obtained by WERTHEIM for different specimens of glass (crystal) were,—

M 3481 to 4429, mean 4039,
$n$ 1288 to 1687, mean 1518,
$k$ 3569 to 4476, mean 3968;

and for different specimens of brass,

M 9665 to 10645, mean 10054,
$n$ 3600 to 3973, mean 3745,
$k$ 10216 to 11058, mean 10631.

SAVART's experiments on the torsion of brass wire lead to the result $n=3682$.

KUPFFER's values of M for nine different specimens of brass range from 8112 to 11617, the value generally increasing with the specific gravity, and the two specimens which agree most nearly with our own in specific gravity show the following results:—

| | |
|---|---|
| Specific gravity 8·4465. | Value of M 10783 |
| Specific gravity 8·4930. | Value of M 11421. |

The values of M found by the same experimenter for steel range from 20569 to 21842.

The values of $\sigma$ found by KIRCHHOFF, WERTHEIM, and MAXWELL have already been given. They all differ widely from our own except in the case of steel.

In conclusion we may state that, as our present form of apparatus is found extremely convenient, it is intended to use it for continuing the series of experiments which have been begun, with, however, an important modification, which will be made for the purpose of diminishing or removing the "mechanical correction."

## IX. *On the Structure of the Optic Lobes of the Cuttle-Fish.*

*By* J. Lockhart Clarke, *F.R.S.*, *&c.*

Received September 26,—Read November 15, 1866.

The brain of the Cuttle-fish is enclosed in a cartilaginous case or skull, which is pierced by foramina for the passage of the nerves which it gives off. It consists of several ganglia closely aggregated and united around the upper end of the œsophagus, as shown in fig. 1, Plate X. *Above* the œsophagus, the foremost or pharyngeal ganglion (*a*), which is much the smallest, is bilobed, somewhat heart-shaped, and closely applied, at the central line, to the junction of that tube with the large globular and powerful pharynx (*b*, *b*, *b*). Behind this ganglion, and joined to it by two nervous cords, is a large bilobed ganglion (*c c*), which is broader above than below. It rests on a kind of disk or collar-like layer of nerve-substance, which forms the *roof* of the ring or short canal (*d*) through which the œsophagus is transmitted.

*Below* the œsophagus, and forming the *floor* of the canal (*d*) that transmits it, is a large and broad ganglion (*e*) which extends forward and is partially divided into an anterior and a posterior portion. The latter portion is connected along the sides of the œsophagus with the *supra*-œsophageal ganglia, by means of bands which complete the œsophageal ring. It gives off nerves to the branchiæ, to the viscera, to the mantle, &c. The *anterior* portion of the subœsophageal mass supplies nerves to the feet and tentacles, and a connecting branch to the pharyngeal ganglion (*a*).

From each side of these cephalic masses, and connected as much, apparently, with the ganglia beneath the œsophagus as with those above it, springs the optic peduncle (*f*) which unites them with the large optic ganglion (*g g*). On its upper surface is a small pyriform tubercle attached to it by a short pedicle or neck. Each optic lobe is as large as the rest of the cephalic ganglia on both sides taken together, and bears a striking resemblance in shape to the human kidney. Fig. 2 represents the left one freed from the nerves in which it is enclosed. It is very convex on its outer side, with a deep notch on its inner side where the peduncle is attached. Its upper surface also is a little concave, and its under surface is convex in a corresponding degree. Everywhere it is covered and entirely concealed by a thick stratum of optic nerves disposed in flattened bands, which issue from all parts of its substance and proceed at once to the back of the eye (*h*), in a layer or kind of fan-shaped expansion (*i i*), which, like the optic lobe itself, is somewhat concave at its upper surface and a little convex below. As they enter the back of the eye the bands of the layer decussate; those proceeding form the *upper* surface (*i'*, *right side of figure*) sloping downward over the convex border

of the lobe, to the *under* side of the eye; while those which proceed from the *under* surface of the lobe, ascend in a similar way between the others to reach the *upper* side of the eye at *i''*. Moreover, at the upper surface, this layer of fibres is curiously folded over each end of the lobe, as shown at *g g*.

After removing one of the optic lobes with its peduncle and layer of nerves, if a thin longitudinal section be made through the middle line, and magnified about seven diameters, it presents the appearance delineated in fig. 3. Here it will be seen that the optic nerves at *j* proceed outward in almost straight lines, while those nearer the end and at the opposite side of the lobe (*k*, *k*, *k*) are successively more curved in their course to join the others for the formation of the fan-like expansion which is attached to the globe of the eye. The substance of the lobe consists of two distinct portions, which differ from each other considerably both in structure and general appearance. The outer portion (*l*, fig. 3) resembles a thin rind or shell, and is very delicate and very easily separated from the substance which it encloses. It may be said to consist of three concentric layers,—an external layer (*m*), an internal layer (*n*), and a broad pale layer between them, containing two thin and concentric layers (*o*) of a somewhat darker hue.

When this outer or cortical portion of the lobe is subjected to higher magnifying powers, it is found to possess a very beautiful and a very elaborate structure. Fig. 4, *p* represents a thin vertical section magnified 220 diameters. The first or outer layer (*q*) to which the optic nerves (*r r*) are attached, is composed of small round nuclei of nearly uniform size, together with a few nucleated cells of either an oval or a poly-angular shape. Near the surface the nuclei are comparatively few, but increase in number as they descend, and on approaching the border, which is very sharply defined, they are crowded closely together. Through this layer the nerves enter in separate bundles which diverge as they descend. The majority, at least, of their fibres are connected with the nuclei, and form with them a close network; but whether any of them run directly through to the next subjacent layer I have not determined satisfactorily.

The second layer (*s*) is composed entirely of fine nerve-fibres which run in two different directions at right angles to each other, the one being vertical, and the other horizontal or concentric with the layer. The vertical fibres are by far the more numerous. They issue from the under surface of the first layer as prolongations of the nerve-roots, but they have no fascicular arrangement, being uniformly disposed in parallel lines. Some of them, particularly at the lower part, abandon their original direction to become continuous with the horizontal fibres (*t*, *t'*), while the rest are prolonged across these fibres as far as the next layer (*u*). The horizontal fibres, for the most part, are collected into separate bands. The highest of these (*t*) is very slender, and below it are delicate fibres running in the same direction, but with wider intervals between them. The other two bands are at the lower end of the layer. They are much broader, and are separated only by a narrow interval. The lowest is in contact with the third or subjacent layer (*u*).

This third or internal layer (*u*, fig. 4, and *n*, fig. 3) of the cortical portion of the lobe

is composed of round and closely aggregated nuclei of nearly the same size as those of the first layer. The nuclei are united in a network of fibres, which at the upper border (*w*) are directly continuous with both the transverse and longitudinal fibres of the layer next above. At the lower border (*w'*) is a single row of nucleated cells, which send their processes upwards, inwards, and sideways.

But the cortical substance, consisting of these three layers, forms only a very small portion of the optic lobe, the chief bulk of which has a structure and appearance of quite another kind. From the nuclear network and nucleated cells of the third or inner layer (*u*, fig. 4) of the cortical portion of the lobe, a continuous series of fine nerve-fibres may be seen to issue at its lower border (*w*). At first these fibres are vertical, parallel, and arranged side by side in a nearly uniform series; and between them are scattered without regularity a number of round, oval and triangular nuclei like those of the cortical layer; but immediately after their exit the fibres begin to arrange themselves in bundles, which, as they descend, deviate from their former vertical direction, and decussate each other in a plexus or network (*x*, *x*, *x*); while the nuclei, in corresponding proportion, collect at first into small irregular groups (*y y*) between the bundles, and then into groups that are larger and more compact, within the meshes of the plexus (*y' y'*); they are not, however, completely isolated, but communicate with each other, to a greater or less extent, by means of lateral offsets. At first the meshes and the groups which they contain are more or less fusiform and disposed with their longer axes vertical, so as to constitute a system of communicating rays at right angles to the cortical layers (*y' y'*, figs. 4, 3, 5, & 6); but as they reach the centre of the lobe, where the bundles of the plexus are more divergent and decussate each other, in every direction, at greater angles, the groups contained in the meshes become more globular, more isolated, and at the same time larger; but still they communicate by lateral offsets like a number of stellate cells (figs. 3, 5, & 6). Fig. 5 represents a transverse section of the lobe along the line *a'*, fig. 2, and shows the globular and stellate form of the cell-groups in the centre, with their elongated form and radiate arrangement near the surface*. Fig. 6 represents another transverse section through the peduncle and its tubercle, along the line *b'*, fig. 2. Here the cell-groups in the centre are still more globular, and consequently, in section, many of them appear much larger than the elongated groups which are cut transversely in fig. 5.

Under a magnifying power of 60 diameters, thin sections made in different planes through the central parts of the lobe present the appearances delineated in fig. 7. Here the blank, communicating spaces, so well seen in fig. 5 between the central groups of cells, are occupied by the most intricate part of the plexus, where the bundles interlace in every possible direction. The fibres of each bundle are connected with the cells of different groups by means of their processes, which are turned to every side.

On examining the cell-groups under still higher powers, they were found to consist of round, oval, pyriform, and triangular nuclei, confusedly mingled with small and large

* Fig. 5 should have been reversed from right to left by the engraver.

nucleated cells of every variety of shape, as represented in fig. 8, magnified 220 diameters.

From the plexus on the inner side of the lobe the bundles converge and unite to form the fibrous portion of the peduncle (*z*, figs. 3 & 6). In fig. 3, which represents a longitudinal section of the lobe, they are seen to decussate in a very striking manner; but in planes at right angles to it there is no appearance of decussation, as may be seen in fig. 6, which represents a transverse section through the middle of the lobe and peduncle in the direction of the line *b'*, fig. 2. Moreover, it may be here observed that the fibres occupy only the inferior half of the peduncle, the superior half (*z'*) consisting chiefly of masses of nuclei and *small* cells like those of the lobe, and giving attachment to the tubercle (*z''*) by a short and narrow pedicle or neck. This little body is pyriform, inclined somewhat outward, and composed of closely-aggregated nuclei connected with a multitude of fibres which converge to the neck and then spread through the cells of the peduncle.

Having thus concluded my description of the optic lobes and their peduncles, I will offer a few brief remarks on the general structure and connexions of the remaining cerebral centres, with the view of determining their homologies. The foremost or bilobed pharyngeal ganglion *in situ* is rendered quadrangular by the roots of the nerves which it gives off. In front it gives off from each angle a thick nerve-trunk (*a*, *a*), which soon divides into two branches. One of these is distributed on the outer side of the mouth to the powerful muscles which move the beak, while the other turns inward to the laminæ or turbinated folds of the palate. These folds are muscular, but have a strong resemblance to the nasal laminæ of fishes and other vertebrata, and are everywhere lined with mucous membrane and epithelium*. Between the large nerves given off from the angles of the ganglion, numerous smaller branches proceed directly forward to different parts of the mouth. Some of them may be traced to the muscular bands that descend to the base of the spiniferous tongue, which is situated at the bottom and in front of the turbinated palate. The ganglion itself has a composite structure. A longitudinal section, that is, a section behind-forward, is oval or fusiform. Examined under a sufficiently high power, it is found to consist of two kinds of tissue—(1) a central, oval, and whitish nucleus (1, fig. 9) composed of the closest interlacement or network of the finest nerve-fibres, with some intervening granules; and (2) a surrounding grey layer of nucleated cells (2, 2), in connexion with fibres proceeding from the central nucleus. Above, and especially below, the nucleus reaches nearly to the surface of the ganglion, so that at those parts the layer of nucleated cells is very thin; but in front (2) and behind (*v*) it is much deeper. From the distribution of its nerves to the muscles of the mouth and tongue, and to the turbinated laminæ of the palate, this ganglion would seem to correspond to the centres of the fifth, the ninth or hypoglossal, and perhaps the olfactory and gustatory nerves of vertebrated animals.

The second and larger ganglion (*c c*, fig. 1), which is connected with the former by two

* They communicate with the mouth by openings or fissures in the palate, and appear to be the olfactory organs.

nervous cords, has also a composite structure. Its superficial, smooth, and convex portion consists of two comparatively thin caps or shells, joined in the middle line, but separated in front by a notch (see fig. 10, *c c*). It is composed of a very close interlacement or network of the finest fibres, interspersed with fine granules, with nuclei, and with small cells of different shapes; and in these respects bears a strong resemblance to the cerebral lobes of fishes. Beneath the convex caps is a large mass of an entirely different structure. This consists of a kind of plexus, or interlacement of coarser fibres in every direction, with intervening but irregular and coalescing groups of nucleated cells and nuclei. It has some resemblance to that of the central parts of the optic lobes, but is finer. At its base, where it overlays the canal for the œsophagus, it presents a somewhat diversified appearance, and projects in front ( $_3$, fig. 10) and behind (fig. 1, $_3$) in the form of a kind of collar, as already stated. Whether this central portion of the ganglion constitutes any part of the nervous apparatus for vision, or whether it should be considered as a cerebellum, is not easy to determine. It is certainly in connexion both in front and behind with the optic peduncles and lobes; but then so is the cerebellum in vertebrate animals, especially in fishes; and it can scarcely be expected that a distinct cerebellum would be wanting in an animal whose cerebral development approaches so closely to that of fishes in which that organ is very large.

The posterior part of the subœsophageal mass, as already stated, gives off nerves which supply the branchiæ, some of the viscera, and the auditory apparatus, and may therefore be considered as homologous with the medulla oblongata; while the anterior part, which supplies nerves to the feet and tentacles, may be regarded as the spinal cord, concentrated, like those organs, in the neighbourhood of the head.

X. *Abstract of the Results of the Comparisons of the Standards of Length of England, France, Belgium, Prussia, Russia, India, Australia, made at the Ordnance Survey Office, Southampton. By Captain* A. R. CLARKE, *R.E., F.R.S., &c., under the Direction of Colonel Sir* HENRY JAMES, *R.E., F.R.S., &c., Director of the Ordnance Survey. With a Preface by Colonel Sir* HENRY JAMES, *R.E., F.R.S., &c.*

Received November 15,—Read December 13, 1866.

THE principal triangulation of the United Kingdom was finished in 1851; and the triangulations of France, Belgium, Prussia, and Russia were so far advanced in 1860 that, if connected, we should have a continuous triangulation from the Island of Valentia, on the south-west extremity of Ireland, in north latitude 51° 55′ 20″, and longitude 10° 20′ 40″ west of Greenwich, to Orsk, on the River Ural in Russia.

It was therefore possible to measure the length of an arc of parallel in latitude 52° of about 75°, and to determine, by the assistance of the electric telegraph, the exact difference of longitude between the extremities of this arc, and thus obtain a crucial test of the accuracy of the figure and dimensions of the earth, as derived from the measurement of arcs of meridian, or the data for modifying the results previously arrived at.

The Russian Government, therefore, at the instance of M. OTTO STRUVE, Imperial Astronomer of Russia, invited (in 1860) the cooperation of the Governments of Prussia, Belgium, France, and England, to effect this most important object, and to their great honour they all consented, and granted the necessary funds for the execution of the work.

The portion of the work which was assigned to me was the connexion of the triangulation of England with that of France and Belgium, and I published the results of this operation in 1862*. But this work has been done in duplicate; for when application was made to the French Government to permit the necessary observations to be made in France, they not only consented to allow this, but at the same time volunteered to join in the labour and expense of the work itself.

It would obviously have been wrong to mix up observations made with different kinds of instruments and on different principles, and therefore it was agreed that the work should, in fact, be made in duplicate, both the French and English geometricians using the exact same stations.

The results obtained by the French geometricians is published in the Supplement to vol. ix. of the 'Mémorial du Dépôt Général de la Guerre,' 1865, and the agreement with the results obtained by the English is truly surprising.

But however accurately the trigonometrical observations might be performed, it is obvious that, without a precise knowledge of the relative lengths of the standards used

* Extension of the Triangulation of the Ordnance Survey into France and Belgium. London, 1863.

as the units of measure in the triangulation of the several countries, it would be impossible accurately to express the length of the arc of parallel in terms of any one of the standards.

It was therefore necessary that a comparison of the standards of length should be made; and as we had a building and apparatus expressly erected for the purpose of comparing standards at this office, the English Government, on my recommendation, invited the Governments of the several countries named to send their standards here; and we have had the following compared with the greatest accuracy:—

1. Russian Standard double Toise, P.
2. Prussian Standard Toise.
3. Belgian Standard Toise.
4. Platinum Metre of the Royal Society, compared with the Standard Metre of France by M. Arago.
5. English Standard Yards, A, B, C, 29, 47, 51, 55, 58.
6. Ordnance Survey 10-foot Standard Bar.
7. Indian 10-foot Standard Bars, new and old.
8. Australian 10-foot Standard Bars.
9. In addition to the above, the 10-foot Standard Bar of the Cape of Good Hope was compared here in 1844.

We have invited the Governments of Austria, Spain, and the United States of America, also to send their standards. We have been promised that of Austria, and but for the unfortunate war in which she has been lately engaged, we should have received it before this.

I have entrusted the execution of the work of comparison and the drawing up of the results to Captain Alexander R. Clarke, of the Royal Engineers, who designed the apparatus used. The numerous comparisons to be made entailed a great amount of labour upon him and his assistants, Quartermaster Steel and Corporal Compton, of the Royal Engineers.

Before the connexion of the triangulations of the several countries into one great network of triangles extending across the entire breadth of Europe, and before the discovery of the electric telegraph and its extension from Valentia to the Ural Mountains, it was not possible to execute so vast an undertaking as that which is now in progress. It is, in fact, a work which could not possibly have been executed at any earlier period in the history of the world. The exact determination of the figure and dimensions of the earth has been the great aim of astronomers for upwards of two thousand years; and it is fortunate that we live in a time when men are so enlightened as to combine their labours to effect an object desired by all, and at the first moment when it was possible to execute it.

A full detailed account of the 'Comparisons of the Standards of Length,' with numerous plates, has just been published, and may be obtained from the agents for the sale of the publications of the Ordnance Survey.

Henry James, *Colonel R.E.*

*Ordnance Survey Office,*
*Southampton, 14th November*, 1866.

*On the Comparison of English and Foreign Geodetical Standards with the English Standard Yard. By Captain* A. R. CLARKE, *R.E., F.R.S., &c.*

In the Philosophical Transactions, Part III., 1857, is the Astronomer Royal's "Account of the Construction of the New National Standard of Length and of its Principal Copies." Those who have looked carefully into this paper must have perceived that the difficulties attending the comparisons of standards, where results of a high order of precision are aimed at, are considerable; requiring the very best workmanship in nearly every part of the apparatus, and demanding the greatest patience and circumspection on the part of the observer. But the difficulties which were encountered and so successfully overcome by Mr. SHEEPSHANKS are considerably enhanced, when, as in the operations which have been recently conducted at the Ordnance Survey Office, Southampton, the bars to be compared are of different and incommensurable lengths. It was therefore foreseen that without building a room especially for the purpose, and devising an apparatus that could be adapted to the measurements of all kinds of lengths up to 13 or 14 feet, the comparison of the geodetical standards with the yard could not be undertaken with any prospect of success.

The bar-room is 20 feet in length by 11 in breadth; the walls are double, the outer 2 feet thick, and the inner $4\frac{1}{2}$ inches with an interval of 3 inches. The foundations are very strongly built. The roof is flat, the walls being spanned by iron girders whose lower flanges support large slates, which again are covered uniformly by 9 inches of concrete. The direction of the length of the room is nearly east and west; in the north face are two small windows, sufficient to admit a moderate amount of daylight. The sash frames, which slide in the interval between the two walls, are further protected by wooden shutters, 3 inches in thickness, on the outside. The doorway is at the east end of the room and is closed by double doors, one on the outside of the room, the other on the inside, so that any one entering the room may close the first or outer door before he opens the second or inner door.

An outer building encloses this room, and so protects it from the variations of temperature of the external air. The outer building is of brick, and is 40 feet in length by 20 in breadth, with an ordinary slate roof. Thus shielded from external influences, the temperature of the inner room is exceedingly steady, leaving nothing to be desired in this respect.

Along the southern wall of the room are three stone piers for supporting the micrometer-microscopes. The centre pier, or block, measures on its upper surface 4 feet by 16 inches; the outer blocks are of the same breadth, but only 3 feet 6 inches in length; they are distant 5 feet from centre to centre from the middle block. These stones are close to the wall of the room, but are not actually in contact with it; they have separate

and deep foundations of brickwork; their upper surfaces are about 4 feet 6 inches above the floor of the room. Each stone is so cut as to present along its front horizontal edge a projecting ledge, the upper surface in fact projecting 3 inches beyond the lower part or face of the stone; the vertical depth of the projecting ledge is 3 inches.

Immediately in front of and close to the piers is placed a large mahogany beam, measuring 14 feet in length by 14 inches in breadth, and 9 inches in vertical depth. Its position is horizontal and parallel to the length of the room, and to the faces of the stone piers; to its upper surface (which is about 2 feet above the floor of the room) are fastened a pair of planed cast-iron rails, 11 inches from centre to centre, and extending the whole length of the beam. In the fixing of these rails to the beam, provision is made for any warping which the beam might undergo; so that the rails can always be kept straight and parallel.

This beam being intended to support the standard bars when under the microscopes, is not itself supported by the floor of the room, but has, like the stone piers, its own foundations. The flooring upon which the observer stands has no contact either with the stone piers, or with the foundations by which the beam is supported. Further, the foundations for the beam are entirely disconnected with the stone piers, thus (and it has been repeatedly and severely tested) no movement of the observer can disturb either the microscopes or the bars under observation. Before perfect immunity from disturbance, however, was obtained, it was found necessary to disconnect the wooden flooring entirely from the walls of the room; the flooring is framed in three separate pieces, each being supported by, or simply resting on, four large blocks of india-rubber.

*Micrometer Microscopes.*—The magnifying power of the microscopes is about sixty. The length of the tube from the diaphragm to the object-glass is 12 inches, and from the object-glass to its external focus 3 inches. The value of one division of the micrometer is about the 35,000th part of an inch. Each microscope is held immediately in a strong hollow gun-metal cylinder about 6 inches in length, the axis of which coincides with that of the microscope. At either extremity this cylinder is internally provided with circular Y's, into or against which the tube of the microscope is pushed by springs, the tube having two strong accurately turned collars for this purpose at one-fourth and three-fourths of its length. The upper collar has a flange which determines longitudinally the position of the microscope with respect to the gun-metal cylinder; while at the same time the microscope is free to revolve in the cylinder, but without anything approaching to a shake. From the cylinder, at its mid length, project three arms by which it is held and levelled, each arm having through its extremity a cylindrical hole bored ($\frac{1}{4}$ inch diameter) parallel to the cylinder itself. This gun-metal cylinder, again, is supported by and held firmly to a strong and heavy plate of cast iron, which, having three bosses on its under surface, rests on one of the stone piers, part of the plate projecting beyond the front of the stone towards the room. That part of the iron plate which rests immediately on the stone is a rectangle of 12 inches by 14 inches, and the projecting part may be described as something like an equilateral triangle of 8 inches

side: the plate is rather more than an inch thick. Towards the apex of the projecting triangle there is a circular hole 2 inches in diameter through the plate, through which the gun-metal cylinder holding the microscope passes. Equidistant from the centre of this hole, and equidistant from one another, are three vertical screws strongly bolted into the iron plate and projecting upwards about 2 inches in length. These screws pass freely through the holes in the arms of the gun-metal cylinder. Suppose now two nuts running upon each of these screws, one above and one below each of the arms, and it is clear that we have the means of rendering truly vertical the axis of the microscope, and also of holding the microscope very firm by clamping down the upper nuts. It will also be seen that the microscope is held without the least strain, and that it can be raised or lowered small quantities so as to bring to focus over a given object. The iron plate is not held down to the stone in any way; its own weight gives sufficient stability; it may be shifted to any position on any of the stones.

*Illumination.*—Much depends upon the proper illumination of the divided surfaces under observation. A candle, whose flame is mechanically kept in a constant position, stands behind the microscopes; and its light, condensed by a lens 3 inches in diameter, passes through an aperture in the projecting part of the cast-iron plate, being brought to a focus on the divided surface under observation. Abundance of light is thus obtained, and the candle being above the bar, the heated air is continually carried away from it; besides, the heat of a candle is the least practicable with a sufficiency of light*.

*Carriages.*—The box containing the bar, or bars under observation, is supported by two carriages which run upon the rails that have been described as fixed on the upper surface of the large mahogany beam; one of the rails is flat, the other triangular in section; each carriage runs on three wheels, two of which, being grooved, run on the angular rail, the third on the flat rail. Thus it will be seen that the motion of the carriage is without any possible jamming. Each of the two carriages is double, that is, consists of an upper and lower carriage; the upper carriage runs upon short rails on the surface of the lower carriage, and in a direction perpendicular to the motion of the latter. A slow-motion screw affords the means of communicating, when required, a small motion to the upper carriage. Without going into further details as to the construction of these carriages, it may be sufficient to say that the different parts are so put together that no shake exists, nor can it be introduced by wear. The box containing the bar or bars under comparison has therefore, when resting on the carriages, a perfectly steady bearing; while it can be moved in a longitudinal direction by the running of the carriages along the rails on the mahogany beam, or moved transversely by the movement of the upper carriages on the lower.

* This method of illumination was decided on after a considerable number of experiments with gas, oil lamps, &c., directed and condensed, or reflected in different ways. Had there been only a few different lengths to compare, the light might have been brought in from the outside of the room through horizontal holes in the wall and piers, and in this manner some of the earlier comparisons on different ten-foot bars were made. But this method could not have been applied to the comparisons generally, on account of the large number of holes that would have been required.

Suppose now we have two bars lying alongside one another in a box (the boxes are of a uniform breadth of 8 inches externally), their axes parallel, and about, as usual, 3 inches apart; then by the movement of the upper carriages, the one bar and the other may be brought alternately under the microscopes.

*Bars.*—The various copies of the standard yard are all 1 inch square in section, and about 38 inches in length. At about an inch from either extremity of the bar, a cylindrical well is drilled halfway through the metal; at the bottom of each well is a gold pin let into the bar. On the surfaces of these gold pins, which are, it will be observed, in the neutral axis of the bar, the lines defining the measure are drawn. The bar has also wells in its upper surface for the bulbs of two or four thermometers. That particular copy of the standard yard which has been compared with all the geodetical standards is No. 55 (Swedish Iron); it is supported on rollers at one-fourth and three-fourths of its length.

*Ordnance Survey Standard* $O_1$.—This is a bar of wrought iron 10 feet 2 inches in length, $1\frac{1}{2}$ inch broad, and $2\frac{1}{2}$ inches deep; supported on rollers at one-fourth and three-fourths of its length. The ends of the bar are cut away to half its depth, so that the dots marking the measure of 10 feet are in the neutral axis of the bar. There are two wells for thermometers.

*Ordnance Intermediate Bar* $OI_1$ is of wrought iron, in section having the form of a girder, with equal upper and lower flanges, the extreme breadth and depth being the same as in the last-mentioned bar. On the upper surface are seven disks, $a, b, c, d, e, f, g$: the spaces $a\,b$ and $f\,g$ are each one yard; $b\,c$, $c\,d$, $d\,e$, $e\,f$ are each one foot. Each disk has one transverse, crossed by two parallel longitudinal lines. This bar is supported on a cradle system of eight rollers.

*Indian Standards* $I_3$, $I_B$ are bars similar to one another and to the bar OI described above, differing only in this, that one is of cast steel, and the other of Baily's metal or bronze.

*Ordnance Toise* ($T_o$) *and Metre* ($M_o$).—These bars are of cast steel, similar in section to the last-mentioned bars, but only an inch wide by one and a half deep. The toise has four disks, $a, b, c, d$; the spaces $a\,b$, $b\,c$ are each one yard; $cd$ is approximately 4·74 inches. It is supported on a cradle system of eight rollers. The metre has three disks; the first two are a yard apart, the second and third are 3·37 inches apart.

*Prussian and Belgian Toises* $T_{10}$, $T_{11}$ are flat bars of cast steel, an inch and three-quarters in breadth and four-tenths of an inch thick, terminating in cylinders about half an inch in length, the axis of the cylinder coinciding with that of the bar, and the diameter of the cylinder coinciding with the depth of the bar. At the extremity of each of these cylinders is affixed a smaller (co-axial) cylinder of tempered steel, an eighth of an inch in diameter, and only a sixtieth of an inch long. The faces of these small cylinders, which are perfect planes beautifully polished, and at right angles to the axis of the bar, form the terminal planes of the measure.

*Russian Double Toise* P is also an end measure, but the terminal surfaces are not

planes, being slightly convex; it is a bar of wrought iron two toises in length, and an inch and a half square in section, supported at one-fourth and three-fourths of its length.

*Standard Foot* F is a bar 13 inches long by 1 inch square divided into inches. The extreme inches are further divided on inlaid strips of platinum, into tenths, and some of these tenths into hundredths.

*Thermometers.*—The standard thermometers, to which all others have been referred, have had their errors determined by calibration measurement to every fifth degree, by means of an apparatus constructed for this purpose. The apparatus consists essentially of three parts:—(1) as a base, a heavy rectangular plate of metal 2 feet long lying horizontally, having at its extremities upright pieces 10 inches high; (2) the top of these uprights are joined by a couple of steel rods parallel to one another and at the same height; along these rods there slides a platform carrying a vertical microscope between the rods; (3) a sliding frame, supported immediately by the lower plate, moving in the direction of the length of the lower plate or of the steel rod above, carries the thermometer to be examined. This sliding frame is moved by means of a micrometer screw, which therefore draws the thermometer along backwards or forwards in the direction of its own axis. The bed of the thermometer is rendered truly horizontal, and the microscope has a level attached by means of which its axis may be always kept strictly vertical. The thermometer is protected, as far as possible, from variations of temperature by being closely surrounded (except its upper surface) by metal; and the detached column of mercury can be shifted from one position to another without touching the thermometer with the hand. The error of the mean length of a degree is determined by boiling the thermometer (in a horizontal position), and immediately after placing it in ice.

The thermometers which record the temperatures of the bars are only 5 inches in length, each showing 20° range of temperature; the degree is about a fifth of an inch in length, and subdivided to tenths. These thermometers are compared as often as necessary (and this is very often) with the standard thermometers. The apparatus for the comparison of thermometers consists of a water trough, 29 inches long by 9 inches broad and 9 inches deep (internal measurement), resting on three points, one of which is a levelling-screw. The thermometers rest on cross bars at the mid depth of the water, their tubes truly level. This trough stands on the ground, and the thermometers are read by means of a long microscope, which is mounted on a travelling platform, and of which the axis is made vertical. The degrees on the long standard thermometers are not generally subdivided; and in order to read them accurately the following arrangement was adopted: on a small strip of plate glass are drawn a system of eleven equidistant and slightly converging lines; this strip slides in the diaphragm of the microscope, and can be moved by the hand; then, if it be required to subdivide a degree seen in the centre of the field of the microscope, the glass slide is moved until the outer lines of the system coincide with the bounding lines of the degree. Thus tenths are immediately read, and the hundredths can be estimated.

In order to read the thermometers when lying in the bars, orifices are provided in the

covers of the boxes: the microscopes used here are 12 inches in length, they are mounted on sliding plates so as to traverse the whole length of the thermometer-tube; the verticality of their axes is also ensured.

*Method of Comparing.*—Whenever practicable, the two bars which have to be compared are mounted side by side in the same box. Each bar is capable of being levelled (by raising or lowering the cradles or rollers on which it rests), or brought to focus under the microscopes. Each microscope has attached to it a level whereby the verticality of the axis may be tested. It is usual to arrange a pair of bars for comparison on the afternoon of one day, and to commence observing the next day. The bars are visited three or four times each day; a *series* of comparisons has generally consisted of about ten visits or comparisons; and the bars are then dismounted, to be compared another time. All adjustments are frequently put out and renewed; there is little use in multiplying observations while none of the circumstances of the observations are changed. Consequently, as far as practicable, the comparisons of any two bars have been made in detached series; thus the fear of constant error is diminished. It is generally assumed that the temperatures of two bars lying together in the box are the same, the minute differences which are sometimes found in comparing the readings of the thermometers being attributed to the thermometers themselves, which certainly do not always immediately indicate changes of two or three hundredths of a degree of temperature. The two bars are also made to interchange places, so that either one of them is next to the observer about as often as it is next to the piers. A temporary constant error may also creep in, if great care is not taken that the divided surfaces are clean; minute particles of dust, almost indiscernible to the eye, or one such particle hanging about the edges of the line where it is to be bisected, will give a false result to all observations; and it is of course undesirable to clean the surfaces, or even rub them gently, oftener than can possibly be helped.

The observations made at any one visit to the bar-room are generally as follows:—(1) the two thermometers in each bar are read; (2) the bar A being adjusted to focus under the microscopes, three readings (bisections) of the microscope on the left are taken, and then three readings of the microscope on the right; (3) B being now adjusted under the microscopes, similar readings are made; (4) B is thrown out of focus by the levelling-screws, and being readjusted in focus under each microscope is observed as before; (5) A is observed a second time; (6) the thermometers are read again. A slight disturbance and rise of temperature is almost inevitably caused by the observer's presence and the heat of the two candles. On the average, the second readings of the thermometers are $0^{\circ}\cdot035$ above the first readings.

In comparing any two bars, the greater part of the comparisons have been made at temperatures differing not more than $2^{\circ}$ from the standard temperature of $62^{\circ}$, and a small portion at a temperature as low as possible; thus the expansions of the bars are eliminated.

With respect to personal error in bisections, this has been found to exist in the case

of some particular lines, to the amount of one or sometimes two micrometer divisions. The only way of eliminating personal error is by the employment of as many expert observers as can be commanded.

*Flexure.*—It is of the utmost importance that a bar be supported invariably at the same points; if these points be altered, then (unless the divided surfaces be in the neutral axis) the length of the bar undergoes a change. The proper positions for the supporting rollers of a bar have been investigated by Mr. AIRY in the Memoirs of the Royal Astronomical Society. In order to test the theory of flexure of bars (considered as elastic rods) by actual experiment, a large number of observations as to changes of length corresponding to change of supports were made on three iron bars specially prepared for the purpose. Each bar was 40 inches long by an inch square. It will suffice here to explain generally the process and give the results for one of the bars. If a bar be supported by its extremities, it is clear that the whole of its upper surface will be compressed, while the lower surface will be correspondingly extended; therefore the length of the bar as measured by the distance between two dots at the extremities of the upper surface will be less than if the neutral axis were straight; in fact, if $i$ be the inclination of the bar at either extremity to the horizon, and $k$ the depth of the bar, its curvature will cause the dots to approach each other by the quantity $ik$. This effect of curvature will be greatly exaggerated if the dots be engraved, not on the surface of the bar, but on the tops of bits of strong wire inserted (in a vertical position) into the bar at its extremities; if $h$ be the length of either wire, the approach of the dots to one another by the curvature of the bar will be $i(k+2h)$. Accordingly four such perpendiculars were erected on the upper surface of the bar, one at either extremity, and one 10 inches from either extremity. A box fitted with rollers was prepared to receive the bar; each roller, mounted in a frame, could be fixed in any required position, and could at the same time be moved in a vertical direction up or down by means of a slow-motion screw outside the box. Now suppose four rollers so fixed in the box that one is under each extremity and two more at 2 inches right and left of the centre of the bar. Suppose also four microscopes adjusted over the four dots, their axes vertical, and their outer foci ranging in a straight horizontal line; by the working of the slow-motion screw (which it is to be particularly observed does not require the opening of the box or the touching of the bar with the hand) the centre rollers can be withdrawn, that is lowered until they cease to have contact with the bar, and then the bar resting on the extreme rollers can be adjusted to focus under the microscopes. The microscopes are then read. The extreme rollers are then lowered, and the bar comes in contact with the rollers near its centre, which are then raised until the dots are in focus. The microscopes are now read a second time; and by these readings, compared with the former, are obtained the changes of length of the whole bar, and of its subdivisions resulting from the alteration in the positions of the supports.

The supporting-rollers were placed at different times in the following positions:—(1) at 20 inches right and left of the centre; when in this position the supports are

designated EE′; (2) at 2 inches right and left of the centre designated CC′; (3) at $\frac{40}{\sqrt{3}}$ inches right and left of the centre designated NN′; (4) at $\frac{40}{6}$ inches right and left of the centre designated SS′.

The following Table contains the observed and computed changes of length of one of the bars, of its whole length, and of its subdivisions; the dots are marked in order from left to right, $m$, $n$, $n'$, $m'$; $\Delta(mn')$ is the alteration of the distance $mn'$ due to the change of supports; the unit is the millionth of a yard.

| Changes of supports. | | $\Delta[mn']$. | | $\Delta[mn']$. | | $\Delta[nn']$. | |
|---|---|---|---|---|---|---|---|
| From | To | Observed. | Computed. | Observed. | Computed. | Observed. | Computed. |
| NN′ | EE′ | 57·8 | 58·6 | 70·7 | 71·8 | 60·7 | 58·6 |
| „ | EN′ | 25·0 | 26·3 | 25·9 | 26·4 | 17·4 | 16·8 |
| „ | NE′ | 16·4 | 16·8 | 26·8 | 26·4 | 28·5 | 26·3 |
| „ | CC′ | −36·2 | −35·3 | −35·8 | −35·7 | −35·7 | −35·3 |
| „ | NC′ | −31·8 | −30·0 | −31·8 | −30·4 | −32·2 | −30·1 |
| „ | CN′ | −31·7 | −30·1 | −31·8 | −30·4 | −32·4 | −30·4 |
| CC′ | EE′ | 93·5 | 93·9 | 106·5 | 107·5 | 93·4 | 93·9 |

The modulus of elasticity by which the computed results are obtained is derived from the observations themselves.

*Absolute Expansion.*—The coefficients of expansion of the Indian standards $I_8$, $I_B$ and of the two other 10-foot bars of iron, $OI_1$, $OI_2$, have been obtained by means of an apparatus constructed for the purpose. Theoretically it is a simple matter to determine the coefficients of expansion of two bars A, B; it may be done as follows:—Compare A hot, say at a steady temperature of 100°, with B at the temperature of, say 40°; next compare A at 40° with B at 100°; and lastly compare the bars when both at the same temperature. But the practical difficulty is to maintain a steady temperature for the hot bar, so that it shall not be cooling while under observation. This has been effected in the following manner:—Imagine two closed tanks of copper measuring 124 inches by 5 by 3 inches, and suppose them fixed to the upper surface of a stout mahogany plank of the same length and 8 inches broad; between the tanks there remains a vacant space 2 inches wide and 5 inches deep; into this space the bar goes with its supporting-rollers, which are capable of slight vertical movement for level or focus adjustment. A current of hot water at a steady temperature enters the bar-room from without by a flexible tube; this current is made to subdivide into four equal streams, entering each tank by two orifices in its upper surface, at one-fourth and three-fourths of the length. The water escapes from each tank by an orifice at the bottom of either extremity, and is conducted out of the bar-room through flexible tubes. The supply being purposely greater than can be carried away from the extremities, an overflow pipe is provided at the centre of the tanks, and this overflow is also carried away from the room by flexible tubes. Thus a constant circulation of water is maintained, and no part of the water in the tanks can be still or cooling, nor can the tanks empty or overflow. The whole is well wrapped up in

blankets. The other bar is similarly mounted between tanks which are full of cold water; but no current is required, as the observations are made in the cold weather; this is also carefully covered with blankets. With respect to the interchange of the bars under the microscopes, this is effected with all desirable rapidity by a piece of mechanism whereby each bar (with its appendage of plank, tanks, and water) is simply rolled away from or up to the microscopes; thus the observers do not have to encounter the weight of these masses. Thus arranged the comparisons of a hot and cold bar are effected with almost as much facility as the ordinary comparisons.

The coefficients of expansion obtained from 6500 micrometer and thermometer readings for the four bars are—

| | | |
|---|---|---|
| Baily's metal | $I_B$ | $0{\cdot}0000098277 \pm 0{\cdot}0000000057$. |
| Steel | $I_S$ | $0{\cdot}0000063478 \pm 0{\cdot}0000000056$. |
| Wrought-iron | $OI_1$ | $0{\cdot}0000064729 \pm 0{\cdot}0000000031$. |
| Wrought-iron | $OI_2$ | $0{\cdot}0000064773 \pm 0{\cdot}0000000033$. |

The Indian bars were heated up to very nearly, but not quite 100°.

*Probable Errors of Observation.*—The quantities measured by the micrometers in the observations just specified are large, and require a very accurate knowledge of the values of the screws. The values for the two microscopes H and K are, expressed in millionths of a yard,

| | |
|---|---|
| H : one micrometer division | $0{\cdot}79566 \pm {\cdot}00008$ |
| K : " " | $0{\cdot}79867 \pm {\cdot}00009$ |

These were obtained from repeated measurements of a space of $\frac{4}{100}$ of an inch on F, the scale being readjusted to focus each measurement. There is no appearance of personal error in the observations (of three observers) from whence these values are deduced. It appears that the probable error of a single measurement of a space of $n$ thousand divisions, $m$ bisections on each line being supposed, is

$$\text{for H} \ldots \pm\sqrt{\frac{0{\cdot}20}{m} + 0{\cdot}072n^2} \text{ micrometer divisions,}$$

$$\text{for K} \ldots \pm\sqrt{\frac{0{\cdot}20}{m} + 0{\cdot}116n^2} \quad \text{" "}$$

The greatest space measured by either of the microscopes in the expansion experiments was 1100 divisions; and $m$ being $=2$, the probable error of the measure would be 0·43 or 0·49 of a micrometer division, according to the microscope used. The probable error of a single bisection by either of the observers is about $\pm 0^{d}{\cdot}316$.

*The Standard Foot, and its subdivisions.*—The length of this foot F in terms of $Y_{55}$ was determined as follows:—Four microscopes, H, I, J, K, were mounted on the stone piers at the distance of 12 inches apart, their axes being vertical and their outer foci in a horizontal straight line. The two bars lying side by side in their box, F was brought successively under the microscopes H I, I J, J K; then $Y_{55}$ under the microscopes H K

The resulting value of F from 900 micrometer and 180 thermometer readings, extending over twelve days, is

$$F = \tfrac{1}{3}Y_{55} - 0{\cdot}36 + 0{\cdot}0066(t-62), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the probable error, when $t=62°$, being $\pm 0{\cdot}108$. The unit to which these small quantities are referred is the millionth of a yard.

The inch lines upon the foot are marked $a, b, c, d, e, f, g, h, k, l, m, n, p$; the inch $[a\,.\,b]$ is divided into tenths by lines marked 1, 2, 3, 4, 5, 6, 7, 8, 9; the spaces $[2\,.\,3]$, $[6\,.\,7]$ are subdivided into hundredths; one of the subdividing lines in $[2\,.\,3]$, called the toise-line, is indicated by the letter $\tau$; the metre-line in $[6\,.\,7]$ is known by the letter $\mu$. The values of the different spaces, as derived from 8000 micrometer readings, are as follows:—

$$\left.\begin{array}{ll}
[a\,.\,2] = \tfrac{2}{10}\tfrac{F}{12} + 3{\cdot}71 \pm {\cdot}063, & [a\,.\,8] = \tfrac{8}{10}\tfrac{F}{12} + 1{\cdot}65 \pm {\cdot}070, \\
[a\,.\,\tau] = \tfrac{36}{100}\tfrac{F}{12} + 5{\cdot}30 \pm {\cdot}109, & [a\,.\,b] = \tfrac{F}{12} + 0{\cdot}14 \pm {\cdot}037, \\
[a\,.\,3] = \tfrac{3}{10}\tfrac{F}{12} - 0{\cdot}20 \pm {\cdot}064, & [a\,.\,c] = 2\tfrac{F}{12} + 1{\cdot}74 \pm {\cdot}047, \\
[a\,.\,4] = \tfrac{4}{10}\tfrac{F}{12} - 0{\cdot}77 \pm {\cdot}066, & [a\,.\,d] = 3\tfrac{F}{12} + 1{\cdot}31 \pm {\cdot}038, \\
[a\,.\,6] = \tfrac{6}{10}\tfrac{F}{12} + 3{\cdot}00 \pm {\cdot}068, & [a\,.\,e] = 4\tfrac{F}{12} + 0{\cdot}84 \pm {\cdot}051, \\
[a\,.\,\mu] = \tfrac{63}{100}\tfrac{F}{12} + 2{\cdot}08 \pm {\cdot}086, & [a\,.\,f] = 5\tfrac{F}{12} + 2{\cdot}20 \pm {\cdot}047, \\
[a\,.\,7] = \tfrac{7}{10}\tfrac{F}{12} + 1{\cdot}58 \pm {\cdot}068. & [a\,.\,g] = 6\tfrac{F}{12} - 0{\cdot}87 \pm {\cdot}037.
\end{array}\right\} \quad . \quad . \quad . \quad (2)$$

*Ten-feet Standards.*—The length of the 10-foot bar $OI_1$ was obtained by comparing each of the yard spaces on its surface, $[a\,.\,b]$, $[b\,.\,e]$, $[c\,.\,f]$, $[f\,.\,g]$, with $Y_{55}$, and the two 12-inch spaces $[b\,.\,c]$, $[e\,.\,f]$ with the foot F. The results are, at 62°,

$$\begin{aligned}
[a\,.\,b] &= Y_{55} + 54{\cdot}75 \pm 0{\cdot}130, \\
[b\,.\,f] &= \tfrac{4}{3}Y_{55} - 23{\cdot}44 \pm 0{\cdot}219, \\
[f\,.\,g] &= Y_{55} - 10{\cdot}23 \pm 0{\cdot}156,
\end{aligned}$$

whence the whole length of the bar, by adding these equations is

$$OI_1 = \tfrac{10}{3}Y_{55} + 21{\cdot}08 \pm 0{\cdot}299. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The length of the Indian steel standard, obtained in nearly the same manner, is

$$I_8 = \tfrac{10}{3}Y_{55} + 70{\cdot}62 \pm 0{\cdot}250. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Comparisons have also been made between the following 10-foot bars; $O_1$, $OI_1$, $I_8$, $I_3$, and $I_4$. (This last is the old Indian Standard B, of which the comparisons between it and $O_1$ in 1831 and 1846 are detailed in the "*Account of the Measurement of the Lough*

*Foyle Base*": it is a bar similar to $O_1$, but rather lighter. Since 1846 it was for some time at St. Petersburg in custody of M. STRUVE, who compared it with his own standard.) The results of the comparisons are these:

$$\left.\begin{aligned} OI_1 - O_1 &= 18{\cdot}38 \pm 0{\cdot}26, \\ I_3 - O_1 &= 63{\cdot}28 \pm 0{\cdot}26, \\ I_B - O_1 &= 195{\cdot}36 \pm 0{\cdot}26, \\ I_3 - I_b &= 86{\cdot}50 \pm 0{\cdot}41, \\ I_B - I_b &= 218{\cdot}58 \pm 0{\cdot}22. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

If from the seven last equations we seek by the method of least squares the most probable lengths of the five 10-foot standards in terms of the yard, we get, at 62°,

$$\left.\begin{aligned} OI_1 &= \tfrac{10}{3}Y_{55} + 22{\cdot}32, \\ I_3 &= \tfrac{10}{3}Y_{55} + 69{\cdot}38, \\ O_1 &= \tfrac{10}{3}Y_{55} + 5{\cdot}17, \\ I_B &= \tfrac{10}{3}Y_{55} + 200{\cdot}84, \\ I_b &= \tfrac{10}{3}Y_{55} - 17{\cdot}43. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (6)$$

These values being substituted in the seven equations, the residual errors are the following:—

$$\begin{aligned} &+1{\cdot}24, \\ &-1{\cdot}24, \\ &-1{\cdot}23, \\ &+0{\cdot}93, \\ &+0{\cdot}31, \\ &+0{\cdot}31, \\ &-0{\cdot}31. \end{aligned}$$

Now these errors are considerably larger than the directly computed probable errors of the different sets of comparison; it is clear therefore that constant error has been influencing some or all the different series. After all, the residual errors are as small as could be well expected. As to $I_b$, it appears that at present its relation to $O_1$ is this:

$$I_b - O_1 = -22{\cdot}60.$$

The difference of the same two bars, as determined in 1831, was $-22{\cdot}25$; and in 1847 it was $-24{\cdot}03$; an agreement most satisfactory.

It appears, then, from the above, that the Ordnance Survey Standard Bar is but very slightly in error, being only $\frac{1{\cdot}55}{1{,}000{,}000}$ths of its length too great; this corresponds to barely 6 feet in the length of this kingdom from Scilly to Shetland; or to 32 feet in the earth's radius.

*Ordnance Toise and Metre.*—In the toise, each of the yard spaces $[a\,.\,b]$, $[b\,.\,c]$ were

compared with the yard $Y_{55}$, and the space $[c.d]$ of 4·74 inches was compared with the very nearly equal space $[rf]$ on the foot F: the result of the comparisons is

$$[a.b] = \quad Y_{55} + 5{\cdot}95,$$
$$[b.c] = \quad Y_{55} + 2{\cdot}34,$$
$$[c.d] = \tfrac{474}{1000} Y_{55} + 0{\cdot}16,$$

or

$$T_0 = \tfrac{7674}{1000} Y_{55} + 8{\cdot}45 \pm 0{\cdot}21,$$

both bars being at 62° F. If both be at 61°·25 F., or 16·25 Centigrade,

$$T_0 = \tfrac{7674}{1000} Y_{55} + 9{\cdot}17 \pm 0{\cdot}21. \quad \ldots\ldots\ldots \quad (7)$$

The yard space on the metre was compared with $Y_{55}$, and the small space with $[\mu.e]$ on F; with these results,

$$[a.b] = \quad Y_{55} - \ 1{\cdot}20,$$
$$[b.c] = \tfrac{338}{1000} Y_{55} - 134{\cdot}25,$$

or

$$M_0 = \tfrac{33338}{1000} Y_{55} - 135{\cdot}45 \pm 0{\cdot}16,$$

both bars being at 62°. If both be at 61°·25 F., or 16·25 C.,

$$M_0 = \tfrac{33338}{1000} Y_{55} - 135{\cdot}11 \pm 0{\cdot}16. \quad \ldots\ldots\ldots \quad (8)$$

The length of $T_0$ is the result of 3120 micrometer and 640 thermometer readings; for $M_0$ were made 1680 micrometer and 320 thermometer readings. The observations for the former bar occupied thirty-three days, for the latter sixteen days.

### *The Toise.*

The unit of length in which by far the greater part of the European geodetical measurements are expressed, is the toise; the actual physical representative of this length being the *Toise of Peru**, at the temperature of 16°·25 C., or 61°·25 F. This bar, which was constructed in 1735 for Messrs. Bouguer and Lacondamine as their standard of reference in the operations conducted by them in Peru, is an end measure of polished iron, 1·51 inch in width and 0·40 inch in thickness. In order to trace our direct connexion with this celebrated and important standard, we must refer to two direct copies of it made by Fortin, of Paris, one in 1821 for M. Struve, and the other in 1823 for M. Bessel. The authority of the former rests on a certificate of the *Bureau des Longitudes*, signed by M. Arago†, in which he states that he has compared the copy with the

* We have no precise information as to the present state of this bar, but from report it would appear that at least as far back as 1858 it was so far damaged that comparisons with it were worthless. For a description of the Toise of Peru see 'Base du Système Métrique Décimal,' tome iii. pp. 405, 680, and the work entitled 'Mesure des Trois premiers Degrés du Méridien dans l'Hémisphère Austral,' par M. de la Condamine; à Paris, 1751, pp. 75, 85.

† See 'Arc du Méridien de 25° 20′ entre le Danube et la Mer Glaciale mesuré depuis 1816 jusqu'en 1855 . . . . . .' par F. G. W. Struve, St. Pétersbourg, 1860, tome i. Introduction, p. lxxiv.

toise of Peru and has found them "perfectly equal" in length. The authority of Bessel's toise is stated in a certificate signed by MM. Arago and Zahrtmann*, who assert that they have compared the toise constructed for M. Bessel with the toise of Peru, and have found it too short by $\frac{1}{1575}$ of a "ligne" (the toise = 864 lignes). If, therefore, we put $F_a$ and $F_b$ for the length of these two toises at 16°·25 C., and $\mathfrak{T}$ for the length of The Toise

$$F_a = \mathfrak{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$F_b = \mathfrak{T} - 0'{\cdot}00080 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In Russia the standard to which all their geodetical measures are referred is a bar two toises in length, designated N†. Its length was determined from $F_a$: the comparisons are given in M. Struve's 'Account of the Russian Meridional Arc of 25° 20′,' the deduced length of N being, at 16°·25 C.,

$$N = 2F_a + 0'{\cdot}01249 \pm 0{\cdot}00070. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The Russian double toise P, which has been compared at Southampton with two lengths of $T_9$, is a copy of N; the difference of the two bars at 16°·25 C. being

$$P = N - 0{\cdot}01809 \pm 0{\cdot}00019. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This bar P has been very extensively used in Russian geodesy. In 1850 it was used as the Standard in the Measurement of the Base of Alten, in Finmark, and the next year in the measurement of the Base of Öfver-Tornea, in Lapland. In 1852 it was used at the measurement of the Base at Taschbunar at the southern extremity of the Russian Arc in Bessarabia, and in various other bases for geodetical operations undertaken by the Topographical Depôt. Subsequently it was used in Eastern Russia.

M. Struve also compared the Prussian toise $T_{10}$ described above, with his normal bar N, the result being, at 16°·25 C.,

$$2T_{10} = N - 0{\cdot}01421 \pm 0{\cdot}00020. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The Prussian and Belgian toises $T_{10}$, $T_{11}$ were compared in 1852 by General Baeyer with Bessel's toise. The comparisons will be found in the work entitled "*Compte rendu des Opérations de la Commission instituée par M. le Ministre de la Guerre, pour étalonner les règles qui ont été employées..... à la mesure des bases géodésiques belges,*" Bruxelles, 1855. The results are‡, at 16°·25 C.,

$$T_{10} = F_b - 0'{\cdot}00019 \pm 0{\cdot}00011, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$T_{11} = F_b - 0'{\cdot}00020 \pm 0{\cdot}00012. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The difference of these two toises, according to these comparisons, is a very minute and inappreciable quantity. It appears, then, that although the very valuable standards $T_{10}$, $T_{11}$, P, which have been compared at Southampton with $T_9$, have not been themselves immediately compared with the toise of Peru, yet the connexion with that standard is

* See 'Untersuchungen über die Länge des einfachen Secundenpendels,' von F. W. Bessel, Berlin, 1828, p. 126.

† See 'Arc du Méridien de 25° 20′ . . . .' Introduction, pp. xl, lxxiii–lxxvi, also pp. 36–38. The results of M. Struve's comparisons among different bars are collected in the Table, pp. lxxiii, lxxiv.

‡ See pages 36–52 of the work named.

sufficiently direct, and judging by the expressed probable errors, should be very accurate. Unhappily, however, there is no information as to the precision of the comparisons made by M. Arago between either $F_1$ or $F_3$ and the toise of Peru. But it is to be remarked that, if we eliminate N and $F_4$ between the equations (9), (11), and (13), we get

$$T_{10}=\mathfrak{T}-0'{\cdot}00086,$$

as the result of M. Struve's comparisons; while from General Baeyer's, (10) with (14),

$$T_{10}=\mathfrak{T}-0'{\cdot}00099,$$

the difference between these two entirely independent values of the Prussian toise is only $0'{\cdot}00013$, or less than the six millionth part of a toise. This shows that Arago's assigned lengths of $F_1 F_3$ are at any rate admirably consistent.

We must now explain how the toises *à bouts* have been compared with the toise *à traits*. Suppose for a moment a cube of steel, one-eighth of an inch side, its faces polished, and a fine dot engraved on one of the faces at about one-hundredth of an inch from one of the edges and exactly opposite the middle point of that edge. Suppose the toise lying horizontally, and consequently its terminal planes in a vertical position, and let a cube as described above be applied against each end of the toise, the face carrying the dot being uppermost and horizontal; then the distance $T+\sigma$ between the dots when so held is about two-hundredths of an inch greater than the toise. Next let the cubes be placed in contact under the microscope, and the distance $\sigma$ between the dots measured; we shall then, by subtracting this quantity, know the exact length of the toise. But the mechanical difficulties to be overcome in this theoretically simple arrangement are found to be very great. After numerous experiments in different ways, the following modification was adopted: suppose a sphere of steel, three-quarters of an inch in diameter, to be cut by two parallel planes, one-eighth of an inch apart, on opposite sides of and equidistant from the centre. Taking the central segment, let it be laid on a horizontal plane, and cut in two along a diameter, leaving two semicircles; next let these two pieces, without removing either of them from the horizontal plane, be placed so that their curved surfaces shall come in contact, while their bases or semidiameters are parallel and at the maximum distance apart; then the common tangent plane at the point of contact will be a vertical plane, even if there should have been any error in the cutting of the sphere, so that one of the planes was nearer the centre than the other. Next suppose each of these semicircles to be placed on and fastened to a carefully planed rectangular plate of steel, say 4 inches long, the diameter of the semicircle being perpendicular to the length of the rectangle, and the curved surface projecting slightly beyond the end of the plate: suppose we have the means of levelling this plate, of raising or lowering it small quantities, of giving it a small motion in the direction of its length, and also in the direction perpendicular to its length, and lastly of giving it an azimuthal movement; then it is clear that we have absolute command as to position over the semicircular pieces. On the upper surface of each semicircle suppose a fine line drawn parallel to the base (or perpendicular to the length of the plate), and as near

as possible to the curved edge. Next, let the two semicircles be placed in contact, the plates being in the same horizontal plane and their lengths parallel in direction; the semicircles being kept in contact by pressure of a spring. In this position the lines drawn on the semicircles will be parallel and very close to one another. If we now, by the transverse movement only, slightly alter the position of one of the plates, the distance of these parallel lines will vary, and there is obviously a certain position in which their distance is a maximum; this occurs when the (vertical) tangent plane to the curved surfaces at their point of contact is parallel to the lines. This distance, when measured, is that by which the length of the toise is increased when the contact pieces are adjusted to its extremities.

In the actual apparatus, this distance $\sigma$, from very numerous observations, repeated on various occasions, is found $=565{\cdot}85\pm0{\cdot}108$ millionths of a yard. The toise (Prussian or Belgian) has been invariably supported on four points 21·5 inches apart. For the comparisons at Southampton a stout bar of iron, rather more than a toise in length, was prepared, carrying on its upper surface four rollers fitted with the necessary adjustments for strict alignment; at each of its extremities it carried a horizontal brass plate to which the contact apparatus was attached. The iron bar itself was held at one-fourth and three-fourths of its length; either support being capable of vertical movement for focus or levelling. The four rollers on which the toise lay were adjusted to a horizontal plane by means of a spirit-level. The contact pieces were held in contact with the ends of the toise by spring pressure. To prevent any constant error in the comparisons, the contacts were renewed after each comparison, and *all* the adjustments thrown out and re-made as often as possible.

The number of comparisons between the Prussian toise and $T_0$ is very large; they extend over twenty-five days, involving 2340 micrometer and 520 thermometer readings. The resulting difference of length of the two bars at 61°·25 F. is

$$T_{10}=T_0-154{\cdot}52\pm0{\cdot}15. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

In the case of the Belgian toise, the comparisons extend over eight days, and with the following result:—

$$T_{11}=T_0-156{\cdot}33\pm0{\cdot}27. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

From these comparisons it would appear that there is a sensible difference between the Prussian and Belgian toises, amounting to 1·81 millionth of a yard; whereas General BAEYER found the difference only $0{\cdot}02\pm0{\cdot}40$.

The comparisons of the Russian double toise extend over 14 days, involving 960 micrometer and 480 thermometer readings. Its length in terms of $T_0$ is found to be, at 61°·25 F.,

$$P=2T_0-321{\cdot}52\pm0{\cdot}31. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The ten equations (9) . . . (18) trace the connexion between the Ordnance toise and the toise of Peru through the intervention of six other bars. If we make the unit for

small quantities in the first seven equations the same as in the last three, and put

$$\left.\begin{aligned} F_1 &= \mathfrak{T}+x_1,\\ F_2 &= \mathfrak{T}+x_2,\\ N &= 2\mathfrak{T}+x_3,\\ P &= 2\mathfrak{T}+x_4,\\ T_{10} &= \mathfrak{T}+x_5,\\ T_{11} &= \mathfrak{T}+x_6,\\ T_9 &= \mathfrak{T}+x_7, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (19)$$

these equations, substituted in (9) . . . (18), give the following:—

$$\left.\begin{aligned} x_1 &= 0,\\ x_2 + 1{\cdot}97 &= 0, \end{aligned}\right\} \text{ Comparisons by M. Arago.}$$

$$\left.\begin{aligned} x_3 - 2x_1 - 30{\cdot}81 &= 0,\\ x_4 - x_3 + 44{\cdot}63 &= 0,\\ 2x_5 - x_3 + 35{\cdot}06 &= 0, \end{aligned}\right\} \text{ ,, M. Struve.}$$

$$\left.\begin{aligned} x_5 - x_2 + 0{\cdot}47 &= 0,\\ x_6 - x_2 + 0{\cdot}49 &= 0, \end{aligned}\right\} \text{ ,, General Baeyer.}$$

$$\left.\begin{aligned} x_5 - x_7 + 154{\cdot}52 &= 0,\\ x_6 - x_7 + 156{\cdot}33 &= 0,\\ x_4 - 2x_7 + 321{\cdot}52 &= 0. \end{aligned}\right\} \text{ ,, Southampton.}$$

From these ten equations the values of the seven quantities $x$ have to be determined by least squares. In doing so we shall not make reference to the probable errors attaching to them, as indeed they are not all known, but regard them as of equal weight. The values of $x_1 \ldots x_7$ being found, and substituted in the preceding equations, give

$$\left.\begin{aligned} F_1 &= \mathfrak{T} - 0{\cdot}07,\\ F_2 &= \mathfrak{T} - 1{\cdot}09,\\ N &= 2\mathfrak{T} + 30{\cdot}65,\\ P &= 2\mathfrak{T} - 14{\cdot}33,\\ T_{10} &= \mathfrak{T} - 2{\cdot}05,\\ T_{11} &= \mathfrak{T} - 2{\cdot}65,\\ T_9 &= \mathfrak{T} + 153{\cdot}42, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (20)$$

the weight of $x_7$ being $\frac{74}{80}$. The residual errors of the ten equations are,

$$\left.\begin{aligned} -{\cdot}007\\ +0{\cdot}07 \end{aligned}\right\} \quad \left.\begin{aligned} -0{\cdot}03\\ -0{\cdot}35\\ +0{\cdot}31 \end{aligned}\right\} \quad \left.\begin{aligned} +0{\cdot}32\\ -0{\cdot}26 \end{aligned}\right\} \quad \left.\begin{aligned} -0{\cdot}95\\ +0{\cdot}26\\ +0{\cdot}35 \end{aligned}\right\}$$

showing that the different series of comparisons are remarkably consistent, and the value of the toise most satisfactorily determined; in fact

$$T_0 = \mathfrak{T} + 153{\cdot}42 \pm 0{\cdot}30. \quad \ldots \quad (21)$$

Now let $T_0$ be eliminated between equations (7) and (21), and we get

$$\mathfrak{T} = \tfrac{7614}{5000}\, Y_{55} - 144{\cdot}25 \pm 0{\cdot}37. \quad \ldots \quad (22)$$

Here $Y_{55}$ is supposed at the temperature 61°·25. Now the expansion of the yard for 1° FAHRENHEIT is 6·5145; therefore if we wish $\mathfrak{T}$ in terms of $Y_{55}$ at 62° we must substitute in the above equation instead of $Y_{55}$, $Y_{55} - 4{\cdot}886$. Thus it becomes

$$\mathfrak{T} = (2{\cdot}13151201 \pm 0{\cdot}00000037) Y_{55}, \quad \ldots \quad (23)$$

which is the length of *the toise* in terms of $Y_{55}$ at 62° FAHRENHEIT.

### *The Metre.*

The metre being by definition 443·296 "lignes" of the toise of Peru, its true length as inferred from the above value of the toise, is

$$\mathfrak{M} = (1{\cdot}09362355 \pm {\cdot}00000019) Y_{55}. \quad \ldots \quad (24)$$

The result of the comparisons made in August and December 1864, extending over eight days, is that

$$M = M_0 + 9{\cdot}98, \quad \ldots \quad (25)$$

where M is the length of the platinum metre, both bars being supposed at 61°·25 F. This equation, combined with (8), gives

$$M = \tfrac{3938}{5000}\, Y_{55} - 125{\cdot}13,$$

where $Y_{55}$ is at 61°·25; but if $Y_{55}$ be at 62°,

$$M = \tfrac{3938}{5000}\, Y_{55} - 130{\cdot}47. \quad \ldots \quad (26)$$

It would appear from what is stated in the '*Base du Système Métrique Décimal*'*, that the platinum bars which were to represent the metre at 32° F. were laid off from the toise of Peru at 16°·25 C. or 61°·25 F., allowance being made for the contraction of the bars according to the rate of expansion ascertained by BORDA. At page 326, tom. iii. BORDA states his results thus, that the expansion of platinum for one degree REAUMUR is $\frac{1}{9180}$. According to this, the correction to the length of the platinum metres at 61°·25 F. would be $\frac{14}{9180}$, or 153·20 millionths of a yard. Hence the supposed length of the Royal Society's platinum metre at 32° F. would be

$$\tfrac{3938}{5000}\, Y_{55} - 283{\cdot}67.$$

Finally, according to M. ARAGO†, this particular metre is too short by 17·59 thousands

* See tom. iii. p. 681.

† The only authority on this point is the statement by Captain KATER in the Philosophical Transactions for 1818, pp. 103, 104.

of a millimetre, or 19·24 millionths of a yard. Adding this, we obtain for the length of the true metre

$$\mathfrak{M}=1{\cdot}09362446\,Y_{55},$$

the yard being at 62° F. The difference between this result and that derived through the Prussian, Belgian, and Russian toises is less than a millionth of a yard. We shall adopt the first-determined value, as the probable error of the second is not assignable.

### *The Yard.*

From numerous comparisons among different copies of the Standard Yard, it has been found that at 62°, $Y_{55}$ is too short by 0·40; or if $\mathfrak{Y}$ be the true *yard*,

$$Y_{55}=\mathfrak{Y}-0{\cdot}40.$$

If we substitute this value in our equations we have the following

### *Final Results.*

| Measures. | Expressed in Terms of the Standard Yard. $\frac{}{\mathfrak{Y}}$. | Expressed in inches. Inch $=\frac{}{36}\mathfrak{Y}$. | Expressed in lines of the Toise. Line $=\frac{}{864}\mathfrak{T}$. | Expressed in Millimetres. Millimetre $=\frac{}{1000}\mathfrak{M}$. |
|---|---|---|---|---|
| **The Yard** | 1·00000000 | 36·000000 | 405·34622 | 914·39180 |
| Copy No. 55 of the Yard at its Standard Temperature of 62·00 F. | 0·99999960 | 35·999986 | 405·34606 | 914·39143 |
| Ordnance Standard Foot " " " 62·00 | 0·33333284 | 11·999982 | 135·11521 | 304·79681 |
| Indian Standard Foot " " " 62·00 | 0·33333611 | 12·000100 | 135·11653 | 304·79980 |
| Ordnance 10-foot Bar $O_1$ " " " 62·00 | 3·33333717 | 120·000138 | 1351·15563 | 3047·97616 |
| Ordnance 10-foot Bar $O1_1$ " " " 62·00 | 3·33335432 | 120·000755 | 1351·16259 | 3047·99184 |
| Indian 10-foot Bar $I_B$ " " " 62·00 | 3·33340138 | 120·002450 | 1351·18166 | 3048·03488 |
| Indian 10-foot Bar $I_B$ " " " 62·00 | 3·33353284 | 120·007182 | 1351·23495 | 3048·15508 |
| Indian 10-foot Bar $I_A$ " " " 62·00 | 3·33331457 | 119·999324 | 1351·14647 | 3047·95550 |
| Australian Standard $OI_1$ " " " 62·00 | 3·33330427 | 119·998954 | 1351·14230 | 3047·94608 |
| Australian Standard $OI_2$ " " " 62·00 | 3·33333747 | 120·000149 | 1351·15576 | 3047·97644 |
| Ordnance Toise " " " 61·25 | 2·13166458 | 76·739925 | 864·06219 | 1949·17660 |
| Ordnance Metre " " " 61·25 | 1·09374800 | 39·374928 | 443·34662 | 1000·11420 |
| Royal Society's Metre à traits " " " 32·00 | 1·09360478 | 39·369772 | 443·28857 | 999·98324 |
| Prussian Toise No. 10 " " " 61·25 | 2·13150911 | 76·734328 | 863·99917 | 1949·03444 |
| Belgian Toise No. 11 " " " 61·25 | 2·13150851 | 76·734306 | 863·99893 | 1949·03390 |
| Russian Double Toise P " " " 61·25 | 4·26300798 | 153·468287 | 1727·99419 | 3898·05952 |
| **The Toise** | 2·13151116 | 76·734402 | 864·00000 | 1949·03632 |
| **The Metre** | 1·09362311 | 39·370432 | 443·29660 | 1000·00000 |

### Table of Logarithms for converting Geodetical Distances.

| Distances to be converted. | Logarithmic multipliers to convert into | | |
|---|---|---|---|
| | Feet. | Metres. | Toises. |
| Distances given in the "Account of the Principal Triangulation of Great Britain" | 0·00000050 | 9·48401156 | 9·19419163 |
| Distances expressed in Metres | 0·51598894 | ..... | 9·71018007 |
| Distances expressed in Toises | 0·80580887 | 0·28981993 | ..... |

## XI. The Bakerian Lecture.—*Researches on Gun-cotton.*—Second Memoir. *On the Stability of Gun-cotton.* *By* F. A. Abel, *F.R.S., V.P.C.S.*

Received March 19,—Read April 4, 1867.

The earlier of the published researches into the composition and properties of gun-cotton were speedily followed by accounts of the spontaneous decomposition which the substance was, in many instances, observed to undergo upon more or less protracted exposure in confined spaces to strong or diffused light. These indications of instability, in conjunction with the occurrence of several serious explosions during the manufacture of gun-cotton in France and England, afforded apparently good grounds for the general conclusion,—arrived at within a brief period after the announcement of Schönbein's discovery, and adhered to until quite recently in all countries except Austria,—that this remarkable explosive agent did not in itself possess the quality of uniform permanence essential to its safe manufacture, or to its employment with any degree of security from accident, in warlike or industrial operations.

It is unnecessary to refer in detail to the results of the numerous observations published before 1860 upon the nature of the spontaneous changes which particular specimens of gun-cotton had suffered. In the brief prefatory review of published investigations upon the production and composition of gun-cotton, contained in the paper on those subjects which I communicated to the Royal Society last year, it has been shown that the products obtained by individual operators in submitting cotton to the action of nitric acid varied greatly in composition, and that, with only one or two exceptions, these could not be viewed as representing the definite substance producible by the most complete action at a low temperature of a mixture of the strongest nitric and sulphuric acids upon purified cotton-wool (or nearly pure cellulose). The behaviour and results of the decomposition of such specimens, or of others of more recent date prepared (for lectures or similar experimental purposes) without special regard being paid to their composition or purity, afford but little information that can be accepted as bearing upon the question of stability of gun-cotton when produced by a system of operation which is now known to furnish uniform products in a condition of comparative purity.

There can be no question that the variations in composition of the different specimens of gun-cotton, the decomposition of which has received investigation at different hands, exerted a most important influence upon the period for which they withstood the destructive effects of heat and light, and upon the *degree of rapidity* with which chemical change, when once established, proceeded from stage to stage. The *products* of change described by different observers have also varied somewhat in their characters, partly

on account of the variations in the gun-cotton itself, and partly because different experimenters have examined the products of its metamorphosis at different stages.

The accounts published by De Luca, Bonet, and Blondeau, between 1861 and 1865, of their investigations into the changes which gun-cotton undergoes spontaneously, include nearly all the results previously described in one or other of the published papers on this subject.

The following is a general statement of the changes which gun-cotton, preserved in bottles partly or perfectly closed, has been observed to undergo by exposure to light, and of the nature of the products of decomposition.

In the first instance nitrous vapours make their appearance, the atmosphere in the vessel becoming sometimes of a deep orange tint. The gun-cotton acquires considerable acidity, exhibits a peculiar pungent odour, and gradually contracts, so that it eventually occupies only a small proportion of the original volume. During this period a considerable proportion of nitric acid accumulates in the mass, and the decomposition proceeds after a time with increased rapidity, especially if the vessel be exposed to sunlight. The contracted gun-cotton gradually becomes more or less friable, its explosiveness is notably reduced, it yields a highly acid extract to water, in which, besides nitric acid, small proportions of glucose, of formic and oxalic acids, and of cyanogen have been detected. The material sometimes contracts to such an extent as to form a very compact somewhat hard mass, but in general it ultimately passes over with more or less rapidity into a brownish gum-like mass, which at first is rendered very porous by the evolution of gas-bubbles, and which becomes lighter in colour and friable after a time. This ultimate product of the decomposition of gun-cotton has been found to contain glucose and oxalic acid in considerable proportions, besides a gum-like substance, formic acid, cyanogen, and an organic acid which by some observers is considered to possess novel characters, while Divers believes that he has identified pectic and parapectic acids in the product of a decomposed specimen*. The amorphous mass has also been found to evolve ammonia when heated with a solution of potassic hydrate.

In some instances the gun-cotton is described as having undergone other intermediate changes, but the greatest discrepancies exist between the observations of even the most practised experimenters regarding the periods within which the decomposition of gun-cotton has become manifest, and the conditions under which the changes have occurred.

* The observations of Divers regarding the occurrence of the pectic acids among the products of decomposition of gun-cotton, have been confirmed by the results of examination of a very large number of specimens obtained by the decomposition of gun-cotton at high temperatures under various conditions. The reactions of pectic and of para- and meta-pectic acids have been so frequently obtained that these substances must be regarded as general products of the gradual decomposition of gun-cotton. On the other hand, although the most careful search has frequently been made for glucose, only two instances of its existence were established by the fermentation test. It appears probable that the reduction of cupric salt from an alkaline solution has in many instances been accepted as a sufficient indication of the presence of glucose, while, in reality, this reaction has been furnished by the pectic acids produced. Small quantities of cyanogen have on several occasions been detected among the products of very gradual decomposition of gun-cotton by heat.

In some instances the first signs of decomposition were observed after exposure of the gun-cotton to daylight for several years, in others a few days' exposure sufficed to establish the change. Some observers state that the material has been preserved in the dark for very protracted periods without change, others (*e. g.* quite recently De Luca and Blondeau) show that, even in the dark, gun-cotton undergoes decomposition within a comparatively short period. Such conflicting observations afford convincing proof of great variations in the composition or degree of purity of the materials experimented upon.

The exposure of gun-cotton to heat has, by most observers, been found to accelerate its decomposition considerably; but here again great discrepancies are presented by different accounts of the behaviour of the material under the influence of different temperatures; thus, its spontaneous explosion has been brought about in some instances by brief exposure to a degree of heat which, in others, has only produced a comparatively very gradual decomposition.

The most interesting and important of recent observations upon the influence of heat on the stability of gun-cotton are those described by Pélouze and Maury in their recent report upon Baron Von Lenk's system of manufacturing gun-cotton, and upon the composition and properties of the products which it furnishes. They describe a number of results obtained with specimens of gun-cotton which, it is to be inferred, were all produced according to Von Lenk's directions, and which, therefore, provided these were strictly adhered to, and such an adherence ensured the uniformity of the products, should have furnished reliable data regarding the powers of purified gun-cotton to resist the destructive effects of heat. The principal results arrived at by Pélouze and Maury are as follows: they found that all specimens which were heated to 100° C. became decomposed in more or less time; a few minutes' exposure to that temperature sufficed in every instance to determine the evolution of nitrous vapours. They describe the results of decomposition as susceptible of variation at will; either the gun-cotton might be brought to explode, or the various forms of decomposition already described by other chemists might be established; or finally, it might be made to furnish simply a small black residue presenting the appearance of carbon, from which ammonia might be disengaged. Identically the same results were obtained by exposing specimens of gun-cotton to temperatures of 90° and 80° C., with this difference, that the phenomena of decomposition, instead of appearing in a few minutes, were not exhibited until after the lapse of several hours. It is further stated that pyroxylin is decomposed at 60° C. (140° F.), and even at 50° C. (122° F.); after the lapse of several days dense vapours filled the vessel containing the specimens, but no explosions of gun-cotton occurred in the experiments conducted at those temperatures. Great stress is laid, however, upon an instance of explosion which occurred with a specimen of gun-cotton prepared according to Von Lenk's process, immediately upon its coming into contact with the metal of an oil-bath, the temperature of which was only 47° C. (116°·6 F.) at the time. Pélouze and Maury afterwards refer to the instances of spontaneous decomposition of gun-cotton at ordinary temperatures observed by other chemists, and to certain specimens, among a number prepared at

Bouchet in 1847, which had undergone alterations such as have already been described. These were examined for sulphuric acid, and none was detected; hence the conclusion is drawn that these samples had been perfectly washed, and that their spontaneous change could not be ascribed to imperfect purification. It is argued that instances of change have been observed to occur under ordinary atmospheric conditions, similar to those established in gun-cotton at higher temperatures; that, because exposure to the latter had occasionally brought about spontaneous explosion, it is possible for instances of spontaneous decomposition at *ordinary* temperatures to result in explosions, and that, consequently, it is right to conclude that the storage of large quantities of gun-cotton is attended by great risk of explosion. In further support of this conclusion the observations are recorded, that the most perfectly washed gun-cotton becomes acid by long exposure to sunlight; that some pyroxylin, which was alkaline at first, after exposure for several weeks to light, in contact with the sides of a glass flask, exhibited an acid reaction; and that, even when gun-cotton is preserved in the dark, this acidity invariably becomes manifest in course of time. Finally, without referring to any single instance in which an explosion or even an appreciable development of heat has been observed as resulting from protracted exposure of gun-cotton to strong daylight or sunlight, PÉLOUZE and MAURY conclude that the indications of gradual decomposition furnished by certain specimens of gun-cotton under those conditions, are sufficient proof of the liability of this material, as now manufactured, to explode spontaneously, when stored in considerable quantities.

The researches into the manufacture, composition, and properties of gun-cotton, upon which, as a member of the Government Gun-cotton Committee, I have been engaged for nearly four years, have included, from their very commencement, careful observations and a great variety of experiments, with both small and large quantities of material, bearing upon the influence exerted by light and heat, and by various modifications introduced into the system of manufacture, upon the stability of gun-cotton produced in accordance with the general directions laid down by VON LENK. It is obvious that, although most of these experiments have furnished very decisive results within a comparatively brief period, there are others which become the more valuable and the more fully conclusive in their character, the longer the period of their duration. It is considered, however, that the data which even the latter class of experiments has already furnished possess sufficient scientific interest and practical importance to warrant their present publication, in addition to those obtained by numerous experiments instituted with the view to ascertain whether and to what extent the results of researches recently published in France upon the spontaneous changes of gun-cotton, apply to the material manufactured in this country during the last four years.

The experiments and observations carried on at Woolwich may be classed as having for their objects,—

(*a*) The determination of the influence of light and of long protracted storage, under ordinary conditions as to temperature, upon the stability of gun-cotton;

(*b*) The investigation of the behaviour of gun-cotton upon exposure, under varied conditions, to artificial temperatures, and to such elevated natural temperatures as are occasionally experienced in particular localities;

(*c*) The examination of the influence exerted upon the stability of gun-cotton by special modes of preparing and preserving it.

A few observations have been made upon specimens of gun-cotton which either were prepared by myself or came into my possession previous to the commencement of the present inquiry, but all actual experiments have been instituted with samples of products of manufacture obtained at Hirtenberg, Stowmarket, and Waltham Abbey, some modifications having been introduced, in special instances, in the ordinary system of operation at the last-named manufactory, with the view to ascertain the nature and extent of their influence upon the stability of the product.

## Part I.—Action of Light upon Gun-Cotton.

The want of uniformity in power to resist the destructive action of light, exhibited by different specimens of gun-cotton with which chemists have experimented, has been additionally exemplified by the behaviour of numerous specimens of gun-cotton which have from time to time come into my hands or were prepared by me, previous to 1862. I will limit my notice of such specimens to two examples.

In the autumn of 1846 a small quantity (one or two pounds) of gun-cotton was prepared by me at the Royal College of Chemistry according to the directions which had been made public in Germany a short time previously. The product, which was insoluble in mixtures of ether and alcohol, was obtained by immersing carded and purified cotton-wool of very high quality for a few minutes in the prescribed mixture of nitric and sulphuric acids, afterwards exposing it for several hours to a current of water, then digesting it in a cold dilute solution of potassic carbonate, and finally washing it in pure water. The larger proportion of the product was gradually expended in lecture-experiments, but a specimen has been preserved by me up to the present time. For sixteen years it was simply enveloped in paper and kept in a drawer much used; at the expiration of that period, when it was found to be perfectly unchanged, not exhibiting the slightest acidity or odour, it was transferred to a stoppered bottle, in which it has been since exposed to diffused daylight for four years. This specimen still remains perfectly unchanged.

Messrs. Hall of Faversham had the goodness, about three and a half years ago, to disinter at my request a sample of a large quantity of gun-cotton manufactured by them in 1847, and which they buried upon the occurrence of the disastrous explosion at their works in that year. This sample was much discoloured when received, but the fibre was strong, and the material did not appear to have undergone any change. Its explosive properties were, however, considerably inferior to those of gun-cotton prepared according to Schönbein's or Von Lenk's directions; and, upon analysis, it furnished

results corresponding very nearly with those required by the formula $C_{18}H_{23}O_{15}7NO_2$, the compound C, or collodion gun-cotton, of which the composition was determined by HADOW. It was, moreover, readily soluble in a mixture of ether and alcohol, and furnished a good collodion-film. It is most probable, therefore, that a deficiency in the strength of the acids employed in its production had led to the manufacture, in this instance, of soluble and less explosive gun-cotton by Messrs. HALL. A specimen of this material, after having been very carefully washed, was dried and enclosed by me in a stoppered bottle, in which it has remained exposed to diffused daylight for upwards of three years. A piece of litmus-paper, enclosed with the gun-cotton, exhibited faint signs of reddening within three months after the first exposure, and within twelve months it was bleached. At this time the gun-cotton possessed a faint but decided cyanic odour; no nitrous vapours were perceptible within the bottle, either then or at any more recent period up to the present time, though the odour of the gun-cotton has now become more pronounced, and is indicative of nitrous acid. The substance has at present a marked acid reaction; it has not as yet altered either in explosiveness, strength of fibre, or other properties, but the odour and slight development of acid are undoubted indications that the material which for sixteen years was preserved in a moist condition in the dark without any apparent change, has during three years' exposure to light furnished slight indications of spontaneous change. It was a specimen of gun-cotton prepared by Messrs. HALL in 1847, and preserved by PERCY since that year in a stoppered bottle, exposed to light, which had gradually become converted into a light-brown semifluid gum-like mass, described by HOFMANN as having exhibited all the properties of ordinary gum, and as being interspersed with crystals of oxalic acid. It is therefore not improbable that the specimen of Messrs. HALL'S manufacture above referred to may, by long-continued exposure to light, eventually furnish more important indications of spontaneous change than have hitherto been developed in it.

There can be little doubt that the quality of cotton operated upon by Messrs. HALL in the production of the specimens above referred to (and certainly in the instance of that examined by me), was considerably inferior to that of the material employed by me in 1846, and the character of the gun-cotton produced demonstrates that the conditions essential to the production of the most explosive material were not fulfilled by the method of manufacture pursued by those gentlemen in 1847. It is equally certain that the great importance of as complete a purification as possible of the cotton employed and of the product obtained was not fully recognized at that period, and that consequently, although a small laboratory operation carefully conducted according to the prescribed directions might furnish a pure product of great stability, the operations of manufacture had not been established with the precision essential to the attainment of satisfactory results.

The following are the results obtained up to the present time by exposure to light, under various circumstances, of gun-cotton prepared and purified according to Von LENK'S directions.

Exposure to strong daylight and to sunlight, either in the open air or in confined spaces for a few days (two to four), developes in the gun-cotton a very faint aromatic odour; and if litmus-paper be allowed to remain in close contact with the confined material, it acquires a rose-coloured tinge similar to that produced by carbonic acid, and recovers its original colour after brief exposure to air. If, after exposure to light in open air for some days, the gun-cotton be placed in the dark, in cases which are not air-tight, the odour becomes gradually fainter, and the effect upon litmus-paper slighter; if the packages containing the gun-cotton are air-tight, the odour and action upon litmus do not increase during storage for several years (the actual experience gained at Woolwich extends over nearly four years).

If the gun-cotton be exposed for protracted periods to daylight with free access of air, it speedily loses all odour and power of affecting litmus. If exposure to diffused daylight in confined spaces be continued, the first results of the action of light are, of course, retained; but up to the present time no single indication of their increase has been observed; indeed, the very faint acid reaction described, which was developed at first, has frequently disappeared, probably in consequence of the neutralizing action of small quantities of earthy carbonates contained in the gun-cotton.

But if the material be exposed continuously in a perfectly confined space to the action of sunlight or strong daylight, it furnishes, after a time, much greater evidence of change than that already described. The acidity gradually becomes more manifest; the odour increases, and becomes in time somewhat pungent and indicative of the presence of very small quantities of nitrous acid; and litmus-paper, if confined in the vessel with gun-cotton thus exposed, becomes entirely bleached after two or three months. Although specimens of gun-cotton always undergo some spontaneous change under these very special circumstances, the decomposition proceeds with extreme slowness; and the results of the observations instituted by me are, therefore, in this respect quite at variance with those recently published by De Luca, who states that the specimens operated upon by him decomposed upon exposure to sunlight, some on the first day of the experiment, others after several days' exposure.

The following account of special experiments instituted with gun-cotton manufactured at Waltham Abbey, will serve to illustrate the rate and nature of the decomposition suffered by this material when exposed to the action of sunlight in confined spaces.

Experiments 1 and 2.—16·37 grms. of air-dry gun-cotton were introduced into a large bulb blown at the extremity of a barometer tube of 10 millims. internal diameter, the length of which below the bulb was 812 millims. The bulb-tube was supported with the open extremity dipping into mercury.

19·12 grms. of the same gun-cotton were placed in a bulb-tube with a stem of the same length. The extremity of this tube was also dipped into mercury; but a small quantity of water was afterwards passed up into the end of the tube, so that the gun-cotton in the bulb might receive the maximum proportion of moisture which it was capable of absorbing. On the 20th of October, 1863, these two samples of gun-cotton were

placed in a very exposed position out of doors in front of a brick wall, where sunlight had access to them during the greater part of the day; moreover, during the summer months, the heat radiated from the wall immediately behind the bulbs was considerable. The appearance of the gun-cotton and of the atmosphere in the bulbs was carefully examined periodically, but the former retained its original appearance, and no coloration by nitrous acid was ever observed in the latter. On the 1st of December, 1864 (more than thirteen months after the commencement of the experiments), the tube enclosing the gun-cotton in a moist atmosphere was accidentally broken. The contents of this bulb were therefore removed and examined. The gun-cotton was found to have a powerful acid reaction and a somewhat pungent odour, its fibre had become tender, but its explosiveness had not sustained any important diminution. The aqueous solution yielded upon evaporation only a very small amorphous residue, which consisted partly of alkaline and earthy salts derived from the gun-cotton. By exhausting one gramme of the material with water, neutralizing the solution with sodic carbonate and concentrating by evaporation, abundant evidence of the presence of nitric acid was obtained. The aqueous extract acquired a yellowish colour upon addition of potassic hydrate, and a small quantity of cupric salt added to the alkaline liquid was reduced when heat was applied.

After treatment with water the gun-cotton furnished about 18 per cent. of matter soluble in a mixture of ether and alcohol. The solution, upon evaporation, did not furnish a tough film, but a horny brittle residue, which contracted and split up into small particles upon complete desiccation. This residue was explosive, and appeared to possess the characters of the lower nitrocellulose, $C_{18}\left\{\begin{matrix}H_{22}\\8NO_2\end{matrix}\right\}O_{15}$. The acidity of the gun-cotton was determined in 5 grms. of the specimen, and was found to correspond to 2 per cent. of $HNO_3$.

A portion of this altered gun-cotton was placed in a stoppered bottle immediately upon removal from the bulb, and set aside in a cupboard to which light had imperfect access. After the lapse of two years (upwards of three from the commencement of the experiment) the specimen was again examined. Upon opening the bottle a faint odour of nitrous acid was perceived. A number of small very hard crystals were found firmly attached to the glass; and similar crystals were interspersed through the material itself, which still retained the appearance of the original gun-cotton, having, however, contracted to some extent. On removal from the bottle it was found to be quite pulverulent; it speedily attracted moisture from the air, passing over into a very adhesive mass; it was soluble in water; the solution, which was turbid only from suspended mineral matter, reduced cupric salts abundantly, and gave the reactions of parapectic acid, but was not found to contain glucose.

A portion of the specimen of air-dry gun-cotton which had been enclosed with ordinarily dry air in a bulb-tube at the same time as the sample just referred to, was also examined after it had been exposed for thirteen months, the remainder being left in

the bulb, and its exposure continued for a further period of two years. The effects upon this specimen of more than one year's exposure in a confined space to strong daylight, frequent sunlight, and considerable heat radiated from the wall during the summer months, were as follows:—the gun-cotton had a somewhat pungent odour, in which, however, nitrous acid could not be recognized; its reaction was decidedly, though not powerfully, acid; the strength of its fibre and its explosiveness had not become affected. By exhaustion with boiling water it furnished a somewhat acid liquid, which contained a very small quantity of organic matter; a yellowish tinge was imparted to it by addition of potassic hydrate, but it did not reduce cupric salts.

After treatment with water, the gun-cotton was carefully exhausted with the usual mixture of ether and alcohol, and was found not to furnish a higher proportion of extract than the original material.

The remainder of this sample, after further exposure in the bulb-tube for two years (*i. e.* upwards of three years from the commencement of the experiment), had not altered in appearance, and was found to be in the following condition.

There was scarcely any perceptible odour in the tube on removing the extremity from the mercury; and the gun-cotton itself, when extracted from the bulb, had decidedly less odour than when examined two years previously.

It was still highly explosive, and the toughness of its fibre had very slightly diminished. Water extracted only 1·3 per cent. of soluble matter; the extract had a faint acid reaction; a minute quantity of nitric acid was detected in it, but no oxalic acid; and it exerted to a very slight extent a reducing action upon cupric salts. The gun-cotton yielded 25·5 per cent. of soluble product to the mixture of ether and alcohol.

Experiments 3 and 4.—It was considered very probable that the gradual metamorphosis sustained by gun-cotton upon exposure to sunlight would be attended by the disengagement of gaseous products, and that the rate of generation of these might furnish an indication of the comparative rapidity with which different specimens were affected. With this view two bulb-tubes, similar to those used in the experiments just described (the stems being 812 millims. long and 10 millims. in diameter), were respectively charged with 15 grms. of air-dry gun-cotton of the same manufacture as used in the preceding experiments. These tubes were then carefully exhausted over mercury, by means of a long narrow glass tube inserted into them, and extending from the mouth of the tube to within the bulb. The column of mercury in each tube was thus raised to within 16 millims. and 17 millims. of the height of the barometer. A small quantity of water was afterwards passed up into one of the tubes, so as to form a layer upon the mercury 3·5 millims. in height. These bulb-tubes were exposed in the middle of October 1863, side by side, in the position already described.

During exposure to light for the first four (autumn and winter) months, the depression of the mercury was not considerable in either instance, but it was nearly twice as great in the bulb-tube which enclosed water as in the other (88 millims. in the latter and 31 millims in the former). During the next six months (from spring to autumn) the

depression was more considerable; but at the expiration of twelve months' exposure to strong daylight and sunlight, it was only 115 millims. in the dry bulb-tube, and 220 millims. in the bulb-tube enclosing water. During the ensuing winter months the development of gas was again very trifling, though it continued to be greater in the tube which enclosed water. After two years' exposure to light the mercury was expelled from this tube, but the depression of the mercury in the dry tube amounted only to 340 millims., and after the third year's exposure the extent of depression in the latter was 660 millims. Neither of the specimens had undergone any alteration in appearance after exposure for three years and four months. They were then removed from the bulbs, and the following were the results of their examination.

*Gun-cotton from the dry tube.*

Slightly altered in explosiveness and strength of fibre. Slightly pungent odour, 0·77 per cent. extracted by water. Aqueous solution very faintly acid, contained a minute quantity of nitric acid, no oxalic acid, reduced cupric salts slightly: 27 per cent. of soluble gun-cotton.

*Gun-cotton from the tube which enclosed water.*

Not greatly changed in explosiveness or strength of fibre. Odour more pungent than in the other sample, 1·4 per cent. extracted by water. Aqueous solution faintly acid, contained a small quantity of nitric acid, no oxalic acid, reduced cupric salts: 39·7 per cent. of soluble gun-cotton.

These four experiments show that—

1. Gun-cotton in an ordinarily dry condition undergoes very slow change indeed when freely exposed in closed vessels (either containing air or with air excluded) to strong daylight and to the light and heat of the sun, the effects upon the material, during upwards of three years' exposure, being to diminish its explosiveness somewhat by the reduction of a portion of the trinitrocellulose to lower cellulose-products. The material, when purified by washing in alkaline water after this very severe exposure to light, is still gun-cotton possessing useful explosive properties, and exhibiting no greater tendency to change than the original material.

2. If the space in which the gun-cotton is enclosed be kept saturated with aqueous vapour, the substance undergoes decidedly more rapid and considerable change, though, even under these circumstances, gun-cotton prepared according to the system now in use is much less rapidly decomposed by severe exposure to light than has been the case with specimens of gun-cotton previously experimented upon.

Samples of gun-cotton which had been submitted to a less perfect purification than usual, afforded indications of being somewhat more rapidly affected by prolonged exposure to strong daylight and sunlight.

Experiment 5.—A quantity of gun-cotton, after removal from the acids, was exposed to the purifying effects of flowing water for one day only, and was afterwards submitted to treatment with the hot alkaline bath as usual. 19·85 grms. of this sample were introduced into a dry bulb-tube from which the air was exhausted over mercury, and

were then exposed to light, and occasionally to radiated heat, in the locality selected for these experiments. For the first few months the depression of the mercury-column was only very slight; in seven months it amounted to 15 millims.; after that period (during bright spring weather) it became more considerable, being 155 millims. after ten months' exposure. During the following year the depression of the mercury continued steadily; in one year and nine months it amounted to 497 millims.; and at the expiration of the three following months (about two years from the commencement) the mercury was expelled from the tube, a result corresponding to that obtained with the gun-cotton confined with aqueous vapour. After exposure for $2\frac{1}{2}$ years the condition of the gun-cotton was as follows: it had a somewhat pungent odour, its explosiveness and strength of fibre were slightly reduced; water extracted 0·6 per cent. of soluble matter; the solution had a slight acid reaction, contained a small quantity of nitric acid, no oxalic acid, and reduced cupric salt to a slight extent. The proportion of soluble matter amounted to 27 per cent.

Experiment 6.—A sample (19·8 grms.) of gun-cotton which had been submitted to long-continued purification in flowing water, but had not been digested in an alkaline bath, was exposed to light in a dry globe, like the other specimens, and by the side of it was placed another globe containing an equal quantity of the same gun-cotton, but covered with black calico, so as to have light excluded from it but to be subject to the effects of considerable heat during summer weather. After the first six (autumn and winter) months' exposure, the depression of the mercury-column in the perfectly exposed tube amounted to 51 millims. (being therefore considerably greater than in the case of the specimen washed with alkali). During this period the gun-cotton in the dark globe exhibited no signs of evolving gas. After nine months' exposure the depression of mercury in the uncovered tube amounted to 320 millims. (against 155 millims. in ten months produced by the sample which had been washed in an alkaline bath). The covered bulb-tube which had been frequently exposed to the heat of the sun during the spring months, exhibited at this time a depression of 38 millims. After the expiration of twelve months the depression of the mercury-column in the perfectly exposed tube amounted to 585 millims., and the mercury was completely expelled from this tube after the lapse of sixteen months.

Although, however, gas had been more rapidly disengaged from this tube than from the one referred to in the previous experiments, the change which the gun-cotton had sustained at the expiration of about $2\frac{1}{2}$ years was not as great as that observed in the specimen which had been treated with the alkaline bath but washed for a short time only. The specimen had only a very faint odour, its explosiveness and strength had not undergone any appreciable change; water extracted only 0·3 per cent. of soluble matter; a trace of nitric acid was detected in the extract, but no oxalic acid and no reaction upon cupric salts could be obtained. The proportion dissolved by ether and alcohol amounted to 16 per cent.

Neither of these specimens, in the preparation of which the complete system of puri-

fication had not been pursued, were found to be as injuriously affected by the very severe exposure as was anticipated. The specimen which had been exposed in the blackened bulb has up to the present time exhibited but slight indications of change (by development of gas), and only when the heat radiated upon it from the wall behind, and absorbed from the sun's rays, has been very considerable. It has not been disturbed for examination, as there is no reason whatever to believe that it would differ in any respect from other portions of this imperfectly purified sample which have been preserved in the dark in well-closed boxes, and which only exhibit a slight acidity after $2\frac{1}{2}$ years' preservation.

Experiment 7.—A sample (19·8 grms.) taken from a large quantity of gun-cotton which, for purposes to be hereafter described, had been impregnated with 0·5 per cent. of sodic carbonate, was exposed to strong light in the same way as the preceding specimens. During six months (between August and March) only a very slight indication of alteration was obtained; the depression of the mercury amounted to 15 millims. at the end of that period. Soon afterwards gas was more abundantly evolved, the depression amounting to 133 millims. after ten months' exposure. After eighteen months' exposure the amount of depression was 324 millims., which had increased to 432 millims. when the sample had been exposed for two years. After the lapse of $2\frac{1}{3}$ years the mercury had not been entirely expelled from the tube. The gun-cotton was not altered in appearance or toughness of fibre, nor did it exhibit any appreciable diminution of explosiveness. It had a slight odour and faint acid reaction; the aqueous extract amounted to 1·5 per cent. (a portion of which was due to soda); it contained no oxalic acid, a small quantity of nitric acid, and reduced cupric salt slightly; ether and alcohol extracted 10 per cent. of soluble matter. This sample had therefore suffered less change than any of the others. The disengagement of gas was manifestly due in part to the decomposition of the sodic carbonate, by small quantities of acid developed after a time by the exposure of the gun-cotton as described. This sample, after having been washed in water, exhibits no difference whatever in character from specimens of freshly prepared gun-cotton, in which the proportion of soluble cellulose-products is above the ordinary average.

The observations made in experiments 1–4, that the preservation of gun-cotton in an atmosphere saturated with moisture rendered it somewhat more prone to alteration by long-continued exposure to light, have been confirmed by other experiments still in progress, in which known quantities of moist and wet gun-cotton are exposed to light in confined spaces, in comparison with dry gun-cotton. Thus, in one of these experiments 43·71 grms. of perfectly dry gun-cotton and 40·045 grms. of gun-cotton in a damp condition have been enclosed in large stoppered bottles and exposed side by side to strong daylight and sunlight. After the lapse of two (summer) months they were carefully dried and their weights determined. The sample which had been exposed to light saturated with water had lost 0·33 per cent., the weight of the dry sample indicated a loss of only 0·02 per cent. They were then again exposed in the wet and dry condition for four months; the total loss in weight of the sample exposed in a wet condition was

then found to amount to 0·6 per cent., that of the dry sample only to 0·14 per cent. (after six months' exposure).

A trifling oxidation at the expense of oxygen in the water, established by the agency of sunlight, is doubtless the cause of the slight but decided influence which, under these circumstances, water has been observed to exert upon the permanence of gun-cotton; an influence which is quite opposed to that exerted by the presence of water in gun-cotton stored in the dark, or exposed to high temperatures, as will be presently demonstrated.

The statement made by De Luca*, that when once decomposition has been established in gun-cotton, resulting in the development of nitrous acid, the progress of the change cannot be arrested, is not borne out by the results of numerous observations made by me. Many specimens of gun-cotton which, by exposure to high temperatures (100° and 90° C.) or by very long-continued exposure to lower temperatures (50° to 65°), have suffered considerable change, resulting in the development of nitric peroxide and of other products, have been afterwards preserved in glass bottles, both tightly closed and partially open, and freely exposed to light for periods ranging from one to three years, without undergoing additional change. In a few exceptional instances, further decomposition has after a time been established by the influence of light; but in those the gun-cotton was impregnated to a considerable extent with free (nitric) acid. Such specimens, in case they were then thoroughly washed, a slightly alkaline solution being employed in their first purification, have afterwards not been found, up to the present time, to exhibit any greater tendency to decomposition by exposure to light, than the original gun-cotton.

## Part II.—Effects of Heat upon Gun-Cotton.

The behaviour of gun-cotton under exposure even to comparatively high temperatures is subject to very considerable modifications, which may be in great measure determined by the conditions of treatment. Illustrations of this were obtained at an early period of these investigations, in experiments instituted with the view to ascertain the average temperature at which gun-cotton explodes.

The following is a summary of the observations made on this head.

*Exploding-point of gun-cotton.*—The apparatus employed in the experiments on this subject consisted of a small air-bath fitted with a thermometer and closed with a mica-plate, having a little circular opening in the centre, through which the gun-cotton might be introduced, and which was kept closed when not in use. The mode of operating was modified in various ways. In the first instance the gun-cotton was combed out into a very loose condition, and allowed to rest upon metal in the air-bath. The temperature of the latter was then raised very gradually from 15° C. to 204°, or 205 C°. When the time occupied in the passage to the maximum temperature was two hours and upwards, the gun-cotton did not explode at all (in six experiments), but gradually

* Comptes Rendus, vol. lix. p. 487.

became dark brown, quite friable, and deprived of all explosive properties. When a considerably shorter time (about one hour) was occupied in the attainment of the maximum temperature, the gun-cotton exploded on one or two occasions, but not until its temperature had reached 205° C.

In the next experiments, the gun-cotton was employed in very small compact masses, and, resting upon a wooden support, was exposed to a continuously increasing temperature. The passage from 15° C. to the exploding-point ranged in these experiments from forty-five minutes to two hours.

*Temperature at commencement of Experiment* =26° C.

| No. of experiment. | Time occupied. | Exploding-point. |
|---|---|---|
| 8 | Forty-five minutes | 137°·5 C. |
| 9 | One hour twenty-five minutes | 136° „ |
| 10 | One hour | 137°·5 „ |
| 11 | One hour | 138°·5 „ |
| 12 | Two hours | 138° „ |

Another series of experiments was instituted for ascertaining in what particular mechanical condition the gun-cotton exploded most readily and at most uniform temperatures; and ultimately the material was employed in the form of pieces of loosely-twisted strand about 20 millims. long, and its exploding-point was determined by first raising the atmosphere of the air-bath to 105° C., then allowing the specimen to fall upon a diaphragm of wire gauze in the air-bath, at once increasing the temperature as rapidly as possible, and carefully reading the thermometer until the explosion occurred. The results of eight observations thus conducted were as follows:—

| No. of experiment. | Exploding-point. |
|---|---|
| 13 | 151°·5 C. |
| 14 | 151° „ |
| 15 | 151° „ |
| 16 | 150°·5 „ |
| 17 | 150°·5 „ |
| 18 | 148°·5 „ |
| 19 | 151° „ |
| 19*a* | 147° „ |

These last experiments, which appear the most trustworthy, indicate that the average temperature at which the gun-cotton explodes when in a condition most favourable to its rapid heating, is about 150° C. In two observations, in which the gun-cotton was in a very open condition, the temperature being raised more rapidly than usual, the explosions occurred when the thermometer indicated 145° and 148°·5 C.; and in the experiments preceding these, which were differently conducted, compact gun-cotton being exposed to heat for a considerable time, the point of ignition ranged between 136° and

138°·5 C. SCHRÖTTER, REDTENBACHER, and SCHNEIDER, in their report upon VON LENK'S gun-cotton, mention that 136° C. is the *lowest* temperature fixed by VON EBNER at which this material explodes.

### EFFECTS UPON GUN-COTTON OF EXPOSURE TO 100° C.

PÉLOUZE and MAURY, in their accounts of the effects of heat upon gun-cotton, describe several kinds or stages of decomposition as occurring, or producible at will, by its exposure to a temperature of 100° C., and state that in every instance they found a few minutes' exposure to that temperature sufficient to produce a disengagement of nitrous vapours.

A large number of experiments has been instituted with gun-cotton prepared at Waltham Abbey and Stowmarket according to VON LENK'S direction, and also with some specimens of Austrian gun-cotton, with the view of ascertaining the effect upon them of exposure to 100° C. The gun-cotton was exposed to heat in sealed tubes and in open vessels arranged in different ways. The quantities operated upon and other conditions in the experiments were varied, as will be presently particularized, the objects contemplated being, in the first instance, to examine into the effects of exposure of gun-cotton to heat, and, afterwards, to ascertain if possible by what circumstances those effects might be subject to modification.

The following is a condensed account of the observations made.

I. *Experiments in sealed tubes.*—Experiment 20. Air-dry gun-cotton (coarse yarn, manufactured in 1863), enclosed in a stout glass tube hermetically sealed, was maintained at 100° C. in a water-bath. The tube was filled with deep orange vapours in about three hours. The vapours gradually diminished in intensity, after a time, until the gun-cotton was converted into a gum-like mass, the transformation occurring most rapidly at the upper end of the tube, where the water produced during the change condensed and returned, charged with acid, upon the gun-cotton. When the sealed tube was opened, after continuation of the heat for three or four days (seven hours daily), nitric oxide escaped under considerable pressure. Upon closing the tube again, after the escape of gas, and continuing the application of heat, the gun-cotton was gradually converted into a black pitch-like mass.

This experiment, several times repeated, always furnished closely similar results.

Experiment 21.—A tube containing fine gun-cotton thread, manufactured in 1863, was exhausted and sealed. After four hours' exposure to 100° C., it exploded with great violence, tearing open the stout copper water-bath in which it was heated. Portions of unburned gun-cotton were scattered about.

Experiment 22.—Another tube, containing some of the same gun-cotton, was opened after seven hours' heating, to allow the gas to escape, and again sealed. On the second day, after heating for three or four hours, it exploded violently.

Experiment 23.—Several experiments were made with perfectly dry gun-cotton, and furnished results quite similar to those obtained with the air-dry material.

Experiment 24.—Fine gun-cotton thread was introduced into a tube sealed at one

end; the other extremity of the tube was constricted, then exhausted and filled with nitrogen, these operations being repeated three times; the tube was afterwards sealed and heated to 100° C. in a water-bath. After forty-five minutes faint red vapours were observed. In another quarter of an hour the colour of the vapours was very deep; in a short time nitrous acid began to condense in the cool part of the tube. After continuing the heat for 1¼ hour longer, the coloured vapours had entirely disappeared. The gun-cotton had become highly bleached, and in the upper extremity of the tube it was partially converted into the gummy substance. Nitric oxide escaped when the tube was opened.

Experiment 25.—A sample of gun-cotton impregnated with about 0·4 per cent. of alkaline carbonate, was exposed to 100° in an exhausted sealed tube, for the purpose of collecting the gases evolved. When the tube had been heated six hours daily for five days, it was opened under mercury, and the gas, which escaped under considerable pressure, collected. The tube was again closed and heated for two days, when gas was once more collected from it. The experiment was interrupted, after the gun-cotton had been further heated for two days, the tube being fractured by the effects of an explosion in its vicinity. The collected gases were found to consist of 50·2 per cent. of carbonic acid, 4·7 per cent. of nitric oxide, and 45 per cent. of nitrogen.

These experiments, in which the gun-cotton was submitted to the influence of 100° C. under the most severe conditions, appear to indicate that nitric peroxide or nitrous acid is liberated by the first decomposition of the gun-cotton, and at once establishes a further destructive action upon the substance, becoming reduced to nitric oxide, nitrogen being eventually liberated by complete reduction of the latter*. The extent of surface of gun-cotton presented to the action of heat, and of the liberated acid, appears to exert, as might be anticipated, an important influence upon the change. Exposure of fine gun-cotton thread to heat under the same conditions as those which were safe with coarse yarn gave rise to explosions, due possibly to the increased pressure of gas in the tubes, but more probably, judging from their great violence, to the sudden decomposition of the gun-cotton at a particular period. The characters exhibited by the products of decomposition of gun-cotton obtained in experiments 20 and 24, were similar to those already described by other chemists, and have been referred to in the preceding parts of this paper.

II. *Experiments in vessels open to the air.*—The following experiments, conducted with considerably larger quantities of gun-cotton than before employed, were made with the view of obtaining, at one time, several distinct data regarding the decomposition of gun-cotton at 100° C. Direct evidence was sought of the development of heat in gun-cotton upon continued exposure to that temperature. The period was carefully noted when decomposition was first indicated by the disengagement of nitrous acid, after commencement of the experiment. In some instances, the loss of weight sustained by the

* Similar results were observed in experiment 109.

gun-cotton was determined at intervals (*e. g.* at the close of six hours, or one day's exposure to heat), the nitrous acid contained in the vessel being first displaced.

The vessels employed in these experiments were globe-flasks fitted by means of perforated corks, with long narrow glass tubes, and in most instances with thermometers graduated from 100° C. upwards. The flasks were of a size to receive the gun-cotton in a compact condition, and the thermometer-bulbs were inserted into the centre of the mass. Continuous observations were made in safety during the experiments, through a small glass let into a wooden screen, which was placed in front of the water-bath containing the heated flask. The results obtained are tabulated for convenience of comparison.

TABLE I.

| No. of experiment. | Description of gun-cotton. | Quantity employed. | Total exposure to 100° C. | Interval between first exposure to 100° C., and first signs of decomposition. | Loss of weight sustained by the gun-cotton. | Temperature observations. | Other observations. |
|---|---|---|---|---|---|---|---|
| 26 | Coarse yarn made in 1864, Waltham. | grms. 15·55 | 20 hours (in 4 days). | 4 hours ..... | 1 per ct. in 6h, 12·91 per ct. in 12h 30m. 13·91 p. c. = total loss in 18 hours. | Not made in this experiment ..... | The gun-cotton *exploded* violently soon after commencement of the 4th day's heating. Nitrous acid was copiously evolved just before the explosion. |
| 27 | Coarse yarn, Waltham. | 22 | 5h 30m (30 minutes on the second day). | 2 hours ..... | 8·35 per cent. in 5 hours. | The thermometer remained almost stationary during the first 4½ hours. It then rose continuously and reached to 109° C. in 25 minutes, when the experiment was interrupted. On 2nd day, when the thermometer reached 100° C., it continued to rise; within 30 minutes it indicated 122° C., and the gun-cotton *exploded* very shortly afterwards. | The evolution of nitrous acid, when it once commenced, continued copious throughout the experiment. The *explosion* of the gun-cotton was very violent. |
| 28 | Coarse yarn, Waltham. | 6·5 | 24 hours (in 4 days). | 8 hours ..... | 23·7 per cent. in 24 hours. | The thermometer did not rise above 100° C. on the first day. On the 2nd day, after 1½ hour's heating, it rose slowly till it reached 109° C. It remained nearly stationary at that point for 1½ hour, and then gradually fell to 100° C. towards the close of the 2nd day. No rise of temperature was observed on the 3rd and 4th days. | At the conclusion of the experiment the sample had contracted somewhat, and assumed a brown colour. It was quite friable, and had lost the properties of gun-cotton. |
| 29 | Fine yarn, Waltham. | 6·5 | 21 hours (in 3 days). | 3 hours ..... | 30 per cent. in 21 hours. | A slight increase of temperature was indicated at the close of the 3rd hour, the highest temperature, 110° C., being indicated 3h 50m after commencing the experiment. Shortly afterwards the thermometer began to fall; after the lapse of 1 hour it indicated 103° C., and had fallen to 100° C. before the close of the 1st day. No rise in temperature was observed on the 2nd or 3rd days. | The gun-cotton began to darken on the 2nd day. At conclusion of experiment it had contracted considerably, was dark brown, yielded a considerable proportion to water, in which the usual products of change were detected. The residue was nearly soluble in alcohol, and completely so in ether and alcohol. |
| 30 | Coarse yarn from Hirtenberg. | 6·5 | 33 minutes ... | 20 minutes ..... | ..... | The thermometer began to rise 25 minutes after first exposure, and continued to do so very rapidly up to the time of explosion. | The gun-cotton *exploded* violently after 33 minutes' exposure to 100° C. |
| 31 | Coarse yarn from Austria (another specimen). | 6·5 | 1 hour ..... | 15 minutes .. | ..... | The thermometer began to rise 45 minutes after first exposure, and continued to rise rapidly, indicating 129° shortly before the explosion. | The gun-cotton *exploded* violently after exposure to 100° C. for one hour. |

These results indicated,—

(1) That sufficiently protracted exposure to 100° C. under conditions unfavourable to the rapid expulsion of the nitrous acid developed by the first action of the heat upon the gun-cotton, ensures the complete destruction of the original properties of this substance, and its conversion into a variety of volatile and fixed products.

(2) That the rapidity and violence of the decomposition resulting from the combined action of heat and of the acid generated, is regulated by the quantity of gun-cotton operated upon.

(3) That, as shown by experiments 28 and 29, conducted with coarse and fine yarn manufactured in precisely the same manner, the mechanical condition of the gun-cotton exerts an important influence over the rapidity of decomposition at 100° (a point also indicated by the results of experiments in sealed tubes).

(4) That a very important difference may exist between the behaviour of different samples of gun-cotton, even if operated upon in precisely the same manner, quantities, and mechanical conditions. This is illustrated by comparing experiments 30 and 31 (conducted with Austrian gun-cotton), with experiment 28, and with a considerable number (18) of precisely similar experiments instituted with different samples of Waltham Abbey gun-cotton, in not one of which was an explosion brought about by long-continued exposure of equal quantities (6·5 grms.) to 100° C. The two specimens of Austrian gun-cotton differed very greatly in composition from all the products of manufacture prepared at Waltham, according to Von Lenk's system; and it will be shown presently that this circumstance may serve to account for the exceptional proneness of these specimens to very violent decomposition under the particular conditions of the above experiments.

It need perhaps scarcely be stated that the temperature-observations in these experiments (and others still to be described) were instituted more with the view to afford a good means of registering the comparative rapidity of decomposition of different specimens of gun-cotton operated upon under equal conditions, than with the idea of attempting to ascertain the actual moment of development of heat and progressive rise of temperature in a mass of gun-cotton. Such observations could only be correctly made with much larger quantities of gun-cotton, so confined as to prevent the escape of heat from the interior, and are therefore impracticable on the score of danger. A considerable number of these thermometric observations, which unquestionably recorded close approximations of the actual rise in temperature of the interior of the mass of badly conducting gun-cotton, showed that, when the temperature passes 110° to 112° C., the development of heat proceeds with great rapidity, so that very speedily the rise of the thermometer does not keep pace with the heating of portions of the gun-cotton in close proximity to it, and therefore the explosion of the mass appears to occur at a temperature considerably lower than the actual exploding point of gun-cotton.

In continuation of the heat-experiments, several samples of gun-cotton from Waltham Abbey and Stowmarket, weighing 3 grms. each, in an air-dry condition, were exposed to

100° C. in conical assay flasks, into which long quill-tubes were fitted. The following results were observed.

TABLE II.

| Number of experiment. | Nature of gun-cotton. | Nitrous vapours first observed after commencement of experiment. | Other observations. |
|---|---|---|---|
| 32 | Coarse yarn from Waltham Abbey, made in 1863. | In 5 hours, very faint colour. | The gun-cotton was heated 5 hours daily for 6 days. The nitrous fumes were never more than very faint, and were no longer visible after the close of the third day's experiment. At the close of the 6th day the gun-cotton had scarcely altered in appearance. The specimen was destroyed on the 7th day by the explosion of a neighbouring vessel. |
| 33 | Fine yarn from Waltham, made in 1864. | In 2 hours, faint colour. | The atmosphere in the flasks was deeply coloured at the expiration of the 6th hour; the experiments were therefore interrupted. |
| 34 | Fine yarn from Waltham, made in 1866. | In 2 hours. | |
| 35 | Medium sized yarn, Waltham, 1864. | In 1½ hour, very faint colour. | The atmosphere in the flask became deeply coloured after 5 hours' exposure to 100° C. |
| 36 | Coarse yarn made at Waltham, 1864. | In 30 minutes. | The vessel was filled with deep-coloured vapours after 5 hours' exposure. The experiment was continued on the next day, when, after further exposure to 100° C. for 4 hours, the specimen *exploded*. |
| 37 | Stowmarket, coarse yarn, 1864 (early manufacture). | In 10 minutes. | Nitrous vapours were abundantly evolved within 10 minutes, and continued to increase until the experiment was arrested. |
| 38 | Stowmarket, coarse yarn (another specimen). | None observed during 9 days' exposure, 5 hours daily. | After 9 days' exposure no nitrous vapours were observed in the vessel. The specimen had an acid reaction and somewhat pungent odour, but was not otherwise altered. |
| 39 | A portion of the same sample as 38. | Very faint after 15 hours' treatment. | The nitrous vapours became more evident on the 4th day of the experiment. |

These eight experiments, conducted precisely alike, point to very important differences in the powers of different specimens of gun-cotton to resist destruction by exposure to 100° C. Of five samples manufactured at Waltham Abbey, only one exhibited the effects of such exposure described by PÉLOUZE and MAURY as invariable, namely, the disengagement of nitrous vapours within a few minutes. One specimen did not exhibit this sign of change until after five hours' exposure, and then only to a very slight extent. Of two specimens of gun-cotton from Stowmarket, one decomposed with very considerable rapidity at 100° C., and the other did not, in one experiment, evolve any visible amount of nitrous acid during forty-five hours' exposure, in nine days, and exhibited very slight signs of change at the expiration of this severe treatment; while in a second experiment, with a portion of the same sample, slight decomposition became apparent at the close of the third day's exposure of five hours.

The cause of the latter difference in the behaviour of one and the same sample, upon different occasions, was traced to the circumstance that the specimen, in the condition in which it was first employed, contained a somewhat larger proportion of moisture than when the experiment was repeated with it, in consequence of its having been in a damp locality for a short time before the first portion was operated upon. Thus one possible reason for the different behaviour of several samples of gun-cotton prepared by one and the same process was indicated. In confirmation of the influence exerted by moisture in retarding the decomposition of gun-cotton exposed to a high temperature, the results of a preliminary experiment may be here recorded, which was instituted with a sample of gun-cotton found to be very readily affected by exposure to heat.

Experiment 40.—Three specimens, each of one grm., were exposed side by side in small long-necked flasks to 100° C., in three different conditions. The one was air-dry (and contained therefore about 2 per cent. of water), the second was dried immediately before the experiment by sufficient exposure to 50° C., and the third was saturated with water and pressed between bibulous paper. The dry sample showed signs of decomposition in ten minutes, the air-dry sample began to decompose in forty-five minutes, and the moistened specimen exhibited no acidity after exposure to 100° C. five hours daily for three days (further experiments on the protective power of water will be presently described). In all subsequent experiments upon the comparative effects of exposure of different samples to elevated temperatures, the gun-cotton was employed in a dry condition.

The discrepancies noticed above in the behaviour of different samples of gun-cotton under exposure to 100° C., led to a searching investigation into the composition of products of manufacture obtained from Waltham Abbey, Stowmarket, and Hirtenberg, the results of which were laid before the Royal Society last year.

It was established by this inquiry that gun-cotton manufactured at those establishments contains variable proportions of the following substances foreign to the most explosive gun-cotton, *trinitrocellulose*, or $C_6 \begin{matrix} H_7 \\ 3NO_2 \end{matrix} O_5$.

(1) Hygroscopic moisture, the proportion of which amounts very uniformly to about 2 per cent., except in special instances, when mineral impurities in the material exert an influence upon its hygroscopic properties*.

(2) Mineral matters, varying in amount with the character of water used in purifying the material, with the duration of its treatment with water, and with the circumstance whether the purified gun-cotton has been submitted to the treatment with soluble glass, recommended by Von Lenk. These mineral matters include calcic and magnesian carbonates, silica, clay, and occasionally small quantities of sand and alkaline salts.

(3) Products of the less complete action of nitric acid upon cellulose, the nature of which has been investigated by Hadow. These products, which are less explosive than trinitrocellulose, and are more or less readily soluble in mixtures of ether and alcohol, were found to exist in very considerable proportion in some samples obtained from Hirtenberg and Stowmarket, and have also been found to the extent of about 2 per cent. in the most perfect products of manufacture. Their formation may be due to insufficiently protracted treatment of the cotton with the mixed acids, to the employment of acids of insufficient strength, and to the presence of hygroscopic moisture in the cotton at the time of its conversion. An elevation of temperature during the treatment of the cotton with the acids would also give rise to the production of soluble gun-cotton.

(4) Products of the partial oxidation, by nitric acid, of organic impurities existing in the cotton, even after the preliminary purification to which it is subject. These products,

* Several of the earlier products of manufacture obtained at Stowmarket were found to absorb from the atmosphere, under ordinary conditions, from 0·5 to 1·5 per cent. more moisture than the average proportion (2 per cent.) contained in Von Lenk's gun-cotton.

which are formed from portions of seed-husk adhering to the cotton, and from small quantities of gum-like and other substances still retained within the fibre, escape complete removal from the gun-cotton, although the larger proportion passes into solution during the treatment with acids and upon the subsequent digestion in an alkaline bath. It need scarcely be stated that the proportion of these substances, existing in the finished gun-cotton, varies with the description and quality of cotton employed, with the duration of the digestion in acids, and the degree of perfection of the purifying processes to which the material has been submitted. They are discovered by treatment of the gun-cotton with alcohol, and no specimen has yet been examined by me which has been found entirely free from them, while comparatively considerable proportions have been found to exist in a few specimens from Hirtenberg and Stowmarket.

A description of the nature of these impurities, as far as it could be determined, has been given in the previous memoir*. They possess acid characters, and their origin leaves no room to doubt that they are less simple and definite, and therefore less stable in their characters, than are the products of the action of nitric acid at low temperatures upon pure cellulose.

It has been argued by SCHRÖTTER, REDTENBACHER, and SCHNEIDER in their official report upon Von LENK'S gun-cotton, that an incomplete conversion of cellulose into the most explosive gun-cotton may be one cause of the want of stability observed in the early products of manufacture (at Bouchet, &c.); and consequently the existence in gun-cotton of a proportion of the third class of impurities above specified should, according to these chemists, give rise to, or promote a tendency to spontaneous change in the material. On the other hand, PÉLOUZE and MAURY consider it probable that a gun-cotton will be the more liable to spontaneous change the further it is removed in composition from the cellulose type, and that products prepared by prolonged immersion in large proportions of very concentrated acids will therefore be more liable to spontaneous ignition than the gun-cotton prepared by a brief immersion in less concentrated acids. No experimental data are given in support of either opinion.

The discordant results furnished by the heat-experiments just described, and the facts established by investigating the composition of the gun-cotton operated upon, led to the institution of a very considerable number of experiments with the view of ascertaining, if possible, whether the establishment of change in gun-cotton by its exposure to high temperatures has to be ascribed to the instability of trinitrocellulose itself, or whether it is to any extent ascribable to the injurious influence of less permanent bodies existing as impurities in the ordinary product of manufacture.

A careful comparative examination was instituted of the effects of exposure, under equal conditions, to 100° C. upon a number of samples in portions of which the matters soluble in ether and alcohol had been determined. One gramme of each sample was first dried in a water-bath at a temperature of 50° C.; it was then introduced loosely into a small flask having a neck about 220 millims. in length, and immersed in boiling water, the first indications of the disengagement of nitrous acid being afterwards care-

* Transactions Royal Society, vol. clvi. p. 285.

fully noted. The specimens were uniformly exposed to 100° C. for thirty hours (six hours daily for five consecutive days), unless, as was the case in a few instances, the gun-cotton had suffered complete change within a shorter period.

TABLE III.

| No. of experiment. | Description of gun-cotton. | Percentage of matter extracted by ether and alcohol. | Period when nitrous vapours were first observed after commencement of experiment. | Other observations. |
|---|---|---|---|---|
| 41 | Coarse yarn, manufactured at different periods at Waltham Abbey, not affected. | 1·80 | 4 hours. | The atmosphere in the flask was only faintly coloured during the whole term of the experiment. At the conclusion the gun-cotton had darkened slightly in a few places, but had not altered in explosiveness or strength of fibre. |
| 42 | | 1·83 | 1 hour. | The atmosphere in the flask became only faintly coloured on the second day; the gun-cotton darkened and contracted, and its explosiveness was much reduced. |
| 43 | | 1·91 | 45 minutes. | The nitrous vapours were not considerable throughout the experiment, colour of the gun-cotton not altered, but the fibre weakened and explosiveness reduced. |
| 44 | | 1·99 | 2h 30m. | A faint coloration of the atmosphere in the flask was only observed on the first day. The colour and strength of fibre were not altered, but the explosiveness was greatly reduced. |
| 45 | | 2·00 | 14 hours. | Only very faint indications of nitrous acid at any time. After 30 hours' exposure the gun-cotton had not suffered any change in colour, strength of fibre, or explosive qualities. |
| 46 | | 2·00 | 1h 30m. | Towards the close of the second day, the atmosphere in the flask was only faintly coloured. Colour and strength of fibre not altered, but explosiveness notably reduced. |
| 47 | | 2·12 | 14 hours. | Only very faint indications of nitrous acid at any time. The gun-cotton sustained no change in colour, strength of fibre, or explosiveness. |
| 48 | | 2·21 | 4 hours. | Only faint indications of nitrous acid observed in either of these experiments. Neither of the samples were altered in appearance or strength of fibre, but the explosiveness of both was diminished. |
| 49 | | 2·22 | 1h 20m. | |
| 50 | | 2·25 | 45 minutes. | Nitrous vapours were somewhat more abundantly evolved in experiment 50 than in 51, only faint indications being obtained in the latter case. Neither sample was altered in colour, but the strength of fibre and explosiveness were reduced in both instances. |
| 51 | | 2·30 | 45 minutes. | |
| 52 | | 2·31 | 1 hour. | The indications of nitrous acid were very faint after the first day's exposure. The gun-cotton was very slightly darkened in a few places, but strength of fibre and explosiveness were not affected. |
| 53 | Coarse yarn, manufactured at Stowmarket, affected. | 2·6 | 3h 30m. | Very slight reduction in explosiveness at conclusion of experiment; no change in appearance. |
| 54 | | 2·85 | 45 minutes. | Faint indication of nitrous acid throughout the experiment; the gun-cotton was darkened, and its explosiveness somewhat reduced. |
| 55 | | 3·0 | 45 minutes. | Faint indications of nitrous acid. Fibre not altered in colour or toughness, but explosiveness much reduced. |
| 56 | | 3·41 | 10 minutes. | Nitrous acid evolved abundantly. The gun-cotton converted into a pulverulent, quite inexplosive substance. |
| 57 | | 3·34 | 17 hours. | Very faint indications of nitrous acid. Strength of fibre not altered, but explosiveness much reduced. |
| 58 | | 3·68 | 5h 45m. | Nitrous acid not considerable. Gun-cotton discoloured and friable, explosiveness destroyed. |
| 59 | | 4·0 | 2h 15m. | Nitrous acid abundant. Fibre of gun-cotton and explosiveness destroyed. |
| 60 | | 4·1 | 45 minutes. | Nitrous acid not abundant. Gun-cotton darkened in parts; strength of fibre and explosiveness slightly reduced. |
| 61 | | 4·15 | 5 hours. | Nitrous acid not abundant. Gun-cotton not darkened, but fibre rotten, and explosiveness very greatly reduced. |
| 62 | | 4·24 | 5 hours. | Nitrous acid considerable. Gum-like mass produced. |
| 63 | | 4·3 | 10 minutes. | Nitrous acid very abundant. Gun-cotton converted into friable non-explosive material. |
| 64 | | 5·1 | 5h 45m. | Nitrous acid very abundant. Gum-like mass produced. |
| 65 | | 8·5 | 15 hours. | Very faint indications of nitrous acid. Gun-cotton slightly discoloured; strength of fibre and explosiveness very slightly reduced. |
| 66 | | 11·78 | 15 hours. | Faint indication of nitrous acid. Gun-cotton darkened; strength of fibre slightly diminished, and explosiveness much reduced. |
| 67 | Coarseyarn, Austria. | 3·02 | 1h 15m. | Nitrous acid considerable on the second day, diminished afterwards. Gun-cotton darkened in some parts; strength of fibre and explosiveness much reduced. |
| 68 | Fine yarn, Austria. | 3·66 | 30 minutes. | Nitrous vapours considerable, disappeared entirely towards close of experiment. Strength of fibre and explosiveness very greatly reduced. |
| 69 | Fine yarn, Austria. | 4·50 | 3h 30m. | Faint indications of nitrous acid. Strength of fibre and explosiveness scarcely affected in either instance. |
| 70 | Fine yarn, Austria. | 5·02 | 1 hour. | |
| 71 | Coarseyarn, Austria. | 7·44 | 15 minutes (total exposure 12 hours). | Nitrous vapours very abundant. Gum-like mass produced, with a charred appearance in some parts. |
| 72 | Coarseyarn, Austria. | 8·6 | 10 minutes (total exposure 12 hours). | Nitrous vapours very abundant. Gun-cotton quite pulverulent and passing into gum-like mass. |
| 73 | Coarseyarn, Austria. | 14·2 | 10 minutes (total exposure 12 hours.) | |
| 74 | Coarseyarn, Austria. | 14·1 | 12 minutes (total exposure 15 hours). | |
| 75 | Fine yarn, Austria. | 15 | 8 minutes. | Nitrous acid vapours considerable; strength of fibre and explosiveness considerably diminished. |
| 76 | Coarseyarn, Austria. | 70 (about) | 30 minutes. | Nitrous acid only faint after the second day. Fibre and explosiveness destroyed. |

The following are the principal facts demonstrated by the foregoing experiments:—

1. The results furnished by the samples of Waltham Abbey gun-cotton demonstrate that different samples of the material, manufactured as far as possible in the same manner, are not alike affected by exposure for a fixed period to 100° C. under uniform conditions. Of thirteen samples of Waltham Abbey gun-cotton, four resisted in a remarkable manner the destructive effects of heat, and remained unchanged in physical properties and explosiveness after thirty hours' exposure to 100° C. This treatment only developed acid to a slight extent in these particular samples; but in the other nine specimens it produced somewhat greater alteration; nitrous acid was disengaged in more considerable proportions, the fibre of the gun-cotton was rendered more or less rotten, and its explosiveness was diminished in different degrees.

2. The comparative celerity with which nitrous acid is disengaged from different specimens of gun-cotton upon exposure to 100° C., does not afford a reliable indication of their relative susceptibility to rapid decomposition at that temperature. As illustrations of this the following instances may be selected from among those furnished by the results detailed in the preceding Table. Those specimens of Waltham Abbey gun-cotton which exhibited uniform powers of resisting the destructive effect of heat (experiments 41, 45, 47, and 52), furnished the first faint indications of development of acid vapours at one hour, four hours, and fourteen hours, respectively, after first exposure to 100° C. Three samples (experiments 44, 48, 49) which were altered alike, and slightly, by uniform exposure to 100° for thirty hours, exhibited the first symptoms of decomposition at $1^h\ 20^m$, $2^h\ 30^m$, and four hours after commencement of the experiment. Again, a sample which exhibited no sign of change beyond slight acidity at the close of the experiment (experiment 52) afforded faint indications of the development of acid in one hour, while another specimen which sustained comparatively considerable change (experiment 48) did not evolve any acid vapour until four hours after its first exposure to heat. An inspection of the results obtained with Stowmarket gun-cotton shows that, out of thirteen samples, five did not furnish signs of disengagement of acid vapours until after the lapse of five hours and upwards, while among the thirteen Waltham Abbey samples only three furnished no signs of change for four hours and upwards. On the other hand, these samples of Waltham Abbey gun-cotton were, after thirty hours' exposure, only very slightly affected, while the Stowmarket samples just referred to, all exhibited important signs of change. Two samples from Stowmarket (experiments 54 and 60), though they evolved acid vapours within forty-five minutes of their first exposure, were not very greatly changed by the thirty hours' treatment, while other two samples (experiments 62 and 64), which evolved no acid for five hours and $5^h\ 45^m$, were completely decomposed by the close of the experiment.

A careful examination into the possible causes of these differences showed that they were to be ascribed, at any rate in very great measure, to variations in the proportion and character of the mineral matters contained in the specimens. Some few of the Waltham Abbey samples contained larger proportions of calcic and magnesian carbonates

(deposited upon the fibre by the hard water in which the material had been washed) than other samples. There was consequently present in such specimens a larger amount of matter capable of neutralizing acid, if liberated by the action of heat, than in others; and therefore the period would be proportionately delayed, in those instances, when the development of free acid would first become evident. The Stowmarket samples had been submitted to the "silicating" process, which consists in impregnating the gun-cotton with a dilute solution of soluble glass, afterwards drying it, and finally washing it in spring- or rain-water. The result of this treatment is that small proportions of alkaline and earthy carbonates are deposited upon the fibre in addition to what it would acquire by simple long-continued exposure to running water. This circumstance tends to explain why the Stowmarket gun-cotton experimented with, though generally much more seriously affected by protracted exposure to 100° C. than the Waltham Abbey samples, appeared to resist change in several instances for much longer periods than the latter.

The proportion and nature of the mineral matters in gun-cotton may, therefore, as shown by those experiments, exert a very notable effect upon the behaviour of the material when exposed to high temperatures. But the results of subsequent experiments have demonstrated most decisively that the influence which the presence of earthy or alkaline carbonates, mechanically distributed in small proportion through a mass of gun-cotton, exerts upon the effects produced by exposure to heat, is in many instances not confined to a simple delay of the *indications* of change furnished by the development of acid; it may also manifest itself in much more important directions, namely, by actually retarding and even considerably limiting, if not altogether preventing, the spontaneous decomposition of the gun-cotton itself. These effects are of such evident importance in connexion with the question of the stability of gun-cotton, that they have been made the subject of extensive experimental inquiry, the results of which will be given under a special head.

3. The different behaviour of the samples of gun-cotton operated upon in the foregoing experiments cannot be ascribed to differences in the *proportions* of matter soluble in ether and alcohol present in them. The four samples (experiments 41, 45, 47, 52) which withstood to the greatest extent the action of heat, contained 1·8, 2, 2·1, and 2·31 per cent. of soluble matter, which numbers represent the lowest, the mean, and almost the highest proportions of soluble matter in the Waltham Abbey gun-cotton. Again, some samples of Waltham Abbey products, containing identical proportions of soluble matter, behaved very differently, as may be seen by comparing experiments 41 and 42, 45 and 46, 51 and 52. The want of connexion between the proportion of matter soluble in ether and alcohol, and the stability of the sample, is perhaps even more strikingly demonstrated by results obtained with specimens of Stowmarket and Hirtenberg products. Samples containing equal proportions of soluble matter, as in experiments 56 and 57, 59 and 60, 65 and 72, behaved very differently, while others, in which the amount of soluble matter differed very considerably, exhibited similar behaviour upon

exposure to heat, as demonstrated by a comparison of experiments 57 and 66, 41 and 70, 56, 63 with 72, 73, and 74.

A comparison of the general results furnished by the Waltham Abbey and Austrian samples might be considered to afford some foundation for the conclusion that the gun-cotton which contains the largest proportion of the less explosive cellulose-products is the most susceptible of change, but it has already been shown that this conclusion is not supported by comparison of the individual experiments; and the following additional illustrations may be pointed out. A sample of Waltham gun-cotton containing 1·83 per cent. of soluble matter sustained decidedly greater change than Austrian samples containing 4·5 and 5·02 per cent., or than a Stowmarket sample containing 8·5 per cent.; and a specimen of Austrian cotton containing 3 per cent. of soluble matter did not sustain less alteration, and was much more rapidly affected than one from Stowmarket containing 11·78 per cent. Again, the specimen of Austrian gun-cotton which consisted chiefly of the lower nitro-products, was not so rapidly or completely changed as another Austrian specimen which contained only 7·4 per cent. of soluble matter (experiment 71), or a Stowmarket sample which contained but 4·3 per cent. (experiment 63).

4. A comparison of the *characters* exhibited by the matters which ether and alcohol extracted from different samples employed in these experiments, appeared to throw much greater light upon the causes of their different behaviour, than the comparison of the *proportions* of soluble matter which they furnished. Reference has already been made (p. 201) to the matters soluble in alcohol alone, which have been discovered in small but variable proportions in all samples of gun-cotton hitherto examined. Both the quantity and character of these substances extracted from different specimens of gun-cotton exhibit variations, as might be anticipated, when it is remembered that they are derived from impurities retained by the cellulose to an extent determined by the particular description and degree of purity of the cotton operated upon. In the Waltham Abbey specimens employed, the proportion of matter varied only slightly (between 0·72 and 0·9 per cent.); yet, although the comparatively slight differences in the effects of heat upon the different samples were in part ascribable to variations in the proportions of mineral matters present, indications were obtained that the gun-cotton which resisted the action of heat to the greatest extent contained the smallest proportion of nitrogenized organic matter not derived from cellulose. The Stowmarket samples afforded much more decided evidence of the influence of these foreign products upon the stability of the gun-cotton. Two specimens (experiments 56 and 63) from which nitrous acid-vapours were disengaged within ten minutes of their first exposure to 100° C., and three others (58, 62, and 64) which, owing apparently to the influence of mineral matters, did not furnish acid-vapours until after five hours' exposure, yielded extracts with ether and alcohol decidedly different in character from the other specimens; a comparatively large proportion consisted of nitrogenized acid matter of a resinous character, soluble in alcohol.

With two or three exceptions, the samples of Austrian gun-cotton exhibited decided



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signs of less complete purification of the cotton previous to conversion, than the generality of samples of English manufacture. It should also be observed that the extract by ether and alcohol after treatment of the samples with alcohol, possessed in several instances the characters of photographic collodion (the solutions furnishing tough transparent films upon glass), which was not the case with any of the specimens of Waltham Abbey gun-cotton, and in only one or two instances among the samples from Stowmarket. The ordinary ethereal extract from the English samples furnished a horny brittle residue, contracting greatly upon perfect desiccation, and appearing to consist chiefly of the product "B" described by HADOW as having the formula $C_{18}H_{22}O_{15}8NO_2$. No decided evidence was obtained in support of the conclusion that this difference in the character of the ethereal extract affected the stability of the gun-cotton. On the contrary, the Austrian samples used in experiments 71 and 72, which did not furnish an extract having the properties of good collodion, and only yielded 7·5 and 8·5 per cent. of total soluble matter, decomposed far more rapidly and completely than the specimen (experiment 76) which consisted chiefly of collodion gun-cotton.

The destructive effect upon the structure of the fibre produced by the long-continued digestion of the gun-cotton in warm solvents, which is necessary for ensuring the extraction as completely as practicable of the soluble matters, renders it very difficult to obtain reliable indications of the effects of heat upon gun-cotton deprived of those substances. The following experiments appear, however, to afford considerable support to the inference drawn from some of the results of the heat-experiments just referred to, that the existence in gun-cotton of small proportions of organic impurities, resulting from partial oxidation of foreign matters enclosed in the cotton fibre, exerts a very prejudicial influence upon the stability of the material, and that there is no sound foundation for the opinion that any such influence is exerted by the lower cellulose-products, when associated in small or large proportion with trinitrocellulose.

Experiments 77–80.—Four specimens of gun-cotton were extracted with ether and alcohol, by being twice digested for periods of twenty-four hours, in a considerable volume of the mixture, and afterwards washed. The dry specimens were then exposed for twelve hours (in two days) to 100° C., side by side with portions of the original samples.

No. 1.—The *original* gun-cotton exhibited faint indications of disengagement of nitrous acid in ten minutes after first exposure; the vapours did not become more abundant throughout the experiment, and, at the conclusion, the gun-cotton, which was strongly acid, had sustained a loss of 18·5 per cent. The *extracted* sample did not, throughout the experiment; afford any indication of the disengagement of nitrous acid; its acidity at the close was comparatively very slight, and it had lost only 3·5 per cent.

No. 2.—This sample, as well as Nos. 3 and 4, contained considerably more soluble matter than No. 1 sample; the original gun-cotton behaved quite similarly to No. 1, but the portion treated with the solvent evolved nitrous vapours in about fifteen minutes, and sustained much more rapid and considerable decomposition than the gun-cotton in its original condition.

No. 3.—The *extracted* gun-cotton first exhibited signs of change, and nitrous vapours were evolved more abundantly than from the original sample, which only afforded faint indications of nitrous acid after several hours' heating. The latter had lost only 5·5 per cent. at the conclusion of the experiment, while the extracted gun-cotton had sustained a loss of 10 per cent.

No. 4.—Signs of change were exhibited by both samples at the same time; faint vapours made their appearance after eight hours' exposure to 100° C. Nitrous acid was afterwards somewhat more abundantly evolved from the extracted sample. The loss sustained by the latter after twelve hours' heating was 5 per cent., that of the original gun-cotton was 4·1 per cent.

In these experiments No. 1 sample, which contained the average proportion of matter soluble in alcohol existing in Waltham Abbey gun-cotton, and a comparatively very small proportion of matter soluble in ether and alcohol, was rendered very much less susceptible of change at 100° C., by extraction with the mixed solvents, while the other samples, apparently in consequence of the injury to the structure of the fibre resulting from the extraction of a comparatively considerable proportion of imperfectly converted gun-cotton by the ether and alcohol, were rendered somewhat more prone to change by the treatment received.

Experiments 81, 82.—Two specimens of gun-cotton, selected from those which in preceding experiments had been found most susceptible of decomposition at 100° C., were digested for some time in dilute acetic acid, afterwards thoroughly washed, first in slightly alkaline water, and then in distilled water. By this treatment such mineral impurities as might have an influence upon the rapidity of decomposition of the gun-cotton were removed. One-half of each sample was digested for twenty-four hours with ether and alcohol, afterwards washed with the mixture, and dried. The samples thus treated were exposed to 100° C. in a water-bath, side by side with corresponding quantities of the same specimen which had simply been extracted with acetic acid. The results observed were as follows:—

| | No. 1. | | No. 2. | |
|---|---|---|---|---|
| | Treated with acetic acid only. | Treated with ether and alcohol. | Treated with acetic acid only. | Treated with ether and alcohol. |
| Indications of change observed after first exposure to 100° C. | 35 minutes. | 7½ hours (3 hours on the second day). | 30 minutes. | 3h 45m, very faint. |
| Loss of weight after 4½ hours' exposure to 100° C. | 17·9 per cent. | 2·3 per cent. | 20 per cent. | 1 per cent. |

The very marked difference in the stability of these specimens (which contained only very small proportions of soluble gun-cotton), when exposed to 100° C. in the two different conditions, appears to afford strong evidence that the abstraction of the matters soluble in ether and alcohol greatly increases the stability of the gun-cotton. The next-following experiments show, on the one hand, that this difference does not appear ascribable to the removal of the soluble gun-cotton from the trinitrocellulose, and indicate, on the

other hand, that the closing up of the fibres resulting from the solution (or gelatiniza tion), but imperfect removal from the gun-cotton of the soluble portions, may, in the above experiment, have imparted to the material increased powers of resisting decomposition at a high temperature.

Experiments 83–87.—Four specimens of gun-cotton, containing small proportions of imperfectly converted material, were thoroughly saturated with a very weak solution of pure soluble gun-cotton (or collodion), then at once removed from the liquid and dried. The mechanical condition of the gun-cotton was not perceptibly altered by this treatment. These specimens were exposed to 100° C., together with portions of the original samples (all of them being for this purpose packed lightly and uniformly in small flasks). In every instance the prepared gun-cotton resisted the action of heat for a much longer period than the unprepared material. The former exhibited the first very faint indications of disengaging of nitrous acid between twelve and fourteen hours after the first exposure, while the unprepared specimens evolved nitrous acid after one hour to $1^h\ 30^m$ exposure.

It would appear therefore from these experiments that the addition of less perfectly converted gun-cotton to the ordinary product does not have the effect of promoting its decomposition at 100° C., but that, on the contrary, when applied as indicated above, it renders the material considerably less susceptible of change, probably because the fibres are partially sealed, or in some other way mechanically protected by the treatment with dilute collodion. That the partial or complete closing of the fibre does exert an important influence upon the power of gun-cotton to resist the action of heat was demonstrated by

Experiment 88.—An ordinary sample of dry gun-cotton was allowed to remain for eighteen hours in a confined space together with an open vessel containing the ether- and alcohol-mixture. It was afterwards dried and exposed to 100° C., side by side with a portion of the sample in its original condition. The latter exhibited signs of decomposition within two hours, the sample which had been exposed to the action of the vapour only exhibited faint signs of change after eighteen hours' exposure.

Experiment 89.—A specimen of Stowmarket gun-cotton containing a large proportion of matter extractable by ether and alcohol was washed with alcohol only, and its behaviour at 100° was afterwards compared with the original gun-cotton. The washed gun-cotton resisted the action of heat only slightly longer than the original gun-cotton, but it was observed that the washing had effected the separation of much of the earthy carbonates mechanically attached to the fibre, and hence was deprived, by the alcoholic treatment, both of a protective and a destructive element. A portion of the washed gun-cotton was afterwards exhausted as far as possible with ether and alcohol. By this treatment the fibre was much disintegrated, and upon exposure of the insoluble gun-cotton to 100° C., it exhibited signs of decomposition much more speedily than the original gun-cotton. The ethereal extract was evaporated, and the dry product was exposed for thirty hours (during six days) to 100° C. No indications of nitrous acid

were observed throughout the experiment, and the material was unaltered in character, excepting that it had become decidedly acid. It should be observed that this ethereal extract was not quite free from the matters soluble in alcohol,which the gun-cotton contained, as it is apparently impossible to extract these perfectly by digestion and frequent washing with alcohol.

Experiment 90.—A specimen of Waltham Abbey gun-cotton was in the first instance digested with dilute acetic acid and thoroughly washed, the object of this treatment being to remove any mineral matters from the sample which might exert a neutralizing action and thereby influence the effects of exposure to heat. One half of this gun-cotton was then digested for two days with warm alcohol, and was afterwards repeatedly washed. On evaporation of the alcoholic extract the usual small yellowish resinous residue was obtained.

A small portion of the dry gun-cotton thus purified was heated to 100° C. in a glass tube side by side with some of the same specimen which had only been extracted with acetic acid. Early on the second day of the experiment, the latter specimen began to evolve nitrous acid; and about thirty minutes afterwards the sample extracted with alcohol exhibited faint signs of decomposition.

Eight grammes of each of these samples, and a similar quantity of the gun-cotton in its original condition, were afterwards introduced into very long-necked flasks, the openings of which were loosely closed with corks and exposed for six days, seven hours daily, to 65° C.; as none of the specimens exhibited any sign of change at the expiration of that period, the temperature of the water-bath was maintained at between 88° and 90°. After about nine hours' exposure to this temperature, the original gun-cotton began to decompose, and two hours later an extremely faint indication of nitrous acid was observed in the sample extracted with *acetic acid*. After two days' (twelve hours) further exposure to heat, the first signs of decomposition became apparent in the flask containing the sample which had been extracted with *alcohol*. The coloration of the atmosphere continued, however, to be only faint in the flasks containing both extracted specimens during ten days' exposure to about 90°.

The observation made in this experiment, that the treatment of the gun-cotton with acetic acid decidedly increased its power of resisting the destructive effects of heat, was quite at variance with the anticipated result; for, undoubted evidence having already been obtained of the retarding effect upon the decomposition exerted by the existence of earthy carbonates when deposited upon the gun-cotton during the washing operations, it was considered that the treatment of ordinary gun-cotton with the acid, if it in any way influenced the subsequent action of heat upon the material, would have an accelerating effect. Several additional experiments confirmed, however, the correctness of the above observations; the following results, furnished by different samples of gun-cotton, of which portions were extracted with acetic acid, may be quoted in illustration of this.

TABLE IV.

| No. of experiment. | Nature of material. | Exposed to | Duration of exposure. | Nitrous acid disengaged after first exposure. | Loss of weight. | Other observations. |
|---|---|---|---|---|---|---|
| 91 | Ordinary gun-cotton, air-dry. | 100° | 4h 30m | 20 minutes | 27 per cent.... | The vapours were very deep-coloured in 20 minutes. |
| | Same sample, extracted with acid. | 100 | 4h 30m | 35 minutes | 20 per cent.... | The vapours were not deep-coloured until after 2 hours' heating. |
| 92 | Ordinary gun-cotton, quite dry. | 100 | 4h 30m | 10 minutes | 24·4 per cent... | The vapours very deep-coloured in 15 minutes. |
| | Same sample, extracted with acid. | 100 | 4h 30m | 30 minutes | 17·9 per cent... | The vapours were deep-coloured after 30 minutes' heating. |
| 93 | Ordinary gun-cotton | 65 | 140 hours (in 20 days). | 30 hours, 4th day *very faint*. | In one week 1·66 per cent. | The sample was greatly changed after 20 days' heating, being converted partly into the matter soluble in water, of the usual character, and partly into soluble gun-cotton. |
| | Same sample, extracted with acid. | 65 | 140 hours (in 20 days). | 38 hours (6th day). | 1·0 per cent... | The sample had sustained a trifling change; it had contracted somewhat, and the soluble matter had increased a little. |

Upon examining the extract obtained by digesting gun-cotton with acetic acid in the cold, it was found to contain a small quantity of organic matter of resinous character, insoluble in water, but soluble in alcohol alone and in ether and alcohol, containing nitrogen and deflagrating when heated; it was evident therefore that this treatment of gun-cotton had the effect of purifying it to some extent from the organic impurities more perfectly removed by the extraction with alcohol.

The treatment of gun-cotton with dilute hydrochloric acid was not found to effect the removal of any proportion of these organic impurities, while the carbonates were, of course, readily extracted thereby. When gun-cotton, thus purified and very thoroughly washed, was exposed to heat side by side with the material in its original condition, the latter exhibited decidedly greater power of resisting change, thus furnishing an important indication of protective power exerted by carbonates if present in gun-cotton, which will be presently examined into more fully.

It is considered that the foregoing experiments afford good grounds for the following conclusions:—

(1) That the invariable existence in gun-cotton of small proportions of organic impurities, resulting from the partial oxidation of foreign matters enclosed within the cotton fibre, exerts a very prejudicial influence upon the stability of trinitrocellulose.

(2) That there is no sound foundation for the opinion that any such influence is exerted by the lower cellulose-products, when associated in large or small proportions with trinitrocellulose.

The following additional experimental data may be quoted in support of the latter conclusion:—

A considerable quantity of perfectly soluble gun-cotton was prepared at Waltham in

1865, the ordinary process of manufacture having been strictly followed, with the exception of the necessary difference in the strength of the acid-mixture used. This gun-cotton was found to correspond closely in composition to the formula of HADOW's compound "C" ($C_{18}H_{23}O_{15}7NO_2$), as shown in my first Memoir*. It has exhibited no tendency whatever to change by long-continued exposure to diffused daylight; several comparative experiments have been instituted with it and with samples of ordinary Waltham-Abbey products, and it has never exhibited any indications of greater susceptibility to change than the most stable of these. Indeed, the following results would appear to indicate that any difference in stability which may exist between the different members of the nitrocellulose group is not in the direction assumed by REDTENBACHER, SCHRÖTTER, and SCHNEIDER.

Experiment 94.—A portion (about 1 grm.) of the soluble gun-cotton, which had been reduced to a fine state of division in a pulping-machine, was introduced into a wide-bulb tube, and a similar tube was charged with a corresponding quantity of ordinary gun-cotton in the same mechanical condition. Both specimens were air-dry. They were exposed in the same water-bath seven hours daily to 100° C. Not the slightest indication of change was observed in either specimen until towards the close of the seventh day, when a faint coloration by nitrous acid was observed on looking down the tube containing the ordinary gun-cotton. Very shortly afterwards a still fainter coloration was noticed in the tube containing the finely-divided *soluble* gun-cotton. Both samples continued from this time to undergo slow decomposition; but for several hours after the first commencement of change, the ordinary gun-cotton evolved nitrous vapours more abundantly than the soluble sample.

Experiment 95.—Larger quantities (11 grms. and 33 grms.) of ordinary gun-cotton and of the soluble gun-cotton, both in a fine state of division, were exposed day and night uninterruptedly to 60° C. After a period of one month the soluble gun-cotton had sustained not the slightest loss of weight, the ordinary gun-cotton having lost 0·058 per cent. At the expiration of another month's uninterrupted heating, the weight of the soluble gun-cotton was still found to have sustained no change, while the ordinary gun-cotton had only sustained a further loss of 0·02 per cent.†

These and other similar results appear to demonstrate satisfactorily that the lower nitrocellulose compounds, when prepared in a condition of equal purity with the ordinary gun-cotton, are certainly not more prone to change at high temperatures.

### EXPOSURE OF GUN-COTTON TO 90° C.

It is stated by PÉLOUZE and MAURY that in their experiments the exposure of gun-cotton to 90° furnished identically the same results as those produced by the tempera-

* Philosophical Transactions, vol. clvi. p. 297.

† After continuous exposure of these samples at 60°, for a further period of four months, the soluble gun-cotton is found not to have sustained any loss in weight, while the total loss sustained by the ordinary gun-cotton amounts to 0·19 per cent.—June 8th, 1867.

ture of 100° C., excepting that the phenomena of decomposition, instead of appearing after a few minutes, only became manifest after several hours. This statement has been confirmed, by the results of comparative experiments which I have instituted at 90° and 100°, to this extent, that in all instances the first signs of change become manifest in the specimens heated to 100°, and that sometimes the interval of time between the first exposure to heat and the first indications of decomposition is much greater at the lower than the higher temperature. The following results furnished by exposure of equal quantities of the same gun-cotton, in the same condition, to 90° and 100° C., may be quoted in illustration of the comparative effects of the two temperatures.

TABLE V.

| No. of experiment. | Total period of exposure to | | First indications of change observed after commencement of exposure to | | Observations. |
|---|---|---|---|---|---|
| | 90° C. | 100° C. | 90° C. | 100° C. | |
| 96 | 48 hours ..... | 6 hours ..... | 4 hours, *very faint.* | 2 hours ..... | At 90° only faint indications of nitrous acid were obtained up to the close of experiment; at 100° the vapours were abundantly evolved after 5¼ hours' exposure. |
| 97 | 26 hours ..... | 26 hours ..... | 6 hours ..... | 5 hours ..... | In both instances the coloration of the atmosphere was very faint throughout the experiment. |
| 98 | 40 hours ..... | 30 hours ..... | 10 hours ..... | 45 minutes ... | The coloration by nitrous acid was only faint throughout in both instances, but the specimen exposed to 100° sustained a more considerable change than the other. |
| 99 | 56 hours ..... | 20 hours ..... | 22 hours, *very faint.* | 14 hours ..... | The disengagement of nitrous acid was only very slight throughout the experiment at 90°; in the other experiment it was more copious, though not abundant at any period. |
| 100 | 42 hours ..... | 10 hours ..... | None observed | 50 minutes ... | The specimen heated to 100° was decomposing rapidly after the lapse of 9¼ hours; the other specimen exhibited no sign of change beyond a slight acidity. |
| 101 | 46 hours ..... | 6 hours ..... | None observed | 2 hours, very faint. | The specimen heated to 100° commenced to evolve nitrous acid abundantly at the expiration of 6 hours, the other specimen showing no sign of change beyond a slight acidity. |

It will be observed from these experiments, which are quoted as representing numerous others of a similar description, that in most instances the decomposition of the gun-cotton was not only slower but also much less serious at 90° than at 100°. Exceptional specimens, exhibiting either a very unusual want of stability (*e. g.* some of the specimens from Hirtenberg and Stowmarket which have already been referred to), or a remarkable power of resisting decomposition at 100°, generally showed but little difference in behaviour when subject to the influences of the two temperatures.

### EXPOSURE OF GUN-COTTON TO TEMPERATURES RANGING FROM 50° TO 60° C.

Several experiments, corresponding in their nature to those described in the first part of the account of the action of light upon gun-cotton, have been instituted for the purpose of obtaining data regarding the influence upon the material of very long-continued exposure to the above-named temperatures. The air-dry gun-cotton was introduced into large bulbs blown at the extremities of barometer-tubes, the latter being placed with

their openings over mercury and exhausted in the manner already described, so that the height of the mercury-column in these tubes, at the commencement of the experiments, was very nearly that of the barometer at the time. Sufficient gun-cotton was employed to fill the globes pretty compactly. The bulbs were enclosed in metal water-baths, in which they were always perfectly surrounded by water maintained for definite daily periods at a constant temperature by gas-flames, accurately adjusted by self-acting regulators. Daily observations were made, before heat was again applied, of the height of mercury in the tube (with the necessary corrections), of the appearance of the gun-cotton through the glass, and of any other points worthy of note.

Experiment 102.—18 grms. of air-dry gun-cotton, manufactured at Waltham Abbey in 1863, were heated for six days, seven hours daily, to a temperature ranging between 36° and 38° C. The column of mercury was not permanently affected to the slightest extent during this period. It was afterwards intended to maintain the temperature for a long period at 49°, but upon the second day of this treatment, the heat was accidentally raised to 55°, it was therefore afterwards maintained at that point for a considerable period. After the first day of this treatment, the column of mercury continued to fall daily, to an extent ranging between 8 and 16 millims., during nine days' further exposure for six hours daily to 55°. For three subsequent days the column fell 18, 20, and 18 millims.; on the thirteenth day the fall amounted only to 11 millims. On the fourteenth day the temperature rose accidentally to 60°, and remained at that point about thirty minutes; on that day the fall of the mercury-column amounted to 23 millims. The temperature was afterwards maintained at 55°, but the daily depression of mercury did not correspond with the observations made before the temperature had accidentally reached 60°; on three successive days it was 43 millims., 24 millims., and 37 millims. It appeared from these results that the increase of temperature to 60° had established a greater tendency to change in the gun-cotton, which afterwards continued, although the temperature was reduced to 55°.

After this exposure of the gun-cotton to heat, from six to seven hours daily, for twenty-four days, during seventeen of which the heat applied was 55°, and for a short time 60°, the specimen was removed from the globe. It had not altered in appearance, but was found to be strongly acid to test-paper; it had a peculiar pungent odour, the fibre had become tender, and its explosiveness had diminished somewhat. A portion of the specimen was washed thoroughly, first in distilled water and afterwards in slightly alkaline water, then dried and placed in a bottle, in which it has been exposed to diffused light for three years without undergoing further change. Nitrous acid vapours could not be distinguished in the globe or tube at any time during the experiment, but soon after the temperature was raised to 55°, a few small yellowish crystals of mercury-salt (mercurous nitrite) appeared upon the surface of mercury in the tube, and were added to a little as the experiment proceeded.

Experiment 103.—16 grms. of the sample of gun-cotton used in the preceding experiment, and 14·75 grms of another sample, were exposed side by side, in one and the

same water-bath, in the manner already described, to 55° C. for six to seven hours daily, during seventy days. After the first day's heating, the column of mercury in each tube was slowly and uniformly depressed, the volume of gas evolved being somewhat greater from the second, smaller sample. At the conclusion of the seventy days' treatment it was calculated, from the capacity of the tubes and the amount of total displacement, that the larger sample had evolved 172·88 cub. centims., and the other sample 189·1 cub. centims. of gas; as, however, a few small crystals of mercury-salt had been produced in each tube by the action of nitrous acid disengaged, those quantities are of course only proximate. Upon removal from the bulbs, the colour of both specimens was unchanged; their odour was decidedly less pungent than that of the preceding specimen; both were acid to test-paper, the smaller sample being the most strongly so; in neither instance was the strength of fibre impaired, the explosiveness diminished, or the solubility in ether and alcohol appreciably increased. The specimens were divided, put into stoppered bottles without any previous purification, and one bottle of each was preserved in the dark, the other being exposed to diffused light. None of the specimens have up to the present time (a period of 3½ years) undergone any further change.

Experiment 104.—13·8 grms. of gun-cotton were exposed in an exhausted bulb-tube, as already described, to 65° for six to seven hours daily, during a period of three months (eighty-four days). The depression of the mercury proceeded uniformly, but much more rapidly than in the preceding experiment. After several days' exposure, a notable quantity of mercurous salt was deposited in crystals within the tube. At the termination of the experiment the gun-cotton was not altered in colour and appearance, but upon removal from the bulb the fibre was found to be considerably weakened; a pungent odour and strong acidity were exhibited by the specimen, its explosive properties were notably reduced, and it dissolved to a large extent in ether and alcohol, the solution furnishing a collodion-film. A portion of the same gun-cotton enclosed in a smaller bulb-tube, sealed at both extremities, was exposed to heat for an equal period in the same water-bath. There was some pressure of gas upon opening the tube, and the gun-cotton exhibited the same appearance and properties as the sample heated over mercury. Both samples were placed in closed glass vessels, and have since been exposed to light for upwards of three years, without undergoing any further change.

Experiments 105–108.—Four specimens of gun-cotton, each weighing 19·5 grms., taken from different samples, were introduced into bulb-tubes of almost the same capacity and with stems of equal length. The bulbs were all enclosed in one water-bath, and the open extremities of the tubes were immersed in a mercury-bath, over which they were exhausted as in the preceding experiments. The water-bath was maintained at 65° seven hours daily, the uniformity of the temperature being ensured by the employment of a self-acting gas-regulator. Before commencing the application of heat each morning, the extent of depression of the mercury in the tubes was noted; and the volume of gas contained in each at stated periods was calculated from the observations made. The following Table shows the effects of uniform exposure to 65° upon these samples:—

TABLE VI.

| Description of gun-cotton. | Volume of gas evolved. | | | Gas escaped from the tube on | Condition of the gun-cotton after 3 months' exposure 7 hours daily to 65° C. |
|---|---|---|---|---|---|
| | 5th day. | 13th day. | 28th day. | | |
| | cub. centims. | cub. centims. | cub. centims. | | |
| No. 1. Fine yarn, Waltham. | 57·6 | 213·3 | ..... | 13th day ..... | Strong acid reaction; strength of fibre and explosiveness diminished. Aqueous extract contained nitric and oxalic acids, and reduced cupric salts readily. Proportion of matter soluble in ether and alcohol = 15 per cent. |
| No. 2. Coarse yarn, Waltham. | 9·6 | 16·1 | 47·3 | 65th day ..... | Acid reaction; strength of fibre and explosiveness not appreciably diminished. Aqueous extract contained nitric acid and a trace of oxalic acid; reduced cupric salts to a very slight extent. Matter soluble in ether and alcohol = 49·1 per cent. (2 per cent. in original sample). |
| No. 3. Fine yarn, Waltham, washed and impregnated with 0·4 per cent. of sodic carbonate. | 27·3 | 67·7 | 221·7 | 30th day ..... | Strongly acid reaction; strength of fibre and explosiveness only slightly reduced. Aqueous solution contained nitric acid and a small quantity of oxalic acid; reduced cupric salts slightly. Soluble matter = 10 per cent. |
| No. 4. Coarse yarn, Stowmarket. | 192·2 | ..... | ..... | 6th day ..... | Acid, friable; explosiveness very much diminished. Large proportion soluble in water. Solution contained a very small quantity of nitric acid, but a large proportion of oxalic acid; reduced cupric salt very abundantly. Portion insoluble in water; dissolved in ether and alcohol. |

The sample of Stowmarket gun-cotton which decomposed so readily at 65°, was an early specimen of manufacture from that establishment; it had evidently been prepared from inferior or very imperfectly purified cotton, and contained a considerable proportion of foreign matter extractable by alcohol. The two specimens of fine yarn (Nos. 1 and 3) were portions of the lowest class of products obtained at Waltham Abbey. Its impregnation with a small proportion of sodic carbonate imparted to it greater power of resisting the effects of heat; this result was not demonstrated, however, to its full extent, because the sample of the gun-cotton (No. 1) in its original condition contained some earthy carbonates, which had been at any rate partially separated from the sample impregnated with sodic carbonate.

The specimen of coarse yarn (No. 2), which was found to be but little changed by the three months' exposure to 65°, was an average specimen of the products obtained at Waltham Abbey.

Specimens 1, 2, and 3 have been preserved in stoppered bottles in the condition in which they were removed from the bulbs, and have been exposed to strong daylight for two years. They have not undergone further change.

Experiment 109.—A sample (20 grms.) of perfectly dry Waltham Abbey gun-cotton, representing the ordinary product of manufacture, was exposed in a bulb-tube exhausted over mercury to 65° C., for seven hours daily. The mercury-column was very gradually depressed, as in the preceding experiments with gun-cotton of this kind, and after the experiment had continued for ten weeks, the gas escaped from the opening of the tube. In about one week more, sufficient gas was collected for examination; it was found to consist of—

| | |
|---|---|
| Carbonic acid | 45·6 per cent. |
| Nitric oxide | 10·8 „ |
| Nitrogen | 43·5 „ |

Some water had become deposited in the sides of the tube, and a small quantity of mercury-salt had formed. The application of heat to the tube was continued with the view of collecting a further quantity of gas, but the experiment was carried on uninterruptedly for a further period of nearly twelve months before a sufficient amount of gas (about 150 cub. centims.) could be collected for analysis. This second product contained—

| | |
|---|---|
| Carbonic acid | 55·7 per cent. |
| Hydrogen | 6·4 „ |
| Carbohydrogen | traces |
| Nitric oxide | 2·1 „ |
| Nitrogen | 35·7 „ |

The total volume of gas evolved during fourteen months' exposure of the gun-cotton to 65° C., for seven hours daily, was about 660 cub. centims.

On removing the specimen from the bulb-tube, it exhibited no alteration in colour, appearance, or explosiveness; the strength of fibre had slightly diminished; nitric acid was detected in small quantity in the aqueous extract, but no oxalic acid. A very slight reducing action was exerted upon cupric salts, but the proportion of matter soluble in ether and alcohol had not appreciably increased. At the conclusion of the experiment the specimen was washed in slightly alkaline water, dried, and exposed to strong daylight and occasional sunlight, in a stoppered bottle. Up to the present time (after nine months' exposure) the specimen has suffered no change whatever.

It would appear from these experiments, which were always commenced *in vacuo*, but continued after a short time in an atmosphere of the gaseous and volatile products formed, that gun-cotton, prepared and purified according to the system now in use, manifests some slight but undoubted symptoms of gradual change, if maintained for several hours at as low a temperature as 55° C. in a confined space;—that a very long-continued exposure to that temperature does in some instances produce a notable alteration in the composition and explosive properties of the substance;—that a similar change is somewhat more rapidly developed if the gun-cotton be exposed to temperatures of 60° and 65°; but that the exposure of the substance several hours daily, even for months, to the highest of those temperatures does not so seriously affect the ordinary products of manufacture as to prevent their being afterwards restored, by the ordinary process of purification from acid, to a condition differing but little, practically, from that of the original material. Although these experiments were instituted with comparatively considerable quantities of gun-cotton (14–20 grms.), it must be at once admitted that, if the material were exposed in large compactly packed masses (5–10 kilos.) to the temperatures ranging between 55° and 65° for the periods given in the preceding experiments,

it would be more seriously affected, and that the changes which would ultimately be developed by the free acid accumulating in the gun-cotton might give rise to spontaneous heating of the mass. On the other hand, it must be borne in mind that even the lowest of those temperatures occurs in nature only under exceptional circumstances, *and for brief periods.*

It may perhaps be considered that the arrangement of heating the gun-cotton over a column of mercury, adopted in the foregoing experiments with the view of obtaining continuous records of the progress of change, was of a nature somewhat favourable to the material operated upon, because a small surface of mercury was exposed in direct contact with the gases or vapours evolved, and might, by its own oxidation, remove a portion of the generated acid which would otherwise have reacted injuriously upon the gun-cotton. It has indeed been stated, in the description of the experiments, that a few crystals of mercurous salt were always formed upon the exposed surface of the mercury, the production of the salt being favoured by the condensation over the metal of a small quantity of water, produced as the experiment proceeded. But it must be borne in mind that the surface of mercury exposed was always very small (only from 78–100 sq. millims.), while the quantity of cotton operated upon was considerable, and that, between each consecutive period of exposure to heat, the gun-cotton absorbed, as it cooled during the night, the water impregnated with acid which had been previously expelled from it. Experimental proof was, however, obtained that ordinary gun-cotton, when exposed to 65° in vessels not closed by mercury, and so arranged that any liberated acid would not escape from contact with the material, was more rapidly and seriously affected than was the case in the globe-experiments.

Experiment 110.—Four specimens taken from different samples of gun-cotton, exposed in a dry state to 65° in very long and narrow-necked flasks, seven hours daily for seven days, sustained no loss of weight. From the tenth to the fourteenth day after the first exposure all showed slight signs of decomposition, which proceeded with somewhat different rapidity in the several samples; two of them were completely decomposed in three weeks after first exposure, the other two resisted for very considerably longer periods.

Experiment 111.—Two other samples were similarly exposed to heat side by side; nitrous vapours became distinctly apparent six days after first exposure to 65°, and continued visible until the twelfth day's exposure. After three weeks' exposure, the specimens had lost 30 per cent. in weight, and were converted chiefly into soluble gun-cotton.

Experiment 112.—The protracted exposure of air-dry gun-cotton to a temperature ranging between 60° and 55° in a vessel *to which air had access*, did not effect any greater alteration in the material than was observed in the globe-experiments. Thus 53·968 grms. of air-dry gun-cotton, after exposure to heat seven hours daily for ten days, weighed 53·902 grms.; after a further exposure for seven days it weighed 53·882 grms., and after a third exposure for five days it weighed 53·881 grms. The sample had therefore only sustained a loss of 0·10 per cent.

Experiment 113.—Two other specimens, weighing 44 and 34 grms., were exposed day and night in a hot-air chamber, to a temperature ranging between 35° and 50°. The specimens were weighed periodically, in an air-dry condition. After the lapse of ten weeks one sample had lost 1·18 per cent., and the other 1·56 per cent.

In conducting these quantitative experiments, it was observed that the exposure of gun-cotton for a protracted period to a moderately elevated temperature had the effect of reducing the hygroscopic power of the fibre, so that upon exposure of gun-cotton which had been thus heated to the atmosphere, the maximum proportion of moisture absorbed by it was very notably lower than that contained in the original sample. The actual loss sustained by the above samples, which were always weighed after exposure to air for definite periods, was therefore somewhat less than indicated by the numbers given.

## Part III.—Influence exerted upon the stability of gun-cotton by special modes of preparing and preserving it.

I. *Reduction of gun-cotton fibre to a fine state of division.*—Abundant proofs have been obtained that the long-continued washing and the treatment with an alkaline liquid to which gun-cotton is submitted, do not completely separate from it products of the partial oxidation of organic impurities retained by the cotton up to the time of its conversion. This is unquestionably due in great measure to the tubular structure of the fibre. If the impurities were merely upon the surface of the fibre, their perfect removal by the action of solvents should be accomplished without difficulty, but it does not appear that even long-continued digestion of gun-cotton in alcohol has the effect of completely freeing it of the impurities soluble in that liquid which are locked within the fibre. The action of a warm or cold alkaline liquid upon the material might perhaps eventually result in the complete removal of these bodies, but the loss of product and destructive effect upon the fibre, resulting from any other than a brief digestion in a very dilute alkaline bath, are too considerable to admit of such a treatment. The following experiments may be quoted in illustration of this.

Experiment 114.—A quantity of gun-cotton which had already been submitted to the usual purification with water and a hot alkaline bath, was boiled for ten minutes in a solution of potassic carbonate of the strength usually employed (of specific gravity 1·02). By this treatment the material sustained a loss of 3·7 per cent., the bath having assumed an amber colour. Upon being again boiled for twenty minutes in the same alkaline bath, which thereby became considerably deepened in colour, the sample sustained a further loss of 12·09 per cent. The strength of the fibre had been considerably reduced by this treatment.

Experiment 115.—6·5 grms. of gun-cotton and 0·4 grm. of sodic carbonate were placed together with 50 cub. centims. of water in a flask to which a vertical condenser was attached, and were heated to 100° for twelve hours. The alkali was then found to have become nearly neutralized, and the dark brown liquid contained sodic nitrate in abun-

dance. The gun-cotton was washed and twice treated in the same manner, the alkali being neutralized on each occasion, as in the first instance.

But though it is evident that the treatment of gun-cotton with warm alkaline baths cannot be advantageously extended, satisfactory proof has been obtained that the stability of gun-cotton which has been purified as far as is possible by the present system, may be importantly increased by submitting the material to a special process of washing.

In the experiments instituted upon the application of gun-cotton as a substitute for gunpowder, some very advantageous results have attended the conversion of the material into homogeneous masses of any desirable form or density, by preparing it according to the method commonly employed for converting rags into paper. In reducing the material to a very fine state of division by means of the ordinary beating- and pulping-machines, the capillary power of the fibres is nearly destroyed, and the gun-cotton is, for a considerable period, very violently agitated in a large volume of water. It would be very difficult to devise a more perfect cleansing process than that to which the gun-cotton is thus submitted; and the natural result of its application is that the material thus additionally purified acquires considerably increased powers of resisting the destructive effects of heat. Samples of the pulped gun-cotton even in the most porous condition have been found to resist change perfectly upon long-continued exposure to temperatures which developed marked symptoms of decomposition in the gun-cotton purified only as usual (experiments 94 and 95 may be referred to in illustration of this).

The pulping process applied to gun-cotton affords therefore important additional means of purifying the material, the value of which may be further enhanced by employing a slightly alkaline water in the pulping-machine.

II. *Impregnation of gun-cotton with substances capable of neutralizing free acid.*—The slightest change sustained by gun-cotton is attended by the development of free acid, which, if it accumulates in the material, even to a very trifling extent, greatly promotes decomposition. Numerous experimental data have been collected with respect to the establishment and acceleration of decomposition in gun-cotton exposed to light or elevated temperature by free acid, which either is present in the imperfectly purified material, or has been developed by decomposition of gun-cotton or its organic impurities.

Experiment 116.—Samples of gun-cotton which, by exposure to elevated temperatures or for considerable periods to strong daylight, had sustained changes resulting in a considerable development of acid, have afterwards been thoroughly purified by washing and exposed to light for months, and in some instances for two and three years (up to the present time) without undergoing further change, while corresponding samples, confined in closed vessels without being purified, have continued, in some instances, to undergo decomposition, and the original substance has been completely transformed into the products repeatedly spoken of. Instances have, however, occurred in these experiments (and have already been quoted) in which gun-cotton has resisted further change, even under these circumstances.

Experiment 117.—Gun-cotton, purified as usual, has been confined in stoppered glass bottles, having previously been rendered slightly acid with nitric acid. In these instances the gun-cotton has always undergone decomposition upon exposure to light, the rapidity of its change varying with the quality of the material.

Experiment 118.—Two specimens of Waltham Abbey gun-cotton (coarse and fine yarn) were introduced into well-stoppered bottles, and pernitric oxide was then passed into those for a short time. The bottles were then tightly closed and placed in a dark cupboard, being inspected from time to time. The gas was rapidly absorbed by the gun-cotton, which assumed a green tinge and gradually contracted, the colour of the vapours in the bottles slowly becoming deeper again. After the lapse of two months both samples had contracted into compact masses, occupying less than one-fourth the original volume. Both were coloured green, and dark orange vapours filled the vessels. From this period the pernitric oxide diminished in quantity very gradually, until, about eighteen months after commencing the experiment, the atmosphere in the bottles was perfectly colourless. The coarse gun-cotton had passed into a viscid mass, exhibiting the usual characters; the fine gun-cotton, though it contracted to about one-tenth of its original volume, still retained to some extent its original appearance; crystals of oxalic acid were dispersed through the mass.

Experiment 119.—Two other samples of gun-cotton employed in the preceding experiment were placed in bottles into which nitrous acid, produced by means of starch, was passed. These bottles were afterwards also placed in the dark. The gas was gradually absorbed by the gun-cotton, the atmosphere in the bottles became colourless, and both samples were highly bleached. After the lapse of two months, a faint orange colour was exhibited, but the specimens of gun-cotton had undergone no apparent change whatever. Three months later, the bottle containing the coarse yarn exhibited deep orange vapours, the gun-cotton had contracted somewhat and assumed a green tinge. The other sample exhibited no signs of change, but a faint orange tinge was manifest in the bottle, which did not increase afterwards. Twenty-eight months after the commencement of the experiment this sample exhibits no signs of change beyond a very slight contraction. The coarse yarn has contracted to about one-third its original volume, is friable, and partly soluble in water.

Pernitric oxide, if left in contact with gun-cotton, is therefore much more rapid in its destructive action than nitrous acid; gun-cotton when confined together with either of them, undergoes gradual decomposition even in the dark.

Experiment 120.—A sample of gun-cotton which had been found to decompose very readily at 100°, was placed in a retort suitably fitted with a delivery-tube, and the gases disengaged from it were passed into four bottles containing different samples of gun-cotton. These were then perfectly closed and exposed to strong daylight.

The first sample soon began to exhibit signs of change. The colour of the vapours increased in depth, and in one month's time had become very deep-coloured, the gun-cotton having assumed a greenish tinge from absorption of gas. The sample was then placed in the dark, after which it underwent further change very gradually, first cor-

tracting considerably and afterwards, after the lapse of upwards of one year, becoming converted into a somewhat hard gum-like mass.

In the case of the second and third samples, the coloured vapours disappeared at first almost entirely, but the atmosphere in the bottles became coloured again after one month's exposure to light. The depth of colour increased so rapidly in one instance that two months after first exposure to light the bottle was placed in the dark. The change in the two bottles then proceeded at about the same rate. The specimens contracted very slowly, and the nitrous vapours disappeared gradually. At present, $2\frac{1}{4}$ years after commencement of the experiment, the specimen which has been exposed to light is scarcely as much changed as the one which after a time was placed in the dark; both have contracted to about one-half their original volumes, but have preserved their normal appearance; a few very minute crystals (probably oxalic acid) are perceptible upon the sides of the bottle which has been kept in the dark.

The fourth sample (prepared at Waltham Abbey in 1863) has resisted change to a remarkable extent. The vapours were at first entirely absorbed, and the gun-cotton has become slightly bleached. A faint orange tinge was first observable in the bottle after it had been exposed to strong daylight for six months. Nitrous vapours were then slowly evolved until the depth of colour was somewhat considerable. After the lapse of several months they gradually diminished again, and ultimately disappeared once more, after about eighteen months' exposure. After the lapse of $2\frac{1}{4}$ years the gun-cotton has contracted only slightly, but exhibits no other signs of change. There can be no doubt, however, judging from this contraction and from the evolution of vapours at one period of exposure, that this sample has suffered change which would not have occurred had it been exposed to light under ordinary conditions.

The experiments in sealed tubes which have been described,—the results of examination of gases collected from gun-cotton which has been exposed to heat for long periods in contact with them,—and the general existence of nitric acid in samples of decomposed gun-cotton, appear to show that the first effect of exposure of the ordinary material to sufficient heat is the disengagement of pernitric oxide and the production of water, by which the former is converted into the nitric and nitrous acids; the latter, if allowed to remain in contact with the heated gun-cotton, is gradually reduced to nitric oxide, and finally the nitrogen becomes deoxidized at the expense of hydrogen and carbon, oxalic and carbonic acids being eventually furnished by the latter. The nitric acid produced attacks the gun-cotton at the same time; the presence of very small quantities of this substance in gun-cotton greatly accelerates the decomposition of the material by heat.

Experiment 121.—One drop of concentrated nitric acid introduced into vessels containing 2 or 3 grms. of gun-cotton, invariably brought about rapid decomposition at comparatively low temperatures (55° to 65°); and by operating at temperatures between 70° and 100° with small samples of gun-cotton to which as small a quantity as possible of dilute nitric acid was added, they were in this way generally converted into the gum-

like product, in a few hours, though instances occurred occasionally in which, even under these severe conditions, the material resisted decomposition to a remarkable extent.

In many experiments instituted with ordinary gun-cotton at 90° and 100° it was found that, if the mode of operating were such as to facilitate the escape from the apparatus of any acid vapours evolved, the gun-cotton would frequently resist decomposition in a remarkable manner, being only very gradually converted into the final products; the two following experiments demonstrate how greatly decomposition at 100° C. can be retarded by impeding the destructive action of acid generated by the exposure to a high temperature.

Experiment 121*.—Weighed quantities (about 1·5 grm.) of dry gun-cotton were loosely packed into two narrow U-tubes, which were immersed in a water-bath and connected with an aspirator. The gun-cotton was in both instances maintained during the day at 100° C. A moderately rapid current of air was passed through one tube, and air was allowed to circulate very slowly through the other. At night both tubes were closed up with corks. Their weight was determined at intervals; the following were the results obtained:—

| Duration of exposure. | Loss sustained by gun-cotton exposed to slow current. | Loss sustained by gun-cotton exposed to rapid current. |
|---|---|---|
| 24 hours (4 days) . . . . . | 17·47 per cent. | 4·8 per cent. |
| 40 hours additional (7 days) . | 32·43 „ | 4·12 „ |
| 196 hours additional (28 days). | —— | 5·04 „ |
| Total loss of weight, in 11 days | 50·90 „ | In 39 days 13·96 „ |

The results of these and the preceding experiment warrant the conclusion, that as soon as acid becomes liberated in gun-cotton changes are developed in the material which would not be brought about by its simple exposure to heat, provided no free acid were present in it. If therefore it be possible to neutralize, at the instant of its liberation, any acid which may be produced by the effect of elevated temperatures upon the comparatively unstable impurities contained in small quantities in gun-cotton, the latter might be expected to resist alteration under circumstances which, if the first acidity were not counteracted, must determine the decomposition of the material.

Reference has been made in this paper, on more than one occasion, to the influence which certain mineral impurities of general occurrence in gun-cotton (earthy carbonates) were observed to exert upon the rapidity with which the substance sustained alteration, upon exposure to heat. Some samples, which were heated for comparatively considerable periods without exhibiting signs of change, were found to contain much larger proportions of calcic and magnesian carbonates than specimens with which, in other respects, they were identical.

This observation led, at an early stage of these investigations, to careful observations of the comparative effects of high temperatures (100° and 90° C.) upon a variety of samples, which were known to vary as regards the proportions of earthy and alkaline carbonates distributed through them. Some specimens were repeatedly rinsed in distilled

water (whereby the larger proportion of earthy carbonates attached to the fibre was removed) and exposed to heat in comparison with corresponding specimens not thus treated. Some other samples were extracted with dilute acid and washed; but as acetic acid was used in most of those experiments, the results of their exposure to heat (experiments 90–93) were not of the nature anticipated, for the reason, as already explained, that the treatment in question removed not merely carbonates but also a proportion of the organic impurities. The extraction of the gun-cotton with dilute hydrochloric acid does not remove organic impurities; and, as has already been shown, this treatment has the effect of rendering ordinary gun-cotton more susceptible to the destructive effects of high temperatures.

A considerable quantity of Waltham gun-cotton, after having been purified in the usual way, was saturated with a solution of sodic carbonate of sufficient strength to deposit from 0·4 to 0·5 per cent. of the salt in the gun-cotton, after the latter had been expressed in the hydro-extractor and dried. Portions of the material thus prepared were carefully washed out in distilled water and submitted to heat in comparison with corresponding samples of the "alkalized" gun-cotton. The washing process did not merely extract the sodic carbonate, it also effected the mechanical removal of a large proportion of the earthy carbonates deposited upon the gun-cotton during the long-continued washing in spring- or river-water.

The results observed in this series of experiments are given in the two following Tables.

TABLE VII.

| No. of experiment. | Description of gun-cotton. | Quantity employed. | Total exposure to 100° C. | Interval between first exposure and first signs of decomposition. | Loss of weight sustained by the gun-cotton. | Temperature observations. | Other observations. |
|---|---|---|---|---|---|---|---|
| | | grms. | | | | | |
| 122 | Fine yarn, Waltham, impregnated with 0·4 per cent. of sodic carbonate. | 22 | 11 hours in 3 days (1½ hour on the 3rd day). | 7 hours ..... | Not observed. | During 4½ hours' heating on the 1st day no indications of a rise of temperature were obtained. After 2 hours' heating on the 2nd day, the temperature rose very slowly, reaching 106° C. by the close of the day's experiment; an interval of 1 day elapsed before the experiment was resumed. Upon exposure to heat on the 3rd day, the temperature speedily exceeded 100°; in 1½ hour it had reached 113° C., and soon afterwards the gun-cotton *exploded*. | The coloration of the atmosphere in the vessel was only very faint until after 3 hours' treatment on the 2nd day, and the decomposition was very gradual when compared with that in experiments made with corresponding quantities of ordinary gun-cotton. |
| 123 | The same description as used in experiment 122. | 22 | 16½ hours in 3 days (2½ hours on 3rd day). | 8 hours ..... | 2·6 per cent. in 14 hours (2 days). | At the close of the 1st day (after 6½ hours) the temperature rose slowly to 102° C.; on the 2nd day it rose gradually to the same temperature and remained stationary during 6 hours. On the 3rd day it rose gradually to 104°·5, and remained stationary for 1½ hour; the thermometer then rose somewhat rapidly to 113° C., and the gun-cotton *exploded* almost immediately afterwards. | The coloration of the atmosphere in the flask was very faint on the first day, and there was only a slight increase in colour until the thermometer passed 104° C. on the 3rd day. |

TABLE VII. (continued.)

| No. of experiment. | Description of gun-cotton. | Quantity employed. | Total exposure to 100° C. | Interval between first exposure and first signs of decomposition. | Loss of weight sustained by the gun-cotton. | Temperature observations. | Other observations. |
|---|---|---|---|---|---|---|---|
| | | grms. | | | | | |
| 124* | The same as in experiment 122. | 6·5 | 16 hours in 3 days. | None observed. | None sustained. | No increase of temperature indicated. | It was observed about 1 hour after commencing the experiment that a very small quantity of water had penetrated into the flask through the cork, which had been accidentally immersed in the water of the bath for a short time. The gun-cotton remained perfectly unchanged; it was dried at the close of the experiment, and found to have sustained no loss in weight. |
| 125 | The same as in experiment 122. | 6·5 | 22 hours in 4 days. | 7 hours ..... | 14 per c. after 3rd day, 4·5 per c. after 4th day, total loss 18·5 per c. | No rise of temperature on the 1st day. After 1 hour's heating on the 2nd day, the thermometer indicated 101°, and remained stationary throughout the day. On the 3rd day the thermometer rose to 104° in 1 hour, but had fallen again to 100° in 15 minutes; no further change occurred up to termination of the experiment. | The nitrous-acid vapours were only faint up to the rise of temperature on the 3rd day, and were never abundant. The colour of the gun-cotton was not altered. It had an acid reaction and slightly pungent odour after the treatment; its explosiveness did not appear diminished. The aqueous extract gave a faint indication of nitric acid, did not reduce cupric salts, nor furnish a precipitate with calcic chloride. The solubility of the gun-cotton in ether and alcohol had notably increased. |
| 126 | The same as experiment 122, but washed repeatedly in distilled water. This treatment not only extracted the alkali, but separated much of the earthy carbonates attached to the fibre. | 6·5 | 1h 10m ..... | 50 minutes .... | Not observed.. | After 45 minutes' heating the temperature rose rapidly; 116° was indicated shortly before the sample *exploded*. | The results of these two experiments should be compared with those obtained in the preceding experiment, and with those furnished in experiment 29 Table I. by 6·5 grms. of the same sample of gun-cotton, which had not been impregnated with alkali, nor submitted to the extra-washing process. |
| 127† | | 6·5 | 1h 15m ..... | 1 hour ..... | " " | At the close of the 1st hour's heating the temperature rose rapidly. In 10 minutes it had reached 113°, and in 5 minutes more it was 118°; the gun-cotton *exploded* almost immediately afterwards. | |
| 128 | Coarse yarn, Waltham, impregnated with 0·5 per cent. of sodic carbonate. | 6·5 | 22 hours in 4 days. | 10 hours ..... | " " | The first rise in temperature was observed after 3 hours' heating on the 3rd day. The thermometer rose gradually to 110°, but soon began to fall again, and had returned to 100° before the close of the 3rd day. No change occurred on the 4th day. | Nitrous acid was abundantly evolved for a short time on the 3rd day. The gun-cotton after the experiment had a pale brownish colour, but the small portion surrounding the upper part of the thermometer had become blackened and hard, in consequence of the destructive action exerted by the moisture charged with acid, which condensed upon the thermometer-stem and fell upon that part of the sample. The aqueous extract contained nitric acid, no oxalic acid, and reduced cupric salts slightly. The gun-cotton dissolved to a large extent in ether and alcohol. |

* This experiment furnished an important indication of the preservative effect of moisture. The gun-cotton was not wet but only thoroughly damp.

† A third experiment, corresponding to these two, furnished precisely similar results.

TABLE VII. (continued.)

| No. of experiment. | Description of gun-cotton. | Quantity employed. | Total exposure to 100° C. | Interval between first exposure and first signs of decomposition. | Loss of weight sustained by the gun-cotton. | Temperature observations. | Other observations. |
|---|---|---|---|---|---|---|---|
| | | grms. | | | | | |
| 129 | The same as experiment 128, but washed repeatedly in distilled water. | 6·5 | 20 hours in 3 days. | 4 hours ..... | Not observed.. | The temperature began to rise on the 1st day after 4 hours' heating. It rose to 109° in 30 minutes, and remained stationary till the close of the day. On the 2nd day it rose gradually to 104°, and after remaining stationary for some time, gradually returned to 100°. No change observed on the 3rd day. | Nitrous vapours were very abundant after 5 hours' heating during 1 hour. The gun-cotton assumed a brown colour. The aqueous extract contained nitric acid, a little oxalic acid, and reduced cupric salts abundantly. The insoluble portion was feebly explosive, and almost perfectly soluble in ether and alcohol. |
| 130 | The same as experiment 129, but very carefully washed in distilled water. | 6·5 | 22 hours in 4 days. | 2h 30m ..... | " " | After 2 hours' heating the temperature rose, and had reached 103° by the close of the 3rd hour, and 109° in another hour. On the 2nd day the thermometer rose to 102° in 30 minutes after commencing, and reached 106° in 15 minutes more. It soon afterwards began to fall, and had returned to 100° by the close of the 6th hour. No rise of temperature occurred on the 3rd and 4th days. | The substance had contracted considerably, was brown and pulverulent, soluble to a considerable extent in water, the solution exhibiting the usual reactions. |
| 131 | Medium-size yarn, Waltham. | 6·5 | 21 hours in 3 days. | 3 hours ..... | 20 per cent.... | The temperature began to increase slightly at the close of the 3rd hour; the maximum temperature 110° was reached in 3h 50m. The thermometer then soon began to fall, and after nearly 7 hours' total heating, had returned to 100°. No change of temperature on the 2nd and 3rd days. | The gun-cotton became dissolved on the 2nd day. At the conclusion it was brown and pulverulent, partially soluble in water; the solution contained a minute quantity of oxalic acid, and reduced cupric salts abundantly. The residue dissolved in ether and alcohol, furnishing a horny brittle substance, which deflagrated when heated. |
| 132 | Same as experiment 131, but carefully washed in distilled water. | 6·5 | 21 hours in 3 days. | 1h 30m ..... | 31 per cent.... | The temperature rose gradually above 100° after 1¼ hour's heating. In 2h 50m it had reached 111°; the thermometer fell slowly soon afterwards; in 3h 30m from the commencement it had returned to 105°, and indicated 100° in 6 hours after commencement. No change on the 2nd and 3rd days. | The substance had sustained the same alteration as in the preceding experiment, but was soluble to a larger extent in water. |
| 133 | Same as experiment 131, but extracted with dilute acid and afterwards carefully washed. | 6·5 | 21 hours in 3 days. | 1 hour ..... | 34·6 per cent.. | The temperature began to rise before expiration of the 1st hour. In 2h 15m it had reached 110°; the thermometer then fell and returned to 100° by the close of the 1st day's experiment. | The colour of the product was deeper than the two preceding, and a very large proportion was soluble in water. |
| 134 | Medium-size yarn, another sample. | 6·5 | 20 hours in 3 days. | 2 hours ..... | 20·3 per cent.. | After 2 hours' heating the temperature rose gradually and reached 111°·5 at the expiration of the 3rd hour. It then returned slowly to 100° by the close of the 1st day's heating. No change on the 2nd and 3rd day. | Both samples had assumed a buff colour, were strongly acid and friable. The unwashed gun-cotton furnished the largest proportion soluble in water. The aqueous solutions furnished the usual reactions. |
| 135 | Same as experiment 134, but repeatedly soaked and washed in distilled water. | 6·5 | 20 hours in 3 days. | 1h 45m ..... | Not recorded.. | The temperature began to rise after the lapse of 1h 40m; in 3h 7m it had reached 118°·5. In another hour it had fallen to 103°·5, and it returned to 100° by the close of the 1st day. No subsequent rise. | |

TABLE VII. (continued.)

| No. of experiment. | Description of gun-cotton. | Quantity employed. | Total exposure to 100° C. | Interval between first exposure and first signs of decomposition. | Loss of weight sustained by the gun-cotton. | Temperature observations. | Other observations. |
|---|---|---|---|---|---|---|---|
| | | grms. | | | | | |
| 136 | Coarse yarn, "silicated". | 6·5 | 22 hours in 4 days. | 3h 25m ..... | 24 per cent.... | After 3h 30m the thermometer began to rise. In 4 hours it indicated 107°, and in 4½ hours 110°. It soon afterwards began to fall, returning to 107° by the close of the 5th hour. On the 2nd day it rose to 104°·5 in 30 minutes, but soon fell again slowly to 100° after the lapse of 4 hours. No further change. | The sample had sustained the usual changes, but did not yield a very large proportion of soluble matter to water. |
| 137 | Coarse yarn, "not silicated". | 6·5 | 1 hour, exploded. | 30 minutes .... | Not observed.. | The temperature began to rise in 35 minutes; it reached 107° in 55 minutes, rising very rapidly. | This sample was of the same date of manufacture as the silicated specimen used in experiment 15. |

TABLE VIII.

| No. of experiment. | Nature of gun-cotton. | Quantity employed. | Duration of exposure to 90° C. | First indication of decomposition after commencement of experiment. | Loss of weight. | Other observations. |
|---|---|---|---|---|---|---|
| | | grms. | | | | |
| 138 | Fine yarn, Waltham, impregnated with 0·5 per cent. of sodic carbonate. | 8·5 | 36 hours in 6 days. | 26 hours; 6th day. | In 6 days; 2·22 per c. | A very small quantity of carbonic acid escaped from the flask on the 2nd day; on the 3rd day the quantity was a little more considerable. The temperature having accidentally risen to 95° on the 6th day, nitrous acid was for the first time disengaged. At close of the experiment, the gun-cotton was perfectly white; it had an acid reaction, but the strength of fibre and explosiveness were scarcely affected. |
| 139 | Coarse yarn, Waltham, impregnated with 0·4 per cent. of sodic carbonate. | 3 | 77 hours in 12 days. | 9 hours; very *faint*. | In 6 days; 3·9 per cent. | The nitrous vapours were never disengaged abundantly. The gun-cotton became of a buff colour by the close of the experiment; it was friable and soluble to a considerable extent in water, and in ether and alcohol. |
| 140 | Fine yarn, containing 0·5 per cent. of sodic carbonate. | 2 | 32 hours in 6 days. | 30 hours; 6th day, very *faint*. | In 6 days; 2·73 per c. | The nitrous vapours were never more than faint. The gun-cotton furnished a slightly acid aqueous extract, in which neither nitric nor oxalic acids could be detected, and which did not reduce cupric salts. The solubility in ether and alcohol was scarcely affected. |
| 141 | Coarse yarn, containing 0·4 per cent. of sodic carbonate. | 2 | 32 hours in 6 days. | 15 hours; 3rd day, very *faint*. | In 6 days; 1·17 per c. | The gun-cotton was almost odourless, only very faintly acid, and exhibited no change in explosiveness or other properties. |
| 142 | Same as experiment 4, but repeatedly washed in distilled water. | 4·5 | 32 hours in 6 days. | 1 hour..... | In 6 days; 40·6 per c. | Nitrous acid was abundantly evolved 1 hour after commencement of experiment. The gun-cotton became brown, friable, soluble to a considerable extent in water, and in ether and alcohol. |
| 143 | Same as experiment 3, but repeatedly washed in distilled water. | 2 | 32 hours in 6 days. | 3 hours. | In 6 days; 10·5 per c. | The gun-cotton was strongly acid, and a small proportion was dissolved in water, the solution reducing cupric salts; the proportion soluble in ether and alcohol had considerably increased. |

It will be seen that, in the experiments instituted at 100° C., with very considerable quantities of the "alkalized" gun-cotton (experiments 122 and 123) seven and eight hours elapsed before any symptom of decomposition was observed, while in operating with similar quantities of ordinary gun-cotton (experiments 26 and 27) decomposition became manifest in two hours and four hours. A comparison of experiment 27 with experiments 122 and 123, also shows that when once the gun-cotton began to undergo

considerable change, the decomposition proceeded much more rapidly in that material than in the alkalized samples.

The influence of the small proportion of alkaline carbonate in retarding the decomposition of the gun-cotton becomes still more evident when smaller quantities of the material are experimented with. It is well illustrated by the results of experiment 29 (Table I.), and those furnished by an equal quantity of the same gun-cotton impregnated with 0·4 per cent. of sodic carbonate. The former exhibited the first indications of change in three hours, the latter in seven hours, after first exposure. The temperature of the ordinary gun-cotton afterwards rose much more rapidly and considerably, and the loss of weight sustained by it in three days' exposure was nearly double that which the "alkalized" sample suffered in four days' treatment, and the alteration sustained by the latter specimen was comparatively slight.

But the protective power exerted by small proportions of carbonates is even more strikingly demonstrated by a comparison of experiment 125 with experiments 126 and 127. While the "alkalized" sample exhibited no signs of change until after the lapse of seven hours, equal quantities of the same sample, purified from carbonates by washing, exploded after exposure to 100° for 1·25 hour. Experiment 128, made with another description of gun-cotton, also impregnated with a small proportion of sodic carbonate, is similarly illustrative of this protective power when compared with experiment 28, Table I., and with experiments 129 and 130, conducted with equal quantities of the original gun-cotton and with the same material freed from carbonates by washing. The comparison of experiment 131 with 132, and of 134 with 135, affords proof that the small proportion of earthy carbonates ordinarily existing in gun-cotton exert a decided protective action, and experiment 133, conducted with a specimen from which these carbonates had been completely extracted by acid treatment, furnished further confirmation of this point.

In the experiments instituted at 90° C., Nos. 138–141, conducted with samples of "alkalized" gun-cotton, also afford important proof of the protective effect of small quantities of carbonates, upon comparing the results with those furnished by experiments 142 and 143. The first of these samples scarcely sustained any alteration by exposure for thirty-two to thirty-six hours to 90° C. during six days, while the specimens of ordinary gun-cotton were considerably altered by similar treatment.

Experiments 136 and 137 afforded a very decided proof that the silicating process prescribed by Von Lenk exerts some amount of protective influence upon gun-cotton when exposed to heat, though this result is not due, as supposed by him, to the closing up of the fibre by an insoluble silicate, but simply to the deposition of a small quantity of earthy (and possibly of alkaline) carbonate upon the fibre when the silicate undergoes decomposition during the drying and the subsequent washing process. The amount of protection thus afforded to the gun-cotton is, however, obviously as liable to variation as that resulting from the deposition of calcic and magnesian carbonates upon the material during the long-continued immersion in flowing water. Numerous specimens of gun-

cotton which had been silicated, varied greatly in their powers of resisting change at high temperatures, but they were invariably rendered decidedly more prone to change if thoroughly washed in distilled water previous to their exposure to heat.

It is remarkable how very small a proportion of a carbonate deposited upon the fibre of gun-cotton, exerts a notable influence upon its power of resisting the effects of heat. Thus, a portion of a sample of gun-cotton which had been carefully freed from carbonates, was saturated with perfectly clear lime-water, wrung out and dried. Upon exposure to 100° in comparison with an equal weight of the sample purified from carbonates, the very small quantity of calcic carbonate which had been deposited upon the gun-cotton proved sufficient to delay to a notable extent the period of first decomposition, and to modify somewhat the results of change produced by exposure for a definite period at 100° C.

In experiment 138, conducted at 90° C., the alkalized gun-cotton was heated in a flask to which a delivery-tube was attached, and the gas which escaped was examined. It was then observed that, upon the second day's exposure, a very small quantity of carbonic acid was continuously evolved; that the quantity increased somewhat upon the third day; and that no nitrous vapours escaped until the sixth day, when the temperature was accidentally raised to 95°. Carbonic acid was then still evolved in small quantity. Similar evidence of the slow decomposition of the carbonate, which always preceded any disengagement of nitrous acid, was obtained in other experiments.

Some experiments were instituted in *sealed tubes* with the gun-cotton impregnated with 0·4 per cent. of sodic carbonate.

Experiment 144.—A specimen was enclosed in a tube with air at the atmospheric pressure, and exposed to 100° C. for $3\frac{3}{4}$ hours, on the first day, during which period no trace of nitrous vapour was visible. It was afterwards heated to 100° six hours daily for thirteen days, and no coloration of the air in the tube was observed at any time during this treatment. After four days' exposure, the tube was opened and gas escaped under somewhat considerable pressure. The tube was sealed up again, and at the close of the experiment it was opened once more, when gas escaped only under slight pressure. The specimen became discoloured in a few places after some time (which is invariably the case when gun-cotton containing alkaline matter is exposed to a high temperature), but exhibited no other signs of change. In similar experiments conducted with ordinary gun-cotton, deep nitrous vapours were observed in the tubes within three hours from the commencement of the experiment, and the material was always converted into a gum-like mass; in some instances the tube exploded violently after a time.

Experiment 145.—An experiment similar to the preceding was instituted with alkalized gun-cotton in a more closely packed condition, the tube being exhausted, filled with nitrogen, and re-exhausted before sealing. This tube was heated to 100° for ten hours (in two days), and afterwards left exposed to light for twenty-four hours, without the slightest coloration by nitrous vapours being observed. On being again heated,

very faint nitrous vapours were observed after two hours' exposure; these did not increase at all, but disappeared entirely after continuation of the heating for four hours longer. When the tube had been heated for six days the point was opened, and gas escaped under considerable pressure. The tube was again closed and heated for nine days, six hours daily; upon afterwards opening it, gas escaped only under slight pressure. It was again heated to 100° for seven days, when the tube was accidentally fractured by the effects of a neighbouring explosion. At this time the gun-cotton had become darkened in some places and had an acid reaction, but exhibited little other signs of change.

Several experiments were conducted with gun-cotton containing considerably larger proportions of sodic carbonate than the samples previously employed, for the purpose of obtaining data with regard to the progressive changes resulting from the protracted exposure of "alkalized" gun-cotton to heat. The results obtained are fairly represented by the following selected experiments.

Experiment 146.—6·5 grms. of Waltham Abbey gun-cotton were impregnated with 0·45 grm. of sodic carbonate. The air-dry specimen was exposed to 100° (for five to six hours daily), being loosely packed in a narrow-necked flask. Small weighed quantities of the sample (from 0·07 to 0·1 grm.) were removed from the flask for examination each morning before heat was again applied. The darkening, always produced by heating gun-cotton with an alkalized substance, commenced upon the first day's exposure to heat, but no other effect was noticed; after six days' exposure the gun-cotton was still slightly alkaline, the coloration having gradually increased, until some parts exhibited a brown tint. In other respects the substance was unchanged. After the ninth day's heating the gun-cotton was found to be quite neutral. About one-fifth of the specimen was then removed for examination. Water extracted the colouring-matter entirely, and a brown solution was obtained, in which sodic nitrate and nitrite were readily detected. The liquid also reduced cupric salts slightly. The gun-cotton itself had not suffered any change in explosiveness or strength of fibre, nor had the solubility in ether and alcohol increased appreciably.

The exposure of the sample to 100° C. was continued for seven days longer. On the sixth day a small portion was examined and found still to be neutral. On the seventh day, the atmosphere in the flask was found to have an acid reaction, though no nitrous vapours were perceptible.

Upon determining the weight of the gun-cotton (with deduction of the portions used during the experiment), it was found by calculation that the loss which it had sustained during exposure to heat for 100 hours (in sixteen days) was greater than would have been caused by the complete expulsion of carbonic acid from the carbonate employed. The aqueous extract was highly coloured, the gun-cotton being left almost colourless. Sodic nitrate and nitrite existed plentifully in the liquid, and the latter reduced cupric oxide in an alkaline solution. The washed gun-cotton was found, as might have been anti-

cipated, to have sustained a greater loss than would have been caused by the extraction of the alkaline base entirely in the form of nitrate. The gun-cotton soluble in ether and alcohol now contained in the specimen amounted to 6·98 per cent.; the material originally contained 2·3 per cent., the increase amounted therefore only to 4·68 per cent. The proportion of nitrogen-oxides which would have been liberated by the reduction to that extent of trinitrocellulose even to the lowest of the substitution-products would not have sufficed to decompose the sodic carbonate present. It would appear from these results that the principal effect of the very long-continued exposure of this "alkalized" gun-cotton was to establish a very gradual action of the alkaline carbonate upon the gun-cotton (resulting in the production of glucic acid, &c.), and that even the *first* stage of decomposition (consisting in the reduction to soluble gun-cotton) caused simply by the action of heat upon the ordinary material, only proceeded to a very slight extent during the sixteen days' treatment.

Experiment 147.—6·5 grms. of gun-cotton were impregnated with 0·38 grm. of sodic carbonate. The sample was thoroughly dried and exposed to 100°, as in the preceding experiment. After the lapse of three days a weighed sample of the gun-cotton was examined. It had darkened somewhat, was alkaline, and exhibited no change of properties. After six days' exposure it was still alkaline, and its solubility in ether and alcohol had not increased appreciably; upon the ninth day the sample was found to be neutral. The experiment was then stopped, the gun-cotton was extracted with water, and the proportion of nitrogen-acids existing in it as sodium-salts was determined by means of nascent hydrogen. The result showed that less than four-tenths of the sodic carbonate employed had been neutralized by those acids, the remainder existing in combination with organic acids. Traces of ammonia were evolved during the treatment of the gun-cotton in this and the preceding experiment, and the loss in weight sustained by the material was greater than would have been occasioned by the simple expulsion of carbonic acid from the carbonate. The solubility in ether and alcohol of the sample had only increased to about double the original proportion.

It was of course impossible actually to demonstrate by experiment whether the small proportion of organic acid produced in these experiments, which exhibited the properties of reducing cupric oxide in an alkaline solution, was glucic acid, resulting from the action of the alkali upon the gun-cotton, or whether it consisted of the pectic acids found in the products of spontaneous decomposition; but as abundant proof exists that the latter are only the products of a *secondary* change resulting from the action upon gun-cotton of liberated nitrogen acids (see especially experiments 155 and 156), there appear to be very good grounds for the conclusion that the results observed in these experiments were mainly ascribable to the action of the alkaline carbonate upon the gun-cotton and the organic impurities present, and that the effects exclusively due to the protracted exposure of the substance to 100° were limited to the liberation of a very small proportion of nitrogen-acid, which was at once neutralized, the only change produced in the gun-cotton consisting therefore in the decomposition of the small quantities of compara-

tively unstable organic impurities, and in a slight increase of the proportion of soluble gun-cotton*.

The power possessed by carbonates to prevent or arrest the decomposition of gun-cotton when exposed to high temperatures, has been demonstrated in a striking manner by some experiments which have, at the same time, furnished evidence in support of the conclusion that the organic impurities contained in gun-cotton constitute the primary cause of its susceptibility to change under the influence of heat and light.

Experiment 148.—Specimens of gun-cotton were exposed in flasks to 90° and 100° until decomposition was established to such an extent that the vessels were filled with deep-coloured vapours; a small quantity of calcic or magnesian carbonate was then introduced into the flask, or the gun-cotton was removed from the vessel, dusted over with a carbonate, and immediately replaced. Decomposition was at once arrested by these means; moreover, the gun-cotton no longer exhibited any susceptibility of decomposition even if exposed to 90° and 100° for several successive days.

Experiment 149.—About 5 grms. of ordinary gun-cotton were dusted over with finely pulverized potassic bicarbonate† and exposed to about 95° in a long-necked flask, side by side with a corresponding quantity of the same specimen of gun-cotton in its ordinary condition. The "alkalized" gun-cotton gradually darkened upon the second day's exposure, eventually assuming a brown colour. After exposure to the above temperature during five days for seven hours daily, the unprepared gun-cotton began to evolve nitrous vapours abundantly. The examination of a small specimen showed that the sample had undergone very little change, the solubility in ether and alcohol having slightly increased.

It was now removed from the flask, dusted over with the powdered carbonate, and immediately re-exposed to heat. All decomposition ceased, the gun-cotton behaving exactly like the specimen which was originally alkalized, except that the usual darkening took place very slowly indeed. When it had been maintained at 95°–100°, seven hours daily for eight days, it was still alkaline; after treatment with water, it exhibited all the original properties of the gun-cotton; the solubility in ether and alcohol had undergone no increase since the alkaline salt was applied‡.

The other specimen to which the carbonate had been applied in the first instance, was examined after exposure to 95° for seven days. The aqueous extract had a brownish colour, and reduced cupric salt to a very slight extent. The extracted gun-cotton was perfectly white; the strength of fibre, explosiveness, and solubility in ether and alcohol had undergone no change. It was now dried and again exposed in a clean flask to 95°–

* The analysis of a specimen of soluble gun-cotton into which the ordinary material had been entirely converted by the gradual action of heat, has furnished results which appear to indicate that trinitrocellulose may become soluble in ether and alcohol without undergoing any modification in its ultimate composition. This point is now being made the subject of further investigation.

† The object of employing this salt was to exclude any protective effect which might be exerted in the course of the experiment by the absorption of moisture, if a calcic, magnesian, or sodic carbonate were employed, and a small quantity of the corresponding nitrate were produced.

‡ The exposure of this sample to 100° was afterwards continued for three weeks without any effect.—*June* 1867.

100°. During six days it has not exhibited the slightest indication of decomposition; no acidity has yet been developed in it, and it is still unchanged in all its properties*.

It would appear from these experiments that a carbonate applied in the most simple manner to gun-cotton in which decomposition has been established, will effectually arrest the change; and that if gun-cotton containing a carbonate be exposed for some time to heat, the latter promotes the transformation of the unstable organic impurities into products fixed by the base, the result being that the gun-cotton undergoes a searching purification from these substances, and afterwards exhibits, in consequence, remarkable stability under very severe conditions of exposure to heat.

A number of experiments has been instituted on the comparative effects of exposure to 90° and 60° C. of samples of gun-cotton impregnated with different proportions of alkaline carbonates ranging from 1 to 10 per cent., the object being to ascertain whether the first-named proportion would suffice to afford security against the development of free acid in the gun-cotton, even under conditions of exposure to heat much more severe than could ever be met with in actual practice, or to determine what proportion might be necessary for that purpose. The results of these experiments showed that 1 per cent. of sodic carbonate, uniformly distributed through gun-cotton, produced but very little darkening effect upon the material, even when the latter was exposed seven hours daily for several weeks (three weeks and upwards) to 95° or 100°, and that no free acid was developed by a continuation of exposure to 95°–100° for three weeks. Gun-cotton containing 2 per cent. of the carbonate did not change colour to a much greater extent, and exhibited still a distinct alkaline reaction after exposure to 95°–100° for four weeks. Samples containing from 4 per cent. upwards of alkaline carbonate darkened very considerably upon long-continued exposure to heat; and a trifling loss in weight was sustained by them, in instances when the treatment was continued between four and five months. This loss somewhat exceeded that which would have been simply occasioned by expulsion of the entire carbonic acid contained in the sodic salt, and furnished evidence of the escape of small proportions of volatile matter. The strength of the fibre was not appreciably affected even in the experiments continued for a considerable period at 90°–100° with the samples containing the highest proportion of carbonates. The colouring matter produced by the action of the alkali was entirely extracted by water; and the properties of the gun-cotton were unchanged.

It therefore appears that, although the presence of somewhat considerable proportions of alkaline carbonate (even as much as 10 per cent.) in gun-cotton does not exert any important action which can be pronounced prejudicial even under very severe conditions of exposure to heat, a proportion as low as 1 per cent. suffices to protect the material, for a longer period than ever could occur in actual practice, from the destructive action of such acid as may be liberated by the decomposition of the organic impurities or by

* The exposure of this sample to 100° was continued for a total period of thirty days, after extraction of the carbonate, without any change resulting. A portion of the specimen was then exposed, seven hours daily for three days, to 115°, and it did not exhibit any symptom of change until the close of the third day.—*June* 1867.

the very gradual effect of a high temperatures (90°–100°) upon the pure gun-cotton. The introduction of considerable quantities of saline matter into gun-cotton necessarily gives rise to the production of smoke and to some deposition of solid residue, upon the explosion of the substance, and although the amount of both these products would then still be very trifling as compared with those of a corresponding character resulting from the explosion of gunpowder, it is inadvisable that they should be unnecessarily increased. It therefore appears preferable to limit the extent of impregnation of gun-cotton with sodic carbonate to 1 per cent.* It has been abundantly demonstrated by the experiments detailed and by one instituted upon a more considerable scale, to be presently described, that even the introduction of one-half that proportion of sodic carbonate into gun-cotton serves to afford it sufficient protection under conditions of exposure to heat exceeding in severity and duration any which the material would have to encounter if substituted for gunpowder in all directions.

III. *Protective action of Water.*—In one of the earlier experiments on the effects of exposure of gun-cotton to 100° (experiment 124, Table VII.), it was found that the accidental introduction of a very small quantity of water into the vessel containing the gun-cotton, afforded most perfect protection to the material, which exhibited no signs of change during sixteen hours' exposure to 100°, and had not sustained any loss in weight at the close of the experiment. This power possessed by water (or aqueous vapour) of preserving gun-cotton from decomposition at high temperatures† is remarkably at variance with the influence exerted by moisture, if confined together with gun-cotton under protracted exposure to bright daylight and sunlight, in which case there appears no doubt, from the results which have been described, that the aqueous vapour operates in determining to a slight extent the decomposition of the material.

Further illustrations, though less striking than the one above quoted, were furnished of the protective effect of aqueous vapour, by the comparative tardiness with which certain samples of gun-cotton containing more than the ordinary proportion of hygroscopic moisture underwent change by exposure to high temperatures (*vide* experiments 38, 39, and 40). This preservative power of water has received the fullest demonstration from the results of a considerable number of experiments, the nature of which is fairly represented by the following examples.

Experiment 150.—A hank of gun-cotton was suspended in the upper part of a capacious flask containing distilled water tinted with litmus. The water was maintained in rapid ebullition for several hours, the greater portion of the steam condensed in the neck of the flask and upon the gun-cotton, returning to the body of water. At the conclusion of the experiment the tint of the litmus (compared with a standard) had not been affected in the slightest, and the gun-cotton was perfectly neutral and unaltered.

* It is scarcely necessary to observe that this carbonate is selected for introduction into the gun-cotton because, while its solubility affords the means of its uniform distribution through a mass of material, it possesses no tendency to increase the hygroscopic properties of the latter.

† See also pp. 199 and 200.

Experiment 151.—A long wide glass tube was loosely filled with gun-cotton. One extremity was drawn out to a beak which was immersed in water tinted with litmus, the other end was connected with a small boiler from which a rapid current of steam was passed over the gun-cotton, uninterruptedly, seven hours daily for three days. Not the slightest alteration was produced in the colour of the litmus, and the gun-cotton was unchanged.

Experiment 152.—A stout glass tube, closed at one end, was partly filled with gun-cotton; sufficient water was introduced to cover the latter, and the tube was then sealed and exposed to 100° seven hours daily for six days. When the tube was opened no gas escaped, the water was not acid, and the gun-cotton exhibited no signs of alteration. The open tube was afterwards exposed to strong daylight and sunlight; after the lapse of eight months the gun-cotton was found to have a very faint acid reaction, and a minute trace of nitric acid was detected in the water. The proportion of matter soluble in ether and alcohol had very slightly increased.

Experiment 153.—Some gun-cotton was saturated with water, which was afterwards expressed to such an extent that the specimen was difficultly combustible when held in a flame. In this condition the sample was exposed in a sealed tube to 100° seven hours daily for twenty-four days. When the tube was opened, no gas issued from it; the gun-cotton exhibited a very faint acid reaction, but no other signs of change. The open tube containing the moist specimen was afterwards exposed to strong daylight and sunlight for six months; the gun-cotton was then found to have a decided acid reaction. It was digested with a small quantity of water; the aqueous extract was acid to test-paper but not to the taste; nitric acid was detected in it, but no oxalic acid; potassic hydrate imparted to it a faint yellowish tinge, and the alkaline liquid reduced cupric salts to a slight extent. The proportion of matter extracted by ether and alcohol was about double the amount originally existing in the sample. The strength of fibre was unaltered, and there was no appreciable diminution in the explosiveness of the gun-cotton.

Experiment 154.—7 grms. of gun-cotton were thoroughly moistened by being suspended for some time in an atmosphere of steam; the sample was then placed in a flask the sides of which were previously moistened. This flask was connected by a delivery-tube with another containing water, and was also fitted with a straight narrow glass tube. A small quantity of steam was passed into the flask from time to time as the moisture became partially expelled during the experiment. After three days' exposure to 100° (six hours daily), a weighed sample was removed for examination. The gun-cotton had become slightly discoloured in a few places, where it was in immediate contact with the sides of the flask. Water extracted a minute quantity of colouring-matter; no other effect of the exposure to heat was observed. The gun-cotton was again heated to 100° for five days (six hours daily); it was then removed, the flask dried and weighed. By these eight days' exposure to 100° in a moist atmosphere it had sustained a loss of only 1·7 per cent. A weighed sample was again examined, it furnished a very faint indication of acidity. The reactions of nitric acid could not be

obtained with the aqueous extract; the proportion of matter extractable by ether and alcohol had very slightly increased.

The sample was again moistened, and exposed to 100°, as before, for nine days; at the expiration of this period it had sustained a further loss of 1·01 per cent. The total loss during the seventeen days' treatment amounted therefore to 2·71 per cent. The condition of the sample was now as follows:—it had darkened in a few places where in close contact with the glass, and possessed a faint odour, such as is always observed in gun-cotton which has been stored for some time in a warm locality; its acidity had not increased, but a faint reaction of nitric acid was obtained in the aqueous extract after concentration to a small bulk. A trace of lime was also found in solution (evidently as calcic nitrate produced from carbonate in the sample). The proportion of matter extracted by ether and alcohol amounted to 4·1 per cent.; in its original condition the sample contained 2·3 per cent.

Experiment 155.—6·5 grms. of gun-cotton were saturated with moisture and placed in a flask fitted with a straight narrow glass tube of considerable length, for the purpose of rendering the expulsion of water very gradual. After exposure to 100° six hours daily, for three days, the specimen was still moist. The examination of a weighed sample did not furnish the slightest indications of change. The same negative result attended the examination of a second sample after further exposure of the gun-cotton to 100° for three days. The apparatus now contained but very little moisture; after a renewed exposure for three hours to 100°, a very faint coloration by nitrous vapours was observed in the flask; a sample was examined, but beyond a faint acidity no signs of change were detected. The heat was continued for four hours more on the same day, at the expiration of which the coloration in the flask was somewhat more distinct; but there were no signs of nitrous vapours on the following morning. The indication of change in the gun-cotton was still limited to a very faint acidity. The sample was once more heated for six hours, during which period no trace of moisture was deposited upon the cool portions of the glass. Nitrous vapours appeared again in very small quantity, and did not increase up to the termination of the experiment; but on the following morning the apparatus contained deep-coloured vapours. The gun-cotton was now extracted with water; the liquid contained a small quantity of nitric acid, but did not reduce cupric salt. The washed gun-cotton was almost perfectly soluble in ether and alcohol; the insoluble portion amounted only to 1·25 per cent.

Experiment 156.—6·5 grms. of gun-cotton in an air-dry condition were placed in a capacious flask (the interior surface of which had been previously moistened) fitted with a long narrow glass tube. A piece of litmus was suspended in the neck of the flask. Shortly after the gun-cotton had been first exposed to 100° the litmus gradually assumed a wine-red tint, and when the heating had been continued for five hours, the paper had become bleached. There was no other indication of change. At the expiration of the second day's heating, the small quantity of water which had condensed in the neck of the flask exhibited an acid reaction, and the gun-cotton possessed the peculiar odour

which has repeatedly been referred to. Towards the close of the third day no trace of moisture was visible in the cool portion of the flask; a very small quantity was deposited some distance up the quill-tube. Soon afterwards a faint coloration by nitrous vapours was observed, which had very considerably increased by the following morning. The gun-cotton was then found to be acid, the strength of fibre and explosiveness had both diminished; the aqueous extract contained nitric acid, but not oxalic acid, nor did it reduce cupric salt even when highly concentrated. About three-fourths of the washed gun-cotton dissolved in ether and alcohol, a portion being rapidly dissolved, the remainder more slowly; the more soluble part yielded a tough collodion film*.

The following points of interest and importance are established by the results of these and other similar experiments.

(1) Gun-cotton immersed in water perfectly resists decomposition when exposed for long-continued periods to 100° C.; and this severe treatment has no effect upon the material, even if it is only in a moist condition, or confined in an atmosphere of aqueous vapour. But if the moist specimens are exposed to bright daylight and sunlight for a considerable period, the water or aqueous vapour does not exert the same protective power (see experiments 2 and 4, and p. 192).

(2) If the water or aqueous vapour is allowed to escape during exposure to 100° until the gun-cotton and the atmosphere surrounding it have become almost dry, decomposition commences very gradually; and by arresting the change at a particular period the material is found to be completely reduced to soluble gun-cotton, without the formation of any appreciable amount of the secondary products which result from the action of liberated acid upon the cellulose-products (see experiments 155 and 156).

The perfect protection afforded by moisture to gun-cotton at 100° C., under severe conditions, rendered any experiments in this direction at somewhat lower temperatures unnecessary. The following experiment furnishes, however, interesting confirmation of the results obtained by operating at higher temperatures.

Experiment 157.—69·706 grms. of gun-cotton, in an air-dry condition, and 52·196 grms. of the same sample, soaked in distilled water, which was afterwards expressed as completely as possible, were introduced into large bottles, into which the stoppers were loosely inserted. These were then placed in a water-oven, the temperature of which was continually maintained at 60°–65° day and night. At the expiration of two months both samples were weighed in an air-dry condition. The results indicated a loss of weight of 12·8 per cent. in the dry sample, and of only 0·13 per cent. in the other. After further exposure of the samples as before, for five weeks, the vessel containing the dry one was filled with very deep-coloured vapours, and the experiment was

* This product was submitted to analysis. The proportion of carbon found corresponded more nearly to that contained in trinitrocellulose than to that required by the formula of the next lower cellulose-product (dinitrocellulose). It would appear as though the former had been rendered soluble in ether and alcohol, only a small proportion having suffered reduction. This point is still under investigation.

therefore interrupted, and the samples were again weighed. The dry specimen had sustained a loss of 34 per cent., was quite friable, and had become converted partly into soluble gun-cotton and partly into the products soluble in water. The sample which had been exposed in a moist condition had sustained a total loss of 0·89 per cent., and did not exhibit the slightest signs of acidity. This sample was again submitted in a moist state to a warm atmosphere, ranging from 55° to 65°, day and night for four calendar months. The total loss which it had then sustained after continuous exposure to heat for between seven and eight months, amounted to 1·47 per cent. The gun-cotton exhibited no acid reaction, and the moisture condensed upon the sides of the bottle gave only a faint indication with litmus paper.

Careful observations have been instituted upon the storage of considerable quantities of the material in a wet or merely moist condition. Ordinary gun-cotton has been immersed in sufficient distilled water just to cover it, and has been kept in that condition in closed vessels, with light excluded, for $2\frac{1}{2}$ years. It is perhaps scarcely necessary to state that the material has not sustained the slightest change, and that the distilled water in which it has been preserved is perfectly neutral, the only impurity found in the latter being a small quantity of saline matter extracted from the gun-cotton. A portion of this sample was transferred to a large glass bottle twelve months ago, and has been left exposed to diffused daylight. This difference in the mode of preservation has been quite without effect upon the gun-cotton.

The principal stock of gun-cotton manufactured at Waltham Abbey for experimental purposes, amounting to about 3000 lbs., has been preserved in a moist condition (just as obtained from the centrifugal hydro-extractor) in closed cases until required for use, some of the packages having been kept for about two years; a few, specially set apart for periodical examination, have been preserved for about three years. I have to record no indications of the slightest change except in instances where the gun-cotton had been preserved in close contact with the tinned-copper linings of some gunpowder cases used for its storage. It was found, after some time, that the surfaces of these became oxidized where they were in contact with the moist material, and that this oxidation determined the development of an acid reaction in the gun-cotton, which, however, was, and has continued to be, confined to the portions in immediate contact with the surface of metal*.

Gun-cotton in the condition above referred to contains sufficient water to render it quite uninflammable, it may therefore be preserved with perfect safety in this convenient condition, and may be at any time prepared for use by desiccation. As far as can be

* The readily oxidizable character of tin and the tendency of a metal to establish, by its own oxidation, that of readily oxidizable substances which are in contact with it, are well known. If gunpowder containing even the ordinary proportion of moisture remain in contact with a surface of tin, the metal becomes oxidized and the gunpowder acquires an acid reaction, sulphuric acid being detected after some time. Similarly, some remarkable instances of the oxidation of iron by contact with somewhat damp gunpowder, attended by the production of sulphuric acid, have come under my notice.

concluded from three years' experience, the close packing of gun-cotton in this damp state is not even in the slightest degree injurious to the structure of the fibre, no tendency whatever of the material to become rotten when thus preserved has yet been discovered. On the contrary, most decided evidence has been obtained that gun-cotton when kept in a damp condition is very considerably more permanent than ordinary cotton, or vegetable substances of similar nature. Thus, many hanks of the gun-cotton stored in the damp state were tied with tape and string for purposes of distinction. Upon examining these hanks ten months after they had been packed, the tape was in all instances found to be almost if not entirely destroyed, crumbling away when touched, and being transformed principally into fungoid bodies; the strings were also quite rotten and covered with vegetable growth, but the gun-cotton even in close proximity to them was unaffected. Similar results were observed in the case of a number of samples of gun-cotton which had been packed in a dry condition in paper envelopes and placed in a small very damp chamber. About twelve months after they were stored the paper wrappings and strings were found to be covered with vegetable growth and partly destroyed, while no vestiges of similar growth or other signs of change were detected in the gun-cotton. A wooden reel having some gun-cotton yarn wound upon it which had been kept in the same locality was also found to be covered in all exposed parts with fungoid growth, but the gun-cotton in immediate contact with the latter upon the wood was unaffected and perfectly free from mildew. This specimen has been preserved for another year in a damp atmosphere upon the reel and exposed to light. In some parts the mildew has extended to the gun-cotton immediately in contact with the wood, but the rest of the material is unaffected.

PART IV.—OTHER OBSERVATIONS, INSTITUTED UPON LARGE QUANTITIES OF GUN-COTTON.

A series of observations has been conducted for about three years under my direction and in accordance with a programme approved of by the Committee on Gun-cotton, for the purpose of ascertaining the effects of storage in considerable quantites, under the ordinary atmospheric conditions of this country, and under conditions, as regards temperature, representing the extremes likely to be met with in tropical climates or in warm localities (*e. g.* the magazines of ships). The gun-cotton, which, with these objects in view, was closely packed in large ammunition-boxes, comprised not simply parcels of the material as obtained by following the present system of manufacture, but also others in the production of which modifications had purposely been introduced with the view of determining the influence which might be exerted, by possible accidental departure from one or other of the fixed regulations of manufacture, upon the stability of the material. The following is a summary of the observations made up to the present time.

I. *Storage of Gun-cotton under ordinary conditions of temperature.*

(a) *The gun-cotton being closely packed in a damp condition.*—Attempts have been made to ascertain whether the close packing of gun-cotton in a damp condition would

render it liable to spontaneous heating. Comparative experiments were instituted with gun-cotton, and with cotton-wool in its original unpurified condition. The materials were prepared in two ways; in one experiment they were exposed to an atmosphere saturated with moisture until about 5 or 6 per cent. of water had been absorbed; they were then closely packed in boxes; in another a small portion was moistened (the excess of water being expressed) and packed in the centre of a considerable quantity of dry material. These packages were first occasionally exposed to the sun, they were afterwards preserved for several weeks in a chamber, the atmosphere of which was artificially heated; but in neither instance could any indication of the development of heat be obtained, although the external temperature frequently reached and sometimes exceeded 50°. The inference drawn from these negative results is that gun-cotton is not more liable to spontaneous heating than ordinary cotton-wool. The latter was exposed in a damp and very closely packed condition, in quantities of from ten to thirty pounds, to a heated atmosphere for several months, in order, if possible, to establish spontaneous heating under conditions to which gun-cotton might afterwards be submitted, but the experiments were without result.

The important evidence which has been collected regarding the perfect preservation of damp gun-cotton, when stored under ordinary conditions of temperature, has already been referred to.

(b) *The gun-cotton being closely packed in an ordinarily dry condition.*—Large ammunition-boxes were closely packed with gun-cotton of the following kinds:—

(1) Prepared strictly in accordance with the directions laid down by Von Lenk (*i. e.* including the "silicating" treatment).

(2) The same, made up into cartridges.

(3) Prepared in the ordinary manner, but not "silicated."

(4) Not "silicated," and packed together with a few skeins (1¼ lb.) of gun-cotton which had only been purified by washing in water (the treatment with alkaline water having been omitted).

(5) Not "silicated," and packed together with some gun-cotton soluble in ether and alcohol.

(6) Ordinary gun-cotton impregnated with about 0·3 per cent. of sodic carbonate.

Pieces of litmus paper were placed in different parts of the various packages.

Cases containing Nos. 1, 3, and 6 were packed in July and September 1864, and stored in a dry locality. During the summer months the maximum temperatures recorded in this store room ranged from 16°·5 to 24°. About six months after the boxes were packed, one of each kind was opened for examination. All the samples had a faint peculiar odour like that of pine-wood, which is always developed in the closely packed material, and was most marked in the unsilicated gun-cotton. Some parts of the litmus paper enclosed in the latter had assumed a pink tinge, and where it had been placed between the gun-cotton and the metal surface of the packing case, it was decidedly

reddened in spots; the tin surface, moreover, exhibited slight signs of oxidation in some places where it was in very close contact with the gun-cotton*.

The litmus in the "silicated" and "alkalized" gun-cotton was unaffected except, again, in one or two small places where it had been interposed between the gun-cotton and the metal surfaces. The boxes were closed immediately after inspection and examined eight months afterwards, when they were found quite unaltered; the litmus paper in the centre of the unsilicated gun-cotton had not been at all affected, but, as before, it was slightly reddened when it had been in contact with the metal.

After further preservation for eighteen months the boxes have recently been again inspected; their condition is precisely what it was on the previous examination. The metal surfaces in the boxes containing the gun-cotton not silicated, exhibited more decided indications of oxidation where they have been in close contact with the gun-cotton than in the other boxes.

Storage in a closely packed condition for nearly three years has therefore not at all affected the gun-cotton in these three states. The material which was not silicated has affected, to a slightly greater extent than the others, the metal surfaces with which it has been in close contact.

Some other cases containing portions of the stock of gun-cotton, "silicated" and not silicated, manufactured in the summer of 1863, and preserved since that time ($3\frac{3}{4}$ years) in a closely packed and ordinarily dry condition, have also been recently inspected, and their contents have been found to be as perfect as those of the cases just described.

The cartridges enclosed in serge bags were packed and stored about $2\frac{1}{4}$ years ago. Their condition is unaltered, and the metal cases containing them are perfectly bright.

The cases containing a proportion of imperfectly purified gun-cotton and of soluble gun-cotton (Nos. 1 and 5) were packed early in January 1866. After storage for fifteen months their contents have been examined and found to present no points of difference from the other packages of gun-cotton above described, the only indications of acidity being discovered where the metal surfaces and the gun-cotton were in very close contact.

Steps are now being taken to substitute simple wooden boxes, rendered impervious to moisture, for the metal-lined ammunition cases in which these stores of gun-cotton are now packed, and which have evidently, in all the experiments with large quantities of gun-cotton, constituted an element unfavourable to the stability of the material, the influence of which it is, however, important to have determined. In storing gun-cotton it is obviously as unnecessary as it is inadvisable to employ receptacles of metal.

II. *Exposure of large packages of gun-cotton, in different conditions, to heat.*

(a) *Preliminary experiment.*—A wooden box holding $4\frac{1}{4}$ lbs. of gun-cotton when closely packed, was fitted with a gutta-percha tube intended to receive a thermometer.

* A piece of bright sheet tin which was packed on this occasion in the centre of the gun-cotton was examined after a period of one year and found to have become corroded in spots, the gun-cotton having a slight acid reaction at the places where this effect was produced.

The sides of the tube were perforated in several places, and it passed horizontally through the centre of the box, the openings being closed with corks. The box was painted black, and having been tightly filled with skeins of gun-cotton, amounting to about 4 lbs., it was placed in the open air throughout each day in the month of August 1864, in a position where it would be most frequently exposed to the sun's rays, and would also receive the heat radiated from a brick wall *. A registering thermometer was enclosed in the tube of the box, and another was placed on the outside, readings being taken of both twice daily. The highest temperature indicated upon the exterior of the box was generally about 6° or 7° above that of the centre of the gun-cotton. The temperature of the latter ranged from 14° (early in the morning) to 38°, while the temperature-indications on the exterior of the box ranged between 15° and 47°. The average temperature in the centre of the box at 5 o'clock in the afternoon was 32°. At the close of the month the box was opened, the gun-cotton possessed the faint odour peculiar to the material when closely packed, but exhibited no signs of change.

The box was immediately reclosed (some litmus paper being introduced) and placed in a chamber artificially heated. The temperature in this chamber was maintained as constantly as possible at 50°, but fluctuating a few degrees on either side. During one month's exposure in this chamber the temperature of the centre was stationary at about 49° for some time on four occasions; but the maximum temperature attained during the day ranged, with those exceptions, between 43° and 47°. At the expiration of the month, the litmus paper was found to be reddened, and the gun-cotton had a somewhat pungent odour. A portion of it, extracted with a small quantity of cold water, furnished a very faintly acid liquid, which, upon being kept in a covered vessel for some hours, was found upon the following day to be distinctly alkaline. No nitric acid could be detected, and with the exception of the peculiar odour the gun-cotton gave no indication of change. It was returned to the box together with litmus paper, and exposed again to heat in the chamber for a few days. The litmus had then changed as before. The gun-cotton was now removed from the box and fully exposed to air for an hour, when it was repacked together with litmus paper and placed in an apartment at the ordinary atmospheric temperature. The contents of the box were inspected weekly; a very trifling reaction was produced upon the litmus in some parts only, and this effect did not increase.

The gun-cotton was afterwards repacked, a very imperfectly purified skein being placed in the centre. It was then kept in the warm chamber for seven months, during which period the temperature of the air surrounding the box ranged between 30° and 50°. When the box was afterwards opened the litmus paper was red and rotten, and the material possessed a pungent odour, but no nitrous vapours were perceptible. One of the skeins in immediate contact with the imperfectly purified gun-cotton was extracted with water, but furnished only a very faintly acid liquid. The box was repacked as quickly as possible, all apertures were perfectly closed up, and it was placed in a

* The box was removed indoors late in the afternoon and replaced at nine in the morning. It was also placed under shelter when rain fell.

magazine. After the lapse of $1\frac{3}{4}$ year it was examined, and its contents were found unchanged, except that the odour was decidedly less pungent than before.

The data furnished by this preliminary experiment appeared to warrant the conclusion that further and more extensive trials of the effects of heat upon gun-cotton might be safely instituted. The following experiments were therefore made.

b. *Exposure of a large case of gun-cotton to the sun's rays.*—A large wooden box with blackened exterior, and fitted with a central tube to receive the recording thermometer, was compactly filled with rather more than 36 lbs. of gun-cotton, *not* silicated, which had been manufactured two years previously. This box was exposed to the open air on all bright sunny days between the middle of April and 1st of October last year; it was placed at a short distance from the brick wall of a hot-air chamber, in a position where it would be exposed to the maximum available amount of sunlight. At night and during wet weather it was placed under shelter. Readings were taken every two hours of the thermometer in the central tube, and of one exposed upon the outside of the box. The extreme temperature attained by the centre of the gun-cotton was 35°, the thermometer upon the box having indicated 49° on that day. Upon several occasions the thermometer in the centre of the box recorded 32°·5. The latter generally attained its maximum temperature from four to six hours after the highest external temperature had been registered. The following are some of the highest readings recorded during the $5\frac{1}{2}$ months' exposure of the box.

| | Inside the box. | Outside the box. |
|---|---|---|
| April | 18·5 | 27·5 |
| | 20 | 27·5 |
| | 21·5 | 30·5 |
| | 26·5 | 40 |
| May | 19 | 27 |
| | 20 | 35·5 |
| | 22·5 | 37·5 |
| | 20·5 | 29 |
| | 29 | 44 |
| | 26·5 | 39 |
| | 25 | 42·5 |
| June | 20·5 | 43·5 |
| | 20·5 | 43·5 |
| | 19·5 | 27·5 |
| | 26·5 | 36·5 |
| | 32·5 | 46·5 |
| | 25·5 | 34·5 |
| | 25 | 37 |
| | 33 | 47·5 |

| | Inside the box. | Outside the box. |
|---|---|---|
| July | 26·5 | 32·5 |
| | 35 | 49 |
| | 31·5 | 40·5 |
| | 31·5 | 40 |
| | 25·5 | 37 |
| | 32·5 | 45 |
| | 26·5 | 36 |
| | 28·5 | 43 |
| August | 22·5 | 40·5 |
| | 24 | 39·5 |
| | 22·5 | 39 |
| | 22·5 | 43·5 |
| | 24 | 41 |
| | 23 | 30·5 |
| | 23 | 36·5 |
| September | 16·5–27 | 20–31 |

When the box was opened, the litmus paper was found to have assumed a pink tinge, and the usual faint odour of confined gun-cotton was somewhat more manifest than if the box had been preserved under ordinary atmospheric conditions, but the material itself was quite unchanged.

c. *Exposure of gun-cotton in different conditions to a heated atmosphere in a confined space.*—A chamber was constructed of brickwork, and suitably fitted for the reception of a number of large ammunition-boxes. A system of iron pipes, standing in the centre of the chambers, was supplied with hot water from a boiler placed in a shed distinct from the chamber and heated with gas. By this arrangement, the atmosphere in the room could be maintained at artificial temperatures without risk of accident.

The boxes in which the gun-cotton was packed were the large ammunition-cases employed in military service, and consisted of thin tinned-copper cases enclosed in stout wooden boxes and very tightly closed with double lids. Experience showed, some time after the experiment was set on foot, that the employment of these metal-lined cases was unquestionably prejudicial to the gun-cotton, as the very slightest development of acid in the latter, where it was in actual contact with the sides of the case, established oxidation of the metal surfaces, whereby in turn the alteration of the gun-cotton at those parts was considerably promoted.

Each case was fitted with a central tube to receive a registering thermometer, in the same way as the black boxes already described. The gun-cotton was closely packed, and the description of material placed in the several boxes was varied (as shown in the following Table) with the view of examining the effects of different modifications in the manufacture upon the power of gun-cotton to resist the effects of heat.

In the first instance, the temperature of the hot-air chamber was raised as rapidly as possible to between 49° and 50°, and then maintained at that temperature (within narrow limits on either side) for several hours daily, periodical readings of a thermometer exposed in the room and of those enclosed in the central tubes of the boxes were recorded. The heating of the chamber was commenced at six in the morning; the maximum temperature was generally attained at about eleven o'clock, and it was maintained (for seven hours) until six in the afternoon.

After the first day of the experiment the temperature of the air in the chamber at six in the morning was always from 8° to 11° lower than the temperatures recorded in the boxes, excepting on the Monday morning, when the difference amounted only to between 1° and 3°. The rapidity with which the temperature rose in the interior of the boxes varied somewhat; the thermometers were stationary, or fell slightly for about two hours after the heating was commenced; at the close of that period the air in the chamber was generally (except on Mondays) 10° or 12° higher than that of the centre of the gun-cotton, the latter then rose gradually, almost reaching the maximum in ten hours, but still rising 2° or 3° in the last two hours. Even after seven hours' exposure to air at the maximum temperature, the contents of the case were, in the centre, from 5° to 9° cooler than the external air. The daily records of temperature obtained from the different

boxes during three months' exposure to a maximum temperature of 50° were very uniform, and not the slightest indication of any development of heat in the mass of the gun-cotton was obtained in any one instance. The maximum temperatures within the boxes were always considerably below the temperatures of the air in the chamber at the time, the difference ranging between 6° and 11°, excepting on Mondays, when the boxes never reached so high a temperature as on other days.

At the termination of three months it was decided to raise the temperature of the chamber to between 54° and 55°. The experiment was conducted as before, and readings of the thermometers were taken every two hours. The temperature of the chamber was generally raised to 54° by about eleven in the morning (sometimes earlier), and was maintained as constantly as possible at that temperature for seven hours. On 79 days out of 195, the temperature of the room reached 55°·5, and continued so from two to four hours. During three months no indication of development of heat was obtained in any one of the boxes; the temperature-records within these at the close of the day were during this period (excluding Mondays) from 6° to 11° below that in the chamber itself, and the highest maximum temperature attained by the boxes up to the termination of that period was 49°. One of the boxes then furnished indications of some development of heat in its contents; it was therefore removed, and the experiment was continued with the remainder of the boxes. During the last three months the maximum temperatures recorded in the several boxes confined in the chamber, more nearly approached that of the air surrounding them; the differences between the readings of thermometers within and outside the boxes at six in the evening ranged between 5°·5 and 1°.

The following is a tabulated statement of the descriptions of gun-cotton operated upon, and the duration and results of their exposure to heat. In the statements given in this Table, of the lowest and the mean of the temperatures recorded in the several packages, the readings obtained on Mondays have not been included, as they were considerably below those of the other five days in the week, in consequence of the chamber and boxes having cooled down during Sundays.

## TABLE IX.

| Description of gun-cotton employed. | Quantity contained in one box. | Exposure to 49°–50°. | | | | | Exposure to 54°–55°5. | | | | | Causes of removal from the hot-air chamber. | Condition of the gun-cotton when removed from the chamber, and other remarks. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Duration of exposure to a maximum temperature of 49°–50° C. | Lowest and highest temperatures recorded in the interior of the boxes at | | | Mean of the temperature recorded in the boxes at 6 p.m. | Duration of exposure to a maximum temperature of 54°·5–55°·5. | Lowest and highest temperatures recorded in the interior of the boxes at | | | Mean of the temperature recorded with boxes at 6 p.m. | | |
| | | | 6 a.m. | | 6 p.m. | | | 6 a.m. | | 6 p.m. | | | |
| | lbs. | | | | | | | | | | | | |
| 1. Made at Waltham Abbey, 1863, *not silicated*. | 11¾ | 3 months... | 21° | 31° | 39° | 44° | 42°·5 | 3 months ..... | 24°·5 (39°·5 when removed from the chamber). | 34°·5 | 44° (52°·5 the night before removal). | 49° | 46°·5 | During the last two hours' heating the temperature in this box rose 4° higher than in all the other boxes (being 52°·5 at 6 p.m.). At 6 the following morning it had fallen to 39°·5. | Upon opening the box, the gun-cotton was found to be impregnated to a large extent with nitric oxide, and the metal of the case was much corroded. An acid extract was obtained with water, in which only nitric acid was detected. The soluble matter in ether and alcohol amounted to 6·8 per cent. (the original gun-cotton contained 2 per cent.). |
| 2. *Silicated* gun-cotton (manufactured in 1863) with 1 lb. of *imperfectly purified* gun-cotton (not treated with the alkaline bath) packed in the centre. | 11¼ | 3 months... | 21·5 | 31 | 37·5 | 42·5 | 42·5 | 3¼ months ... | 24·5 (44° on the morning of its removal). | 35·5 | 44·5 | 50 | 46·5 | At 6 a.m. the temperature within this box was 44°, in the other box was only 34°. | Nitric oxide escaped abundantly from this gun-cotton, when it was removed from the box, and the metal lining of the latter was considerably corroded. The change produced in the gun-cotton was confined to a slight increase in the proportion of matter dissolved by ether and alcohol (which amouted to 3·5 per cent.). |
| 3. *Silicated*, with 12 ozs. of imperfectly converted (*soluble*) gun-cotton, *not silicated*, packed in the centre. | 11¾ | 3 months... | 22·5 | 31 | 39 | 44 | 43 | 4 months..... | 24·5 (58·5 on the morning of its removal). | 35·5 | 43·5 | 51 | 48 | On the evening preceding its removal the temperature within this box was 51°; but after the next morning it was 58°·5, the temperatures in other boxes being 31°·5–32°·5. | The gun-cotton and the case were in the same condition as the two preceding; but the proportion of matter dissolved by ether and alcohol was somewhat higher (8 per cent.). |
| 4. *Silicated*, manufactured at Waltham Abbey, 1863. | 11¾ | 3 months... | 21 | 31 | 38·5 | 43·5 | 41·5 | 5 months..... | 24·5 (40° on the morning of its removal). | 39 | 43·5 | 51 | 47·5 | The temperature within this box was 40° at 6 a.m., while in the other boxes it was 34°·5. | Nitrous vapours escaped less abundantly from this gun-cotton than from the other, and the metal case was decidedly less corroded. The matter extracted by ether and alcohol amounted to 6·5 per cent. |
| 5. *Not silicated*, but impregnated with 0·3 per ct. of *sodic carbonate*. | 13 | 3 months... | 21 | 31 | 37·5 | 43·5 | 41·5 | 7 months..... | 24·5 | 39·5 | 43·5 | 51 | 49 | ..... | No indication of development of heat up to the conclusion of the experiment. |
| 6. Same gun-cotton as No. 1, having been freed from acid by digestion in alkaline water, and subsequent washing in distilled water. | 11 | ..... | | | | | | 3 months..... | 29 | 39 | 50·5 | 52·5 | 51 | ..... | No decided indication of development of heat was recorded in either of these packages, up to the termination of the experiment. |
| 7. Same gun-cotton as No. 2, having been freed from acid by digestion in alkaline water, and subsequent washing in distilled water. | 11 | ..... | | | | | | 3 months..... | 30·5 | 39 | 49·5 | 52 | 51 | ..... | |

*Note.*—The above samples of gun-cotton which had been exposed to heat for 6, 6½, 7, and 8 months, had not altered appreciably in strength of fibre or explosiveness. After having been thoroughly purified from acid, they could not be distinguished from the original gun-cotton, except that they were somewhat whiter, as though prepared from a superior quality of cotton-wool.

The results of these experiments were as follows:—

(1) Gun-cotton closely packed in metal-lined cases, of considerable size, in two of which some imperfectly prepared gun-cotton was purposely included, sustained uniformly a daily exposure for twelve hours during three months to a heated atmosphere, the temperature of which generally ranged from 49° to 51° for a period of seven hours, without furnishing any indication of the development of heat within the mass of the gun-cotton, consequent upon chemical change.

(2) The further exposure of these packages for another period of three months to a heated atmosphere, the temperature of which generally ranged during seven hours daily between 54° and 55°, resulted only in *one* instance in the development of heat in the gun-cotton; and the particular box which, at the expiration of the six months' treatment, furnished this indication that its contents were undergoing decomposition, was filled with gun-cotton in the condition which all previous experiments had indicated as least capable of resisting the effects of prolonged exposure to heat; being, namely, almost free from substances (carbonates) which would exert a neutralizing action upon any acid generated by decomposition of the comparatively unstable impurities existing in the gun-cotton.

(3) The box which next furnished very slight indications of the development of heat, after exposure for $6\frac{1}{2}$ months, contained gun-cotton through which a small proportion of earthy carbonates had been distributed by its submission to the so-called "silicating process," but in the centre of which a specimen of imperfectly purified gun-cotton had been packed. There is no question that this box would have furnished much earlier indications of the occurrence of chemical change in its contents, if the gun-cotton principally composing the latter had not been protected for a considerable period by the presence of carbonates from the destructive effects of acid liberated from the imperfectly purified gun-cotton which was packed in the centre of the box.

(4) A box of "silicated" gun-cotton containing a small quantity of soluble gun-cotton not silicated, was the next to exhibit symptoms of decomposition, after having been exposed to heat for seven months. Numerous experiments have shown that the soluble gun-cotton is not more prone to decomposition than the most perfectly converted material; but the sample packed in the centre of this box was not protected by carbonates, and therefore doubtless sustained change considerably sooner than the chief portion of the contents of this box, promoting an alteration in the latter, after the lapse of some time, when the protective effect of the carbonate had become neutralized.

(5) The box which was entirely filled with gun-cotton, prepared strictly according to Von Lenk's system, including its submission to the "silicating" process, only exhibited a slight indication of internal development of heat after having been exposed for eight months to a heated atmosphere. The protective effect exerted by the small proportion of earthy carbonate deposited in the gun-cotton as a result of the "silicating" treatment was, in this instance, not diminished by the presence of any gun-cotton not thus treated, and consequently the contents of this box resisted change for a longer period than the

"silicated" gun-cotton in the two other boxes (2 and 3). Moreover, this gun-cotton, though exposed to heat for two months longer than the unsilicated gun-cotton (in No. 1 box), was found upon examination to have evolved considerably less acid.

(6) The gun-cotton which had been uniformly impregnated with only 0·3 per cent. of sodic carbonate furnished no signs whatever of development of heat up to the period when the experiment was interrupted, having been, at that time, exposed for ten months to a heated atmosphere, the temperature of which ranged, for seven hours daily during seven months, between 54° and 55°. It is much to be regretted that a careful examination of the contents of this box after so prolonged and severe an exposure to heat was prevented by an accident.

(7) The condition of the gun-cotton after exposure to heat in the three boxes first removed was very similar. Although the material was found to be highly impregnated with nitric oxide and nitrous acid (the development of which there is every reason to believe had been very considerably promoted by the large metal surfaces of the boxes which were in close contact with the gun-cotton), the decomposition had not proceeded in any one of the boxes to such an extent as to produce an alteration in the explosive and other properties of the gun-cotton. When the latter had been purified from the free acid developed in it, no difference could be discovered between it and the original material, except that it had become slightly bleached. The gun-cotton from boxes 1 and 2, after being purified by digestion in alkaline water and subsequent repeated washing in distilled water, was dried, repacked and returned to the hot-air chamber. It now contained no carbonates whatever by which the destructive effect of acid, if developed, could be retarded or prevented; but the boxes, each containing eleven pounds of this gun-cotton, were exposed to heat for three months, the temperature of the air ranging from 54° to 55°·5 for seven hours daily, and no indication whatever of development of heat was obtained in either instance. (The purified gun-cotton from boxes 3 and 4 was also repacked and returned to the chamber at later periods.)

After the heat-experiments described above had been continued between eight and nine months, two barrels, fitted with tubes for thermometers, each containing about twenty-three pounds of gun-cotton which was neither "silicated" nor impregnated with sodic carbonate, were placed in the chamber, the object being to obtain direct proof of the extent of influence exerted by the metal surfaces in the cases employed in the other experiments, upon the behaviour of the gun-cotton itself.

Two other much smaller metal-lined cases, each containing about five pounds of gun-cotton, were also placed in the chamber at this time; one of them was filled with a sample which had already been subjected to severe exposure to heat and had subsequently been purified from acid, and the other was filled with disks prepared by compressing gun-cotton which had been reduced to pulp. Lastly, an ammunition case containing twenty-three pounds of gun-cotton, which was impregnated with a more considerable quantity of sodic carbonate than employed in the first experiment, was added to the contents of the chamber. The heating of the latter to 54°–55° was continued

for about six weeks, when an explosion occurred, which destroyed the chamber and the whole of the samples, excepting some of the compressed gun-cotton.

As is generally the case in accidents of this kind, the immediate cause of the explosion could not be traced with certainty. The usual periodical readings of the thermometers enclosed in the packages had been taken shortly before the explosion occurred, and all the temperatures last recorded were below that of the air in the chamber, which had been at 55°·5 from two till six o'clock; not one of the packages had furnished any indication that heat was developed, but the temperature in the two small boxes was considerably higher than in the larger packages; the comparatively small volume of gun-cotton became much more rapidly heated throughout, so that the temperature recorded in these instances at the close of the day's heating was generally within 2° of the maximum external temperature. It appears most probable, therefore, that the small parcel of gun-cotton which had already suffered some change by exposure to heat, and which had since been exposed for six weeks to a heated atmosphere ranging between 54° and 55°·5 for seven hours daily, eventually sustained further alteration, which, though very gradual for a time, at length increased to such an extent that heat was very rapidly developed, raising the gun-cotton to the temperature required for its explosion within a comparatively brief period. The experiments made at 100° C. with small quantities of gun-cotton which have been described in an early part of this paper (p. 197), demonstrated that, when once a considerable decomposition of the substance had set in, the development of heat was very rapid indeed. It was believed, however, that the *first* establishment of decomposition would in all instances be indicated by so gradual a rise of temperature that frequent periodical observations of a thermometer placed in the centre of packages of heated gun-cotton would always afford the means of carrying on experiments of this class with security, a belief which was strongly supported by the results of the experiments carried on for periods of five, six, and ten months with the five large packages of gun-cotton. The power to resist serious decomposition upon continued exposure to a highly heated atmosphere had proved so unexpectedly great in the case of every one of those experiments, that it was considered important to ascertain, if possible, the full extent of those powers; and hence, with what proved to be undue reliance upon the infallibility of the measures adopted to guard against accident, the experiments were protracted and the variety of the tests increased, far beyond the extreme limits necessary for the attainment of their original object, which was to ascertain how far gun-cotton, either of ordinary manufacture, or accidentally defective, or protected by special preparation, would resist change under conditions representing the extremes, both in extent and duration, of heat to which it might be exposed if stored, or used in active military service, in tropical climates.

The Committee on Gun-cotton has endeavoured to collect reliable data with reference to the average and extreme temperatures to which gun-cotton might be exposed in ships' magazines, during the passage of vessels through tropical regions, or to which it might be subject in India if directly exposed to the sun in ammunition-boxes; these being

the most severe natural conditions of exposure to heat which would ever be likely to occur.

A statement was furnished to General SABINE, by the late Admiral FITZROY, of the maximum and minimum temperatures recorded monthly in the chronometer room of Her Majesty's Ship 'Odin' between September 1861 and September 1863, this vessel having been during that period at Japan, in the China Sea, Malacca Strait, Indian Ocean, Bengal Bay, North and South Atlantic, &c. The highest temperatures recorded were in May 1862 and April 1863 (in the Indian Ocean), being 31° (88° F.) on both occasions; the minimum temperature in those months were 29° (84° F.) and 26°·8 (80° F.). Between February 1862 and August 1863 the registered maximum temperatures ranged between 25°·3 and 31°, and the minimum temperatures between 14°·7 and 29°. Admiral FITZROY considered that, except at times when men were continuously at work in a ship's magazine, the temperature within the latter would be regulated by that of the surrounding water, which, at a few feet below the surface, is never warmer than from 26° to 30°. If this is the case, the temperature-records obtained from the 'Odin' afford a fair representation of the maximum and minimum temperatures of the atmosphere in magazines where gun-cotton might be stored on board ship. Steps have, however, been taken to obtain records of the maximum and minimum temperatures actually experienced in ships' magazines.

At the request of General SABINE, Mr. POGSON, the Astronomer at Madras, took daily readings, from May 1 to June 30, 1866, of thermometers placed in boxes, the one painted black and the other white, and both exposed to the sun. The complete account of the observations made by that gentleman have not yet been received, but, in a letter to General SABINE, he states that during the above-named period, which occurred in the hottest and driest season ever experienced at Madras, the highest temperature registered inside the black box was 51°·2 (124°·4 F.), that in the white box being 44° (111°·2 F.), whilst the lowest minimum readings recorded were 26·8 (80° F.) in the black box, and 26°·1 C. (79° F.) in the white box*. The maximum temperature recorded *in the empty black box*, exposed to the sun at Madras, was therefore about 5° C. below the average temperature to which closely packed gun-cotton, in a condition most favourable to change, was exposed for about seven hours daily, during three months (having previously been similarly exposed to an atmosphere at 50° for an equal period), before there was any indication of development of heat, while gun-cotton prepared according to Von LENK's directions resisted a similar exposure for five months, and ordinary gun-cotton containing a small proportion of sodic carbonate furnished no indication of change when it had been stored under the same circumstances for seven months.

General MORIN†, in some observations upon the recent report of PÉLOUZE and MAURY on gun-cotton, referred to the existence of instances in which the atmosphere in the interior of buildings had been raised to a temperature of 38°, 40°, or 42° (the external

* The difference between the maximum records in the black and white boxes is reported as ranging between 3° and 7°.

† Comptes Rendus, vol. lix. p. 374.

atmosphere being only 21°) by the passage of solar heat through glass windows or roofs, and also stated that it was not uncommon to find the interior of powder-wagons, covered with metal, at a temperature of 50°–60°, while the external temperature was only 24°. It is evident from the nature of this statement that the elevation of temperature to this extent in the localities described was only transient; but even if the atmosphere in magazines or ammunition-wagons were occasionally at such temperatures for several consecutive hours during a considerable period, it may be confidently maintained that gun-cotton properly purified and impregnated with a small proportion of sodic carbonate, as has been described, may be preserved in such localities with perfect safety, even in a closely packed condition. It has been shown that gun-cotton, even without the aid of the very decided though variable protection afforded to it by the "silicating" process, sustained no change whatever by continuous exposure to the sun's rays in a black box between April and September, the temperature of the external surface of the box having frequently exceeded 40°, and that the same kind of gun-cotton sustained, without any change, three months' exposure for several hours to an atmosphere of 50°, and did not exhibit any indication of change until after further exposure for nearly three months to an atmosphere maintained for several hours daily at 54°·5–55°·5. In both these instances the gun-cotton was as closely packed as possible, in one mass* (and in the latter it was contained in a case lined with tinned copper, which seriously influenced the effect of heat upon the gun-cotton).

It is therefore considered that the extent and circumstances of exposure to heat which even this perfectly unprotected gun-cotton resisted, may be regarded as exceeding in severity such as it would have to encounter in the actual employment of the material in naval and military service.

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The following are some of the principal conclusions which may be drawn from the results of observations and investigations described in the foregoing pages:—

1. Gun-cotton produced from properly purified cotton, according to the directions given by Von Lenk, may be exposed to diffused daylight, either in open air or in closed vessels, for very long periods without undergoing any change. The preservation of the material for $3\frac{1}{2}$ years under those conditions has been perfect.

2. Long-continued exposure of the substance, in a condition of ordinary dryness, to strong daylight and sunlight produces a very gradual change in gun-cotton of the description defined above; and the statements which have been published regarding the very rapid decomposition of gun-cotton when exposed to sunlight do not therefore apply to the nearly pure trinitrocellulose obtained by strictly following the system of manufacture now adopted.

* In ammunition-wagons, the gun-cotton would be packed in the form of cartridges, enclosed in serge, and with intervening air-spaces. It would therefore be in a condition much less favourable to the accumulation of heat, than the gun-cotton was, in the experimental cases.

3. If gun-cotton in closed vessels is left for protracted periods exposed to strong daylight and sunlight in a moist or damp condition, it is affected to a somewhat greater extent; but even under these circumstances the change produced in the gun-cotton by several months' exposure, is of a very trifling nature.

4. Gun-cotton which is exposed to sunlight until a faint acid reaction has become developed, and is then immediately afterwards packed into boxes which are tightly closed, does not undergo any change during subsequent preservation in ordinary storehouses (as far as the experience of $3\frac{1}{2}$ years has shown).

5. Gun-cotton prepared and purified according to the prescribed system, and stored in the ordinarily dry condition, does not furnish any indication of alteration, beyond the development, shortly after it is first packed, of a slight peculiar odour, and the power of gradually imparting to litmus, when packed with it, a pink tinge.

6. The influence exercised upon the stability of gun-cotton of average quality, as obtained by strict adherence to Von LENK'S system of manufacture, by prolonged exposure to temperatures considerably exceeding those which are experienced in tropical climates, is very trifling in comparison with the results recently published by continental experimenters relating to the effects of heat upon gun-cotton; *and it may be so perfectly counteracted* by very simple means, which in no way interfere with the essential qualities of the material, that the storage and transport of gun-cotton presents no greater danger, and is, under some circumstances, attended with much less risk of accident, than is the case with gunpowder.

7. Perfectly pure gun-cotton, or trinitrocellulose, resists to a remarkable extent the destructive effects of temperatures even approaching 100° C.; and the lower nitro-products of cellulose (soluble gun-cotton) are at any rate not more prone to alteration, when pure. The incomplete conversion of cotton into the most explosive product does, therefore, not of necessity result in the production of a less perfectly permanent compound than that obtained by the most perfect action of the acid-mixture.

8. But all ordinary products of manufacture contain small proportions of organic nitrogenized impurities, of comparatively unstable properties, which have been formed by the action of nitric acid upon foreign matters retained by the cotton fibre, and which are not completely separated by the ordinary or even a more searching process of purification.

It is the presence of this class of impurity in gun-cotton which first gives rise to the development of free acid, when the substance is exposed to the action of heat; and it is the acid thus generated which eventually exerts a destructive action upon the cellulose-products, and thus establishes decomposition which heat materially accelerates. If the small quantity of acid developed from the impurity in question be neutralized as it becomes nascent, no injurious action upon the gun-cotton results, and the great promoting cause of the decomposition of gun-cotton by heat is removed. This result is readily attained by uniformly distributing through gun-cotton a small proportion of a carbonate, the sodic carbonate, applied in the form of solution, being best adapted to this purpose.

9. The introduction into the finished gun-cotton of one per cent. of sodic carbonate affords to the material the power of resisting any serious change, even when exposed to such elevated temperatures as would induce some decomposition in the perfectly pure cellulose-products. That proportion affords, therefore, security to gun-cotton against any destructive effects of the highest temperatures to which it is likely to be exposed, even under very exceptional climatic conditions. The only influences which the addition of that amount of carbonate to gun-cotton might exert upon its properties as an explosive, would consist in a trifling addition to the small amount of smoke attending its combustion, and in a slight retardation of its explosion, neither of which could be regarded as results detrimental to the probable value of the material.

10. Water acts as a most perfect protective to gun-cotton (except when it is exposed to sunlight), even under extremely severe conditions of exposure to heat. An atmosphere saturated with aqueous vapour suffices to protect it from change at elevated temperatures, and wet or damp gun-cotton may be exposed for long periods in confined spaces to 100° without sustaining any change.

Actual immersion in water is not necessary for the most perfect preservation of gun-cotton; the material, if only damp to the touch, sustains not the slightest change, even if closely packed in large quantities. The organic impurities, which doubtless give rise to the very slight development of acid observed when gun-cotton is closely packed in the dry condition, appear equally protected by the water; for damp and wet gun-cotton which has been preserved for three years has not exhibited the faintest acidity. If as much water as possible be expelled from wet gun-cotton by the centrifugal extractor, it is obtained in a condition in which, though only damp to the touch, it is perfectly non-explosive; the water thus left in the material is sufficient not only to act as a perfect protective, but also to guard against all risk of accident. It is therefore in this condition that all reserve stores of the substance should be preserved, or that it should be transported in large quantities. If the proper proportion of sodic carbonate be dissolved in the water with which the gun-cotton is originally saturated for the purpose of obtaining it in this non-explosive form, the material, whenever it is dried for conversion into cartridges, or employment in other ways, will contain the alkaline matter required for its safe storage and use in the dry condition in all climates.

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Although some experiments bearing upon the different branches of inquiry included in this Memoir are still in progress, with a view to the attainment of additional knowledge of the conditions which regulate the stability of gun-cotton, it is confidently believed that the results which have been described amply demonstrate that the objections which have been of late revived, especially in France, against the employment of gun-cotton, on the ground of its instability, apply only in a comparatively slight degree to the material produced by strictly pursuing the system of manufacture perfected by Von Lenk; that, as far as they do exist, they have been definitely traced to certain

difficulties in the manufacture of *pure* gun-cotton which further experimental research may overcome; but that, in the mean time, these objections are *entirely set aside* by the adoption of two very simple measures, against the employment of which no practical difficulties can be raised, and which there is every reason to believe must secure for this material the confidence of those who desire to avail themselves of the special advantages which it presents as an explosive agent.

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The nature of the decomposition of gun-cotton, when exploded under different conditions, is now under investigation by me, and the results arrived at will I trust be communicated before long to the Royal Society.

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NOTE.—Since this Memoir was communicated to the Royal Society, a circumstance of a very unexpected nature has been observed, accidentally in the first instance, which appears to have an important bearing upon the question of the stability of gun-cotton.

A skein of Waltham Abbey gun-cotton has been suspended upon a line in the upper part of my laboratory for about twelve months. It has therefore been freely exposed to air and diffused daylight during that period. A portion of this sample was recently employed in a comparative heat-experiment, with some specially prepared samples, and was found to resist exposure to 100° C. in a very remarkable manner. Several portions have been maintained for many hours at 100°, upon consecutive days, without undergoing the slightest change, although originally a brief exposure to that temperature sufficed to develope symptoms of decomposition in this gun-cotton.

The behaviour of this specimen led to an examination of several samples of Stowmarket gun-cotton, portions of which had been employed in the heat-experiments described in Table III. of this Memoir, and which have since been exposed to diffused daylight, in loosely stoppered bottles, for about $2\frac{1}{2}$ years. Three among them which, when first examined, were found to undergo decomposition after exposure for a few minutes to 100° C., were selected for re-examination, and they all perfectly resisted decomposition upon long-continued exposure to that temperature.

It thus appears definitely established that the stability of gun-cotton is importantly increased by long-continued exposure to diffused daylight. An examination into the cause of this interesting fact is now being prosecuted, and the results promise importantly to strengthen the confidence which is already placed in the permanence of gun-cotton.—*June* 1867.

## XII. *On the Orders and Genera of Ternary Quadratic Forms.* By Henry J. Stephen Smith, *M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford.*

Received February 21,—Read February 27, 1867.

Eisenstein, in a Memoir entitled "Neue Theoreme der höheren Arithmetik"*, has defined the ordinal and generic characters of ternary quadratic forms of an uneven determinant; and, in the case of definite forms, has assigned the weight of any given order or genus. But he has not considered forms of an even determinant, neither has he given any demonstrations of his results. To supply these omissions, and so far to complete the work of Eisenstein, is the object of the present memoir.

Art. 2. We represent by $f$ the ternary quadratic form

$$ax^2+a'y^2+a''z^2+2byz+2b'xz+2b''xy; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we suppose that $f$ is *primitive* (*i. e.* that the six integral numbers $a$, $a'$, $a''$, $b$, $b'$, $b''$ admit of no common divisor other than unity), and that its discriminant is different from zero; this discriminant, or the determinant of the matrix

$$\begin{vmatrix} a, & b'', & b' \\ b'', & a', & b \\ b', & b, & a'' \end{vmatrix}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

we represent by D; by $\Omega$ we denote the greatest common divisor of the minor determinants of the matrix (2); by $\Omega$F the contravariant of $f$, or the form

$$\left.\begin{aligned}(a'a''-b^2)x^2+(a''a-b''^2)y^2+(aa'-b'^2)z^2 \\ +2(b'b''-ab)yz+2(b''b-a'b')zx+2(bb'-a''b'')xy;\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

we shall term F the *primitive contravariant of* $f$, and we shall write

$$\mathrm{F}=\mathrm{A}x^2+\mathrm{A}'y^2+\mathrm{A}''z^2+2\mathrm{B}yz+2\mathrm{B}'xz+2\mathrm{B}''xy. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If $\mathrm{D}=\Delta\Omega^3$, $\Delta$ is an integral number, and the discriminant, contravariant, and primitive contravariant of F are respectively $\Omega\Delta^3$, $\Delta f$, and $f$. The numbers $\Omega$ and $\Delta$ are arithmetical invariants of $f$; *i. e.* they remain unaltered when $f$ is transformed by any substitution of which the determinant is unity and the coefficients integral numbers. We shall accordingly describe the primitive form $f$, and the class of forms containing $f$, as a form, and class, of the invariants $[\Omega, \Delta]$. Similarly, F is a form of the invariants $[\Delta, \Omega]$, and the class containing F is a class of those invariants. The relation between the forms $f$ and F is reciprocal; and this reciprocity extends throughout the whole

* Crelle's Journal, vol. xxxv. p. 117.

theory, the contravariants $f$ and F, and the invariants $\Omega$ and $\Delta$, being everywhere simultaneously interchangeable.

Of definite forms we shall consider only those which are positive; and in the case of such forms we shall suppose $\Omega$, as well as $\Delta$, to be positive, in order that F as well as $f$ may be positive. In the case of indefinite forms we shall always attribute opposite signs to $\Omega$ and $\Delta$; so that in this case the discriminants of $f$ and F will be of opposite signs. Thus the definiteness, or indefiniteness, of a form is indicated by the signs of its invariants; if, for example, $p$ and $q$ are positive numbers, the forms $x^2+py^2+pqz^2$, $x^2-py^2-pqz^2$, $-x^2+py^2+pqz^2$ are respectively of the invariants $[p, q]$, $[-p, q]$, $[p, -q]$; and their primitive contravariants, $pqx^2+qy^2+z^2$, $-pqx^2+qy^2+z^2$, $pqx^2-qy^2-z^2$, are respectively of the invariants $[q, p]$, $[q, -p]$, $[-q, p]$.

Art. 3. A primitive form $f$ is properly primitive when one at least of its three *principal* coefficients $a$, $a'$, $a''$ is uneven; it is improperly primitive when those coefficients are all even. In an improperly primitive form, one at least of the three coefficients $b$, $b'$, $b''$ is uneven (or the form would not be primitive); if, therefore, $f$ is improperly primitive, $\Omega$ is uneven and F properly primitive; and, reciprocally, if F is improperly primitive, $\Delta$ is uneven and $f$ properly primitive. Again, the discriminant of an improperly primitive form is always even. Whenever, therefore, $\Omega$ and $\Delta$ are both even, or both uneven, neither $f$ nor F is improperly primitive. Primitive forms of the same invariants $[\Omega, \Delta]$ are said to belong to the same order when they and their primitive contravariants are alike properly or alike improperly primitive. An order of properly primitive forms of the invariants $[\Omega, \Delta]$ always exists, for the form

$$x^2+\Omega y^2+\Omega\Delta z^2$$

is a form of that order. And we shall show hereafter that, when $\Omega$ is uneven and $\Delta$ even, there is always an improperly primitive order of forms of the invariants $[\Omega, \Delta]$, in which $f$ is improperly and F properly, primitive except when $\Omega$ is an uneven square, and $\frac{1}{2}\Delta$ an even or uneven square. And, reciprocally, when $\Delta$ is uneven and $\Omega$ even, there is always an improperly primitive order of forms of the invariants $[\Omega, \Delta]$, in which $f$ is properly and F improperly primitive, except when $\Delta$ is an uneven square, and $\frac{1}{2}\Omega$ an even or uneven square. These exceptions cannot occur if the forms are indefinite.

For example, there are two orders of forms of the invariants $[1, 12]$. The properly primitive order contains three classes, represented by the forms

$$\begin{pmatrix}1, 1, 12\\0, 0, 0\end{pmatrix}, \quad \begin{pmatrix}1, 3, 4\\0, 0, 0\end{pmatrix}, \quad \begin{pmatrix}2, 3, 3\\1, 1, 1\end{pmatrix}.$$

The improperly primitive order, in which the forms are improperly primitive, but their contravariants properly primitive, contains two classes, represented by the forms

$$\begin{pmatrix}2, 2, 4\\-1, -1, 0\end{pmatrix}, \quad \begin{pmatrix}2, 2, 4\\0, 0, -1\end{pmatrix}.$$

Art. 4. From the identical equations

$$\left.\begin{aligned} f(x_1, y_1, z_1)\times f(x_2, y_2, z_2) - \tfrac{1}{4}\left[x_1\frac{df}{dx_2}+y_1\frac{df}{dy_2}+z_1\frac{df}{dz_2}\right]^2 \\ = \Omega F(y_1z_2-z_1y_2,\ z_1x_2-x_1z_2,\ x_1y_2-y_1x_2), \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\left.\begin{aligned} F(x_1, y_1, z_1)\times F(x_2, y_2, z_2) - \tfrac{1}{4}\left[x_1\frac{dF}{dx_2}+y_1\frac{dF}{dy_2}+z_1\frac{dF}{dz_2}\right]^2 \\ = \Delta f(y_1z_2-z_1y_2,\ z_1x_2-x_1z_2,\ x_1y_2-y_1x_2), \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

we obtain the subdivision of the Orders into Genera. If $\omega$ represent any uneven prime dividing $\Omega$, $\delta$ any uneven prime dividing $\Delta$, these equations imply the theorems—

I. "The numbers, prime to $\omega$, which are represented by $f$, are either all quadratic residues of $\omega$, or all non-quadratic residues of $\omega$." In the first case we attribute to $f$ the *particular generic character* $\left(\frac{f}{\omega}\right)=+1$, in the second we attribute to $f$ the particular generic character $\left(\frac{f}{\omega}\right)=-1$.

II. "The numbers, prime to $\delta$, which are represented by F, are either all quadratic residues of $\delta$, or all non-quadratic residues of $\delta$." We attribute to F the particular generic character $\left(\frac{F}{\delta}\right)=+1$ in the first case, $\left(\frac{F}{\delta}\right)=-1$ in the second.

If $\Omega$ and $\Delta$ are both divisible by any uneven prime, $f$ and F will both have particular characters with respect to that prime. These theorems are due to EISENSTEIN.

Besides its particular characters with respect to uneven primes dividing $\Omega$, $f$, if properly primitive, will have in certain cases particular characters (which we shall call *supplementary*) with respect to 4 and 8. If the uneven numbers represented by $f$ are all $\equiv 1$, mod 4, we attribute to $f$ the particular character $(-1)^{\frac{f-1}{2}}=+1$; if they are all $\equiv 3$, mod 4, we attribute to $f$ the particular character $(-1)^{\frac{f-1}{2}}=-1$. If they are all either $\equiv 1$, or $\equiv 7$, mod 8, we attribute to $f$ the particular character $(-1)^{\frac{f^2-1}{8}}=+1$; if they are all either $\equiv 3$, or $\equiv 5$, mod 8, we attribute to $f$ the particular character $(-1)^{\frac{f^2-1}{8}}=-1$. Lastly, if they are all either $\equiv 1$, or $\equiv 3$, mod 8, $f$ has the character $(-1)^{\frac{f-1}{2}+\frac{f^2-1}{8}}=+1$; if they are all either $\equiv 5$, or $\equiv 7$, mod 8, it has the character $(-1)^{\frac{f-1}{2}+\frac{f^2-1}{8}}=-1$. Similarly, if F is properly primitive, it will, in certain cases, acquire the characters $(-1)^{\frac{F-1}{2}}=+1$, or $=-1$; $(-1)^{\frac{F^2-1}{8}}=+1$, or $=-1$; $(-1)^{\frac{F-1}{2}+\frac{F^2-1}{8}}=+1$, or $=-1$.

The following Table is useful for ascertaining the supplementary characters of any proposed form.

TABLE I.

A.—$f$ and F properly primitive.

| | $\Omega\equiv 1$, mod 2. | $\Omega\equiv 2$, mod 4. | $\Omega\equiv 4$, mod 8. | $\Omega\equiv 0$, mod 8. |
|---|---|---|---|---|
| $\Delta\equiv 1$, mod 2. | $\Psi$ | $(-1)^{\frac{f-1}{2}}\Psi$ | $(-1)^{\frac{f-1}{2}}$, $*(-1)^{\frac{F-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $*(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f'-1}{2}}$ |
| $\Delta\equiv 2$, mod 4. | $(-1)^{\frac{F-1}{2}}\Psi$ | $(-1)^{\frac{f'-1}{2}+\frac{F-1}{2}}\Psi$ | $(-1)^{\frac{f-1}{2}}$, $\dagger(-1)^{\frac{F-1}{2}}$ $*(-1)^{\frac{F-1}{2}+\frac{F-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $*(-1)^{\frac{F-1}{2}+\frac{F-1}{2}}$ $(-1)^{\frac{f'-1}{2}}$, $\dagger(-1)^{\frac{F-1}{2}}$ |
| $\Delta\equiv 4$, mod 8. | $*(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ | $\dagger(-1)^{\frac{f'-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $*(-1)^{\frac{f-1}{2}+\frac{f'-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f'-1}{2}}$ |
| $\Delta\equiv 0$, mod 8. | $*(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{F'-1}{2}}$ | $*(-1)^{\frac{f-1}{2}+\frac{f'-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $\dagger(-1)^{\frac{f'-1}{2}}$, $(-1)^{\frac{F'-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{F'-1}{2}}$ | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f'-1}{2}}$, $(-1)^{\frac{F'-1}{2}}$ |

B.—$f$ improperly, F properly primitive.

$\Omega\equiv 1$, mod 2; $(-1)^{\frac{F-1}{2}}=-(-1)^{\frac{\Omega-1}{2}}$.

| | |
|---|---|
| $\Delta\equiv 2$, mod 4. | $(-1)^{\frac{F-1}{2}}$ |
| $\Delta\equiv 0$, mod 4. | $(-1)^{\frac{F-1}{2}}$, $(-1)^{\frac{F'-1}{2}}$ |

C.—$f$ properly, F improperly primitive.

$\Delta\equiv 1$, mod 2; $(-1)^{\frac{f-1}{2}}=-(-1)^{\frac{\Delta-1}{2}}$.

| | |
|---|---|
| $\Omega\equiv 2$, mod 4. | $(-1)^{\frac{f-1}{2}}$ |
| $\Omega\equiv 0$, mod 4. | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{f'-1}{2}}$ |

In this Table the asterisk, prefixed to a supplementary character of $f$, indicates that that character is attributed to $f$ only when $(-1)^{\frac{F-1}{2}}=(-1)^{\frac{\Omega'-1}{2}}$; prefixed to a supplementary character of F, it indicates that that character is attributed to F only when $(-1)^{\frac{f-1}{2}}=(-1)^{\frac{\Delta'-1}{2}}$, $\Omega'$ and $\Delta'$ denoting the greatest uneven divisors of $\Omega$ and $\Delta$, taken

with the same signs as $\Omega$ and $\Delta$. Similarly, the obelisk prefixed to a character of $f$ or F indicates that that character is attributable to $f$ or F only when $(-1)^{\frac{F-1}{2}}=-(-1)^{\frac{\Omega-1}{2}}$ in the first case, and $(-1)^{\frac{f-1}{2}}=-(-1)^{\frac{\Delta-1}{2}}$ in the second.

The use of the Table is most easily explained by an example. Let the proposed form be

$$f=2x^2+7y^2+7z^2-2yz;$$

its invariants are [2, 24], and its primitive contravariant is

$$F=24x^2+7y^2+7z^2+2yz.$$

Since $\Omega\equiv 2$, mod 4, $\Delta\equiv 0$, mod 8, F has the supplementary characters $(-1)^{\frac{F-1}{2}}$ and $(-1)^{\frac{F^2-1}{8}}$; the values of these characters are found by an inspection of the coefficients, and are $-1$ and $+1$ respectively. Again, since $\Omega'=1$, $(-1)^{\frac{F-1}{2}}=-(-1)^{\frac{\Omega'-1}{2}}$; the character $(-1)^{\frac{f^2-1}{8}}$ is therefore attributable to $f$, and an inspection of its coefficients shows that $(-1)^{\frac{f^2-1}{8}}=+1$.

The demonstration of the assertions implied in the Table (so far as they relate to supplementary characters) is obtained without difficulty from the equations (5) and (6). It will suffice to consider one case as an example of the rest. Let $f$ and F be both properly primitive, and let $\Omega=2\Omega'\equiv 2$, mod 4; $\Delta\equiv 0$, mod 8. If $M_1=F(x_1, y_1, z_1)$, $M_2=F(x_2, y_2, z_2)$ are two uneven numbers represented by F, we infer from equation (6) that $\frac{1}{2}\left(x_1\frac{dF}{dx_2}+y_1\frac{dF}{dy_2}+z_1\frac{dF}{dz_2}\right)$ is an uneven number, and consequently that $M_1\times M_2\equiv 1$, mod 8; $M_1$ and $M_2$ are therefore congruous to one another, mod 8; *i. e.* all uneven numbers represented by F are congruous, mod 8, or F has the characters $(-1)^{\frac{F-1}{2}}$ and $(-1)^{\frac{F^2-1}{8}}$. To prove that $f$ has the supplementary character attributed to it in the Table, we observe first of all that F cannot represent unevenly even numbers; for, if possible, let $F(x_1, y_1, z_1)$ be unevenly even, and let $F(x_2, y_2, z_2)$ be any uneven number represented by F; then in the equation (6) we have a square congruous, mod 8, to an unevenly even number, which is impossible. Now let $m_1=f(x_1, y_1, z_1)$, $m_2=f(x_2, y_2, z_2)$ be any two uneven numbers represented by $f$; the number $\frac{1}{2}\left(x_1\frac{df}{dx_2}+y_1\frac{df}{dy_2}+z_1\frac{df}{dz_2}\right)$ is uneven in equation (5); and considering that equation as a congruence for the modulus 8, we find $m_1\times m_2\equiv 1$, or $m_1\times m_2\equiv 1+2\times(-1)^{\frac{\Omega'-1}{2}+\frac{F-1}{2}}$, according as

$$F(y_1z_2-y_2z_1,\ z_1x_2-z_2x_1,\ x_1y_2-x_2y_1)$$

is evenly even, or uneven. If, then, $(-1)^{\frac{F-1}{2}}=(-1)^{\frac{\Omega'-1}{2}}$, $m_1\times m_2$ is $\equiv 1$, or $\equiv 3$, mod 8; *i. e.* the uneven numbers represented by $f$ are either all of one or other of the linear forms $8k+1$, $8k+3$, or else all of one or other of the linear forms $8k+5$, $8k+7$; so that $f$ has the supplementary character $(-1)^{\frac{f-1}{2}+\frac{f^2-1}{8}}$. But if $(-1)^{\frac{F-1}{2}}=-(-1)^{\frac{\Omega'-1}{2}}$,

$m_1 \times m_2 \equiv +1$, or $-1$, mod 8, and the uneven numbers represented by $f$ are either all of the linear forms $8k \pm 1$, or else all of the linear forms $8k \pm 3$, so that $f$ has the character $(-1)^{\frac{f'-1}{8}}$.

The signification of the symbols $\Psi$, $(-1)^{\frac{f'-1}{8}}\Psi$, $(-1)^{\frac{F'-1}{8}}\Psi$, $(-1)^{\frac{f'-1}{8}+\frac{F'-1}{8}}\Psi$, which occur in the Table, is explained in Arts. 6 and 7. In the next article we shall establish an auxiliary proposition which is frequently useful.

Art. 5. "There exist pairs of forms $\varphi$ and $\Phi$, equivalent to $f$ and F, and satisfying the congruences

$$\left.\begin{aligned} \varphi &\equiv \alpha x^2 + \beta\Omega y^2 + \gamma\Omega\Delta z^2, \\ \Phi &\equiv \beta\gamma\Omega\Delta x^2 + \alpha\gamma\Delta y^2 + \alpha\beta z^2, \\ \alpha\beta\gamma &\equiv 1, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (7)$$

for any proposed modulus $\nabla$; but this modulus must be uneven, if either $f$ or F is improperly primitive."

In the proof of this proposition we shall employ two lemmas of a very elementary character.

(i) A properly primitive form $f$ represents numbers prime to any given number $\nabla$; and an improperly primitive form $f$ represents the doubles of numbers prime to any given number $\nabla$.

Let $p$ be any prime divisor of $\nabla$, and if $f$ is improperly primitive, let $p$ be an uneven prime. If one of the numbers $a$, $a'$, $a''$ is prime to $p$, let $a$ be prime to $p$; then if $x$ is prime to $p$ and $y$ and $z$ are divisible by $p$, $f$ will acquire a value prime to $p$. If $a$, $a'$, $a''$ are all divisible by $p$, one of the three numbers $b$, $b'$, $b''$ must be prime to $p$; let $b$ be prime to $p$; then if $x$ is divisible by $p$, and $y$ and $z$ are prime to $p$, $f$ will acquire a value prime to $p$.

If $f$ is improperly primitive and $p=2$, we may consider $\frac{1}{2}f$ instead of $f$ and $\frac{1}{2}a$, $\frac{1}{2}a'$, $\frac{1}{2}a''$ instead of $a$, $a'$, $a''$; and we may prove in the same way that $\frac{1}{2}f$ represents uneven numbers.

Thus, among the $p^3$ systems of values which can be attributed to $x$, $y$, $z$ for the modulus $p$, there are always some which render $f$ (or $\frac{1}{2}f$) prime to $p$; there are, therefore, among the $\nabla^3$ systems of values which can be attributed to $x$, $y$, $z$ for the modulus $\nabla$, some which render $f$ (or $\frac{1}{2}f$) simultaneously prime to every prime dividing $\nabla$.

(ii) If $\Omega\Delta$ is uneven, $f$ represents numbers of both the linear forms $4k+1$ and $4k+3$.

One at least of the principal coefficients of $f$ is uneven, because its discriminant is uneven: let then $a$ be uneven, and let $a' \equiv \lambda$, mod 2, $a'' \equiv \mu$, mod 2; the substitution $x = x + \lambda y + \mu z$ will transform $f$ into a form $f_1$, in which $a_1$, $a_1'$, $a_1''$ are all uneven, and in which, because the discriminant is uneven, either only one, or else all three, of the coefficients $b_1$, $b_1'$, $b_1''$ are even. The four numbers $a_1$, $a_1'$, $a_1''$, $a_1 + a_1' + a_1'' + 2b_1 + 2b_1' + 2b_1''$ are then all uneven; they are all represented by $f_1$, that is by $f$; but they are not all congruous to one another for the modulus 4; therefore $f$ represents numbers of both the linear forms $4k+1$ and $4k+3$.

It follows from these lemmas (i) that if $f$ is an improperly primitive form, we can find

a form equivalent to $f$, and having one of its principal coefficients unevenly even and prime to any uneven number; (ii) that if $f$ is properly primitive, we can find a form equivalent to $f$, and having one of its principal coefficients prime to any given number; (iii) that if $\Omega\Delta$ is uneven, we may suppose this principal coefficient of either of the two linear forms $4k+1$, or $4k+3$, at our option.

We shall first suppose that the forms $f$ and F, which it is proposed to transform into forms $\phi$ and $\Phi$ satisfying the congruences (7), are properly primitive. Let $\nabla' = \nabla\Omega^3\Delta$, and let us assume that in the form F, $A''$ is prime to $\nabla'$, and also that $A'' \equiv \Omega$, mod. 4, if $\Omega\Delta$ is uneven. Let $\gamma \equiv \frac{1}{A''}$, mod $\nabla'$; the redundant system of congruences

$$\left.\begin{aligned} a\,x+b''y+b' &\equiv 0,\\ b''x+a'y+b &\equiv 0\\ b'x+by+a'' &\equiv \gamma\Omega\Delta, \end{aligned}\right\} \text{mod } \nabla',$$

is resoluble, admitting $\Omega$ incongruous solutions*. Let

$$\left.\begin{aligned} x &\equiv \lambda,\\ y &\equiv \mu, \end{aligned}\right\} \text{mod } \nabla',$$

be any one of these solutions, and let us transform $f$ by the substitution

$$\left.\begin{aligned} x &= x+\lambda z,\\ y &= y+\mu z, \end{aligned}\right\}$$

into an equivalent form $f_1$. The coefficients $a_1$, $b_1''$, $a_1'$ are the same as $a$, $b''$, $a'$; the coefficients $a_1''$, $b_1$, $b_1'$ are respectively congruous for the modulus $\nabla'$ to $\gamma\Omega\Delta$, 0, 0; so that $f_1$ satisfies the congruence

$$f_1 \equiv ax^2+2b''xy+a'y^2+\gamma\Omega\Delta z^2, \text{ mod } \nabla'.$$

The binary form $(a, b'', a')$ is primitive; for if $d$ is a prime dividing $a$, $b''$, $a'$, it divides $-\Omega A''$, the determinant of $(a, b'', a')$, and $\Omega^2\Delta$, the discriminant of $f$; it therefore divides $\Omega$ (because $A''$ and $\Delta$ are relatively prime), and is a common divisor of the coefficients of the primitive form $f_1$, i. e. $d=1$. Again, $(a, b'', a')$ is not improperly primitive; if $\Omega\Delta$ is even, this is manifest, for $f_1$ is not improperly primitive; if $\Omega\Delta$ is uneven, $\Omega A''$ is by hypothesis of the form $4k+1$, and there are no improperly primitive binary forms of the determinant $-\Omega A''$. We may now suppose that, in the properly primitive binary form $(a, b'', a')$, $a$ is uneven and prime to $\nabla'$; let $\beta \equiv \frac{A''}{a}$, mod $\nabla'$; then the congruences

$$\left.\begin{aligned} a\,x+b'' &\equiv 0,\\ b''x+a' &\equiv \beta\Omega, \end{aligned}\right\} \text{mod } \nabla',$$

are resoluble and admit of one solution. Let $x \equiv \lambda$, mod $\nabla'$, be that solution; if $f_1$ be transformed by the substitution $x = x+\lambda y$, the resulting form $\phi$ will satisfy the congruence

$$\phi \equiv ax^2+\beta\Omega y^2+\gamma\Omega\Delta z^2, \text{ mod } \nabla',$$

and the forms $\phi$ and $\Phi$ will satisfy the congruences (7) for the modulus $\nabla$.

* Philosophical Transactions, vol. cli. p. 323.

Attributing to $\nabla'$ the value $\nabla\Omega'^2\Delta'$, we may apply the same demonstration to the case in which either $f$ or F is improperly primitive, the modulus $\nabla$ being supposed uneven.

Art. 6. When neither $f$ nor F is improperly primitive, and neither $\Omega$ nor $\Delta$ evenly even, $f$ and F, considered separately, have no particular characters, properly so called, with respect to 4 or 8. But, considered jointly, they have a certain character with respect to 4 or 8, which we shall term their *simultaneous generic character*, and which it is important to consider here.

If $m=f(x, y, z)$, $\mathrm{M}=\mathrm{F}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$, the representations of $m$ by $f$ and of M by F are said to be *simultaneous* when $x, y, z, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$ satisfy the equation

$$x\mathrm{X}+y\mathrm{Y}+z\mathrm{Z}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This definition of simultaneous representation suffices for our immediate purpose; we add, however, that if the two representations are *primitive** as well as simultaneous, the equations

$$\begin{Vmatrix} x, & y, & z \\ x', & y', & z' \end{Vmatrix} = \mathrm{X}, \mathrm{Y}, \mathrm{Z}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\begin{Vmatrix} \mathrm{X}, & \mathrm{Y}, & \mathrm{Z} \\ \mathrm{X}', & \mathrm{Y}', & \mathrm{Z}' \end{Vmatrix} = x, y, z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

are resoluble in integral numbers $x', y', z', \mathrm{X}', \mathrm{Y}', \mathrm{Z}'$. For, considering the first of these equations, we observe that $[x, y, z]$ is a given solution, in relatively prime numbers, of the equation (8); let $[x', y', z']$ be a solution of the same equation which with $[x, y, z]$ forms a fundamental set; the equation (9) is then satisfied by virtue of the characteristic property of the fundamental set (Philosophical Transactions, vol. cli. p. 297). Thus if the representations of $m$ by $f$, and of M by F, are primitive and simultaneous, $m$ is primitively represented by a binary form of determinant $-\Omega\mathrm{M}$, which is itself primitively represented by $f$†; and, reciprocally, M is primitively represented by a binary form of determinant $-\Delta m$, which is itself primitively represented by F.

In the four cases in which neither $\Omega$ nor $\Delta$ is evenly even, the simultaneous character of $f$ and F is given in the Table of Art. 4. The symbol $\Psi$ in that Table represents the unit $(-1)^{\frac{\Delta'f+1}{2}\cdot\frac{\Omega'\mathrm{F}+1}{2}}$, *i. e.* the unit $(-1)^{\frac{\Delta'm+1}{2}\cdot\frac{\Omega'\mathrm{M}+1}{2}}$, $\Omega'$ and $\Delta'$ denoting the same numbers as in Art. 4, and $m$, M being any two uneven numbers simultaneously represented by $f$ and F. Thus, if $\Omega \equiv \Delta \equiv 1$, mod 2, the simultaneous character $\Psi$ is attributed in the Table to $f$ and F; *i. e.* the uneven numbers simultaneously represented by $f$ and F either all satisfy the equation $\Psi=1$, or else all satisfy the equation $\Psi=-1$.

To demonstrate these simultaneous characters we consider the four cases separately, and for the forms $f$ and F we substitute forms $\varphi$ and $\Phi$ equivalent to them, and satisfying

* The representation of a number by a form is said to be *primitive* when the values of the indeterminates do not admit of any common divisor besides unity.

† See Art. 10.

certain congruences for the modulus 4 or 8. The existence of the equivalent forms thus assumed results, in each case, from the theorem of the last article.

Case (i) Let $\Omega \equiv \Delta \equiv 1$, and let $\varphi$ and $\Phi$ satisfy the congruences

$$\left.\begin{aligned} \Delta\varphi &\equiv \alpha x^2+\beta y^2+\gamma z^2, \\ \Omega\Phi &\equiv \beta\gamma X^2+\alpha\beta Y^2+\alpha\beta Z^2, \\ \text{or} \quad \Omega\Phi &\equiv \alpha X^2+\beta Y^2+\gamma Z^2, \\ \alpha\beta\gamma &\equiv 1, \end{aligned}\right\} \text{mod } 4.$$

Attributing in succession to the indeterminates

$$\begin{matrix} x, & y, & z \\ \mathrm{X}, & \mathrm{Y}, & \mathrm{Z} \end{matrix}$$

all systems of values, mod 2, which satisfy the congruence

$$x\mathrm{X}+y\mathrm{Y}+z\mathrm{Z} \equiv 0, \text{ mod } 2,$$

and which render $m$ and $\mathrm{M}$ simultaneously uneven, we find that in every case $\Delta m$ is congruous, for the modulus 4, to one of the numbers $\alpha$, $\beta$, $\gamma$, and $\Omega\mathrm{M}$ to one of the remaining two. Thus $\frac{\Delta m+1}{2} \times \frac{\Omega \mathrm{M}+1}{2}$ is necessarily congruous, for the modulus 2, to one of the three numbers

$$\frac{(\beta+1)(\gamma+1)}{4}, \quad \frac{(\gamma+1)(\alpha+1)}{4}, \quad \frac{(\alpha+1)(\beta+1)}{4}.$$

But these numbers are all congruous to one another for the modulus 2, because the congruence $\alpha\beta\gamma \equiv 1$, mod 4, implies the congruence $\alpha+\beta+\gamma+1 \equiv 0$, mod 4. Therefore the unit $\Psi$ has always the same value for every pair of uneven numbers simultaneously represented by $f$ and F.

It will be seen that $\Psi = -1$, or $\Psi = +1$, according as the congruences $\alpha \equiv \beta \equiv \gamma \equiv 1$, mod 4, are or are not satisfied.

Case (ii) Let $\Omega \equiv 2$, mod 4, $\Delta \equiv 1$, mod 2, and let

$$\begin{aligned} \Delta\varphi &\equiv \alpha x^2+2\beta y^2+2\gamma z^2, \text{ mod } 8, \\ \Omega'\Phi &\equiv 2\alpha X^2+\beta Y^2+\gamma Z^2, \text{ mod } 4, \\ \alpha\beta\gamma &\equiv 1, \text{ mod } 4. \end{aligned}$$

The admissible combinations of the values of $x, y, z, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$, mod 2, give rise to eight cases,

$$\begin{aligned} &\Delta m \equiv \alpha, && \text{mod } 8; && \Omega'\mathrm{M} \equiv \beta, \text{ or } \gamma, \text{ mod } 4, \\ &\Delta m \equiv \alpha+2\beta, && \text{mod } 8; && \Omega'\mathrm{M} \equiv -\beta, \text{ or } +\gamma, \text{ mod } 4, \\ &\Delta m \equiv \alpha+2\gamma, && \text{mod } 8; && \Omega'\mathrm{M} \equiv \beta, \text{ or } -\gamma, \text{ mod } 4, \\ &\Delta m \equiv \alpha+2\beta+2\gamma, && \text{mod } 8; && \Omega'\mathrm{M} \equiv -\beta, \text{ or } -\gamma, \text{ mod } 4, \end{aligned}$$

and, in all of them, the value of the unit $(-1)^{\frac{\Delta m \mathrm{M}-1}{4}}\Psi$, and therefore of the unit $(-1)^{\frac{f'-1}{4}}\Psi$, is the same, because, by virtue of the congruence $\alpha+\beta+\gamma+1 \equiv 0$, mod 4,

the four numbers

$$\frac{(\alpha+1)(\alpha+2\beta+1)}{8},\ \frac{(\alpha+1)(\alpha+2\gamma+1)}{8},$$

$$\frac{(\alpha+2\beta+2\gamma+1)(\alpha+2\beta+1)}{8},\ \frac{(\alpha+2\beta+2\gamma+1)(\alpha+2\gamma+1)}{8}$$

are all congruous to one another for the modulus 2.

Case (iii) $\Omega \equiv 1$, mod 2, $\Delta \equiv 2$, mod 4. In this case the simultaneous character of the forms $f$ and F may be demonstrated as in case (ii), or may be inferred by reciprocation from the result in that case.

Case (iv) $\Omega \equiv \Delta \equiv 2$, mod 4. Let

$$\Delta'\varphi \equiv \alpha x^2 + 2\beta y^2 + 4\gamma z^2, \text{ mod } 8,$$

$$\Omega'\Phi \equiv 4\alpha X^2 + 2\beta Y^2 + \gamma Z^2, \text{ mod } 8,$$

$$\alpha\beta\gamma \equiv 1, \text{ mod } 4.$$

Here again there are eight cases,

$$\Delta' m \equiv \alpha \quad ; \ \Omega' \mathrm{M} \equiv \gamma, \quad \text{or } \gamma+2\beta \quad , \text{ mod } 8,$$

$$\Delta' m \equiv \alpha+2\beta \ ; \ \Omega' \mathrm{M} \equiv \gamma, \quad \text{or } \gamma+2\beta+4, \text{ mod } 8,$$

$$\Delta' m \equiv \alpha+4 \quad ; \ \Omega' \mathrm{M} \equiv \gamma+4, \text{ or } \gamma+2\beta+4, \text{ mod } 8,$$

$$\Delta' m \equiv \alpha+2\beta+4 ; \ \Omega' \mathrm{M} \equiv \gamma+4, \text{ or } \gamma+2\beta \quad , \text{ mod } 8 ;$$

and in all eight the value of the unit $(-1)^{\frac{\Delta' m^2-1}{8}+\frac{\Omega' \mathrm{M}^2-1}{8}}\Psi$, and therefore of the unit $(-1)^{\frac{f'-1}{8}+\frac{\mathrm{F}'-1}{8}}\Psi$, is the same, because by virtue of the congruence $\alpha+\beta+\gamma+1 \equiv 0$, mod 4, the two numbers

$$\frac{(\alpha+\gamma)(\alpha+\gamma+2)}{8},\ \frac{(\alpha+\gamma+2\beta)(\alpha+\gamma+2\beta+2)}{8}$$

are congruous to one another for the modulus 2.

Art. 7. The following observations will serve to show more clearly the import of the simultaneous character in each of the four cases.

Case (i) Let $\Psi = -1$; then, if $m$ and M are any two uneven numbers simultaneously represented by $f$ and F, $m \equiv \Delta$, mod 4, and $\mathrm{M} \equiv \Omega$, mod 4. Also $f$ cannot represent numbers congruous to $7\Delta$, mod 8, nor F numbers congruous to $7\Omega$, mod 8; for the congruences

$$\frac{(\beta+1)(\gamma+1)}{4} \equiv \frac{(\gamma+1)(\alpha+1)}{4} \equiv \frac{(\alpha+1)(\beta+1)}{4} \equiv 1, \text{ mod } 2,$$

imply that $\alpha \equiv \beta \equiv \gamma \equiv 1$, mod 4; *i. e.* that $\varphi$, or, which is the same thing, $f$ can only represent uneven numbers congruous to $\Delta, 3\Delta, 5\Delta$. And similarly of uneven numbers F can only represent those which are congruous to $\Omega, 3\Omega, 5\Omega$. Numbers congruous to $3\Delta$ are represented by $f$, and numbers congruous to $3\Omega$ are represented by F; but these representations are not simultaneous with the representation of any uneven number by F in the first case, and by $f$ in the second.

Let $\Psi=+1$; then if $m$ and M are uneven numbers simultaneously represented by $f$ and F, one at least of the two congruences $m\equiv-\Delta$, mod 4, $M\equiv-\Omega$, mod 4, must be satisfied. Subject to this restriction, $m$ and M may have any of the four linear forms $8k+1, 3, 5, 7$.

Case (ii) The restrictions imposed on the numbers $m$ and M by the simultaneous characters are exhibited in the annexed Table.

| If | $(-1)^{\frac{f-1}{8}}\Psi=(-1)^{\frac{\Delta^2-1}{8}}$. | $(-1)^{\frac{f-1}{8}}\Psi=-(-1)^{\frac{\Delta^2-1}{8}}$. |
|---|---|---|
| $M\equiv\ \Omega'$, mod 4. | $m\equiv5\Delta, 7\Delta$, mod 8. | $m\equiv\ \Delta, 3\Delta$, mod 8. |
| $M\equiv3\Omega'$, mod 4. | $m\equiv\ \Delta, 7\Delta$, mod 8. | $m\equiv3\Delta, 5\Delta$, mod 8. |

Except when $\Omega$ and $\Delta$ are both uneven it will be found that, in the case of any two properly primitive forms $f$ and F, every representation of an uneven number by either of the two is simultaneous with the representation of uneven numbers by the other. If therefore $(-1)^{\frac{f-1}{8}}\Psi=(-1)^{\frac{\Delta^2-1}{8}}$, $f$ cannot represent numbers congruous to $3\Delta$, mod 8, because it cannot represent them simultaneously with uneven numbers, and if $(-1)^{\frac{f-1}{8}}\Psi=-(-1)^{\frac{\Delta^2-1}{8}}$, $f$ cannot represent numbers congruous to $7\Delta$, mod 8.

Case (iii) In this case, which is the reciprocal of the last, we have the Table,

| If | $(-1)^{\frac{F-1}{8}}\Psi=(-1)^{\frac{\Omega^2-1}{8}}$. | $(-1)^{\frac{F-1}{8}}\Psi=-(-1)^{\frac{\Omega^2-1}{8}}$. |
|---|---|---|
| $m\equiv\ \Delta'$, mod 4. | $M\equiv5\Omega, 7\Omega$, mod 8. | $M\equiv\ \Omega, 3\Omega$, mod 8. |
| $m\equiv3\Delta'$, mod 4. | $M\equiv\ \Omega, 7\Omega$, mod 8. | $M\equiv3\Omega, 5\Omega$, mod 8. |

And F cannot represent numbers congruous to $3\Omega$, or cannot represent numbers congruous to $7\Omega$, according as $(-1)^{\frac{F-1}{8}}\Psi=(-1)^{\frac{\Omega^2-1}{8}}$, or $-(-1)^{\frac{\Omega^2-1}{8}}$.

Case (iv) In this case both $f$ and F represent numbers of all the four linear forms $8k+1, 3, 5, 7$. The Table, in which the modulus is everywhere 8, exhibits the restrictions imposed by the simultaneous character.

| If | $(-1)^{\frac{f-1}{8}+\frac{F-1}{8}}\Psi=(-1)^{\frac{\Delta^2-1}{8}+\frac{\Omega^2-1}{8}}$ | $(-1)^{\frac{f-1}{8}+\frac{F-1}{8}}\Psi=-(-1)^{\frac{\Delta^2-1}{8}+\frac{\Omega^2-1}{8}}$ |
|---|---|---|
| $m\equiv\ \Delta'$ | $M\equiv5\Omega', 7\Omega'$ | $M\equiv\ \Omega', 3\Omega'$ |
| $m\equiv3\Delta'$ | $M\equiv3\Omega', 5\Omega'$ | $M\equiv\ \Omega', 7\Omega'$ |
| $m\equiv5\Delta'$ | $M\equiv\ \Omega', 3\Omega'$ | $M\equiv5\Omega', 7\Omega'$ |
| $m\equiv7\Delta'$ | $M\equiv\ \Omega', 7\Omega'$ | $M\equiv3\Omega', 5\Omega'$ |

Art. 8. The complete generic character of a form or class is the complex of all the particular characters attributable to the form or class, and to its primitive contravariant, including their simultaneous character, if any. And two forms, or classes, which have the same complete generic character are said to belong to the same genus. But not every complete generic character that can be assigned *à priori*, is the character of any really existing genus of forms. The annexed Table will serve, in the case of any given order, to distinguish those complete generic characters, which are possible, *i. e.* to which actually existing genera correspond, from those which are impossible.

In this Table $\Omega_0^2$ and $\Delta_0^2$ are the greatest squares dividing $\Omega$ and $\Delta$; the quotients $\Omega \div \Omega_0^2$, $\Delta \div \Delta_0^2$ are respectively represented, if uneven, by $\Omega_1$ and $\Delta_1$, if even by $2\Omega_1$ and $2\Delta_1$, so that $\Omega_1$ and $\Delta_1$ are always uneven and not divisible by any square; $\omega_1$ and $\delta_1$ are any primes dividing $\Omega_1$ and $\Delta_1$, $\omega_2$ and $\delta_2$ are any uneven primes dividing $\Omega_2$ and $\Delta_2$, but $\omega_2$ must not divide $\Omega_1$, nor must $\delta_2$ divide $\Delta_1$; lastly, $\Psi$ is still the unit $(-1)^{\frac{\Omega f+1}{2}\cdot\frac{\Delta F+1}{2}}$, or, which is the same thing, the unit $(-1)^{\frac{\Omega_1 F+1}{2}\cdot\frac{\Delta_1 F+1}{2}}$, $f$ and F in the exponents of these units denoting uneven numbers simultaneously represented by the forms $f$ and F.

The Table A of properly primitive generic characters contains twenty-five compartments corresponding to the twenty-five cases indicated in its margin; the Tables B and C of improperly primitive genera contain three such compartments each. In each compartment are inscribed all the particular characters which make up the complete generic character of a form coming under the case to which the compartment corresponds; the symbols $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{\mathrm{F}}{\delta_1}\right)$, $\left(\frac{\mathrm{F}}{\delta_2}\right)$ implying that $f$ has a particular character with respect to every prime $\omega_1$ or $\omega_2$, F a particular character with respect to every prime $\delta_1$ or $\delta_2$. Each compartment is divided into two parts by a vertical line, and the particular characters (one of which in Table A either is or contains $\Psi$) placed to the left of this line are subject to the condition that their product is equal in Table A to the unit $(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}$, in Table B to the unit $(-1)^{\frac{\Omega_1^2-1}{8}} \times (-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}$, in Table C to the unit $(-1)^{\frac{\Delta_1^2-1}{8}}\times(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}$. If $\alpha=+1$, or $-1$, according as $\Omega$ is of the form $\Omega_1\Omega_2^2$, or $2\Omega_1\Omega_2^2$, and if, similarly $\beta=+1$, or $-1$, according as $\Delta$ is of the form $\Delta_1\Delta_2^2$, or $2\Delta_1\Delta_2^2$, we may express this condition in Table A by the equation

$$\Psi\times\alpha^{\frac{F-1}{2}}\times\beta^{\frac{F-1}{2}}\times\left(\frac{f}{\Omega_1}\right)\times\left(\frac{\mathrm{F}}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and in Tables B and C respectively by the equations

$$(-\beta)^{\frac{F-1}{2}}\times\left(\frac{f}{\Omega_1}\right)\times\left(\frac{\mathrm{F}}{\Delta_1}\right)=(-1)^{\frac{\Omega_1^2-1}{8}+\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$(-\alpha)^{\frac{F-1}{2}}\times\left(\frac{f}{\Omega_1}\right)\times\left(\frac{\mathrm{F}}{\Delta_1}\right)=(-1)^{\frac{\Delta_1^2-1}{8}+\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The condition distinguishes the possible and impossible genera, every generic character which satisfies it being the character of an actually existing genus, and every

A.—$f$ and F properly primitive.

| | $\Omega=\Omega_1\Omega_2^2$. | $\Omega_2\equiv 1$, mod 2. | $\Omega=\Omega_1\Omega_2^2$. | $\Omega_2\equiv 2$, mod 4. | $\Omega=\Omega_1\Omega_2^2$. | $\Omega_2\equiv 0$, mod 4. |
|---|---|---|---|---|---|---|
| $\Delta=\Delta_1\Delta_2^2$. $\Delta_2\equiv 1$, mod 2. | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ S | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^\gamma$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=3\times 2^{\gamma-1}$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{f-1}{2}}$ $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=3\times 2^\gamma$ |
| $\Delta=\Delta_1\Delta_2^2$. $\Delta_2\equiv 2$, mod 4. | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=3\times 2^{\gamma-1}$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+1}$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+2}$ |
| $\Delta=\Delta_1\Delta_2^2$. $\Delta_2\equiv 0$, mod 4. | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=3\times 2^\gamma$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+2}$ | $\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f^2-1}{8}}$, $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+3}$ |
| $\Delta=2\Delta_1\Delta_2^2$. $\Delta_2\equiv 1$, mod 2. | $(-1)^{\frac{F^2-1}{8}}\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ S | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^\gamma$ | $(-1)^{\frac{F^2-1}{8}}\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+1}$ | $(-1)^{\frac{F^2-1}{8}}\Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{f-1}{2}}$ $(-1)^{\frac{f^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+2}$ |
| $\Delta=2\Delta_1\Delta_2^2$. $\Delta_2\equiv 0$, mod 2. | $\Psi$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=3\times 2^\gamma$ | $\Psi$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+2}$ | $\Psi$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $(-1)^{\frac{f^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma=2^{\gamma+3}$ |

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B.—*f* improperly, F properly primitive.

$(-1)^{\frac{F-1}{2}} = -(-1)^{\frac{\Omega-1}{2}}$; $\Omega \equiv 1$, mod 2.

| | $\Omega = \Omega_1 \Omega_2^2$. | $\Omega_2 \equiv 1$, mod 2. |
|---|---|---|
| $\Delta = \Delta_1 \Delta_2^2$. $\Delta_2 \equiv 0$, mod 2. | $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ |
| $\Delta = 2\Delta_1 \Delta_2^2$. $\Delta_2 \equiv 1$, mod 2. | $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma-1}$ |
| $\Delta = 2\Delta_1 \Delta_2^2$. $\Delta_2 \equiv 0$, mod 2. | $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ |

C.—*f* properly, F improperly primitive.

$(-1)^{\frac{f-1}{2}} = -(-1)^{\frac{\Delta-1}{2}}$; $\Delta \equiv 1$, mod 2.

| | $\Delta = \Delta_1 \Delta_2^2$ | $\Delta_2 \equiv 1$, mod 2. |
|---|---|---|
| $\Omega = \Omega_1 \Omega_2^2$. $\Omega_2 \equiv 0$, mod 2. | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ |
| $\Omega = 2\Omega_1 \Omega_2^2$. $\Omega_1 \equiv 1$, mod 2. | $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma-1}$ |
| $\Omega = 2\Omega_1 \Omega_2^2$. $\Omega_1 \equiv 0$, mod 2. | $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ |

| | $\Omega = 2\Omega_1 \Omega_2^2$. | $\Omega_2 \equiv 1$, mod 2. | $\Omega = 2\Omega_1 \Omega_2^2$. | $\Omega_2 \equiv 0$, mod 2. |
|---|---|---|---|---|
| | $(-1)^{\frac{f-1}{8}} \Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ S | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ | $\dot{\Psi}$ $(-1)^{\frac{f-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ Q | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 3 \times 2^\gamma$ |
| | $(-1)^{\frac{f-1}{8}} \Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{F-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+1}$ | $\Psi$ $(-1)^{\frac{f-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ |
| | $(-1)^{\frac{f-1}{8}} \Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{F-1}{8}}$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ | $\Psi$ $(-1)^{\frac{f-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{8}}$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ |
| | $(-1)^{\frac{f-1}{8}+\frac{F^2-1}{8}} \Psi$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ S | $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^\gamma$ | $(-1)^{\frac{F^2-1}{8}} \Psi$ $(-1)^{\frac{f-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{f-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ |
| | $(-1)^{\frac{f-1}{8}} \Psi$ $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ R | $(-1)^{\frac{F-1}{8}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ | $\Psi$ $(-1)^{\frac{f-1}{8}}$, $(-1)^{\frac{F^2-1}{8}}$ $\left(\frac{f}{\omega_1}\right)$, $\left(\frac{F}{\delta_1}\right)$ P | $(-1)^{\frac{f-1}{2}}$, $(-1)^{\frac{F-1}{2}}$ $\left(\frac{f}{\omega_2}\right)$, $\left(\frac{F}{\delta_2}\right)$ $\Gamma = 2^{\gamma+2}$ |

generic character which does not satisfy it belonging to no forms whatever. The demonstration of this important theorem will occupy the next articles; it is, however, requisite to show in the first place that the enumeration of the supplementary characters in Table II. is in accordance with the Table I. of Art. 4. For the Tables B and C this is evident; in Table A it is necessary to attend to the signification of the symbol $\Psi$, which serves to represent the simultaneous character of $f$ and F (as has been already explained in Arts. 6 and 7) in those cases (marked S in the Table) in which neither $(-1)^{\frac{f-1}{2}}$ nor $(-1)^{\frac{F-1}{2}}$ is a character, but which also appears in every compartment of the Table without exception.

(1) When $(-1)^{\frac{f-1}{2}}$ and $(-1)^{\frac{F-1}{2}}$ are both characters (cases P in the Table), $\Psi$ is not an independent character, because its value is determined by the values of $(-1)^{\frac{f-1}{2}}$ and $(-1)^{\frac{F-1}{2}}$. It is retained in the Table only because it serves to express the criterion of possibility.

(2) When $(-1)^{\frac{f-1}{2}}$ and $\Psi$, but not $(-1)^{\frac{F-1}{2}}$, are incribed as characters, $\Psi$ represents the character $(-1)^{\frac{F-1}{2}}$, if $(-1)^{\frac{f-1}{2}}=(-1)^{\frac{\Delta_1-1}{2}}$, and is simply $+1$ (*i. e.* not a character at all), if $(-1)^{\frac{f-1}{2}}=-(-1)^{\frac{\Delta_1-1}{2}}$. This is in accordance with Table I., according to which, in the cases under consideration, $(-1)^{\frac{F-1}{2}}$ is or is not a character, according as $(-1)^{\frac{f-1}{2}}=(-1)^{\frac{\Delta_1-1}{2}}$, or $=-(-1)^{\frac{\Delta_1-1}{2}}$. Similarly, if $(-1)^{\frac{F-1}{2}}$ and $\Psi$, but not $(-1)^{\frac{f-1}{2}}$, are inscribed as characters, $\Psi$ represents the character $(-1)^{\frac{f-1}{2}}$, or is not a character at all, according as $(-1)^{\frac{F-1}{2}}=(-1)^{\frac{\Omega_1-1}{2}}$, or $=-(-1)^{\frac{\Omega_1-1}{2}}$; which again agrees with Table I.

In these cases, marked Q in the Table, the symbol $\Psi$ supersedes the asterisks and obelisks of Table I., and also serves to express the criterion of possibility.

(3) When $(-1)^{\frac{f-1}{2}}$ and $(-1)^{\frac{F-1}{2}}\times\Psi$, but not $(-1)^{\frac{F-1}{2}}$, are characters in the Table, $(-1)^{\frac{F-1}{2}}\times\Psi$ represents the character $(-1)^{\frac{F-1}{2}+\frac{F-1}{2}}$, or $(-1)^{\frac{F-1}{2}}$, according as $(-1)^{\frac{f-1}{2}}=(-1)^{\frac{\Delta_1-1}{2}}$, or $-(-1)^{\frac{\Delta_1-1}{2}}$. And again, when $(-1)^{\frac{F-1}{2}}$ and $(-1)^{\frac{f-1}{2}}\times\Psi$, but not $(-1)^{\frac{f-1}{2}}$, are characters in the Table, $(-1)^{\frac{f-1}{2}}\times\Psi$ represents the character $(-1)^{\frac{f-1}{2}+\frac{f-1}{2}}$, or $(-1)^{\frac{f-1}{2}}$, according as $(-1)^{\frac{F-1}{2}}=(-1)^{\frac{\Omega_1-1}{2}}$, or $-(-1)^{\frac{\Omega_1-1}{2}}$.

The result in these cases (marked R in the Table) is again in accordance with Table I.; and the use of the symbol $\Psi$ is the same as in the cases Q.

Thus the units $\Psi$, $(-1)^{\frac{f-1}{2}}\times\Psi$, $(-1)^{\frac{F-1}{2}}\times\Psi$, which properly represent simultaneous characters of the forms $f$ and F, are employed, in the cases Q and R of the Table, to represent supplementary characters. This use of these symbols is admissible, because, when $\Omega\Delta$ is even (as it is in the cases Q and R), every representation of an uneven number by $f$ or F is simultaneous with the representation of uneven numbers by F or $f$.

In the lower right-hand corner of each compartment in the Table, the number of possible genera contained in the order to which the compartment refers is represented by $\Gamma$; $\gamma$ is the number of uneven primes dividing $\Omega$, together with the number of uneven primes dividing $\Delta$, so that if the same prime divide both $\Omega$ and $\Delta$, it is to be counted twice. But it is to be observed that, when $\Omega$ and $\Delta$ are both perfect squares (a case which can only arise when the forms are definite), the number of possible genera is two-thirds of the number stated in the Table in the cases (Q), and one half in the cases (P). And again (as has been already stated in Art. 3), in Table B, when $\Omega$ is an uneven square, and $\Delta$ the double of a square, there are no possible genera; and when $\Delta$ is an uneven square, and $\Omega$ the double of an uneven square, there are none in Table C.

Art. 9. It results from the theorem of Art. 5 that if $f$ and F are properly primitive, they simultaneously and primitively represent uneven numbers prime to $\Omega\Delta$. We may therefore suppose that in $f$ and F, $a$ and $A''$ are uneven and prime to $\Omega\Delta$; we may also suppose that these numbers are prime to one another, because $A''$ being prime to $\Omega\Delta$, and $a$ being uneven, the binary form $(a, b'', a')$ is properly primitive (Art. 5), and so represents numbers prime to its determinant. Lastly, we may assume that $a$ and $A''$ are positive. If the forms $f$ and F are definite, $a$ and $A''$ are certainly positive; if they are indefinite, $\Delta$ and $\Omega$ are of opposite signs; supposing, for example, that $\Delta$ is positive and $\Omega$ negative, let $m$ be any positive number primitively represented by $f$, and M any number simultaneously and primitively represented by F, then M is positive as well as $m$; otherwise $mMf$, which is of the type $MX^2+\Omega Y^2+m\Omega\Delta Z^2$, would be a definite form. Positive numbers are therefore simultaneously and primitively represented by $f$ and F; *i. e.* we may suppose $a$ and $A''$ simultaneously positive. The complete generic character of $f$ is then determined by the characters of $a$ and $A''$. But

$$aa'-b^2=\Omega A'', \quad A'A''-B^2=\Delta a,$$

whence it follows that

$$\left(\frac{-\Omega}{a}\right)\left(\frac{A''}{a}\right)=1, \quad \left(\frac{-\Delta}{A''}\right)\left(\frac{a}{A''}\right)=1;$$

multiplying these equations together and observing that, by the laws of quadratic residues,

$$\left(\frac{A''}{a}\right)\left(\frac{a}{A''}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{A''-1}{2}},$$

we find

$$(-1)^{\frac{a-1}{2}\cdot\frac{A''-1}{2}}\left(\frac{-\Omega}{a}\right)\left(\frac{-\Delta}{A''}\right)=1. \quad \ldots\ldots\ldots \quad (14)$$

Let $\alpha$ and $\beta$ retain the significations assigned to them in equation (11), Art. 8; transforming $\left(\frac{-\Omega}{a}\right)$ and $\left(\frac{-\Delta}{A''}\right)$ by the law of quadratic reciprocity, we find

$$\left(\frac{-\Omega}{a}\right)=(-1)^{\frac{a-1}{2}\cdot\frac{\Omega_1+1}{2}}\alpha^{\frac{a''-1}{2}}\left(\frac{a}{\Omega_1}\right),$$

$$\left(\frac{-\Delta}{A''}\right)=(-1)^{\frac{A''-1}{2}\cdot\frac{\Delta_1+1}{2}}\beta^{\frac{A''-1}{2}}\left(\frac{A''}{\Delta_1}\right),$$

and equation (14) becomes

$$(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}+\frac{\Omega_1+A''}{2}\cdot\frac{\Delta_1+a}{2}}\alpha^{\frac{a'-1}{2}}\beta^{\frac{A''-1}{2}}\times\left(\frac{a}{\Omega_1}\right)\left(\frac{A''}{\Delta_1}\right)=1;$$

or observing that

$$(-1)^{\frac{\Omega_1+A''}{2}\cdot\frac{\Delta_1+a}{2}}=(-1)^{\frac{\Omega_1A''+1}{2}\cdot\frac{\Delta_1a+1}{2}}=\Psi,$$

and writing $f$ and F for $a$ and $A''$,

$$\Psi\times\alpha^{\frac{f'-1}{2}}\times\beta^{\frac{F'-1}{2}}\times\left(\frac{f}{\Omega_1}\right)\times\left(\frac{F}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}},$$

*i. e.* the generic character of $f$ satisfies the condition of possibility (11).

Again, if $f$ is improperly and F properly primitive, let $A''$ be prime to $2\Omega\Delta$; then the binary form $(a, b'', a')$ is primitive, because $A''$ is prime to $\Delta\Omega$, and improperly primitive, because $f$ is improperly primitive. We may therefore suppose that $\frac{1}{2}a$ is uneven and prime to $\Omega A''$, and, as before, that $a$ and $A''$ are positive. Multiplying together the equations

$$\left(\frac{-\Omega}{\frac{1}{2}a}\right)\times\left(\frac{A''}{\frac{1}{2}a}\right)=1,\quad\left(\frac{-2\Delta}{A''}\right)\left(\frac{\frac{1}{2}a}{A''}\right)=1,$$

and transforming the result by the law of reciprocity, we find

$$(-\beta)^{\frac{A''a-1}{2}}\times\left(\frac{a}{\Omega_1}\right)\times\left(\frac{A''}{\Delta_1}\right)=(-1)^{\frac{\Omega_1-1}{2}+\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}},$$

*i. e.* the condition (12) is satisfied by the generic character of $f$.

The case in which $f$ is properly and F improperly primitive, is the reciprocal of the preceding.

To show that the conditions (11), (12), (13) are sufficient as well as necessary, other principles are required. These principles relate to the representation of binary by ternary quadratic forms, and will be found in the 'Disquisitiones Arithmeticæ,' arts. 282–284; it will, however, be convenient briefly to restate them here, in a form suited for our present purpose.

Art. 10. A binary quadratic form $(p, q'', p')$ or $\varphi$ is said to be represented by a ternary form $f$ when $f$ is transformed into $\varphi$ by a substitution of the type

$$x=\alpha_1x+\beta_1y,$$
$$y=\alpha_2x+\beta_2y,$$
$$z=\alpha_3x+\beta_3y.$$

The representation is said to be *primitive* when the determinants of the matrix

$$\begin{vmatrix}\alpha_1, & \beta_1\\ \alpha_2, & \beta_2\\ \alpha_3, & \beta_3\end{vmatrix}$$

are relatively prime. If $\varphi$ is primitively represented by $f$, $f$ is equivalent to a form containing $\varphi$ as a part, *i. e.* to a form $f'$ of the type

$$f'=px^2+p'y^2+p''z^2+2qyz+2q'xz+2q''xy,$$

for $f$ is transformed into such a form by a substitution of which the matrix is

$$\begin{vmatrix} \alpha_1, & \beta_1, & \gamma_1 \\ \alpha_2, & \beta_2, & \gamma_2 \\ \alpha_3, & \beta_3, & \gamma_3 \end{vmatrix}$$

$\gamma_1, \gamma_2, \gamma_3$ denoting any three numbers which render the determinant of that matrix equal to $+1$.

Let

$$F' = Px^2 + P'y^2 + P''z^2 + 2Qyz + 2Q'xz + 2Q''xy$$

be the primitive contravariant of $f'$, so that, in particular,

$$\Omega P'' = q''^2 - pp'; \quad \ldots \ldots \ldots \ldots \quad (15)$$

multiplying the equations

$$\left.\begin{aligned} PP'' - \ Q^2 &= \Delta p, \\ QQ' - P'Q'' &= \Delta q'', \\ PP'' - \ Q'^2 &= \Delta p' \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \quad (16)$$

(which result from the contravariance of $f'$ and F') by $x^2$, $2xy$, $y^2$ respectively, we obtain

$$-\Delta \times (px^2 + 2q''xy + p'y^2) = (Q^2 - PP'')x^2 - 2(QQ' - P''Q'')xy + (Q'^2 - PP'')y^2;$$

and this equation, considered as a congruence for the modulus P'', becomes

$$\Delta \times \phi + (Qx - Q'y)^2 \equiv 0, \text{ mod } P'', \quad \ldots \ldots \ldots \quad (17)$$

the coefficients of $x^2$, $2xy$, $y^2$ in the left-hand member being all divisible by P''. If therefore $\phi$ is a binary quadratic form of determinant $-\Omega P''$, admitting of primitive representation by a ternary form of order $[\Omega, \Delta]$, $-\Delta\phi$ is a quadratic residue of P''. And we shall now show that if $\phi$ is a primitive (and not negative) binary form of determinant $-\Omega P''$, P'' being of the same sign as $\Delta$ and prime to $\Delta$, $\phi$ admits of primitive representation by ternary forms of the invariants $[\Omega, \Delta]$, whenever $-\Delta\phi$ is a quadratic residue of P''.

Because $-\Delta\phi$ is a quadratic residue of P'', the congruence (17) admits of solution in integral numbers Q, Q'. Any solution of this congruence supplies a system of five numbers, P, P', Q, Q', Q'', satisfying the equations (16). The greatest common divisor of these five numbers divides $\Delta$, because $p$, $q''$, $p'$ are relatively prime; but P'' is prime to $\Delta$; therefore the six numbers P, P', P'', Q, Q', Q'' are relatively prime. Let $q$ and $q'$ be determined by the equations

$$\left.\begin{aligned} qq'' - q'p' &= \ \Omega Q', \\ qp - q'q'' &= -\Omega Q, \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \quad (18)$$

which are always resoluble because their determinant $q''^2 - pp' = \Omega P''$ is different from zero. Also let $p''$ be determined by the equation

$$q'Q' + qQ + p''P'' = \Omega\Delta. \quad \ldots \ldots \ldots \ldots \quad (19)$$

The values of $q$, $q'$, $p''$ are rational; and, if they are fractions, their denominators, when they are expressed in their lowest terms, are divisors of P''. Substituting in (19) for

$P''$, $Q'$, $Q$ their values derived from the equations (15) and (18), we find that $\Omega^2\Delta$ is the determinant of the matrix

$$\begin{vmatrix} p\ ,\ q''\ ,\ q' \\ q''\ ,\ p'\ ,\ q \\ q'\ ,\ q\ ,\ p'' \end{vmatrix} \quad \ldots\ldots\ldots\ldots \quad (20)$$

Let

$$\begin{vmatrix} \Omega[p\,], & \Omega[q''], & \Omega[q'\,] \\ \Omega[q''], & \Omega[p'], & \Omega[q\,] \\ \Omega[q'], & \Omega[q\,], & \Omega[p''] \end{vmatrix} \quad \ldots\ldots\ldots \quad (21)$$

be the matrix reciprocal to the matrix (20); we know, from the equations (15) and (18), that $[p'']=P''$, $[q']=Q'$, $[q]=Q$. Again, in the reciprocal matrices (20) and (21), we must have

$$\begin{aligned} &[p'][p'']-[q]^2=\Delta p, \\ &[q][q'] \ -[p''][q']=\Delta q'', \\ &[p][p'']-[q'^2]=\Delta p', \end{aligned}$$

or substituting for $[p'']$, $[q]$, $[q']$ their values,

$$\begin{aligned} &[p']P''-Q^2=\Delta p, \\ &QQ'-P''[q'']=\Delta q'', \\ &[p]P''-Q'^2=\Delta p''. \end{aligned}$$

Comparing these equations with the equations (16), and observing that $P''$ is not zero, we infer that

$$[p']=P', \quad [q'']=Q'', \quad [p]=P.$$

The matrix reciprocal to the matrix (20) is therefore

$$\begin{vmatrix} \Omega P\ , & \Omega Q''\ , & \Omega Q' \\ \Omega Q''\ , & \Omega P'\ , & \Omega Q \\ \Omega Q'\ , & \Omega Q\ , & \Omega P'', \end{vmatrix} \quad \ldots\ldots\ldots \quad (22)$$

and, consequently,

$$\begin{aligned} &\Delta q=Q'Q''-PQ, \\ &\Delta q'=QQ''-P'Q', \\ &\Delta p''=PP'-Q''^2. \end{aligned}$$

These equations prove that the denominators of $q$, $q'$, $p''$ are divisors of $\Delta$; *i. e.* that $q$, $q'$, $p''$ are integral numbers, because $P''$ is prime to $\Delta$. The coefficients of the ternary form

$$f'=px^2+p'y^2+p''z^2+2qyz+2q'xz+2''qxy$$

are therefore integral; this form is primitive, and represents primitively the form ($p$, $q''$, $p'$); it is also a form of the given invariants $[\Omega, \Delta]$; for its discriminant is $\Delta\Omega^2$, and the greatest common divisor of the first minors of its matrix is $\Omega$; hence its second invariant is $\Delta$, and its first invariant either $+\Omega$, or $-\Omega$. But when the given invariants $\Omega$ and $\Delta$ are both positive, $\varphi$ is a positive binary form of the negative determinant

$-\Omega P''$; and such a form cannot be represented by an indefinite ternary form of a positive discriminant; $f'$ is therefore definite, and its first invariant is $+\Omega$. When the given invariants $\Omega$ and $\Delta$ are of opposite signs, $\varphi$ is a binary form of the positive determinant $-\Omega P''$; such a form cannot be represented by a definite ternary form; $f'$ is therefore indefinite, and, as its invariants must be of opposite signs, in this case also its first invariant is $+\Omega$.

Also, if $\varphi$ is properly primitive and $P''$ uneven, the forms $f'$ and $F'$ are both properly primitive, one of the principal coefficients of each being uneven. In this case, therefore, $\varphi$ is represented by a form of the properly primitive order $[\Omega, \Delta]$. If $\varphi$ is improperly primitive (a supposition which implies that $\Omega P'' \equiv 3$, mod 4), and if $\Delta$ is even, $f'$ is improperly primitive. For no properly primitive ternary form of even discriminant can represent primitively an improperly primitive binary form, the supposition that ($p$, $q''$, $p'$) is improperly primitive and $p''$ uneven implying that the discriminant is uneven. And the same thing follows from the preceding analysis; for, considering the equations (18) as congruences for the modulus 2, we find on the supposition that $\varphi$ is improperly primitive, $q \equiv Q'$, mod 2, $q' \equiv Q$, mod 2, and substituting in (19), $p'' \equiv 0$, mod 2, so that $f'$ is improperly primitive.

Art. 11. We can now assign a properly primitive form of any given invariants $[\Omega, \Delta]$, and of any given generic character satisfying the condition of possibility. Let M be any number prime to $2\Delta$, of the same sign as $\Delta$, and possessing all the particular characters (except the simultaneous character, if any) which are attributed to F in the given generic character; also if $\Omega$ is uneven, and $\Delta$ uneven or unevenly even, we shall suppose that $M \equiv \Omega$, mod 4. Let $\varphi$ be any properly primitive, and not negative binary quadratic form of determinant $-\Omega M$; and let $m$ be any number prime to $2\Omega M$ which is represented by $\varphi$. By the theory of binary quadratic forms, the generic characters which are attributable to $\varphi$, are (i) its characters with respect to primes dividing M, (ii) its characters with respect to primes dividing $\Omega$, (iii) its supplementary characters. These last are exhibited in the following Table,

| If $-\Omega M \equiv$ | Supplementary characters. |
|---|---|
| 1, mod 4. | None. |
| 3, mod 4. | $(-1)^{\frac{\varphi-1}{2}}$ |
| 2, mod 8. | $(-1)^{\frac{\varphi^2-1}{8}}$ |
| 6, mod 8. | $(-1)^{\frac{\varphi-1}{2}+\frac{\varphi^2-1}{8}}$ |
| 4, mod 8. | $(-1)^{\frac{\varphi-1}{2}}$ |
| 0, mod 8. | $(-1)^{\frac{\varphi-1}{2}}$, $(-1)^{\frac{\varphi^2-1}{8}}$ |

Let $\mu$ be any prime divisor of M, and let us determine the first set of characters by the equations

$$\left(\frac{\varphi}{\mu}\right)=\left(\frac{-\Delta}{\mu}\right), \quad \ldots \quad (23)$$

the second set by the equations

$$\left(\frac{\varphi}{\omega}\right)=\left(\frac{f}{\omega}\right), \quad \ldots \quad (24)$$

$\left(\frac{f}{\omega}\right)$ being a particular character of $f$, of which the value is assigned in the proposed generic character. With respect to the supplementary characters of $\varphi$, it will be found on a comparison of the above Table with Table II. A, that, when the proposed generic character includes no simultaneous character, the supplementary characters attributable to $\varphi$ are the same as those attributable to $f$; we then assign to the supplementary characters of $\varphi$ the same values which are assigned to the supplementary characters of $f$ in the proposed generic character. But when the proposed generic character includes a simultaneous character, there is always a supplementary character (and only one) attributable to $\varphi$, and not to $f$; this character of $\varphi$ we determine so that the value of the simultaneous character of $f$ and F, and the value of the unit similarly formed with $m$ and M, may be coincident. This determination is always possible, as will be seen on a comparison of the cases (S) of Table II. A, with the above Table of supplementary characters of binary forms. As we have now assigned a value to every particular character attributable to $\varphi$, it is necessary to inquire whether a form $\varphi$, possessing such a complete character, actually exists; *i. e.* whether the character that we have assigned to $\varphi$ satisfies the condition of possibility for binary forms of determinant $-\Omega M$.

If, as in art. 8, $\alpha=+1$, or $-1$, according as $\Omega$ is of the form $\Omega_1\Omega_2^2$, or $2\Omega_1\Omega_2^2$, that condition is

$$(-1)^{\frac{\Omega_1 M+1}{2}\cdot\frac{\varphi-1}{2}}\alpha^{\frac{\varphi^2-1}{8}}\left(\frac{\varphi}{\Omega_1 M}\right)=1, \quad \ldots \quad (25)$$

or, since

$$(-1)^{\frac{\Delta_1+1}{2}+\frac{\Delta_1\varphi+1}{2}}=(-1)^{\frac{\varphi-1}{2}},$$

$$(-1)^{\frac{\Omega_1 M+1}{2}\cdot\frac{\Delta_1+1}{2}+\frac{\Omega_1 M+1}{2}\cdot\frac{\Delta_1\varphi+1}{2}}\alpha^{\frac{\varphi^2-1}{8}}\left(\frac{\varphi}{\Omega_1 M}\right)=1, \quad \ldots \quad (26)$$

But $\left(\frac{\varphi}{\Omega_1}\right)=\left(\frac{f}{\Omega_1}\right)$, by the equations (24), and if (again as in art. 8) $\beta=+1$, or $-1$, according as $\Delta$ is of the form $\Delta_1\Delta_2^2$, or $2\Delta_1\Delta_2^2$,

$$\left(\frac{\varphi}{M}\right)=\left(\frac{-\Delta}{M}\right)=(-1)^{\frac{M-1}{2}\cdot\frac{\Delta_1+1}{2}}\times\beta^{\frac{M^2-1}{8}}\times\left(\frac{M}{\Delta_1}\right).$$

Substituting these values in (26), and observing that in every case

$$(-1)^{\frac{\Omega_1 M+1}{2}\cdot\frac{\Delta_1\varphi+1}{2}}\alpha^{\frac{\varphi^2-1}{8}}\beta^{\frac{M^2-1}{8}}=\Psi\times\alpha^{\frac{f^2-1}{8}}\beta^{\frac{F^2-1}{8}},$$

we obtain

$$\Psi\times\alpha^{\frac{f^2-1}{8}}\beta^{\frac{F^2-1}{8}}\left(\frac{f}{\Omega_1}\right)\left(\frac{F}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}.$$

But this equation is the equation (11) of art. 8, which is by hypothesis satisfied by the proposed generic character; therefore the equation (25) is also satisfied; *i. e.* a properly primitive binary form $\varphi$ exists, of determinant $-\Omega M$, possessing the generic character which we have assigned to it. This form, multiplied by $-\Delta$, is a quadratic residue of M; for the equation

$$\left(\frac{-\Delta\varphi}{\mu}\right)=1$$

is satisfied for every prime dividing M, by virtue of the equations (23). Let, then, a ternary form $f$, of the properly primitive order, and of the invariants $[\Omega, \Delta]$, be determined, representing $\varphi$ primitively. The generic character of this form is completely determined by the numbers $m$ and M, which are uneven numbers simultaneously represented by $f$ and F; it is therefore a form of the proposed generic character.

Of the two improperly primitive orders, it will suffice to consider that in which $f$ is improperly and F properly primitive; so that $\Omega$ is uneven and $\Delta$ even. Let M be a number prime to $2\Omega\Delta$, of the same sign as $\Delta$, and satisfying the generic characters of F, including the congruence $M \equiv -\Omega, \mod 4$; also let $\varphi$ be an improperly primitive binary form of determinant $-\Omega M$; the generic characters attributable to $\varphi$ are (i) its characters $\left(\frac{\varphi}{\mu}\right)$, (ii) its characters $\left(\frac{\varphi}{\omega}\right)$. These characters we determine, as before, by the equations (23) and (24). The complete generic character thus assigned to $\varphi$ is possible; for the condition that it should be possible is

$$\left(\frac{\varphi}{\Omega_1 M}\right)=(-1)^{\frac{\Omega_1^2-1}{8}+\frac{M^2-1}{8}},$$

or

$$\left(\frac{f}{\Omega_1}\right)\left(\frac{-2\Delta}{M}\right)=(-1)^{\frac{\Omega_1^2-1}{8}}.$$

Transforming $\left(\frac{-2\Delta}{M}\right)$ by the law of reciprocity, we find

$$(-\beta)^{\frac{F^2-1}{8}}\times\left(\frac{f}{\Omega_1}\right)\left(\frac{F}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}+\frac{\Omega_1^2-1}{8}},$$

an equation which the proposed generic character satisfies by hypothesis (equation (12) Art. 8). An improperly primitive form $\varphi$ of determinant $-\Omega M$ therefore actually exists, having the generic character which we have assigned to it; *i. e.* ternary forms exist having the proposed generic character.

It is evident from the demonstration that if M is of the same sign as $\Delta$, prime to $2\Delta$, and also (when $\Omega$ is uneven and $\Delta$ uneven or unevenly even) congruous to $\Omega$, mod 4, there is always one genus of properly primitive binary forms of determinant $-\Omega M$ capable of primitive representation by a given genus of ternary forms of the properly primitive order $[\Omega, \Delta]$, of which the contravariant characters coincide with the characters of M. And similarly, if $\Delta$ is even, $\Omega$ uneven, M prime to $\Delta$, and $\equiv -\Omega$, mod 4, there is always one genus of improperly primitive binary forms of determinant $-\Omega M$

capable of primitive representation by a given genus of ternary forms of the improperly primitive order $[\Omega, \Delta]$, of which the contravariant characters coincide with the characters of M. And in both cases no other primitive form (if M is prime to $\Omega$, no other form, primitive or derived) of determinant $-\Omega M$ is capable of such representation.

Art. 12. By a rational substitution we shall understand in this article a substitution of which the determinant is unity, and of which the coefficients are rational. If the common denominator of the coefficients is prime to any number $m$, we shall say that the substitution is prime to $m$.

If $f_1$ and $f_2$ are ternary forms, having integral coefficients, of which $f_1$ is a form of the invariants $(\Omega, \Delta)$, and is transformed by a rational substitution, prime to $2\Omega\Delta$, into $f_2$, $f_2$ is a form of the same invariants, of the same order, and of the same genus as $f_1$. This may be proved nearly in the same way in which it is proved that equivalent forms have the same invariants and are of the same order and genus; it is only necessary to observe that $F_1$ and $F_2$, as well as $f_1$ and $f_2$, are transformable into one another by rational substitutions, prime to $2\Omega\Delta$. The converse proposition,

"If $f_1$ and $f_2$ are two forms of the same invariants $(\Omega, \Delta)$, of the same order, and of the same genus, they are transformable, each into the other, by rational substitutions prime to $2\Omega\Delta$," is also true, and is of importance in the present theory, because it establishes the completeness of the enumeration of the generic characters of ternary forms. To avoid the introduction, in this place, of principles relating to quaternary quadratic forms, we shall give an indirect demonstration of it, depending on the following lemma which relates to binary quadratic forms.

"If $\varphi_1$, $\varphi_2$ are two primitive binary quadratic forms of the same determinant, and of the same genus, the resolubility of the equation $\varphi_1(x, y)=M$ implies the resolubility of the equation $\varphi_2(x, y)=Mz^2$; and in the solution of this equation the value of $z$ may be supposed prime to any given number $k$."

Because $\varphi_1$ and $\varphi_2$ are of the same genus, $\varphi_2$ is transformable, by a bipartite linear substitution, into the product $\chi\times\varphi_1$, $\chi$ representing a properly primitive form of the principal genus (Disq. Arith. art. 251). But $\chi$ is transformable, by a quadratic substitution, into the square of a properly primitive form $\psi$ (ibid. art. 287). Therefore, by a mixed quadratic and linear substitution, $\varphi_2$ is transformed into the product $\psi^2\times\varphi_1$. Attributing, in this mixed substitution, to the indeterminates of $\varphi_1$ the values which satisfy the equation $\varphi_1=M$, and to the indeterminates of $\psi$ any values whatever for which $\psi$ acquires a value $z$ prime to $k$, we obtain a solution of the equation $\varphi_2=Mz^2$.

Let us first suppose that the given ternary forms $f_1$ and $f_2$ belong to the properly primitive order of the invariants $(\Omega, \Delta)$; let $M_1$, $M_2$ be two numbers of the same sign as $\Delta$, prime to $2\Omega\Delta$, and primitively represented by $F_1$, $F_2$ respectively; we may suppose that $M_1=M_2$, mod 8; and that the representations of $M_1$ and $M_2$ are simultaneous with the representations of uneven numbers by $f_1$ and $f_2$. Let $\varphi_1$, $\varphi_2$ be two binary quadratic forms, of the determinants $-\Omega M_1$, $-\Omega M_2$ respectively, represented by $f_1$ and $f_2$ simul-

taneously with the representations of $M_1$ and $M_2$ by $F_1$ and $F_2$*. Then $\varphi_1$ and $\varphi_2$ are properly primitive; their generic characters with respect to uneven primes dividing $\Omega$ will coincide, because

$$\left(\frac{\phi}{\omega}\right)=\left(\frac{f_1}{\omega}\right)=\left(\frac{f_2}{\omega}\right)=\left(\frac{\phi_2}{\omega}\right);$$

their supplementary characters will also coincide; for the same supplementary characters are attributable to $\varphi_1$ and $\varphi_2$, and these supplementary characters are determined for $\varphi_1$, in accordance with the supplementary characters of $f_1$, or the simultaneous character of $f_1$ and $F_1$, and for $\varphi_2$ in accordance with characters which are the same with these; lastly, if $\mu$ is any prime dividing both $M_1$ and $M_2$, the characters of $\varphi_1$ and $\varphi_2$ with respect to $\mu$ will also coincide; for

$$\left(\frac{\phi_1}{\mu}\right)=\left(\frac{-\Delta}{\mu}\right)=\left(\frac{\phi_2}{\mu}\right).$$

The remaining characters of $\varphi_1$ and $\varphi_2$ (*i. e.* their characters with respect to primes dividing only one of the two numbers $M_1$ and $M_2$), being characters with respect to different primes, cannot be incompatible. The complete generic characters of $\varphi_1$ and $\varphi_2$ are therefore compatible, and are satisfied by the numbers contained in certain arithmetical progressions. Each of these progressions contains (by the theorem of LEJEUNE DIRICHLET) an infinite number of positive and negative primes. Let $p$ be one of these primes of the same sign as $\Omega$, and not dividing $2\Omega\Delta$; $p$ will satisfy the generic characters both of $\varphi_1$ and $\varphi_2$, and will be represented by some form of determinant $-\Omega M_1$, and of the same genus as $\varphi_1$, and by some form of determinant $-\Omega M_2$, and of the same genus as $\varphi_2$. Therefore, by the lemma of this article, $p\theta_1^2$ will be primitively represented by $\varphi_1$, and $p\theta_2^2$ by $\varphi_2$, $\theta_1$ and $\theta_2$ denoting numbers prime to $2\Omega\Delta$. Let $\Phi_1$, $\Phi_2$ be two properly primitive binary forms represented by $F_1$, $F_2$, simultaneously with the representations of $p\theta_1^2$, $p\theta_2^2$, by $f_1$, $f_2$. The determinants of $\Phi_1$, $\Phi_2$ are $-\Delta p\theta_1^2$, $-\Delta p\theta_2^2$; and it will be found (as in the case of the forms $\varphi_1$, $\varphi_2$) that the generic characters of $\Phi_1$, $\Phi_2$ are compatible; and that a prime P of the same sign as $\Delta$, and not dividing $2\Omega\Delta$, is assignable, such that $P\Theta_1^2$, $P\Theta_2^2$ are primitively represented by $\Phi_1$, $\Phi_2$ respectively, $\Theta_1$ and $\Theta_2$ denoting numbers prime to $2\Omega\Delta$. Thus the numbers $p\theta_1^2$, $P\Theta_1^2$ are simultaneously and primitively represented by $f_1$ and $F_1$; the numbers $p\theta_2^2$, $P\Theta_2^2$ are simultaneously and primitively represented by $f_2$ and $F_2$. We may therefore suppose that $\psi_1$ is a form equivalent to $f_1$, in which $a_1=p\theta_1^2$, $A_1''=P\Theta_1^2$, and that $\psi_2$ is a form equivalent to

* If

$$M=F(\alpha'\beta''-\alpha''\beta',\ \alpha''\beta-\alpha\beta'',\ \alpha\beta'-\alpha'\beta),$$

and if $f$ is transformed into a binary form $\varphi$ by the substitution

$$\begin{aligned} x&=\alpha x+\beta y,\\ y&=\alpha' x+\beta' y,\\ z&=\alpha'' x+\beta'' y, \end{aligned}$$

the representations of M by F, and of $\varphi$ by $f$, are said to be simultaneous, or to appertain to one another (GAUSS, Disq. Arith. Art. 280).

$f_2$, in which $a_2 = p\vartheta_1^2$, $A_2'' = P\Theta_1^2$. The fractional form

$$\frac{1}{Pp}[PX^2+\Omega Y^2+p\Omega\Delta Z^2]$$

is then transformed into $\psi_1$ by the substitution

$$\begin{vmatrix} \frac{a_1}{\vartheta_1}, & \frac{b_1''}{\vartheta_1}, & \frac{b_1'}{\vartheta_1} \\ 0, & \frac{A_1''}{\Theta_1\vartheta_1}, & -\frac{B_1}{\Theta_1\vartheta_1} \\ 0, & 0, & \frac{1}{\Theta_1} \end{vmatrix}$$

of which the determinant is $Pp$, and into $\psi_2$ by a similar substitution of the same determinant. Either of the two forms $\psi_1$, $\psi_2$ (and consequently either of the two $f_1$, $f_2$) is therefore transformable into the other, by a rational substitution prime to $2\Omega\Delta$. It will be found that if the signs of $\Theta_1$, $\Theta_2$, $\vartheta_1$, $\vartheta_2$ are properly determined, the primes P, $p$ will not appear in the denominators of these substitutions.

If $f_1$ and $f_2$ belong to an improperly primitive order, the preceding proof requires very little modification. It will suffice to consider the case in which $f_1$ and $f_2$ are improperly, $F_1$ and $F_2$ properly primitive. We take $M_1 \equiv M_2 \equiv -\Omega$, mod 4; $\varphi_1$ and $\varphi_2$ are then improperly primitive and have compatible generic characters; let $2p\vartheta_1^2$ be represented by $\varphi_1$, and $2p\vartheta_2^2$ by $\varphi_2$; $\Phi_1$ and $\Phi_2$ are properly primitive and of the determinants $-2\Delta p\vartheta_1^2$, $-2\Delta p\vartheta_2^2$; these forms have compatible generic characters (their supplementary characters, in particular, being determined by those of $F_1$ and $F_2$); let, then, $P\Theta_1^2$ be represented by $\Phi_1$ and $P\Theta_2^2$ by $\Phi_2$, and let us suppose that $\psi_1$, $\psi_2$ are forms equivalent to $f_1$, $f_2$, in which $a_1 = 2p\vartheta_1^2$, $A_1'' = P\Theta_1^2$, $a_2 = 2p\vartheta_2^2$, $A_2'' = P\Theta_2^2$; the fractional form

$$\frac{1}{Pp}[\tfrac{1}{2}(P+\Omega)X^2+(\Omega-P)XY+\tfrac{1}{2}(P+\Omega)Y^2+p\Omega\Delta Z^2]$$

is transformed into $\psi_1$ by the substitution

$$\begin{vmatrix} \frac{1}{2}\frac{a_1}{\vartheta_1}, & \frac{1}{2}\frac{b_1''\Theta_1+A_1''}{\Theta_1\vartheta_1}, & \frac{1}{2}\frac{b_1'\Theta_1-B_1}{\Theta_1\vartheta_1} \\ -\frac{1}{2}\frac{a_1}{\vartheta_1}, & -\frac{1}{2}\frac{b_1''\Theta_1-A_1''}{\Theta_1\vartheta_1}, & -\frac{1}{2}\frac{b_1'\Theta_1+B_1}{\Theta_1\vartheta_1} \\ 0, & 0, & \frac{1}{\Theta_1} \end{vmatrix}$$

and into $\psi_2$ by a similar substitution. The determinant of each of these substitutions is $Pp$, and the denominators of their coefficients do not contain the prime 2, because $b_1''$, $b_2''$, $A_1''$, $A_2''$, $\Theta_1$, $\Theta_2$ are all uneven, and because $B_1 \equiv b_1'$, mod 2, $B_2 \equiv b_2'$, mod 2. Each of the forms $f_1$, $f_2$ is therefore transformable into the other by a rational substitution prime to $2\Omega\Delta$.

Art. 13. We have hitherto considered ternary forms of a negative determinant, definite or indefinite; we shall now confine our attention to definite forms. By a binary

form we shall henceforward understand a positive form of negative determinant, by a ternary form a positive and definite form; and we shall occupy ourselves in the remainder of this memoir with the determination of the weight of a given genus or order of such ternary forms.

A ternary form has always 1, 2, 4, 6, 8, 12, or 24 *positive* automorphics, *i. e.* automorphics of which the determinant is a positive unit. The weight of a form is the reciprocal of the number of its positive automorphics; so that a form and its contravariant have the same weight; the weight of a class is the weight of any form contained in the class; the weight of a genus or of an order is the sum of the weights of the non-equivalent classes contained in the genus or order. When a number is represented by a ternary form, the weight of the representation is the weight of the ternary form. The weight of a binary form, or class, is also the reciproeal of the number of its positive automorphics; thus the weight of a binary form is always $\frac{1}{2}$, except when the form either is, or is derived from, a form of determinant $-1$, or an improperly primitive form of determinant $-3$; in these excepted cases the weight of the binary form is $\frac{1}{4}$ and $\frac{1}{6}$ respectively. When a binary form is represented by a ternary form, the weight of the representation is the product of the weights of the two forms.

To determine the weight of a given genus of ternary forms, we avail ourselves of the principles introduced into arithmetic by Gauss and Dirichlet, and employed by them to determine the number of binary forms of any given determinant. Let $(f, \mathrm{F})$ represent a given genus of ternary forms of the invariants $[\Omega, \Delta]$, and either of the properly primitive order, or of that improperly primitive order in which $f$ is improperly and F properly primitive. Let $f_1, f_2, \ldots$ or $(f)$ denote a system of forms representing the classes of the given genus; $\mathrm{F}_1, \mathrm{F}_2, \ldots$ or (F), the primitive contravariants of those forms. Let M represent any positive number, prime to $2\Omega\Delta$ and satisfying the generic characters of F; when $(f, \mathrm{F})$ is a properly primitive genus, $\Omega$ being uneven, and $\Delta$ uneven or unevenly even, we shall also suppose that M satisfies the congruence $\Omega\mathrm{M} \equiv 1$, mod 4: the numbers designated by M will be subject to the restrictions here stated throughout the whole investigation. Lastly, let L be a positive quantity which we shall afterwards suppose to increase without limit; and let T be the sum of the weights of the representations by the forms (F) of all the numbers M which do not surpass L. The quotient $\mathrm{T} \div \mathrm{L}^{\frac{3}{2}}$ approximates to a finite limit, when L is increased without limit. Of this limit, we shall obtain two distinct expressions, the one containing as a factor the weight W of the genus $(f, \mathrm{F})$, the other not containing that factor, and depending on the arithmetical relation which subsists between the sum of the weights of the representations of a given number M by the forms (F), and the sum of the weights of the properly or improperly primitive binary classes of determinant $-\Omega\mathrm{M}$. A comparison of the two expressions will then give the required weight of the genus $(f, \mathrm{F})$.

Art. 14. The first determination of the limit of the quotient $\mathrm{T} \div \mathrm{L}^{\frac{3}{2}}$ depends on the following auxiliary propositions, in which F represents any form of the system (F).

(1) If $\delta$ is an uneven prime dividing $\Delta$, F acquires a value prime to $\delta$ for $\delta^2(\delta-1)$ systems of values of $x, y, z$, mod $\delta$.

As, instead of $f$ and F, we may consider any forms equivalent to $f$ and F, we may suppose that $f$ and F satisfy, for any assigned powers of the uneven primes dividing $\Omega\Delta$, the congruences of Art. 5,

$$f \equiv \alpha x^2 + \beta\Omega y^2 + \gamma\Omega\Delta z^2,$$
$$F \equiv \beta\gamma\Omega\Delta x^2 + \alpha\gamma\Delta y^2 + \alpha\beta z^2,$$
$$\alpha\beta\gamma \equiv 1.$$

The congruence $F \equiv 0$, mod $\delta$, is then satisfied by $\delta^2$ systems of values of $x, y, z$, mod $\delta$; for $z$ must be divisible by $\delta$, but $x$ and $y$ may have any values, mod $\delta$; F is therefore prime to $\delta$ for the remaining $\delta^2(\delta-1)$ systems of values of $x, y, z$, mod $\delta$.

(2) If $\omega$ is an uneven prime dividing $\Omega$, but not $\Delta$, F is prime to $\Omega$, for $\omega(\omega-1)\left(\omega-\left(\frac{-\Delta f}{\omega}\right)\right)$ systems of values of $x, y, z$, mod $\omega$.

For if $F \equiv 0$, mod $\omega$, $x$ may have any value, mod $\omega$, but $y$ and $z$ must have values satisfying the congruence $\gamma\Delta y^2 + \beta z^2 \equiv 0$, mod $\omega$. If $\left(\frac{-\Delta\beta\gamma}{\omega}\right) = -1$, the only values of $y$ and $z$ that satisfy this congruence are $y \equiv 0$, $z \equiv 0$, mod $\omega$; and the congruence $F \equiv 0$, mod $\omega$, is satisfied by $\omega$ systems of values of $x, y, z$, mod $\omega$. If $\left(\frac{-\Delta\beta\gamma}{\omega}\right) = +1$, the congruence $\gamma\Delta y^2 + \beta z^2 \equiv 0$, mod $\omega$, is satisfied by $2\omega-1$ systems of values of $y$ and $z$; in this case therefore the congruence $F \equiv 0$, mod $\omega$, admits of $\omega(2\omega-1)$ solutions. And, observing that $\left(\frac{-\Delta\beta\gamma}{\omega}\right) = \left(\frac{-\Delta\alpha}{\omega}\right) = \left(\frac{-\Delta f}{\omega}\right)$, we find in both cases alike that F is prime to $\omega$ for $\omega(\omega-1)\left(\omega-\left(\frac{-\Delta f}{\omega}\right)\right)$ systems of values of $x, y, z$, mod $\omega$.

(3) It is evident from the congruence

$$F \equiv Ax^2 + A'y^2 + A''z^2, \text{ mod } 2,$$

in which one at least of the numbers A, A′, A″ is uneven, that F acquires an uneven value for 4 out of the 8 systems of values, mod 2, which can be attributed to $x, y, z$.

(4) If $\Omega\Delta$ is uneven, the number of solutions of the congruence $\Omega F \equiv 1$, mod 4, is $8(2-\Psi)$.

For this congruence may be written in the form (art. 6)

$$\alpha x^2 + \beta y^2 + \gamma z^2 \equiv 1, \text{ mod } 4,$$

$\alpha, \beta, \gamma$ representing uneven numbers which satisfy the congruence $\alpha+\beta+\gamma+1 \equiv 0$, mod 4. Of the three numbers $x, y, z$ one must be uneven, the other two even. The number of solutions in which $x$ is uneven, $y$ and $z$ even, is 8 or 0, according as $\alpha \equiv +1$, or $\equiv -1$, mod 4. The whole number of solutions is therefore

$$12 + 4\left[(-1)^{\frac{\alpha-1}{2}} + (-1)^{\frac{\beta-1}{2}} + (-1)^{\frac{\gamma-1}{2}}\right],$$

*i. e.* 24, or 8, according as the congruences $\alpha \equiv \beta \equiv \gamma \equiv 1$, mod 4, are, or are not satisfied; or again (Art. 6), according as $\Psi = -1$, or $\Psi = +1$. The congruence $\Omega F \equiv 1$, mod 4, admits therefore of $8(2-\Psi)$ solutions.

(5) If $\Omega$ is uneven, and $\Delta$ unevenly even, $f$ as well as F being properly primitive, there are 16 solutions of the congruence $\Omega F \equiv 1$, mod 4; for this congruence may be written in the form (Art. 6)

$$2\alpha x^2+2\beta y^2+\gamma z^2 \equiv 1, \text{ mod } 4.$$

For clearness, we shall henceforward represent by $r$ any uneven prime dividing both $\Omega$ and $\Delta$, by $\delta$ any uneven prime dividing $\Delta$, but not $\Omega$; by $\omega$ any uneven prime dividing $\Omega$, but not $\Delta$. Let $\theta=2-\Psi$, if $\Omega \equiv \Delta \equiv 1$, mod 2; $\theta=2$, if, $f$ and F being properly primitive, $\Omega$ is uneven and $\Delta$ unevenly even; $\theta=4$ in every other case; also let

$$\nabla = 4\Pi r \times \Pi\omega \times \Pi\delta,$$

$$\psi(\nabla)=\frac{\theta}{8}\nabla^2\Pi\left[1-\frac{1}{r}\right]\Pi.\left[1-\frac{1}{\omega}\right]\Pi\left[1-\frac{1}{\delta}\right]\Pi\left[1-\left(\frac{-\Delta f}{\omega}\right)\frac{1}{\omega}\right].$$

Combining the lemmas (1)...(5) we obtain the theorem—

"The form F represents numbers of the series M for $\psi(\nabla)$ of the $\nabla^3$ systems of values, mod $\nabla$, that can be attributed to $x, y, z$."

Let $x_0, y_0, z_0$ represent one of these $\psi(\nabla)$ systems of values; it is evident that F represents a number of the series M for every system of values of $x, y, z$ included in the formulæ

$$\left.\begin{aligned} x &= \nabla X + x_0 \\ y &= \nabla Y + y_0 \\ z &= \nabla Z + z_0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (27)$$

in which X, Y, Z represent any integral numbers whatever. It is also evident that there are as many systems of values of $x, y, z$ included in the formulæ (27), for which F acquires a value not surpassing L, as there are points having their rectangular coordinates of the form

$$x=\frac{\nabla X+x_i}{\sqrt{L}},$$

$$y=\frac{\nabla Y+y_i}{\sqrt{L}},$$

$$z=\frac{\nabla Z+z_i}{\sqrt{L}},$$

and lying inside, or on the surface of, the ellipsoid,

$$F(x, y, z)=1. \quad \ldots\ldots\ldots\ldots \quad (28)$$

Let $\nu_i$ be the number of these points, and let L be increased without limit; the limit of the fraction $\frac{\nabla^3 \nu_i}{L^{\frac{3}{2}}}$ is the volume of the ellipsoid (28), or $\frac{4}{3}\frac{\pi}{\Delta\sqrt{\Omega}}$. Extending this result to all the $\psi(\nabla)$ values of $i$, we find

$$\lim \frac{\Sigma \nu_i}{L^{\frac{3}{2}}}=\frac{4}{3}\frac{\psi(\nabla)}{\nabla^3}\cdot\frac{\pi}{\Delta\sqrt{\Omega}}. \quad \ldots\ldots\ldots\ldots \quad (29)$$

Let $\tau$ be the sum of the weights of the representations of the numbers M which do not surpass L by the form F, and let $w$ be the weight of $f$ or F, so that $\tau = w\Sigma\nu_i$; the equation (29) becomes

$$\lim \frac{\tau}{L^{\frac{3}{2}}} = \frac{4}{3} \frac{\psi(\nabla)}{\nabla^3} \cdot \frac{\pi}{\Delta\sqrt{\Omega}} \cdot w; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

or considering in succession all the forms of (F), and observing that $T = \Sigma\tau$, $W = \Sigma w$,

$$\lim \frac{T}{L^{\frac{3}{2}}} = \frac{\vartheta W}{6} \times \Pi\left(1-\frac{1}{r}\right)\Pi\left(1-\frac{1}{\vartheta}\right)\Pi\left(1-\frac{1}{\omega}\right) \times \Pi\left[1-\left(\frac{-\Delta f}{\omega}\right)\frac{1}{\omega}\right], \quad . \quad . \quad (31)$$

which is the first determination of the limit of the quotient $\frac{T}{L^{\frac{3}{2}}}$.

Art. 15. The second determination of the limit of the quotient $T \div L^{\frac{3}{2}}$ depends on the following theorem:—

"The sum of the weights of the primitive representations by the forms (F) of a given number M divisible by $\mu$ unequal primes, is $2^\mu$ times the weight of a genus of binary forms, of determinant $-\Omega M$, and properly or improperly primitive, according as the forms $(f)$ are properly or improperly primitive."

The principles which give the demonstration of this theorem are contained in Arts. 280–284 of the 'Disquisitiones Arithmeticæ,' and have been in part already employed in Art. 10 of this memoir. We have shown in Art. 11 that one genus and only one of binary forms of determinant $-\Omega M$ admits of primitive representation by the forms $(f)$ of the ternary genus $(f, F)$. Let $\varphi_1, \varphi_2, \ldots$ or $(\varphi)$ be a system of forms representing the classes of that binary genus; these forms are properly or improperly primitive, according as the forms $(f)$ are properly or improperly primitive: let $n$ be their number and $\nu$ the sum of their weights; as their weights are all equal, the weight of each of them is $\frac{\nu}{n}$; so that each has $\frac{n}{\nu}$ positive automorphics, and is transformed into any equivalent form by $\frac{n}{\nu}$ positive substitutions. We shall first show that the sum of the weights of the primitive representations of the forms $(\varphi)$ by the forms $(f)$ is equal to $2^\mu \times \nu$; and secondly, that the sum of the weights of the primitive representations of the numbers M by the forms (F) is equal to the sum of the weights of the primitive representations of the forms $(\varphi)$ by the forms $(f)$.

(i) Each of the $n$ congruences

$$-\Delta\varphi \equiv (Qx - Q'y)^2, \text{ mod } M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

in which Q, Q' are the numbers to be determined, is resoluble, and admits of $2^\mu$ incongruous solutions. From each such solution we deduce, by the method of Gauss employed in Art. 10, a ternary form $f'$ of the given genus, containing one of the forms $(\varphi)$ as a part, and having Q, Q', M for the coefficients of $2yz$, $2xz$, $z^2$ in its primitive contravariant. There are $2^\mu \times n$ of these forms $(f')$; none of them is the same as any other, and none of them can be transformed into any other by a substitution of the type

$$\left|\begin{array}{c} 1,\ 0,\ \varkappa' \\ 0,\ 1,\ \varkappa \\ 0,\ 0,\ 1; \end{array}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

for if one of them could be so transformed into another, these two would contain as a part the same form $\phi$, and the values of Q, Q′ in the primitive contravariant of the one would be congruous, for the modulus M, to the values of Q, Q′ in the primitive contravariant of the other; the two forms would thus be derived from the same solution of the same congruence (32). Again, the primitive representations of the forms $(\phi)$ by the forms $(f)$ are equal in number to the positive transformations of the forms $(f)$ into the forms $(f')$. For every positive transformation of a form of $(f)$ into a form of $(f')$ supplies a primitive representation of some form of $(\phi)$ by that form of $(f)$; and these representations are all different, because the same form $f$ cannot be transformed into two of the forms $(f')$, or twice into one of them, by positive substitutions of which the first two columns are the same; otherwise one of the forms $(f')$ could be transformed into another by a substitution of the type (33), or else one of those forms would have an automorphic of that type, whereas no substitution of the type (33), in which $\varkappa$ and $\varkappa'$ are different from zero, can be an automorphic of any ternary form. There are therefore at least as many different primitive representations of the forms $(\phi)$ by the forms $(f)$, as there are positive transformations of the forms $(f)$ into the forms $(f')$. And there are no more; for if

$$\left|\begin{array}{c} \alpha\ ,\ \beta \\ \alpha',\ \beta' \\ \alpha'',\ \beta'' \end{array}\right|$$

is a given primitive representation of $\phi$ by $f$, let $\gamma, \gamma', \gamma''$ be numbers which render the determinant of the substitution

$$\left|\begin{array}{c} \alpha\ ,\ \beta\ ,\ \gamma \\ \alpha',\ \beta',\ \gamma' \\ \alpha'',\ \beta'',\ \gamma'' \end{array}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

equal to $+1$; and let $f_1$ be the form, containing $\phi$ as a part, into which $f$ is transformed by the substitution (34). The coefficient of $z^2$ in the primitive contravariant of $f_1$ is M, and if the coefficients of $2yz$, $2zx$ in that contravariant are $Q_1$, $Q_1'$, these numbers supply a solution of the congruence (32). Let $f'$ be that form of $(f')$ which is deduced from this solution; then $f_1$ is equivalent to $f'$, and is transformed into it by a substitution of the type (33), in which $\varkappa = \frac{Q_1 - Q}{M}$, $\varkappa' = \frac{Q_1' - Q'}{M}$. Therefore $f$ is transformed into $f'$ by the substitution

$$\left|\begin{array}{c} \alpha\ ,\ \beta\ ,\ \gamma + \varkappa'\alpha + \varkappa\beta \\ \alpha',\ \beta',\ \gamma' + \varkappa'\alpha' + \varkappa\beta' \\ \alpha'',\ \beta'',\ \gamma' + \varkappa'\alpha'' + \varkappa\beta'', \end{array}\right|,$$

*i. e.* the given primitive representation of $\varphi$ by $f$ is included among those supplied by the positive transformations of the forms $(f)$ into the forms $(f')$. Thus the number of the primitive representations of the forms $(\varphi)$ by the forms $(f)$ is equal to the number of the positive transformations of the forms $(f)$ into the forms $(f')$: to obtain the sum of the weights of these representations, we consider, in particular, $f$ one of the forms of $(f)$; let $d$ be the number of its positive automorphics, so that $\frac{1}{d}$ is its weight, and let $s$ be the number of the forms $(f')$ which are equivalent to it. Then there are $ds$ primitive representations of the forms $(\varphi)$ by $f$; but the weight of each of these representations is $\frac{1}{d}\times\frac{\nu}{n}$; the sum of the weights of the primitive representations of the forms $(\varphi)$ by $f$ is therefore $s\times\frac{\nu}{n}$. Extending this conclusion to all the forms of $(f)$, and observing that $\Sigma s$ is equal to the number of the forms $(f')$, *i. e.* to $2^{\omega}\times n$, we find that the sum of the weights of the primitive representations of the forms $(\varphi)$ by the forms $(f)$ is $2^{\omega}\times\nu$.

(ii) Let $M=F(\Gamma, \Gamma', \Gamma'')$ be a given primitive representation of M by F; and let

$$\begin{vmatrix} \alpha\,,\ \beta \\ \alpha',\ \beta' \\ \alpha'',\ \beta'' \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

be a matrix, of which the constituents satisfy the equations

$$\alpha'\beta''-\alpha''\beta'=\Gamma,\quad \alpha''\beta-\alpha\beta''=\Gamma',\quad \alpha\beta'-\alpha'\beta=\Gamma''. \quad . \quad . \quad . \quad . \quad (36)$$

All the matrices, of which the constituents satisfy these equations, are then included in the formula

$$\begin{vmatrix} \alpha\,,\ \beta \\ \alpha',\ \beta' \\ \alpha'',\ \beta'' \end{vmatrix} \times |v| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

in which $|v|$ is a square binary matrix of which the determinant is $+1$. Thus the binary forms, which are represented by $f$ simultaneously with the given representation of M by F, are all equivalent to one another, and to some form of $(\varphi)$; let $\varphi$ be that form of $(\varphi)$ to which they are equivalent, and let us suppose (as we may do) that $f$ is transformed into $\varphi$ by the substitution (35). Substituting successively for $|v|$ in the formula (37), the $\frac{n}{\nu}$ positive automorphics of $\varphi$, we obtain $\frac{n}{\nu}$ representations of $\varphi$ by $f$: these representations are all different, and they include every representation of $\varphi$ by $f$ which is simultaneous with the given representation of M by F: the weight of each of them is $\frac{1}{d}\times\frac{\nu}{n}$; the sum of their weights is therefore equal to $\frac{1}{d}$, or to the weight of the given representation of M by F. Hence the sum of the weights of all the primitive representations of M by the forms (F) is equal to the sum of the weights of the simultaneous representations of the forms $(\varphi)$ by the forms $(f)$, or, which is the same thing,

to the sum of the weights of all the primitive representations of the forms $(\phi)$ by the forms $(f)$; because every primitive representation of a form $(\phi)$ by a form $(f)$ is simultaneous with one and only one primitive representation of M by a form (F).

Combining the conclusions (i) and (ii), we obtain the result enunciated at the beginning of this article.

Art. 16. Let $\sigma$ represent the number of uneven primes dividing $\Omega$, counting those which also divide $\Delta$; let $\sigma'=-1$, when $\Omega M\equiv -1$, mod 4*; $\sigma'=+1$, when $\Omega\equiv 0$, mod 8; and $\sigma'=0$ in all other cases. Let also $h(\Omega M)$ and $h'(\Omega M)$ be the weights of the properly and improperly primitive orders of binary forms of determinant $-\Omega M$; then $2^{\sigma}\times_{\prime}=\frac{h(\Omega M)}{2^{\sigma+\sigma'}}$, or $\frac{h'(\Omega M)}{2^{\sigma+\sigma'}}$, according as the forms $(f)$ are properly or improperly primitive. If $\lambda^2$ is any square divisor of M, the sum of the weights of those representations of M by the forms (F), which are derived from primitive representations of $\frac{M}{\lambda^2}$ by the same forms, is $\frac{h\left(\frac{\Omega M}{\lambda^2}\right)}{2^{\sigma+\sigma'}}$, or $\frac{h'\left(\frac{\Omega M}{\lambda^2}\right)}{2^{\sigma+\sigma'}}$. Therefore the sum of the weights of all the representations of M by the forms (F) is $\frac{\Sigma . h\left(\frac{\Omega M}{\lambda^2}\right)}{2^{\sigma+\sigma'}}$, or $\frac{\Sigma . h'\left(\frac{\Omega M}{\lambda^2}\right)}{2^{\sigma+\sigma'}}$, the signs of summation extending to every square divisor of M. Or, if we represent by $H(\Omega M)$ the sum of the weights of those uneven binary classes of determinant $-\Omega M$ which are prime to $\Omega$, and by $H'(\Omega M)$ the sum of the weights of those even classes of determinant $-\Omega M$ which are prime to $\Omega$, the sum of the weights of all the representations of M by the forms (F) is

$$\frac{H(\Omega M)}{2^{\sigma+\sigma'}}, \text{ or } \frac{H'(\Omega M}{2^{\sigma+\sigma'}},$$

according as the forms $(f)$ are properly or improperly primitive.

Art. 17. We now consider the sums

$$\Sigma[xz-y^2=\Omega M], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

$$\Sigma'[xz-y^2=\Omega M]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

In both the sign of summation extends to every solution in integral numbers of the equation

$$xz-y^2=\Omega M,$$

in which the greatest common divisor of $x, y, z$ is prime to $\Omega$, and in which $x, y, z$ satisfy the inequalities

$$\left.\begin{array}{lll} x>0, & y\geqq 0, & z>0, \\ x\geqq 2y\leqq z, & & x\leqq z \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

But, in the first sum, one at least of the two numbers $x$ and $z$ is uneven; in the second, $x$ and $z$ are even, and $y$ is uneven. The symbol $[xz-y^2=\Omega M]$ is 1, or $\frac{1}{2}$, or $\frac{1}{4}$, or $\frac{1}{8}$,

* If this congruence is satisfied by any one number of the series M, it is satisfied by every number of that series.

according as the inequalities (40) are satisfied, excluding all signs of equality, or admitting one, or two, or three such signs. Again, representing by $(2y)$ the absolute value of $2y$, we observe that a *reduced* binary form is a form $(x, y, z)$ of which the coefficients satisfy the inequalities,

$$\left.\begin{array}{lll} \text{(i)} & x>0, & z>0, \\ & x\geqq(2y)\leqq z, & x\leqq z. \\ \text{(ii)} & \text{If } x=(2y), & y>0, \\ & \text{If } x=z, & y\geqq 0. \end{array}\right\} \quad \ldots\ldots\ldots \quad (41)$$

and that, by a fundamental proposition in the theory of binary forms, every class contains one and only one reduced form. Attending only to those uneven classes of determinant $-\Omega M$ which are prime to $\Omega$, and comparing the inequalities (40) and (41), we find that the sum (38) contains (i) an unit corresponding to every pair of reduced forms $(x, y, z)$, $(x, -y, z)$ of which the coefficients satisfy none of the equalities $y=0$, $x=2y$, $x=z$; (ii) one-half of an unit corresponding to every reduced form of which the coefficients satisfy one of them; and (iii) one-fourth of an unit corresponding to a reduced form (if there be such a form of determinant $-\Omega M$ prime to $\Omega$) of which the coefficients satisfy the two equalities, $y=0$, $x=z$, and of which the weight is consequently $\frac{1}{4}$. We thus obtain the equation

$$H(\Omega M)=\Sigma[xz-y^2=\Omega M].$$

Again, attending only to those even classes of the uneven determinant $-\Omega M$ which are prime to $\Omega$, we find that the sum (39) contains units corresponding to pairs of reduced forms, and half units corresponding to single reduced forms; it also contains one-sixth of an unit corresponding to a reduced form (if there be such a reduced form of determinant $-\Omega M$ prime to $\Omega$) of which the coefficients satisfy the three equalities $x=2y$, $2y=z$, $x=z$, and of which the weight is consequently $\frac{1}{3}$. We therefore have the equation

$$H'(\Omega M)=\Sigma'[xz-y^2=\Omega M].$$

Art. 18. According as the forms $(f)$ are properly or improperly primitive, let

$$\Upsilon=\Sigma\,.\,\Sigma[xz-y^2=\Omega M],$$

or

$$\Upsilon=\Sigma\,.\,\Sigma'[xz-y^2=\Omega M],$$

the first sign of summation extending to all values of M not surpassing L; so that, in both cases alike,

$$T=\frac{\Upsilon}{2^{\nu+\nu'}}.$$

To determine the limit of the quotient $\frac{T}{L^3}$, when L is increased without limit, we shall again employ the geometric method of GAUSS. For its application here the following preliminary lemmas are requisite, relating to the arithmetical properties of the function $xz-y^2$.

(1) If $p$ is any uneven prime, and $m$ any given number, the congruence

$$xz-y^2\equiv m, \text{ mod } p, \quad \ldots \quad (42)$$

admits of $p\left[p+\left(\frac{-m}{p}\right)\right]$ solutions.

For if $\left(\frac{-m}{p}\right)=+1$, $y^2+m$ is prime to $p$ for $p-2$ values of $y$, and is divisible by $p$ for 2 values of $y$. When $y^2+m$ is prime to $p$, we may assign to $z$ any value prime to $p$, determining $x$ by the congruence $xz\equiv y^2+m$; we thus obtain $(p-1)(p-2)$ solutions of (42). When $y^2+m$ is divisible by $p$, the congruence $xz\equiv 0$, mod $p$, admits of $2p-1$ solutions; we thus obtain in all $(p-1)(p-2)+2(2p-1)=p(p+1)$ solutions of (42).

If $\left(\frac{-m}{p}\right)=-1$, $y^2+m$ is prime to $p$ for every value of $y$; there are thus $p(p-1)$ solutions of (42).

Lastly, if $\left(\frac{-m}{p}\right)=0$, *i. e.* if $m\equiv 0$, mod $p$, $y^2+m$ is prime to $p$ for $p-1$ values of $y$, and divisible by $p$ for one value of $y$. There are thus $(p-1)^2+2p-1=p^2$ solutions of (42).

We shall have to use the following corollary of this lemma,

If $m$ is prime to $p$, and if we successively attribute to $x$, $y$, $z$ the $p^3$ systems of values, mod $p$, of which they are susceptible, $xz-y^2$ will have the same quadratic character as $m$ for $\frac{1}{2}p(p-1)\left[p+\left(\frac{-m}{p}\right)\right]$ of these systems.

(2) The congruences $xz-y^2\equiv 1, 3, 5, 7$, mod 8, each admit 48 solutions in which $x$ and $z$ are not simultaneously even; of the congruences, $xz-y^2\equiv 3, \equiv 7$, mod 8, the first admits 16, the second 48 solutions in which $x$ and $z$ are simultaneously even.

For example, let the proposed congruence be $xz-y^2\equiv 3$, mod 8. If $y$ has one of its four even values, mod 8, we may give to $z$ any one of its four uneven values, mod 8, and determine the value of $x$ in the resulting congruence; we thus obtain $4\times 4$ solutions in which $x$ and $z$ are uneven. If $y$ has one of its four uneven values, mod 8, the congruence becomes $xz\equiv 4$, mod 8, which admits of 8 solutions in which $x$ and $z$ are not simultaneously even, and 4 in which they are simultaneously even. There are thus $(4\times 4)+(4\times 8)=48$ solutions of the congruence $xz-y^2\equiv 3$, mod 8, in which $x$ and $z$ are not simultaneously even, and $4\times 4=16$ in which $x$ and $z$ are simultaneously even.

(3) If $p$ is any prime, even or uneven, $i$ and $i'$ integral exponents, of which $i>0$, $i'\geq 0$, and $m$ any given number, prime to $p$, or divisible by any power of $p$, the congruence

$$xz-y^2\equiv mp^i, \text{ mod } p^{i+i'} \quad \ldots \quad (43)$$

admits of $p^{2i+2i'}\left(1-\frac{1}{p^2}\right)$ *primitive* solutions, *i. e.* solutions in which $x, y, z$, or, which is the same thing, $x, z$ are not simultaneously divisible by $p$.

(i) If the assertion is true for $i, i'$, and if $j\leq i'$, it is true for $i+j, i'-j$. For, on writing $mp^j$ for $m$ in (43), it becomes

$$xz-y^2\equiv mp^{i+j}, \text{ mod } p^{(i+j)+(i'-j)};$$

if therefore the former congruence admits

$$p^{2i+2i'}\left(1-\frac{1}{p^2}\right)=p^{2(i+f)+2(i'-f)}\left(1-\frac{1}{p^2}\right)$$

primitive solutions, the latter does so too.

(ii) If the assertion is true for $i$, 0, it is also true for $i$, $i'$, where $i'\leqq i$.

For if $x$, $y$, $z$ is a given primitive solution of

$$xz-y^2\equiv mp^i, \mod p^i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$Xp^i+x$, $Yp^i+y$, $Zp^i+z$ is a primitive solution of (43), whenever X, Y, Z satisfy the congruence

$$Xz-2Yy+Zx+\frac{xz-y^2}{p^i}\equiv m, \mod p^{i'}.$$

This congruence admits of $p^{2i'}$ solutions; for the given numbers $x$ and $z$ are not simultaneously divisible by $p$. Thus from each primitive solution of (44) we obtain $p^{2i'}$ primitive solutions of (43). These solutions are all different, and exhaust all the solutions of (43); if therefore (44) admits of $p^{2i}\left(1-\frac{1}{p^2}\right)$ solutions, (43) admits of $p^{2i+2i'}\left(1-\frac{1}{p^2}\right)$ solutions.

(iii) The assertion is true if $i=1$, $i'=0$. For (lemma 1) there are $p^2$ solutions of the congruence $xz-y^2\equiv 0$, mod $p$, and of these one is not primitive.

The proposition is, therefore, true universally. We shall have to employ the following corollaries from it.

(1) The function $xz-y^2$ is divisible by $p^i$, but not by $p^{i+1}$, for $p^{2i}(p-1)^2(p+1)$ systems of values of $x, y, z$, mod $p^{i+1}$; the values of $x$, $y$, $z$ not being simultaneously divisible by $p$.

(2) If $p$ is an uneven prime, the quotients obtained by dividing these $p^{2i}(p-1)^2(p+1)$ values of $xz-y^2$ by $p^i$, are half quadratic residues, and half non-quadratic residues of $p$.

(3) If $p=2$, the function $xz-y^2$ is divisible by $2^i$, but not by $2^{i+1}$, for $3\times 2^{2i+6}$ systems of values of $x$, $y$, $z$, mod $2^{i+3}$, the values of $x$, $y$, $z$ not being simultaneously even. And if these $3\times 2^{2i+6}$ values of $xz-y^2$ be divided by $2^i$, one-fourth part of the quotients is contained in each of the linear forms $8k+1$, 3, 5, 7.

Art. 19. Let $\nabla=8\Omega\times\Pi r\times\Pi\omega\times\Pi\mathfrak{d}$, and let us successively attribute to $x, y, z$ in the function $xz-y^2$ the $\nabla^3$ systems of values, mod $\nabla$, of which they are susceptible; let $\varphi(\nabla)$ represent the number of those systems, in which the greatest common divisor of $x$, $y$, $z$ is prime to $\nabla$, and which give to $xz-y^2$ a value divisible by $\Omega$, and such that the quotient $\frac{xz-y^2}{\Omega}$ is a number of the series M; if the forms $(f)$ are properly primitive, $x$ and $z$ are not to be simultaneously even; if those forms are improperly primitive, $x$ and $z$ are to be simultaneously even. We shall now show that $\varphi(\nabla)$ is determined by the equation

$$\left.\begin{aligned}\varphi(\nabla)=\tfrac{8}{8}\times\eta\times\nabla^3\times\tfrac{1}{\Omega}\times\Pi\left(1-\tfrac{1}{r}\right)\times\Pi\left(1-\tfrac{1}{\mathfrak{d}}\right)\times\Pi\left(1-\tfrac{1}{\omega}\right)\\ \times\Pi\tfrac{1}{2}\left(1-\tfrac{1}{r^2}\right)\times\Pi\left(1-\tfrac{1}{\omega^2}\right)\times\Pi\tfrac{1}{2}\left[1+\left(\tfrac{-\Omega F}{\mathfrak{d}}\right)\tfrac{1}{\mathfrak{d}}\right],\end{aligned}\right\} \quad . \quad . \quad . \quad (45)$$

$\eta$ being a coefficient of which the value is 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{12}$*, as shown in the following Table.

(i) $(f)$ properly primitive.

| | $\Omega\equiv 1$, mod 2. | $\Omega\equiv 2$, mod 4. | $\Omega\equiv 4$, mod 8. | $\Omega\equiv 0$, mod 8. |
|---|---|---|---|---|
| $\Delta\equiv 1$, mod 2. | $\frac{1}{2}$ | 1 | $\frac{1}{4}\left[3+(-1)^{\frac{\Delta f+1}{2}}\right]$ | $\frac{1}{4}\left[3+(-1)^{\frac{\Delta f+1}{2}}\right]$ |
| $\Delta\equiv 2$, mod 4. | $\frac{1}{2}$ | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ |
| $\Delta\equiv 4$, mod 8. | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $\Delta\equiv 0$, mod 8. | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

(ii) $(f)$ improperly primitive.

| | |
|---|---|
| $\Delta\equiv 2$, mod 4. | $\frac{1}{3}$ |
| $\Delta\equiv 0$, mod 4. | $\frac{1}{12}\left[2+(-1)^{\frac{\Omega^2-1}{8}+\frac{F^2-1}{8}}\right]$ |

To establish the equation (45), we consider separately the different primes dividing $\nabla$. And first let us take an uneven prime $\delta$, dividing $\Delta$ but not $\Omega$. Of the $\delta^3$ systems of values of $x$, $y$, $z$, mod $\delta$,

$$\delta^3\times\left(1-\frac{1}{\delta}\right)\times\frac{1}{2}\left[1+\left(\frac{-\Omega F}{\delta}\right)\frac{1}{\delta}\right] \text{ systems}$$

give to $xz-y^2$ a value prime to $\delta$, and satisfying the equation (Lemma i, Cor.)

$$\left(\frac{xz-y^2}{\delta}\right)=\left(\frac{\Omega F}{\delta}\right).$$

Secondly, let us consider an uneven prime $\omega$ dividing $\Omega$ but not $\Delta$; and let $\omega^i$ be the highest power of $\omega$ dividing $\Omega$. Of the $\omega^{3i+3}$ systems of values of $x$, $y$, $z$, mod $\omega^{i+1}$,

$$\omega^{3i+3}\times\frac{1}{\omega^i}\times\left(1-\frac{1}{\omega}\right)\times\left(1-\frac{1}{\omega^2}\right) \text{ systems},$$

in which $x$, $y$, $z$ are not simultaneously divisible by $\omega$, render $xz-y^2$ divisible by $\omega^i$, and also render the quotient $\frac{xz-y^2}{\omega^i}$ prime to $\omega$ (Lemma iii. Cor. 1).

Thirdly, let us consider an uneven prime $r$ dividing both $\Delta$ and $\Omega$, and let $r^i$ be the highest power of $r$ dividing $\Omega$. Of the $r^{3i+3}$ systems of values of $x$, $y$, $z$, mod $r^{i+1}$,

$$r^{3i+3}\times\frac{1}{r}\times\left(1-\frac{1}{r}\right)\times\frac{1}{3}\left(1-\frac{1}{r^2}\right) \text{ systems},$$

in which $x$, $y$, $z$ are not simultaneously divisible by $r$, render $xz-y^2$ divisible by

* It will be seen that $4\eta$ in the Table (i), is in every case the number of the linear forms $8k+1$, 3, 5, 7, in which the numbers $\mathbf{M}$ are contained.

$r^i$, but not by $r^{i+1}$, and also render the quotients $\frac{xz-y^2}{r^i}$ all quadratic residues, or all non-quadratic residues of $r$ (Lemma iii. Cor. 2.).

Lastly, let us consider the even prime 2, and let $2^i$ be the highest power of 2 dividing $\Omega$. Considering separately the eighteen cases of the Tables (i) and (ii), we find that of the $2^{3i+9}$ systems of values of $x, y, z$, mod $2^{i+3}$,

$$\tfrac{3}{8}\times\eta\times 2^{3i+9}\times\frac{1}{2^i} \text{ systems}$$

(in which $x, y, z$ are not simultaneously even, but $x$ and $z$ are or are not simultaneously even, according as the forms $(f)$ are improperly or properly primitive) give to $\frac{xz-y^2}{2^i}$ an integral and uneven value, satisfying the supplementary character (if any) of $\frac{\Omega}{2^i}$ F, and, if the forms $(f)$ are properly primitive, satisfying the congruence $xz-y^2\equiv 1$, mod 4, when $\Omega$ is uneven, and $\Delta$ uneven or unevenly even.

For example, let $i\geqq 1$, $\Delta\equiv 0$, mod 8. Here F, or $\frac{\Omega}{2^i}$ F, has two supplementary characters, and of the $2^{3i+9}$ systems of values of $x, y, z$, mod $2^{i+3}$,

$$\tfrac{3}{8}\times\tfrac{1}{4}\times 2^{3i+9}\times\frac{1}{2^i} \text{ systems,}$$

in which $x$ and $z$ are not simultaneously even, give to $\frac{xz-y^2}{2^i}$ an integral and uneven value, satisfying the supplementary characters of $\frac{\Omega}{2^i}$ F (Lemma iii. Cor. 3).

Again, let $i\geqq 2$, $\Delta\equiv 1$, mod 2. Here F has or has not a supplementary character, according as $(-1)^{\frac{\Delta'+1}{2}}=-1$, or $=+1$. In the former case, of the $2^{3i+9}$ systems of values of $x, y, z$, mod $2^{i+3}$,

$$\tfrac{3}{8}\times\tfrac{1}{2}\times 2^{3i+9}\times\frac{1}{2^i} \text{ systems}$$

(in which $x$ and $z$ are not simultaneously even) give to $\frac{xz-y^2}{2^i}$ an integral and uneven value satisfying the supplementary character of $\frac{\Omega}{2^i}$ F. In the latter case, of the same $2^{3i+9}$ systems of values,

$$\tfrac{3}{8}\times 1\times 2^{3i+9}\times\frac{1}{2^i} \text{ systems,}$$

in which $x$ and $z$ are not simultaneously even, give to $\frac{xz-y^2}{2^i}$ an integral and uneven value. Both results are comprised in the formula

$$\tfrac{3}{8}\times\tfrac{1}{4}\left[3+(-1)^{\frac{\Delta'+1}{2}}\right]\times 2^{3i+9}\times\frac{1}{2^i}.$$

As a third example, let $i=0$, $\Delta\equiv 0$, mod 4, and let the forms considered be of an improperly primitive order. Then $\Omega\mathrm{F}\equiv 3$, mod 4; and either $\Omega\mathrm{F}\equiv 3$, mod 8, or $\Omega\mathrm{F}\equiv 7$,

mod 8. The congruence $xz-y^2\equiv\Omega F$, mod 8, in which only even values of $x$ and $z$ are to be admitted, is satisfied in the former case by 16, in the latter by 48 systems; *i. e.* in either case by

$$\tfrac{3}{8}\times\tfrac{1}{12}\left[2+(-1)^{\frac{\Omega^2-1}{8}+\frac{F^2-1}{8}}\right]\times 2^9 \text{ systems.}$$

The formula (45) results immediately from the combination of these determinations relative to the primes 3, $\omega$, $r$, and 2.

Art. 20. Let $x_i$, $y_i$, $z_i$ be one of the $\phi(\nabla)$ systems of values of $x$, $y$, $z$, mod $\nabla$, defined in the last article; and let us decompose the sum $\Upsilon$ of art. 18 into $\phi(\nabla)$ partial sums $\Upsilon_1$, $\Upsilon_2$, ..., comprising in the sum $\Upsilon_i$ all those terms of $\Upsilon$ in which $x$, $y$, $z$ are of the linear forms

$$x=\nabla X+x_i,$$
$$y=\nabla Y+y_i,$$
$$z=\nabla Z+z_i,$$

X, Y, Z denoting any integral numbers whatever. The sum $\Upsilon_i$ is equal to the number of points having their positive rectangular coordinates of the forms

$$x=\frac{\nabla X+x_i}{\sqrt{\Omega L}},$$
$$y=\frac{\nabla Y+y_i}{\sqrt{\Omega L}},$$
$$z=\frac{\nabla Z+z_i}{\sqrt{\Omega L}},$$

and lying within the hyperboloidal cuneus, bounded by the planes $y=0$, $x=z$, $x=2y$, and the hyperboloid $xz-y^2=1$; points lying on the hyperboloidal boundary are counted as lying within the cuneus; points lying on its plane boundaries are counted as $\frac{1}{2}$ each, and points lying on the intersection of $y=0$ with $x=z$, and with $x=2y$ respectively as $\frac{1}{4}$ and $\frac{1}{8}$. Let V be the volume of the cuneus, and let L be increased without limit; we have

$$\lim\frac{\Upsilon_i}{L^{\frac{3}{2}}}=\Omega^{\frac{3}{2}}\times\frac{V}{\nabla^3};$$

and since this limit is thus ascertained to be the same for all the partial sums $\Upsilon_1$, $\Upsilon_2$, ..,

$$\lim\frac{\Upsilon}{L^{\frac{3}{2}}}=\Omega^{\frac{3}{2}}\times\frac{\phi(\nabla)}{\nabla^3}\times V,$$

or, which is the same thing,

$$\lim\frac{T}{L^{\frac{3}{2}}}=\frac{1}{2^{\sigma+\sigma'}}\times\Omega^{\frac{3}{2}}\times\frac{\phi(\nabla)}{\nabla^3}\times V.$$

The value of V may be determined by dividing the cuneus into laminæ parallel to the plane of $xz$; if A be the area of a section at a distance $y$ from that plane, we find

$$A=(1+y^2)[\tfrac{1}{2}\log(1+y^2)-\log 2y]-\tfrac{1}{2}(1-3y^2);$$

whence

$$V=\int_0^{\sqrt{3}}A\,dy=\frac{\pi}{9}. \qquad (46)$$

Substituting for $\frac{\phi(\nabla)}{\nabla^3}$ and for V their values, given by the equations (45) and (46), we find

$$\lim \frac{T}{L^{\frac{3}{2}}} = \frac{\pi}{24} \times \frac{\eta}{2^{\sigma}} \times \sqrt{\Omega} \times \Pi\left(1-\frac{1}{r}\right) \times \Pi\left(1-\frac{1}{\delta}\right) \times \Pi\left(1-\frac{1}{\omega}\right) \times \Pi\tfrac{1}{4}\left(1-\frac{1}{r^2}\right) \times \Pi\tfrac{1}{2}\left(1-\frac{1}{\omega^2}\right) \times \Pi\tfrac{1}{3}\left[1+\left(\frac{-\Omega F}{\delta}\right)\tfrac{1}{\delta}\right] \Bigg\}, \quad . \quad . \quad . \quad . \quad (47)$$

which is the second determination of the limit of the quotient $\frac{T}{L^{\frac{3}{2}}}$.

Finally, equating the two values of this limit, and denoting the coefficient $\frac{1}{2^{\sigma}} \times \frac{\eta}{\theta}$ by $\frac{1}{2}\zeta$, we obtain the following determination of the weight of the proposed genus,

$$W = \frac{\Delta\Omega}{8} \times \zeta \times \Pi\tfrac{1}{4}\left(1-\frac{1}{r^2}\right) \times \Pi\tfrac{1}{2}\left[1+\left(\frac{-\Delta f}{\omega}\right)\tfrac{1}{\omega}\right] \times \Pi\tfrac{1}{2}\left[1+\left(\frac{-\Omega F}{\delta}\right)\tfrac{1}{\delta}\right], \quad . \quad . \quad (48)$$

the values of $\zeta$ (which are computed from those of $\sigma', \eta, \theta$) being as follows:—

(A).—($f$) and (F) properly primitive.

| | $\Omega \equiv 1$, mod 2. | $\Omega \equiv 2$, mod 4. | $\Omega \equiv 4$, mod 8. | $\Omega \equiv 0$, mod 8. |
|---|---|---|---|---|
| $\Delta \equiv 1$, mod 2. | $\frac{1}{8}[2+\Psi]$ | $\frac{1}{2}$ | $\frac{1}{8}\left[3+(-1)^{\frac{\Delta f+1}{2}}\right]$ | $\frac{1}{16}\left[3+(-1)^{\frac{\Delta f+1}{2}}\right]$ |
| $\Delta \equiv 2$, mod 4. | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| $\Delta \equiv 4$, mod 8. | $\frac{1}{8}\left[3+(-1)^{\frac{\Omega F+1}{2}}\right]$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| $\Delta \equiv 0$, mod 8. | $\frac{1}{16}\left[3+(-1)^{\frac{\Omega F+1}{2}}\right]$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |

(B).—($f$) improperly, (F) properly primitive.

$\Omega \equiv 1$, mod 2; $\Omega F \equiv 3$, mod 4.

| | |
|---|---|
| $\Delta \equiv 2$, mod 4. | $\frac{1}{8}$ |
| $\Delta \equiv 0$, mod 4. | $\frac{1}{16}\left[2+(-1)^{\frac{\Omega^2 F^2-1}{8}}\right]$ |

(C).—($f$) properly, (F) improperly primitive.

$\Delta \equiv 1$, mod 2; $\Delta f \equiv 3$, mod 4.

| | |
|---|---|
| $\Omega \equiv 2$, mod 4. | $\frac{1}{8}$ |
| $\Omega \equiv 0$, mod 4. | $\frac{1}{16}\left[2+(-1)^{\frac{\Delta^2 f^2-1}{8}}\right]$ |

The last of these Tables is obtained by reciprocation from the second.

The result in the case $\Omega \equiv \Delta \equiv 1$, mod 2, is given in the memoir of EISENSTEIN (Crelle, vol. xxxv. p. 128).

Art. 21. The equation (47) may also be deduced from the theorem of Art. 15 by another method. We consider first and principally the case in which the forms $(f)$ and (F) are both properly primitive.

From Art. 16 we obtain the equation

$$T=\frac{1}{2^{\sigma+\sigma'}}\sum_{M\geq 1}^{M\leq L}\left[\Sigma h\left(\frac{\Omega M}{\lambda^2}\right)\right],$$

the interior sign of summation extending to every square divisor of M. Inverting the order of the summations, and designating by $m$ any number prime to $2\Omega\Delta$, we may write this equation in the form

$$T=\frac{1}{2^{\sigma+\sigma'}}\sum_{m=1}^{m\leq\sqrt{L}}\sum_{M\geq 1}^{M\leq\frac{L}{m^2}} h(\Omega M).$$

But, by a theorem of LEJEUNE DIRICHLET,

$$h(\Omega M)=\frac{1}{\pi}\sqrt{\Omega M}\,\Sigma\left(\frac{-\Omega M}{n}\right)\frac{1}{n},$$

the sign of summation extending to all uneven numbers prime to ΩM. The limit of $\frac{T}{L}$ is therefore the limit of the expression

$$\frac{1}{2^{\sigma+\sigma'}}\frac{\sqrt{\Omega}}{\pi}\times\frac{1}{L^{\frac{3}{2}}}\times\sum_{m=1}^{m\leq\sqrt{L}}\sum_{M\geq 1}^{M\leq\frac{L}{m^2}}\sqrt{M}\,\Sigma\left(\frac{-\Omega M}{n}\right)\frac{1}{n},$$

or, leaving the summation with respect to $n$ to be effected last, of the expression

$$\frac{1}{2^{\sigma+\sigma'}}\frac{\sqrt{\Omega}}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{n}\sum_{m=1}^{m\leq\sqrt{L}}\frac{1}{L^{\frac{3}{2}}}\sum_{M\geq 1}^{M\leq\frac{L}{m^2}}\left(\frac{-\Omega M}{n}\right)\sqrt{M}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

In this expression $n$ is uneven and prime to Ω; but $n$ is not necessarily prime to Δ. Let $n=n_1^2n_2$, $n_1^2$ denoting the greatest square dividing $n$, so that $n_2$ is a product of unequal primes; also let $\nu$ represent any prime dividing $n$, other than one of the primes $\delta$; and let $\eta$ represent $\frac{1}{4}$, $\frac{1}{2}$, or 1, according as the numbers M are contained in one, two, or all four of the linear forms $8k+1$, 3, 5, 7; so that $\eta$ has the same value as in Art. 19. The limit of the sum

$$\frac{1}{L^{\frac{3}{2}}}\sum_{M\geq 1}^{M\leq\frac{L}{m^2}}\left(\frac{-\Omega M}{n}\right)\sqrt{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

is zero, or

$$\frac{\eta}{3}\times\left(\frac{-\Omega F}{n_2}\right)\times\Pi\left(1-\frac{1}{\omega}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{r}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{\delta}\right)\times\Pi\left(1-\frac{1}{\nu}\right)\times\frac{1}{m^3}, \quad . \quad (51)$$

according as $n_2$ does or does not contain any primes other than the primes $\delta$. For, in

the sum (50), it is only necessary to consider those numbers M which are are prime to $n$; because $\left(\frac{-\Omega M}{n}\right)=0$, if M is not prime to $n$; and if

$$\nabla=8\Pi\omega\times\Pi r\times\Pi\delta\times\Pi\nu,$$

$$\chi(\nabla)=\nabla\times\tfrac{1}{2}\eta\times\Pi\left(1-\frac{1}{\omega}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{r}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{\delta}\right)\times\Pi\left(1-\frac{1}{\nu}\right),$$

the sum (50) contains $\chi(\nabla)$ numbers M inferior to $\nabla$; let these be represented by $x_1, x_2, \ldots, x_{\chi}$; then all the numbers M, which enter into that sum, are contained in the $\chi(\nabla)$ linear forms $x\nabla+x_i$; and the sum (50) may be decomposed into $\chi(\nabla)$ partial sums, of which the sum

$$\left(\frac{-\Omega x_i}{n_2}\right)\frac{1}{L^{\frac{3}{2}}}\sum_{x=0}^{x\nabla+x_i\leq\frac{L}{m^2}}\sqrt{x\nabla+x_i}$$

is one. The limit of this sum is

$$\tfrac{2}{3}\times\frac{1}{\nabla}\times\frac{1}{m^3}\times\left(\frac{-\Omega x_i}{n_2}\right),$$

so that the limit of the sum (50) is

$$\tfrac{2}{3}\times\frac{1}{\nabla}\times\frac{1}{m^3}\Sigma_i\left(\frac{-\Omega x_i}{n_2}\right).$$

If $n_2$ is divisible by any prime other than the primes $\delta$, the symbols $\left(\frac{-\Omega x_i}{n_2}\right)$ are one half equal to $+1$, and one half equal to $-1$; in this case, therefore, the limit of the sum (50) is zero. But if $n_2$ contain no prime other than the primes $\delta$, the symbols $\left(\frac{-\Omega x_i}{n_2}\right)$ are all equal to one another and to $\left(\frac{-\Omega F}{n_2}\right)$; and the limit of the sum (50) is

$$\tfrac{2}{3}\times\frac{\chi(\nabla)}{\nabla}\times\frac{1}{m^3}\times\left(\frac{-\Omega F}{n_2}\right),$$

in accordance with the formula (51). Substituting in the expression (49) for the sum (50) its limiting value, we find

$$\left.\begin{aligned}\lim\frac{T}{L^{\frac{3}{2}}}=\frac{1}{2^{\varepsilon+\varepsilon'}}\times\frac{\sqrt{\Omega}}{\pi}\times\tfrac{\eta}{3}\times\Pi\left(1-\frac{1}{\omega}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{r}\right)\times\Pi\tfrac{1}{2}\left(1-\frac{1}{\delta}\right)\\ \times\Sigma_{m=1}^{m=\infty}\frac{1}{m^3}\times\Sigma\Sigma\left[\frac{\left(\frac{-\Omega F}{n_2}\right)\Pi\left(1-\frac{1}{\nu}\right)}{n_1^2 n_2}\right]\end{aligned}\right\}\quad\ldots\ldots\quad(52)$$

In the sum $\Sigma\Sigma\left[\frac{\left(\frac{-\Omega F}{n_2}\right)\Pi\left(1-\frac{1}{\nu}\right)}{n_1^2 n_2}\right]$ the summations extend to all values of $n_2$ composed of unequal primes $\delta$, and to all values of $n_1$ prime to $2\Omega$; $\nu$ is any prime divisor of $n_1$, other than one of the primes $\delta$. Thus the two summations are independent, and

$$\Sigma\Sigma\left[\frac{\left(\frac{-\Omega F}{n_2}\right)\Pi\left(1-\frac{1}{\nu}\right)}{n_1^2 n_2}\right]=\Sigma\left(\frac{-\Omega F}{n_2}\right)\frac{1}{n_2}\times\Sigma\frac{\left(1-\frac{1}{\nu}\right)}{n_1^2}.$$

But

$$\Sigma\left(\frac{-\Omega F}{n_2}\right)\frac{1}{n_2}=\Pi\left[1+\left(\frac{-\Omega F}{\delta}\right)\frac{1}{\delta}\right]$$

and

$$\Sigma.\frac{\Pi\left(1-\frac{1}{\nu}\right)}{n_1^2}=\Pi\left[\frac{1}{1-\frac{1}{\delta^2}}\right]\times\Pi\left[1+\frac{1-\frac{1}{\nu}}{\nu^2}+\frac{1-\frac{1}{\nu}}{\nu^4}+\frac{1-\frac{1}{\nu}}{\nu^6}+\ldots\right]=\Pi\left[\frac{1}{1-\frac{1}{\delta^2}}\right]\times\Pi\frac{1+\frac{1}{\nu}+\frac{1}{\nu^2}}{1+\frac{1}{\nu}},$$

the last sign of multiplication extending to all primes $\nu$ which do not divide $2\Omega\Delta$. Also

$$\Sigma_{m=1}^{m=\infty}\frac{1}{m^2}=\Pi\frac{1}{1-\frac{1}{\nu^2}}=\Pi\frac{1}{1-\frac{1}{\nu}}\times\Pi\frac{1}{1+\frac{1}{\nu}+\frac{1}{\nu^2}},$$

so that the product

$$\Sigma\frac{1}{m^2}\times\Sigma\Sigma\left(\frac{-\Omega F}{n^2}\right)\frac{\Pi\left(1-\frac{1}{\nu}\right)}{n_1^2n_2}$$

is equal to

$$\Pi\left[1+\left(\frac{-\Omega F}{\delta}\right)\frac{1}{\delta}\right]\times\Pi\frac{1}{1-\frac{1}{\delta^2}}\times\Pi\frac{1}{1-\frac{1}{\nu^2}},$$

or to

$$\frac{\pi^2}{8}\Pi\left[1-\frac{1}{\omega^2}\right]\times\Pi\left[1-\frac{1}{\nu^2}\right]\times\Pi\left[1+\left(\frac{-\Omega F}{\delta}\right)\frac{1}{\delta}\right], \quad . \quad . \quad . \quad . \quad (53)$$

because

$$\Pi\frac{1}{1-\frac{1}{\omega^2}}.\Pi\frac{1}{1-\frac{1}{\nu^2}}.\Pi\frac{1}{1-\frac{1}{\delta^2}}.\Pi\frac{1}{1-\frac{1}{\nu^2}}$$

is equal to the sum of the squares of the reciprocals of the uneven numbers, that is to $\frac{\pi^2}{8}$. Substituting for the product (52) its equivalent (53) in the equation (51), we obtain the formula (47).

If the forms $(f)$ are improperly primitive, we have to employ the equation

$$h(\Omega M)=\tfrac{1}{8}\left[2+(-1)^{\frac{\Omega^2-1}{8}+\frac{M^2-1}{8}}\right]\frac{\sqrt{\Omega M}}{\pi}\Sigma\left(\frac{-\Omega M}{n}\right)\frac{1}{n};$$

and the proof is the same as in the former case. Only, if $\Delta\equiv 2$, mod 4, it is convenient, on account of the factor $2+(-1)^{\frac{\Omega^2-1}{8}+\frac{M^2-1}{8}}$, separately to determine the limit $T\div L^{\frac{1}{2}}$ for the numbers M which satisfy the congruences $M\equiv 3\Omega$, $M\equiv 7\Omega$, mod 8; and then to add the results.

Art. 22. The weight of an order (Art. 13) is the sum of the weights of the genera contained in the order. The determination of this sum may in every case be effected by means of the formulæ

$$R=\Sigma\left\{\Pi\tfrac{1}{2}\left[1-\tfrac{1}{r^2}\right]\times\Pi\tfrac{1}{2}\left[1+\left(\tfrac{-\Delta f}{\omega}\right)\tfrac{1}{\omega}\right]\times\Pi\tfrac{1}{2}\left[1+\left(\tfrac{-\Omega F}{\delta}\right)\tfrac{1}{\delta}\right]\right\}$$

$$=\Pi\left(1-\tfrac{1}{r^2}\right),$$

$$R'=\Sigma\left\{\left(\tfrac{f}{\Omega_1}\right)\left(\tfrac{F}{\Delta_1}\right)\Pi\tfrac{1}{2}\left[1-\tfrac{1}{r^2}\right]\Pi\left[1+\left(\tfrac{-\Delta f}{\omega}\right)\tfrac{1}{\omega}\right]\times\Pi\tfrac{1}{2}\left[1+\left(\tfrac{-\Omega F}{\delta}\right)\tfrac{1}{\delta}\right]\right\}$$

$$=0, \text{ or } =-\frac{(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}}{\Omega_1\Delta_1}\times\alpha^{\frac{\Delta_1-1}{2}}\times\beta^{\frac{\Omega_1-1}{2}}\times\Pi\left(1-\tfrac{1}{r^2}\right),$$

according as $\Omega_1\Delta_1$ is or is not divisible by any of the primes $r$; *i. e.* according as $\Omega_1\Delta_1$ is not, or is prime to the greatest common divisor of $\Omega$ and $\Delta$. In the expressions of R and R′ the signs of summation extend to every combination of the equations

$$\left(\tfrac{f}{r}\right)=+1, \text{ or } -1;\quad \left(\tfrac{F}{r}\right)=+1, \text{ or } -1;\quad \left(\tfrac{f}{\omega}\right)=+1, \text{ or } -1;\quad \left(\tfrac{F}{\delta}\right)=+1, \text{ or } -1;$$

*i. e.* the value of the continued product is to be determined on each of these suppositions, and the sum of these values is to be taken. From this definition it is evident that in the sum R, we may substitute for any factor of the form

$$\tfrac{1}{2}\left[1+\left(\tfrac{-\Delta f}{\omega}\right)\tfrac{1}{\omega}\right],$$

or

$$\tfrac{1}{2}\left[1+\left(\tfrac{-\Omega F}{\delta}\right)\tfrac{1}{\delta}\right],$$

a factor of the form

$$\tfrac{1}{2}\left\{\left[1+\left(\tfrac{-\Delta}{\omega}\right)\tfrac{1}{\omega}\right]+\left[1-\left(\tfrac{-\Delta}{\omega}\right)\tfrac{1}{\omega}\right]\right\}, \quad \ldots\ldots\ldots \quad (54)$$

or

$$\tfrac{1}{2}\left\{\left[1+\left(\tfrac{-\Omega}{\delta}\right)\tfrac{1}{\delta}\right]+\left[1-\left(\tfrac{-\Omega}{\delta}\right)\tfrac{1}{\delta}\right]\right\} \quad \ldots\ldots\ldots \quad (55)$$

outside the sign of summation. And similarly for any factor $\tfrac{1}{2}\left(1-\tfrac{1}{r^2}\right)$ we may substitute the factor

$$1-\frac{1}{r^2}$$

outside the sign of summation. Observing that the factors (54) and (55) are all positive units, we obtain immediately

$$R=\Pi\left(1-\tfrac{1}{r^2}\right).$$

Again, if a prime $r$ divide $\Omega_1$ or $\Delta_1$, the sum R′ vanishes, being composed of pairs of terms equal in absolute magnitude and opposite in sign; if, for example, $r$ divide $\Omega_1$, the two terms in one of which $\left(\tfrac{f}{r}\right)$, contained in $\left(\tfrac{f}{\Omega_1}\right)$, is $+1$, and in the other $-1$, but which are in other respects identical, will destroy one another. But if none of the primes $r$ divide $\Omega_1$ or $\Delta_1$, we replace those factors of the general term of R′, which

contain primes not dividing $\Omega_1$ and $\Delta_1$, by factors placed outside the sign of summation; we thus find

$$R'=\Pi\left(1-\frac{1}{r^3}\right)\Sigma\left\{\Pi\tfrac{1}{2}\left[\left(\frac{f}{\omega_1}\right)+\left(\frac{-\Delta}{\omega_1}\right)\frac{1}{\omega_1}\right]\times\Pi\tfrac{1}{2}\left[\left(\frac{F}{\delta_1}\right)+\left(\frac{-\Omega}{\delta_1}\right)\frac{1}{\delta_1}\right]\right\},$$

where only primes $\omega_1$ which divide $\Omega_1$ and primes $\delta_1$ which divide $\Delta_1$ occur after the sign of summation. We then substitute for each factor containing $\omega_1$ or $\delta_1$ a factor of the form

$$\tfrac{1}{2}\left\{\left[1+\left(\frac{-\Delta}{\omega_1}\right)\frac{1}{\omega_1}\right]+\left[-1+\left(\frac{-\Delta}{\omega_1}\right)\frac{1}{\omega_1}\right]\right\}=\left(\frac{-\Delta}{\omega_1}\right)\frac{1}{\omega_1},$$

or

$$\tfrac{1}{2}\left\{\left[1+\left(\frac{-\Omega}{\delta_1}\right)\frac{1}{\delta_1}\right]+\left[-1+\left(\frac{-\Omega}{\delta_1}\right)\frac{1}{\delta_1}\right]\right\}=\left(\frac{-\Omega}{\delta_1}\right)\frac{1}{\delta_1}$$

outside the sign of summation; and observing that by the law of reciprocity

$$\left(\frac{-\Delta}{\Omega_1}\right)\left(\frac{-\Omega}{\Delta_1}\right)=-(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}\times\alpha^{\frac{\Delta_1^2-1}{8}}\times\beta^{\frac{\Omega_1^2-1}{8}},$$

we find

$$R'=-\frac{(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}}{\Omega_1\Delta_1}\times\alpha^{\frac{\Delta_1^2-1}{8}}\times\beta^{\frac{\Omega_1^2-1}{8}}\times\Pi\left(1-\frac{1}{r^3}\right).$$

As an example of the application of these formulæ, let us consider the properly primitive order in the case in which $\Delta\equiv1$, mod 2, $\Omega\equiv4$, mod 8. We may determine separately the weights of those genera for which $(-1)^{\frac{\Delta f+1}{2}}=-1$, and of those for which $(-1)^{\frac{\Delta f+1}{2}}=+1$. In a genus of the former kind the characters

$$\left(\frac{f}{r}\right),\ \left(\frac{F}{r}\right),\ \left(\frac{f}{\omega}\right),\ \left(\frac{F}{\delta}\right)\quad\ldots\ldots\ldots\quad(56)$$

may have any assigned values because the condition of possibility is

$$(-1)^{\frac{\Omega_1 F+1}{2}}\left(\frac{f}{\Omega_1}\right)\left(\frac{F}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}.$$

Therefore the sum of the weights of these genera is $\frac{\Omega\Delta}{8}\times\zeta\times R$, or $\frac{\Omega\Delta}{32}\times R$, because $\zeta=\frac{1}{4}$. But in a genus of the latter kind the characters (56), or some of them, are subject to the condition

$$\left(\frac{f}{\Delta_1}\right)\left(\frac{F}{\Delta_1}\right)=(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}};\quad\ldots\ldots\ldots\quad(57)$$

we have therefore to consider a sum of which the general term is the same as that of R, but into which only those terms are admitted which are formed with values of $\left(\frac{f}{\omega_1}\right)$ and $\left(\frac{F}{\delta_1}\right)$ satisfying the condition (57). This sum is expressed by the formula

$$\tfrac{1}{2}\left(R+(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}R'\right);$$

so that the sum of the weights of the genera of the latter kind is

$$\frac{\Omega\Delta}{16}\times\tfrac{1}{2}\times\left(\mathrm{R}+(-1)^{\frac{\Omega_1+1}{2}\cdot\frac{\Delta_1+1}{2}}\mathrm{R}'\right).$$

Adding the two sums together, and substituting for R and R′ their values, we find for the weight of the proposed order the expression

$$\frac{\Omega\Delta}{16}\Pi\left(1-\frac{1}{r^2}\right),\text{ or }\frac{\Omega\Delta}{32}\left(2-\frac{1}{\Omega_1\Delta_1}\right)\Pi\left(1-\frac{1}{r^2}\right)$$

according as $\Omega_1\Delta_1$ is not or is prime to the greatest common divisor of $\Omega$ and $\Delta$.

If, in general, we represent the weight of any proposed order of the invariants $[\Omega, \Delta]$ by the expression

$$\frac{\Omega\Delta}{8}\times Z\times\Pi\left(1-\frac{1}{r^2}\right),$$

the following Table (with which we shall conclude this memoir) will assign the value of the coefficient Z, and will thus serve to ascertain the weight of the order*. The determinations contained in it have been obtained by the method just described; $\lambda$ is $\frac{1}{\Omega_1\Delta_1}$, or 0, according as $\Omega_1\Delta_1$ is or is not prime to the greatest common divisor of $\Omega$ and $\Delta$; $I_1$, $I_2$ are the exponents of the highest powers of 2 dividing $\Omega$ and $\Delta$ respectively.

(A).—($f$) and (F) properly primitive.

| | $I_1=0$. | $I_1$ even. | $I_1$ uneven. |
|---|---|---|---|
| $I_2=0$. | $\frac{1}{3}(2-\lambda)$ | $\frac{1}{4}(2-\lambda)$ | $\frac{1}{2}$ |
| $I_2$ even. | $\frac{1}{4}(2-\lambda)$ | $\frac{1}{4}(2-\lambda)$ | $\frac{1}{2}$ |
| $I_2$ uneven. | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

(B).—($f$) improperly, (F) properly primitive.

| | $I_1=0$, $I_2>0$. |
|---|---|
| $I_2$ even. | $\frac{1}{12}(2-\lambda)$ |
| $I_2$ uneven. | $\frac{1}{6}(1-\lambda)$ |

* For the case of uneven invariants, the result has been given by Eisenstein (Crelle, vol. xxxv. p. 123); there is, however, a slight discrepancy. According to Eisenstein, $\lambda$ is not zero, when the greatest common divisor of $\Delta$ and $\Omega$ is a square; according to the definition in the text, $\lambda$ is always zero, except when the exponent of every uneven prime common to $\Delta$ and $\Omega$ is even both in $\Delta$ and $\Omega$. For the invariants ($p^2$, $p^3$) the weight assigned by the formula of Eisenstein is $\frac{p^3}{24}\left(2-\frac{1}{p}\right)\left(1-\frac{1}{p^3}\right)$, $p$ denoting an uneven prime; a result which can hardly be right, because the weight of each genus separately is $\equiv 0$, mod $p^3$.

(C).—($f$) properly, (F) improperly primitive.

| | $I_2=0$, $I_1>0$. |
|---|---|
| $I_1$ even. | $\frac{1}{1^2 3}(2-\lambda)$ |
| $I_1$ uneven. | $\frac{1}{6}(1-\lambda)$ |

## XIII. *On the Appendicular Skeleton of the Primates.* By St. George Mivart, *F.L.S.*, *Lecturer on Comparative Anatomy at St. Mary's Hospital. Communicated by Professor* Huxley, *F.R.S.*

Received November 22, 1866,—Read January 10, 1867.

The interesting question regarding the number and value of the anatomical resemblances and differences existing between Man and the rest of the Primates, has led to complete and detailed descriptions and comparisons such as those of Professors Owen *, Duvernoy †, and Gratiolet ‡. But the valuable treatises of these authors yet leave much to be desired, because they relate only to the highest forms of the Order, and some distinctions resulting from such limited comparisons are apt to disappear, and the anatomical value of others to decrease when the survey is considerably extended.

The memoir of Professor Vrolik § gives a somewhat more extended view, and Professor Huxley ‖ has carried his observations and comparisons much further; but for the thorough investigation of the skeleton of the limbs of the Primates, nothing less than the careful examination of every bone throughout the whole series of forms is requisite, while man's peculiarities can be justly appreciated only after a similarly extensive comparison.

Dr. J. Ch. G. Lucae ¶ has recently published an elaborate paper, with careful and minute comparisons, on the limbs of Man, Apes, and Marsupials, but he confines himself almost entirely to the terminal segments of the limbs, the *manus* and the *pes* **, and besides he does not appear to have had at his disposal a sufficient supply of specimens, as the very remarkable genera *Indris*, *Loris*, *Nycticebus*, *Perodicticus*, *Arctocebus*, *Tarsius*, and *Cheiromys* are not noticed by him.

The rich collections of the British Museum and of the Royal College of Surgeons have supplied me with abundant materials, and I should be wanting in duty if I omitted to express my acknowledgments for the great facilities afforded me, at both those Institutions, for studying the skeletons therein preserved. To Mr. W. H. Flower espe-

* "Osteological Contributions to the Natural History of the Chimpanzees and Orangs," Trans. Zool. Soc., vols. i. to v.; and "Memoir on the Gorilla," 1865. † Archives du Museum d'Hist. Nat. Paris, 1855.

‡ Nouvelles Archives du Mus., 1866, vol. ii. § Recherches d'Anat. Comp. sur le Chimpansé. Amsterdam, 1841.

‖ 'Man's Place in Nature,' 1863; and 'Hunterian Lectures,' reported in Medical Times, 1864.

¶ Abhandl. Senckenb. Naturforsch. Ges., 1865, v. pp. 275 to 332, with four plates.

** On account of the ambiguity arising from the as yet unsettled connotation of the terms "hand" and "foot," I think it better in a scientific treatise to disuse them altogether, and to follow the example set by Professor Owen (in his memoir on Cheiromys) and by Mr. W. H. Flower (in the labels placed on his recent additions to the Museum of the Royal College of Surgeons), by adopting for the anterior extremity (the carpus, and all beyond it) the term *manus*, and for the homotypal posterior segment the term *pes*. The all but necessity for distinct *homological* terms for such parts is obvious.

cially my thanks are due; nor can I refrain from expressing my admiration of the liberal spirit in which the magnificent collection placed under his zealous supervision is made available to cultivators of natural science.

After considering the skeleton of each entire limb, and of every segment of each, and describing the several bones in some detail, after also giving the dimensions and proportions of these parts, I propose to consider the number and value of the peculiarities presented by the more aberrant forms, and especially by Man, and finally to enumerate some of the more obvious characters of the several groups (as deducible from their appendicular skeleton), and the relations thence derivable of such groups to each other.

The arrangement here adopted as to the families, subfamilies, and genera of the Order is as follows:—

Suborder I. Anthropoidea.

| | | | |
|---|---|---|---|
| Family. | I. Hominidæ | | Homo. |
| II. Simiidæ | Subfamily 1. *Simiinæ* | Troglodytes. | |
| | | Simia. | |
| | | Hylobates. | |
| | Subfamily 2. *Semnopithecinæ* | Semnopithecus. | |
| | | Colobus. | |
| | Subfamily 3. *Cynopithecinæ* | Cercopithecus. | |
| | | Macacus. | |
| | | Cynocephalus. | |
| III. Cebidæ | Subfamily 1. *Cebinæ* | Ateles. | |
| | | Lagothrix. | |
| | | Cebus. | |
| | Subfamily 2. *Mycetinæ* | Mycetes. | |
| | Subfamily 3. *Pitheciinæ* | Pythecia. | |
| | | Brachyurus. | |
| | Subfamily 4. *Nyctipithecinæ* | Callithrix. | |
| | | Chrysothrix. | |
| | | Nyctipithecus. | |
| IV. Hapalidæ | | | Hapale. |

Suborder II. Lemuroidea.

| | | | |
|---|---|---|---|
| V. Lemuridæ | Subfamily 1. *Indrisinæ* | Indris.* | |
| | | Propithecus. | |
| | | Microrhynchus. | |
| | Subfamily 2. *Lemurinæ* | Lemur. | |
| | | Hapalemur. | |
| | | Microcebus. | |
| | | Lepilemur. | |
| | Subfamily 3. *Nycticebinæ* | Nycticebus. | |
| | | Loris. | |
| | | Perodicticus. | |
| | | Arctocebus. | |
| | Subfamily 4. *Galagininæ* | | Galago. |
| VI. Tarsiidæ | | | Tarsius. |
| VII. Cheiromyidæ | | | Cheiromys. |

* Since this paper was read I have had, through the great kindness of Professor Peters, an opportunity of examining a skull of the species for which the genus *Propithecus* was instituted. I am now convinced that the three above-mentioned genera of *Indrisinæ* constitute but a single natural genus—*Indris*. See Proceed. ool. Soc. 1867, p. 247.

Specimens have been examined of all the above genera except Propithecus, Hapalemur, Microcebus, and Lepilemur, no skeleton belonging to any of these genera existing, to my knowledge, in this country. There is, however, every reason to believe that the skeleton of Propithecus closely resembles that of Indris, and the other three are probably very similar to Lemur or Galago.

### The Pectoral Limb.

The entire pectoral limb (measured from the summit of the head of the humerus to the distal end of the longest digit) attains its greatest absolute length in the Gorilla and Orang, after which come the Chimpanzee and Man. If the manus, however, be excluded, the rest of the limb of Man exceeds that of the Chimpanzee in the specimens examined.

The proportion borne by the entire limb to the spine, measured as before mentioned, is greatest in Hylobates, namely, as much as about 203, or even 222, to 100. Next come Tarsius, in which it is about 187 to 100; Ateles, 174; Simia, 170; the Gorilla, 150; and the Chimpanzee, 142*. The rest range from 128 (Cheiromys) to a little less than the spine in length (Man being about 107 to 100), except certain forms in which the proportion is much less; thus in Chrysothrix and Hapale it is less than 85 to 100, while in Perodicticus and Lemur it is under 80, and in Arctocebus as little as 75·3 to 100.

The length of the limb without the manus, compared with that of the spine, is again by far greatest in Hylobates, then in Tarsius, Ateles, Simia, and the Gorilla; in all the rest, except the Chimpanzee, the pectoral limb without the manus is shorter than the spine, and shortest of all in Perodicticus.

### Scapula.

This bone throughout the Order has a well-developed spine, and more or less large acromion and coracoid processes.

Estimating its size by a line drawn from the anterior (in Man upper) end of the glenoid surface to the posterior (in Man inferior) vertebral angle, this bone is seen to attain its greatest absolute size in the Gorilla. Man follows next, with the Chimpanzee and Orang, which two Apes more nearly equal him in the size of this bone than he does the Gorilla.

This dimension, compared with the length of the vertebral column, is again greatest in the Gorilla, namely, about 35·5 to 100; then in the Orang and Chimpanzee about 30, and in the Gibbons and Ateles about 25. In Man it is about as 22·8 to 100, and in most of the other forms it is less, and least in Perodicticus, namely about 15·6.

* Dr. Lucae, *loc. cit.* p. 279, makes the proportional length of the limb greater in the Chimpanzee than in the Gorilla, as also does Dr. G. M. Humphry (Human Skeleton, p. 106). In all the adult, or nearly adult specimens in the Museum of the Royal College of Surgeons I find the pectoral limb longer compared with the spine in the Gorilla than in the Chimpanzee.

As to the relative proportions of the several margins of the bone, if the axillary margin be taken as a standard, then the vertebral border exceeds it by a fourth or a fifth of its (the axillary margin's) length in Man and Perodicticus. It considerably exceeds it in the Gorilla, and decidedly so, though to a less extent, in the Chimpanzee and in Arctocebus. In Nycticebus the two dimensions are about equal, but in other forms the vertebral margin is the shorter, though only slightly so in Mycetes, Ateles, and Pithecia, and sometimes in Cynocephalus. In the Orang and Gibbons it is about as 86 or 71 to 100, while in Lemur and Galago the vertebral margin is only about half the length of the axillary one, and the proportion is even less in Tarsius.

If the anterior (in Man superior) margin be compared in length to the axillary one, estimating it by a straight line drawn from the glenoid surface to the anterior vertebral angle, it will be found to attain its greatest relative size in the lowest Simiidæ, being in Cynocephalus sometimes as 107·6 to 100. Its proportional length is also great (91) in Perodicticus; in the rest it varies from near this to 61 (Man and Indris about 64), except in the Simiinæ and Ateles, where it is less, being least in the Chimpanzee, *i. e.* sometimes only as 40 to 100.

The proportion borne by the anterior margin (superior in Man) to the vertebral one is greatest in Tarsius, more than 2 to 1; but it is more or less in excess also in Cheiromys, the Lemurinæ, and the lowest Simiidæ, Nyctipithecus, and Chrysothrix. The anterior margin is the shorter of the two in Man, Ateles, Mycetes, Pithecia, Indris, the Nycticebinæ (except Loris) and the Simiinæ, and is shortest of all in the Chimpanzee.

The posterior vertebral angle is most acute in Troglodytes niger, where it is sometimes as small as 22°. In the other Simiinæ and in Ateles, it is more acute than in Man, in whom it is about 35° or 40°; but in the rest of the Order it is more obtuse, even reaching to 75° in some of the lowest Simiidæ.

The anterior vertebral angle is most marked in Man *, the Simiinæ, Ateles, Pithecia, the Nycticebinæ, Tarsius, and Cheiromys. In the other forms the vertebral margin passes into the anterior one without any marked prominence (Plate XI. fig. 2).

The direction of the spine of the scapula, with regard to the blade of that bone, may perhaps be best estimated by the angles it forms with the vertebral and axillary margins.

The angle formed by it with the vertebral margin is greatest in the Chimpanzee, the Siamang, and in Ateles, where it amounts to about 125°, or even rather more; and in Galago and Lemur, where it is about 120°. In the rest of the order it ranges between this and a right angle (Man being about 95°), except in some of the lower Simiidæ, where it falls below a right angle, being sometimes in Cynocephalus as small as 74°.

* In the Museum of the Royal College of Surgeons are skeletons of a male and female (Nos. 5357 and 5357A) from South Africa, in which this angle is rounded off, as has been noticed by Professor Owen, Osteol. Catalogue, vol. ii. p. 832. In another female of the same race, however, this angle is exceedingly produced.

The angle formed by the spine of the scapula with the axillary margin of that bone is most obtuse in Man, namely about 55° or 60°. It approaches the human proportion most closely in the Orang, aged Semnopithecinæ, Mycetes, Pithecia, Nycticebus tardigradus, and in Perodicticus, in which it is 40° or upwards; in the rest it varies between this and 20° (it being sometimes as small as 20° in the Chimpanzee, Ateles, Lemur, and Cheiromys), except in Galago, where it may be as small as 17°, and in Hylobates, where it is at its minimum, namely 15°, or even sometimes only 12°.

The glenoid surface is broadest in proportion to its antero-posterior (vertical) extent in Man and Ateles, namely, about 73 or 75 to 100. In Troglodytes it is about 68 to 100; in the rest of the Anthropoidea it is less; but the breadth always exceeds half the length; this is not the case in some Lemuroidea, *e. g.* Indris, Nycticebus, and Cheiromys; and in the Nycticebinæ the anterior part becomes remarkably twisted inwards towards the midline of the body, and the long axis of the glenoid surface forms an angle with the prevailing plane of the blade of the scapula. The angle formed by this (glenoid) surface with the axillary margin varies generally between 130° and 144°. In the Simiinæ, the Pitheciinæ, Tarsius, Mycetes, Ateles, and Hylobates it is 125° or less, sometimes in the last-mentioned genus being as small as 93°.

The size of the supraspinous fossa, as compared to the infraspinous one, attains its maximum in the Gorilla and Mycetes (Plate XI. fig. 4), then in Hylobates and Arctocebus. The Orang, Man, the Pitheciinæ, Nycticebus tardigradus, Tarsius, and Cheiromys have the supraspinous fossa exceptionally small (Plate XI. figs. 5 & 6).

The anterior (in Man superior) margin is often much produced, so as to be strongly convex forwards *, and to much increase the size of the supraspinous fossa. This production does not exist in Man † or in the Simiinæ, in which this margin is more or less concave, as also in Ateles (external to the suprascapular foramen), Pithecia, and Nycticebus. In the other forms the anterior margin is generally more or less decidedly convex, and attains its maximum of convexity in aged Cynocephali (Plate XI. fig. 2).

A suprascapular notch is not well defined in the great majority of the order, only, indeed, in Man, the Chimpanzee, and the Cebidæ, except Pithecia and Chrysothrix; but in some of the last-named family (*e. g.* Ateles ‡ and Mycetes) it is constantly, and in others (*e. g.* Lagothrix) it is often so enclosed by bone as to become a foramen. In Mycetes a peculiar flat process § springs from the anterior surface of the bridge of bone

* That this prominence really answers to the anterior margin of Man, and is not produced by a bending downwards and forwards of the anterior vertebral angle, is shown by the specimens numbered 4756 and 4822A in the Museum of the College of Surgeons, in which the true anterior vertebral angle is distinguishable. Another reason for this determination is that, in Cynocephalus, Cercopithecus, and Lemur, the *levator anguli scapulæ* is not inserted into the convex prominence, but only extends forwards a little in front of the vertebral end of the spine, while the *omohyoid* is inserted into the projecting part of the convex prominence.

† Except the two South-African skeletons in the Museum of the Royal College of Surgeons, in which the anterior margin is as strongly convex as in Macacus (see Plate XI. fig. 1).

‡ De Blainville, 'Ostéographie,' Cebus, p. 12.

§ Mentioned by De Blainville, 'Ostéographie,' Primates, Cebus, p. 16.

bounding this foramen anteriorly, a process existing in no other genus (Plate XI. fig. 4*x*).

The supraspinous fossa is almost always deepest, from before backwards, at its vertebral end. Not so, however, in the Orang *, and scarcely so in Pithecia.

The axillary margin, apart from the production for the *teres major*, is generally straight, especially in Troglodytes, Hylobates, Ateles, Mycetes, Loris, and Nycticebus javanicus. It is markedly concave in Perodicticus and Nycticebus tardigradus (Plate XII. fig. 1, and Plate XI. fig. 6); on the other hand, it is convex in Simia and Indris. This margin is generally more or less grooved longitudinally, but only in Indris is this groove so placed as to be visible on the dorsum of the scapula.

The surface for the *teres major* projects out very strongly in the lower Simiidæ, Cebus, and Chrysothrix. On the other hand, in the Simiinæ, Ateles, Indris, and the Nycticebinæ it is less marked than in Man, and, indeed, in Indris, Loris, and Nycticebus it does not project at all (Plate XI. fig. 6).

The vertebral margin is generally more or less convex, but sometimes in Man, the Gorilla and Orang, Ateles and Chrysothrix, it presents a sigmoid curve. Sometimes it is nearly straight, as in Indris; sometimes it is very strongly convex, as in Perodicticus (Plate XII. fig. 1).

The convexity of the middle part of the infraspinous fossa, which is present in Man and, more or less, in the Simiinæ, does not generally exist. In Mycetes a projection, like a faintly-marked second spine, traverses the outer surface of this fossa midway between the spine and the axillary margin.

The subscapular fossa is particularly deep in Hylobates; in Mycetes it is traversed by strongly-marked ridges, in Indris its posterior part is strongly convex.

The spine generally extends from quite the vertebral margin to a point more or less near the border of the glenoid surface. In the Gorilla, however, it rarely attains the vertebral margin†, and it scarcely does so in Hylobates, where the depth of the spine subsides with great rapidity, as also in Ateles and Nycticebus javanicus.

The superior (in Man posterior) end of the spine is almost always much nearer to the anterior than to the posterior end of the vertebral margin; but in the Chimpanzee it is (generally at least) nearer to the latter, and in Hylobates, alone of all primates, it is considerably nearer to the latter than to the former. In the Gorilla, Ateles, and Arctocebus it is more remote, relatively, from the anterior end of the vertebral margin than in Man; in all the others, including Simia, it is relatively nearer to it. A smooth, flat, triangular surface at the vertebral end of the spine, and extending thence downwards (forwards in Man) along its margin for a greater or less extent, exists in Man, the Orang, Mycetes, Loris, and Arctocebus.

The spine, except at its acromial end, always stands out more or less at right angles with the outer surface of the blade of the scapula, but sometimes it inclines forwards over

* Owen notices this condition of the supraspinous fossa in the Osteological Catalogue of Coll. of Surg., vol. ii.

† Noticed by Duvernoy in Archiv. du Mus. tome viii. p. 40.

the supraspinous fossa; sometimes it is produced backwards over the infraspinous one It is much antroverted in Simia * and Nycticebus, and rather so in Ateles, Brachyurus, Callithrix, Chrysothrix, and the other Nycticebinæ. On the other hand, in Lemur, Galago, and Tarsius, and also in Nyctipithecus, it is more or less produced over the infraspinous fossa.

Generally the spine approaches very nearly to the border of the glenoid surface, but it remains rather distant from it in Man and the Chimpanzee, the Orang and Ateles, and still more so in the Gorilla and Hylobates.

The spine also may or may not closely approach the axillary margin towards the glenoidal end of the latter. It does so in the great majority of forms; but in Man, Perodicticus, Pithecia, Loris, and Nycticebus it recedes from it so as to produce a greater width in the infraspinous than in the supraspinous fossa at that part. In the Orang and Mycetes it recedes also, and would produce a similar predominance of the infraspinous fossa but for the peculiar development of the supraspinous fossa which alters the proportion. The two fossæ are about equal in breadth, near the border of the glenoid surface, in the Gorilla, Indris, and Arctocebus; in all the others the supraspinous one is in excess (except Tarsius and Cheiromys), especially in Cebus, Chrysothrix, and the lower Simiidæ.

The base of the spine is generally grooved behind (below in Man) at its glenoidal end, most so in Cynocephalus and Mycetes. In Man, the Simiinæ, Ateles, Indris, Loris, Tarsius, and Cheiromys, this groove, as far as I have seen, is absent.

The acromion is long and narrow in Simia, Ateles, Mycetes, Pithecia, Chrysothrix, and Loris. It is short and ends very bluntly in the Semnopithecinæ and Cynopithecinæ, especially the latter (Plate XI. fig. 2). Sometimes the acromion expands, so as to send back a metacromion-like process, before reaching its distal end. This is the case in Man, Nyctipithecus, Hapale, and Troglodytes, and sometimes in Hylobates, and also in Lagothrix, Lemur, and Galago. In the two last-mentioned genera, unlike the higher forms, this expansion projects backwards over the infraspinous fossa, instead of over the head of the humerus. The expanded part is very large, and it is concave externally.

The coracoid process is large in Man and in all the Simiinæ, Ateles, and the Lemuroidea. It is short in Mycetes and the lower Simiidæ, especially in Cynocephalus (Plate XI. fig. 3). It advances much forward at its distal end in Man, the Simiinæ, Ateles, and some of the lower Cebidæ (*e. g.* Callithrix), and in the Lemuroidea. In the others its distal end scarcely, if at all, advances in front of the glenoid surface.

The ridge or process for the attachment of the coraco-clavicular ligament is *very* little marked in Man, the Orang and Gorilla, Lemur, Loris, and Cheiromys. It is very small, though distinct, in Indris, moderate in the Chimpanzee and Hylobates, and larger in the other forms, though in Mycetes it is small, apart from the peculiar flat process of that genus, with which process it comes ultimately to unite.

* Mentioned by Professor Owen, Trans. Zool. Soc. vol. i. p. 364, and by Vrolik, Cyclop. Anat. & Phys. vol. iv. p. 203.

In Ateles and Lagothrix, with a suprascapular foramen, it is of course large, but as compared with the rest of the coracoid, it attains its maximum in the lower Simiidæ, in some of which it sometimes almost equals in size the latter (Plate XI. fig. 3, *a*, *b*). It is sometimes well developed in the Nycticebinæ.

The point of attachment for the long head of the *Biceps* becomes in many a prominent tubercle. This is particularly developed in the lower Simiidæ and the Nycticebinæ, and rather so in the lower Cebidæ. In Simia it is much more developed than in Homo and Troglodytes, and more so in Ateles than in Hylobates, in which last it is peculiar in projecting rather over the infraspinous fossa than over the glenoid surface.

When the scapula is so placed that the long axis of the glenoid surface is vertical, if that surface be placed opposite the eye of the observer, then the acromion process generally does not rise nearly so high as the summit of the coracoid. In Simia, Lemur, and Galago, however, it about equals it, and almost always exceeds it in Man, Troglodytes, and Hylobates, and sometimes in Ateles, and, indeed, sometimes also in Lemur. In the Nycticebinæ it is much below it, because of the peculiar production inwards of the summit of the glenoid surface in that subfamily (Plate XII. fig. 2).

The approximation of the end of the acromion to the prolongation upwards of a vertical line traversing the long axis of the glenoid surface is very close in Man, the Simiinæ, Ateles, and Mycetes, but it diverges widely in the other genera.

The extremity of the coracoid diverges from the glenoidal margin in Man and the Lemuroidea (Plate XII. fig. 2); it approaches it much more nearly in the other Anthropoidea.

### The Clavicle.

The absolutely largest clavicle of the Order is that of the Orang, and then follow those of Man and the Gorilla.

Its relative length, as compared with that of the vertebral column, is greatest in Hylobates * and Simia, in which genera only it exceeds one-fourth the length of that column. The proportion exceeds one-fifth in Man and Troglodytes, and does not fall much below in Lagothrix, Ateles, and Mycetes. In most other forms it is as about 14 or 16 to 100, but in Colobus the Nyctipithecinæ, Hapale, Arctocebus, Lemur, and Indris, it is about an eighth or less; in Lemur being sometimes as little as 9·7 to 100.

The length of the clavicle, in proportion to that of the scapula (the latter being measured from the anterior end, or summit, of the glenoid surface to the posterior vertebral angle), is in excess (111·8 to 100) only in Hylobates. It is next longest in Simia and Man, where alone it is nine-tenths the length of the scapula. It is shortest in the lowest Simiidæ, Hapale, Lemur, and Tarsius, in all of which it but little exceeds half the length of that bone.

This bone is of very exceptional slenderness, in Mycetes† its breadth, near the middle,

* Its unusual length in Hylobates is noticed by Prof. Owen, Comp. Anat. of Vertebrates, vol. ii. p. 544.

† As remarked by De Blainville, *loc. cit.* Cebus, p. 16.

being only about one-twentieth of its length (Plate XII. fig. 3). It is also narrow in proportion to its length in Hylobates and Simia.

It is very broad relatively in Troglodytes, the Cynopithecinæ, Lemur, Perodicticus, Arctocebus, Tarsius, and Cheiromys, where its thickness near the middle is about equal to one-tenth of its length.

The amount of expansion of the acromial end is liable to considerable individual variation, but it appears to be greatest in Man, the Simiinæ, and Ateles; on the other hand, it is remarkably small in the Nycticebinæ.

The expansion of the sternal end is subject to even greater variation. In Simia, Mycetes, the lower Cebidæ, and Lemuroidea it often exceeds the acromial end in breadth.

A sigmoid vertical (in Man horizontal) curvature is not generally well marked; it is most so in Man, and next perhaps in Ateles, some other Cebidæ, and the Nycticebinæ.

The sternal vertical (in Man horizontal) curvature, concave backwards, is so extended in some forms as almost to obliterate the acromial curve. This is the case in many Lemuroidea and, sometimes at least, in Hylobates. On the other hand, the sternal curvature is much less than in Man, even in Troglodytes and Simia, and in the lowest Simiidæ it disappears, as also in Indris.

The acromial vertical (in Man horizontal) curvature, concave forwards, is more constant, only disappearing in those forms, above referred to, in which the sternal curvature is so extensive. It is very strongly marked in the Nycticebinæ and in the Indrisinæ, but in no other Lemuroidea.

The antero-posterior (in Man vertical) curvature is generally slight; most marked perhaps sometimes in Ateles. It is this curvature which gives a sigmoid appearance to the clavicle in Lemur and Cheiromys.

The tubercle and ridge for the attachment of the coraco-clavicular ligament are generally at the margin of the bone, or nearer to it than in Man.

Very commonly there is no distinct process or ridge other than the superior (in Man posterior) margin of a subclavicular fossa, as is the case in the lower Simiidæ. There is, however, a marked tubercle in Troglodytes, and a large process in Simia and in Ateles and Logothrix. It is faintly marked in Mycetes and Indris.

The acromial end of the bone has its anterior (in Man superior) surface almost always more or less convex, but there is a marked concavity there in Hylobates.

The posterior (in Man under) surface of the acromial end is convex and roughened in Man *, and more or less so in the Simiinæ; in all the others, except the Nycticebinæ, it is concave, and in the lower Simiidæ this concavity becomes a very deep fossa. In all the Simiidæ other than the Simiinæ this part is close to the acromial end of the bone, but in the Simiinæ the clavicle is more prolonged outwards, and most so in the Siamang. In Man, however, this prolongation is carried still further.

Rarely, as in Simia, there is a very prominent deltoidal ridge. Sometimes a distinct

* In a skeleton of a male African negro (No. 5372 in the Museum of the College of Surgeons) there is a distinct, though small, subacromial fossa. This is wanting in all the Boschismen.

prominence marks the attachment of the costo-clavicular ligament, but this is very inconstant.

### The Humerus.

This bone, throughout the order, presents the same main features, the same fossæ and prominences as those existing in Man. It is absolutely longest in the Gorilla and Orang; Man, the Siamang, and the Chimpanzee successively follow as regards this dimension.

Its length, as compared with the spine, is, as might be expected, greatest in Hylobates, namely, as 70 or 80 to 100. In the Gorilla and Orang it is from about 60 to near 65 to 100; in Ateles about 60; in the Chimpanzee and Lagothrix about 53; in Man about 47; and in the bulk of the order from between 45 and 30 to 100. In Indris, Lemur, and Perodicticus it is still shorter, though still more than a quarter the length of the spine *.

Compared with the scapula (the latter being measured, as before, from the anterior end of the glenoid surface to the posterior vertebral angle) it is nearly three times as long in Hylobates, considerably more than twice in Ateles, Lagothrix, and Loris, and slightly more than twice in Man, Cercopithecus, and the Pitheciinæ. All the rest have it less than twice as long (unless possibly sometimes in Simia); and in Hapale, Galago, and Tarsius its length is less than once and a half that of the scapula.

The breadth of the middle of the shaft to the length of the bone is mostly as between 6 and 7½ to 100. In most Cynopithecinæ, Perodicticus, Tarsius, and Cheiromys it is more than 8; on the other hand, in the Pitheciinæ, Loris, and Arctocebus it is between 5 and 6, less than 5 in Ateles, and less than 4, at least sometimes, in Hylobates.

The width of each extremity of the bone is greatest, relatively, in Cheiromys and least in Ateles and Hylobates. But the width of the proximal part (between the tuberosities) is very great, relatively, in Cynocephalus, and of the distal portion, in Galago, Perodicticus, and Tarsius.

The head of the humerus is generally less wide than the extreme width of the tuberosities; but in the Gorilla they are about equal, and sometimes in Simia † and Ateles, and always, apparently, in Hylobates, the head is the wider, being therefore at its relative maximum.

The shaft is often almost quite straight, as in Indris; often it is curved, as in Man and Lemur; in some it is somewhat convex forwards, as sometimes in Hylobates.

The articular surface of the head is always directed backwards and inwards ‡, but in Lemuroidea it is almost exclusively backwards, while in Man it is almost as exclusively

* The bone is measured from the summit of the head to the bottom of the ulnar margin of the trochlea.

† De Blainville says of the head of the humerus in the Orang, it is "surtout singulière par son enormité, son diamètre étant bien supérieur à celui de la tête du fémur" (*l. c.* p. 30).

‡ Professor Huxley, in his Hunterian Lectures for 1864, called attention to the greater backward direction of the head of the humerus in the lower Apes as compared to its condition in the Simiinæ and Man. See 'Medical Times' for 1864, vol. i. p. 672.

inwards. In the other Anthropoidea it is intermediate, approximating to Man in the higher forms, though still differing considerably from him.

The tuberosities often project upwards slightly above the articular head, as in the Cynocephali, and very slightly in Cheiromys. Generally they are about on a level with its top, or slightly below it, but they are decidedly below it in Lagothrix and Ateles, and still more so in Simia and Hylobates.

The insertion of the *infraspinatus* is generally very marked, especially in Indris. There is sometimes a projecting tubercle for the insertion of the *teres minor*.

The ulnar tuberosity is almost always so placed as to hide the neck of the bone when its front surface is looked at, the long axis being vertical. This is not the case, however, in Man, nor in Hylobates, the Chimpanzee, nor sometimes Ateles. Very often not only the neck but a great part of the head is also hidden by it, as in the lower Simiidæ and Cebidæ, Hapale, Lemur, Indris, Tarsius, and Cheiromys. In Indris this tuberosity is bent backwards in a peculiar way (Plate XII. fig. 6). In Cheiromys it is almost as large as the radial one.

The bicipital groove is sometimes more or less strongly overlapped by its lateral margins. It is much so in the Simiinæ, especially in the Chimpanzee, in which it is sometimes spanned by a bridge of bone *.

The radial border of the bicipital groove attains its maximum of development in the Lemuroidea, especially in Indris and Perodicticus, but it is also very prominent in the Cynocephali (Plate XII. fig. 4). Its ulnar border is generally faintly marked.

The surface for the insertion of the deltoid is generally more or less defined by the radial margin of the bicipital groove on one side, and by a ridge (giving origin to the external head of the triceps) on the other. It attains its maximum in the Cynopithecinæ (Plate XII. fig. 4).

The position of the foramen for the nutrient artery presents even individual variations, being at, above, or below the middle of the bone; its direction, as far as I have observed, is always distad.

The supinator ridge is generally well developed, especially in Cynocephalus, Cebus, Hapale, Lemur, Perodicticus, and Galago; but above all in Cheiromys and Microrhynchus. In Man and the Simiinæ it is only slightly developed.

The external condyle is distinct in Man and the Simiinæ, being more prominent in them than in him; and it is also marked in Indris (Plate XII. fig. 6), Loris, Nycticebus, and Perodicticus. In the other genera it is closely applied to the capitellum, and in all of them, except Ateles and Lagothrix, looks more or less entirely outwards instead of forwards.

The internal condyle projects inwards and more or less backwards. It is least backwardly directed in Man, the Simiinæ, Hapale, and Indris (Plate XII. fig. 6). In almost all the Cebidæ (Pl. XII. fig. 5) it is so bent downwards that its extremity is quite or

* As in a mounted specimen in the Osteological Collection of the British Museum.

almost as low as the margin of the innermost border of the trochlea; it is also very much bent down in Lemur, Tarsius, and Cheiromys.

A supracondyloid foramen is only present in the Anthropoidea, in Cebus, the Pithecinæ, Chrysothrix, Callithrix, sometimes in Nyctipithecus, and sometimes in Hapale *; on the other hand, it is present in all the Lemuroidea except Arctocebus.

The surface of the humerus between the internal condyle and the innermost border of the trochlea extends in front of and behind that condyle in all the Anthropoidea. This surface is largest relatively and absolutely in Man and the Simiinæ, where it extends in front of, beneath, and behind the condyle, though in Hylobates it is largely developed only in front of it. In the lower Simiidæ its posterior part either disappears or only exists as a very narrow groove overlapped by the condyle. In the Cebidæ this surface is smaller, and scarcely ever projects below the end of that process. In Indris and Cheiromys it is not developed in front of the condyle, but in the other Lemuroidea it is developed both in front of it and behind it, and pretty equally so in the Nycticebinæ.

The coronoid fossa is generally shallower in the Lemuroidea than in the Anthropoidea. A perforation extends into the olecranal fossa in some. This is very large and constant in Loris, but it is also present in Troglodytes † and Simia, and sometimes in Hylobates, Man, Cercopithecus, Macacus, and Arctocebus.

The olecranal fossa is sometimes deep, as in the Simiidæ, especially the Cynopithecinæ. It is less so in Man, and still less so in the Lemuroidea, especially in Indris.

The capitellum is largest relatively in the Lemuroidea, where it often (as in Indris and the Nycticebinæ) occupies half, or more than half, of the articular surface of the distal end of the humerus (Plate XII. fig. 6). It is next largest in the Cebidæ, smaller in the Simiidæ other than the Simiinæ, still smaller relatively in Hylobates and Simia, then in Man and the Chimpanzee, and relatively smallest of all in the Gorilla. Sometimes in Cynocephalus, as also in Mycetes, Lagothrix, Indris, Nycticebus, and Perodicticus, the smooth surface is prolonged outwards externally to the convexity of the capitellum.

The projection of the radial margin of the trochlea is most prominent in Hylobates, the Chimpanzee, Man, the Gorilla, Indris, Lemur, and Hapale. It all but or quite disappears in the Cynopithecinæ (Plate XII. fig. 4), many Cebidæ (especially Ateles and Lagothrix), Loris, Nycticebus, and Arctocebus.

The innermost margin of the trochlea projects downwards below its radial margin in Man, the Chimpanzee, Simia, and the Cynopithecinæ, and *very* much so in the Cynocephali (Plate XII. fig. 4). It projects below the radial margin, but very slightly (or not at all) below the capitellum in the Semnopithecinæ, Cebidæ, and some Lemuroidea, *e. g.* Galago and the Nycticebinæ.

* In the Tamarin, according to De Blainville, *l. c.* p. 22, he adds, "ce qui n'a pas lieu cependant ni chez le Pinche ni chez les Ouistitis."

† In the specimens of T. Niger, Nos. 5177 c and 5177 d in the College of Surgeons, this perforation exists.

It scarcely projects below the radial margin, and *à fortiori* not below the capitellum in the Gorilla, Hylobates, Hapale, Indris (Plate XII. fig. 6), and Lemur.

### Radius and Ulna.

Both these bones are always distinct and separate in the Primates, and being approximated at their extremities, diverge more or less from each other midway, the divergence being relatively most extreme in the Gorilla and Indris (Plate XII. fig. 7).

### Radius.

The radius is absolutely longest in the Gorilla and Orang, but in the Chimpanzee and Siamang it is also longer absolutely than in Man.

Its length, as compared with that of the spine, is greatest in Hylobates, being more than four-fifths of the length of the latter. It is more than three-fifths in the Orang and Ateles, half, or a little more, in Troglodytes and Tarsius. In the great bulk of the order it is between three-tenths and two-fifths (Man being about as 35 to 100) of the length of the spine. It is a little less in the Nyctipithecinæ, and only a quarter of its length in Hapale and Lemur.

The total length of the radius rather more frequently falls short of than exceeds that of the humerus. It exceeds it most in Tarsius and the Indrisinæ. It also exceeds it, though not to such an extent, in Hylobates*, Loris, Perodicticus, Arctocebus, and, sometimes, at least, in Ateles†, Cynocephalus, Semnopithecus, and Simia. In all the others it is more or less shorter than the extreme length of the humerus, though in none so much so as in Man‡.

The radius is thickest relatively in Cynocephalus, and then in Man. It is very slender in Ateles and Loris, but most so in Hylobates.

The radius is always more or less curved, most so perhaps in the Gorilla and Indris. It expands laterally at its distal end, but this expansion is least marked in Hylobates, Ateles, and the Nycticebinæ. It is perhaps as marked in Man as in any other primate.

The ulnar margin is sometimes sharp as in Man, the lowest Simiidæ and others; sometimes it is rounded, as in Troglodytes, Hylobates, some Cebidæ, Indris (Pl. XII. fig. 6), and Loris.

The outer margin is rather marked in Man, less so in Troglodytes and Simia, and

* De Blainville says, "L'avant-bras est encore plus long que le bras d'un septième au moins" (*loc. cit.* p. 26).

† Dr. Lucae, *loc. cit.* p. 286 (Table of Measurements and Proportions), makes the humerus longer than the fore-arm in all the American apes, but in no others, and both equal in Colobus; yet at p. 287 he says that the humerus is the smaller in all the long-tailed apes, except Colobus and Ateles.

‡ In Brachyurus (British Museum specimen) I have found the radius to be to the humerus as 75·9 to 100, and in the Boschisman as much as 81 to 100, so that in some exceptional cases the human proportion is surpassed.

more rounded still in the other forms, except the Cynopithecinæ, where sometimes it is better defined than in Man, as sometimes also in Cebus, Lemur, and Galago.

The degree of distinctness of the ridges and depressions for muscular attachment is subject to much individual variation.

In the Cynopithecinæ, Ateles, and sometimes in Lemur and Nycticebus, there is a marked depression for the *supinator brevis*. I have not observed it elsewhere.

The ridge giving origin to the *flexor sublimis digitorum* is marked in Man and the lower Simiidæ (especially Cynocephalus). I have also found it marked in Chrysothrix, Indris, and Arctocebus; less, or not at all so in other forms.

The excavation in which the *flexor longus pollicis* takes origin is marked in Man, sometimes in Hylobates, in the Simiidæ other than the Simiinæ, in Cebus, Pithecia, Lemur, Nycticebus, and Arctocebus. In others I have found the surface flat or rounded. A similar depression for the *extensor pollicis* is sometimes very marked in the lower Simiidæ; it is also marked in Man (sometimes) and in Chrysothrix, and slightly so in Indris. In the other genera I have only observed a flattening of the bone at the most.

The insertion of the *pronator teres* is sometimes marked by a roughness of the surface. This I have seen in Man, the Orang, Cynocephalus, Mycetes, Chrysothrix, Hapale, and Nycticebus. A decided fossa is occasionally present at that spot as, sometimes at least, in Macacus, Callithrix, Brachyurus, Lemur, Galago, Perodicticus, Arctocebus, Tarsius, and Cheiromys.

The inferior margin of the anterior, or flexor, surface is now and then much produced, as in Man and Cynocephalus; sometimes only the ulnar side of the inferior margin is prominent, as in Ateles, Mycetes, Lemur.

In the Nycticebinæ there is a process, at the lower end of the radius, projecting ulnad and articulating with the head of the ulna. A rudiment of this process exists in Indris.

The lower end of the posterior surface is generally traversed by a median longitudinal ridge, which appears to attain its maximum in Cynocephalus.

The styloid process is constant. It is large in Man and the Simiinæ, shorter in the Cynopithecinæ, Cebidæ, and Hapale, very short in Indris, Lemur, and Galago, but longer again in Nycticebus and Arctocebus.

A prominence for the insertion of the *supinator longus* is more or less marked in Troglodytes*, Simia, and Hylobates. It is much so in some of the Cynopithecinæ, and in Cebus, Lemur, and Galago.

The foramen for the medullary artery is situated above the middle of the bone, and is always directed upwards, except in Ateles† and Arctocebus. In Ateles the long groove which the artery makes on the surface of the bone is remarkable.

The groove for the tendon of the *extensor ossis metacarpi pollicis* is almost always very marked. In Hylobates it equals in size that for the tendons of the radial extensors,

* Owen, Trans. Zool. Soc. vol. v. p. 7.

† Not always downwards in Ateles, however; for in a skeleton of A. Geoffroyii in the British Museum it is directed upwards in one arm and downwards in the other.

and it is very marked in Ateles, in spite of the rudimentary condition of the pollex. On the other hand, it is not distinctly marked in Indris, and it is small in the Nycticebinæ.

The groove for the radial extensors I have found to be sometimes double in Macacus, Ateles, and some Nycticebinæ.

That for the *extensor secundi internodii pollicis* I have only found distinct in Man, the Chimpanzee, and Orang.

The groove for the *extensor communis digitorum* is smaller relatively in the lower Simiidæ than in Man and the Simiinæ. It is small also in Ateles and Mycetes (thus differing from Hylobates), and generally in the Lemuroidea.

### Ulna.

In absolute and relative length this bone varies almost as does the radius, being, however, always somewhat longer.

It is thickest (in the shaft), in proportion to its length, in Cynocephalus, most slender in Hylobates and Indris.

The bone is much curved in the Chimpanzee, less so in the Gorilla, Orang, and still less so in Man. It is generally more curved than in Man in the Cebinæ*, but straighter than in him in the other Cebidæ. It is sometimes very straight in Hylobates and the lower Simiidæ; also in Loris and Tarsius, but in the other Lemuroidea it is much as in Man.

The greater sigmoid cavity is exceptionally broad, in proportion to its length, in Man, Troglodytes, and Simia; in the other forms it is narrower, and turned more outwards, towards the radius.

The lesser sigmoid cavity looks outwards in Man, Troglodytes, Simia, and the Nycticebinæ. It looks more forwards in the lower Simiidæ.

The coronoid process (and surface for the insertion of the *brachialis anticus*) is at its maximum of breadth in Man. In Troglodytes and Simia this part is already narrower than in him, and more excavated. In the Hylobates it is still narrower, and yet more so in the lower Simiidæ, the Cebidæ, Hapale, and the Lemuroidea, especially in some of the Nycticebinæ.

The olecranon is broadest in Man, Simia, and Troglodytes. In Hylobates it is still very like that of Man; but in the lower Simiidæ it is much longer, extending further up (*i. e.* in the direction of the bone's length) beyond the sigmoid cavity, being at its maximum of development in this respect in Cynocephalus. The Cebidæ, including Ateles, resemble in this the lower Simiidæ, and the olecranon is very long in Mycetes. It is also long in Lemur and Galago, Tarsius, and Cheiromys, but is less so in Indris (Plate XII. fig. 7) and the Nycticebinæ.

There is a distinction between the anterior and inner surfaces of this bone in Man, Troglodytes, and Simia; but thence, downwards through the order, there may be said to be but one surface answering to these two of Man and the highest Apes.

* Its curved condition in Cebus is noticed by De Blainville, *loc. cit.* p. 8.

The anterior surface is generally not so depressed for the origin of the *flexor profundus* as in Man and Simia. The surface answering to the anterior and inner ones of Man is concave in Ateles and some other Cebidæ, and very much so in Lemur.

There is a deep depression for this last-mentioned muscle (inside the olecranon, or beneath the greater sigmoid cavity) in the Gorilla and Orang, the Simiidæ other than the Simiinæ, the Cebidæ (except Ateles), and the Lemuroidea.

In Loris and Indris there are no excavations or marked depressions for muscular attachment on the shaft of the bone, which is very rounded and cylindrical (Pl. XII. fig. 7).

The fossa for the origin of the *extensores pollicis* is marked in Man, and much so below the surface which gives attachment to the *anconeus*; as also in Simia and Hylobates. In Troglodytes it is much less so, and still less in the lower Simiidæ, except at the proximal end of the radial surface of the ulna. It is very slightly marked in Ateles and Mycetes, but more so in Lagothrix and Cebus; but slightly in the other Cebidæ, except Chrysothrix, in which, as also in Hapale, it is very marked. It is very marked in Lemur and Galago, less so in the other Lemuroidea.

The surface of the *supinator brevis* is marked and deep in Man and Troglodytes, less so in Simia and Hylobates. It is longer, but narrower relatively, in the Simiidæ other than the Simiinæ. In the Lemuridæ* it is absent.

The place of attachment of the *anconeus* is more marked in Man and Simia than in Troglodytes. In Hylobates, for the first time in descending from Man, it does not extend so high up as the upper margin of the lesser sigmoid cavity; and in the other lower forms of the order I have not found any fossa marked off from that for the *extensores pollicis*.

The ridge for the attachment of the *pronator quadratus* is very slightly marked indeed in Man, and very little more so in Troglodytes and the lower Simiidæ; while in the lower Cebidæ, Indris, Galago, and most Nycticebinæ it is rudimentary or absent. On the other hand, it is sometimes marked in Simia, and occasionally still more so in Hylobates, the Cebinæ, Mycetes, and Hapale, and also more or less in Cheiromys, but it attains its maximum of development in Lemur.

The head of the ulna is large and rounded in Man and Troglodytes. It is more transversely extended in Simia, and in all below is much smaller as compared with the styloid process, especially in Ateles, Hapale, Lemur, and Galago. It is rather larger again in Indris and the Nycticebinæ.

The styloid process is of moderate length in Man; it is shorter in the Gorilla, and still more so in the Orang. In the Chimpanzee it is longer and more curved; also in Hylobates, where it developes a peculiar prominence from its hinder side (for the internal lateral ligament of the wrist), which prominence also sometimes exists in the Cynopithecinæ and in Indris. The styloid process is very elongated in the Cebidæ generally; but in Ateles it becomes enormous, having a rounded articular head placed, as it were, at the end of a peduncle, and being really much more the continuation of the shaft of

* This muscle has no connexion with the ulna in Lemur.

the ulna than is the comparatively rudimentary head of the bone. In Lemur it is formed on the same type. In Hapale and Indris it is long, but in the Nycticebinæ it is very so, and curved, except in Perodicticus, in which it is exceedingly long but nearly straight, attaining perhaps the maximum of relative length in the whole Order*.

The foramen for the medullary vessels is always situated more or less above the middle of the bone, and is, as far as I have observed, always directed upwards.

### Manus.

This segment attains its greatest bulk in the Gorilla; its absolute length, however, is greatest in the Orang, then in the Gorilla and Chimpanzee, and afterwards in Man. In Ateles and Indris it is longer than in any of the lower Simiidæ, except the Cynocephali.

The proportion borne by the whole length of the manus to that of the spine is greatest in Tarsius, Cheiromys, and Hylobates, where it is more than half, and then in Simia, where it is but little less. In the rest it varies between this and one-fourth, except in Cercopithecus, the Nycticebinæ, Lemur, and Chrysothrix; being shortest in Arctocebus, where it is rather less than one-fifth.

The length of the manus, as compared with that of the rest of the pectoral limb, is far greatest in Cheiromys, where the first is more than four-fifths of the latter; then in Indris, Nyctipithecus, Galago, and Tarsius, where the proportion is as much as, or more than, 45 to 100; in the rest it varies between this and three-tenths, except in Loris, where it is scarcely more than one-quarter.

The length of the manus, as compared with that of the radius, is far greatest in Cheiromys, where the former is much more than once and a half the length of the latter. In Tarsius the manus is considerably longer than the radius, and in Brachyurus and Hapale the two segments are about equal. The manus is always more than half the length of the radius, except in Loris, where it is a little less†. In Man it appears to be generally a little more than three-fourths of its length.

### Carpus.

The largest carpus is that of the Gorilla; that of the Orang about equals Man's, while the Chimpanzee's is slightly smaller.

This segment, excluding sesamoids, consists of eight or nine bones, except where an extra ossicle exists in the transverse carpal ligament‡.

Its length (measured from the summit of the semilunare to the distal end of the

* See Van Campen's representation, plate 1, fig. 3, in the Verhandelingen der Koninklijke Akademie van Wetenschappen. Zevende Deel, 1859.

† As mentioned by De Blainville, *loc. cit.* Lemur, p. 17.

‡ Only the case, as far as I have been able to ascertain, in Perodicticus (see Plate XIV. fig. 5). It is described and figured by Van Campen in his Memoir on the Potto of Bosman in the Verhandelingen der Koninklijke Akademie van Wetenschappen. Zevende Deel, 1859, p. 18, and plate 1, figs. 4, 10.

magnum), as compared with that of the spine, appears to be greatest in Cheiromys, the Orang, and Hylobates, and least in Loris and Indris.

Its length, as compared with that of the entire manus*, I have found to present no important differences, but it generally appears not to more than very slightly exceed one-fifth, and in the long-handed forms (Ateles, Hylobates, and the Chimpanzee) it is but little more than one-tenth.

The breadth of the carpus almost always exceeds its length, being often, as in Man, half as broad again as long. In Hylobates, however, the length equals or slightly exceeds the breadth; as also in Perodicticus and Arctocebus, while in many genera there is little difference between the two dimensions.

The carpal bones, at their proximal end, always form a double arch, as in Man. The arch which has its convexity turned towards the fore-arm, is in general rather more acute than in him† or than in the Gorilla. This is especially the case in the Nycticebinæ; but in most of the other forms its outline is generally more or less interrupted by the projection of the pisiforme.

The arch which has its concavity towards the palm, is sometimes not much marked, as, *e. g.*, in Ateles; but generally it is more so than in Man, and in the Nycticebinæ it becomes remarkably deep, and, finally, in Perodicticus is changed, by the introduction of a supernumerary ossicle into a complete ring of bone (Plate XIV. fig. 5).

Rarely, *i. e.* only in Man, Troglodytes, and Simia, the carpus articulates directly with the radius alone; in all other forms it does so with the ulna as well as with the radius —to a very slight extent, however, in the Nycticebinæ.

*Scaphoides.*—This bone has always much the same shape throughout the Order. On its radial side is a tuberosity which sometimes, as in some of the lower Simiidæ, the Gorilla‡, Indris, and the Nycticebinæ, is much enlarged. It is generally received into a concavity formed by the trapezium and a sesamoid.

In Man, Troglodytes, and the Indrisinæ the connexions of the scaphoid are as in Man; but in all other forms an os intermedium separates it from the bones of the distal row, except the trapezium. In Man and Troglodytes there is a transverse dorsal groove.

*Intermedium.*—This bone, which in most cases might from its shape be termed a

* Sometimes, however, there appears to be considerable variation as to this proportion, as Lucae gives the proportions of the carpus to the manus at 100, as follows:—Inuus, 10·5; Macacus gelada, 20; Semnopithecus entellus, 12·03; S. comatus, 9·4!

† Dr. Lucae in a note (*loc. cit.* p. 289) criticizes Professor Huxley's remark "The bones of the first row with the bones of the fore arm form the wrist-joint, and are arranged side by side, no one greatly exceeding or overlapping the rest" (Man's Place in Nature, p. 87), saying that such a condition of the parts would limit the motion of the wrist to flexion and extension. It is surely obvious that several conjoined bones, no *one* of which "greatly exceeds or overlaps the rest," may together produce a very considerable convexity. Professor Lucae writes as if Professor Huxley had asserted that the proximal surfaces of the proximal carpals were all in one plane, instead of stating the undoubted fact that the projection of any *one* is small.

‡ Owen, Trans. Zool. Soc. vol. v. plate 10, fig. 1, and pp. 9 & 10.

second semilunare *, exists in all the genera of the Order below Troglodytes, with the exception of those forming the subfamily Indrisinæ †.

It has a flattened proximal surface, which joins the ulnar part of the concave distal articular surface of the scaphoides.

Its distal surface is generally deeply concave antero-posteriorly, and embraces the radial side of the head of the magnum, and sometimes (as in the Nycticebinæ and Cheiromys) the unciforme also, which last, however, is excluded from it in the Anthropoidea, and apparently also in Tarsius ‡.

At the ulnar side of the bone is a narrow surface, which articulates with the radial side of the semilunare.

At the radial side of its distal surface is a concavo-convex surface, which joins the trapezoides, and on its palmar side is a narrow surface, which joins the radial side of the distal surface of the scaphoides.

This bone appears to answer to part of the scaphoid of Man, as De Blainville §, Professor G. M. Humphry ‖, Professor Huxley ¶, and others have regarded it, and not to be a dismemberment of the os magnum, as Cuvier ** seems to have been inclined to consider it. Indeed, if that part of the scaphoides of Man which is on the distal side of the dorsal groove were cut away, it would answer tolerably well to the intermedium. Nevertheless, the united scaphoides and intermedium of any ape together form a mass which is much more disto-proximally extended than is the ulnar part of the human scaphoides.

In one manus of a Chimpanzee ††, however (Plate XIV. fig. 1), I have found the scaphoides develope a large process, embracing the magnum dorsally, while at the same time the part passing beneath the trapezium is much developed, so that in this case it, I think, evidently and completely responds to both the scaphoides and the intermedium of the Orang (Plate XIV. fig. 2).

Again, in Indris, in which the intermedium is wanting, the outer part of the scaphoides is enlarged, and has a more or less marked projection over the dorsum of the os magnum ‡‡. It would be a fact of much interest if it should turn out that in the

* Its form in Cynocephalus is very well described by Dr. Johann Georg Ilg, 'Monographie der Sehnenrollen.' Zweiter Abschnitt. Erste Abtheilung, 1824, p. 4.

† I make no doubt but Propithecus resembles in this Indris and Microrhynchus.

‡ Burmeister's 'Tarsius,' Table 2, fig. 5, *b*; and Blanchard's 'Règne Animal,' Mammifères, Primates, pl. 22, fig. 9, *b*. In Cheiromys it joins the unciforme, and extends between it and the lunare.—Owen, Trans. Zool. Soc. vol. v. pl. 21, figs. 17 & 18, *i*.

§ *Loc. cit.* Pithecus, p. 16.

‖ In his very interesting and valuable memoir on the limbs of Vertebrates. 1860, p. 4.

¶ Hunterian Lectures. See 'Medical Times,' 1864, vol. i. p. 565.

** Leçons d'Anat. Comp. 2nd edit. 1835, vol. i. p. 425.

†† The skeleton No. 5083A in the Museum of the Royal College of Surgeons. M. Gratiolet has, I find, noticed the same thing in his recent treatise, 'Troglodytes Aubreyi,' in the Nouvelles Archives du Muséum, vol. ii. 1866.

‡‡ De Blainville, *loc. cit.*, Lemur, pl. 10.

Chimpanzee or Indris the distal part of the scaphoid is developed from a separate centre of ossification.

If FISCHER's observation be correct,—that the intermedium, which he found separate in the young Macaco, was in an adult one united with the scaphoides *,—these separate centres may not improbably exist in the young, at least, of Indris †.

*Semilunare.*—This bone has much the same shape throughout the Order. In the Gorilla, and also in other Simiidæ, it is larger and broader relatively than in Man. In some forms, where it is relatively small, as in Lemur and Indris ‡, it yet unites slightly with the unciforme, from which in higher forms it is, although relatively larger, sometimes excluded. It is large in the Nycticebinæ, projecting much proximally in Loris, as also in Tarsius §. It is very large in Cheiromys, where it is separated from the unciforme by the extension of the intermedium as far as the cuneiforme ‖.

*Cuneiforme.*—This is longer and narrower in Troglodytes than in Man, and it is exceedingly elongated in Simia. It is large also in the lower Simiidæ. Compared to the semilunare, it is large in Lemur, and still more so in Indris ¶, but in Loris it is much smaller than the last-mentioned bone.

In all but Man, Troglodytes, and Simia, it articulates directly with the ulna, though very slightly so in the Nycticebinæ. In all the rest of the Order it has an articular surface, for the reception of the styloid process of the ulna, which is contiguous to a similarly destined articular surface of the pisiforme.

Sometimes in the lower Simiidæ the outer end of the cuneiforme is produced into a rounded process or tubercle, projecting into the palm in front of the pisiforme.

The cuneiforme is small in Tarsius **.

*Pisiforme.*—The pisiforme is very small, relatively, in Man, Simia, and the Nycticebinæ. It is rather small in the other Lemuroidea and in Ateles; larger in the rest of the Cebidæ (especially in Mycetes, where its distal end is much expanded) and in Hapale; larger still in the Simiidæ other than Simia, and very large indeed in the Gorilla, where it attains its greatest absolute size, though relatively, perhaps, it is yet greater sometimes in Cynocephalus. In Hylobates it is long, but slender.

The pisiforme of the Simiidæ resembles that of Man, enlarged and pulled out at its free end, so as to change the little transverse groove which exists in him into a very long and slightly concave surface.

In all, except Man, Simia, and the Nycticebinæ, it developes an articular surface for the ulnar side of the styloid process of the ulna, and contributes to form, with the

* His words are, "Dieses wäre später gewiss mit dem Kahnbein ganz verwachsen."—Anatomie der Maki, p. 142. And the intermedium is represented as united with the scaphoides in his plate, Tab. 15. fig. A, 7.

† In January 1867 (therefore since this paper was communicated), Dr. WENGEL GRUBER published an account of a divided human scaphoides. See REICHERT and DU BOIS REYMOND's Archiv, 1866, p. 565, Tab. 16.

‡ Its smallness in these genera is noticed by DE BLAINVILLE, *loc. cit.* pp. 10 & 21.

§ BURMEISTER's 'Tarsius,' Tab. 2. fig. 5, *c*.

‖ OWEN, *loc. cit.* pl. 21. fig. 18, *l*, *i*, *u*.

¶ DE BLAINVILLE, *loc. cit.* Lemur, pl. 10.

** BURMEISTER, *loc. cit.* Tab. 2. fig. 5, *d*.

cuneiforme, a cup for the reception of the end of that process. This cup may be well seen in the lower Simiidæ, the Cebidæ, and Lemur.

In the Nycticebinæ this bone is very small (as has been already remarked), and bending distad, is applied to the outer surface of the unciforme, and has its extremity united to the palmar process of that bone.

*Trapezium.*—This is often a very irregularly-shaped bone. It is most nearly quadrate in the lower Simiidæ and Cebidæ; it is most compressed disto-proximally in Man, Troglodytes, and the Lemuroids, but especially in Indris and the Nycticebinæ (Plate XIV. fig. 4).

The radial tuberosity varies as to its development. It is sometimes enormous in the Gorilla * and large in the Chimpanzee, but in Man, Simia, the Simiidæ below the Simiinæ, and in the American Anthropoidea it is slight. In Lemur this process projects ulnad, and in Arctocebus it is large, and approaches within a short distance of the unciforme, to which it is united by a strong ligament. In Indris this process is conspicuous, and rather acute; it is bent distally and rather ulnad. In Hylobates it is sometimes developed to an extent rivalling that of the unciform process, being very long and projecting distad and ulnad. In Perodicticus it is very much developed, and is somewhat like that of Hylobates, only that its width from disto-proximally is less compared to its transverse extent, and that it is directed more transversely and less distally than in Hylobates. It approximates to the unciform process, but is separated from it by an extra ossicle (Plate XIV. fig. 5).

Very commonly a sesamoid exists (from Simia † downwards) at the radial border of the trapezium, and helps to complete the concavity for the reception of the radial end of the scaphoid. This sesamoid cannot be, as Dr. Lucae suspects ‡, a separated tuberosity, because it exists separately (*e. g.* Hylobates, Perodicticus) when that process is at its maximum, and in Loris and Nycticebus it is present, together with two processes (Plate XIV. fig. 4).

The surface for the reception of the first metacarpal is convex from the dorsal to the palmar surface of the bone, and sometimes it is more or less concave in the reverse direction, *i. e.* radiad from the trapezoides. In Man alone is this concavity constantly and strongly developed.

In the highest Apes there appears to be much irregularity as to its development. Thus in the Gorilla it is sometimes very well marked, sometimes § very slightly. In the Chimpanzee it is generally developed slightly, but sometimes absolutely

* Owen, Trans. Zool. Soc. vol. v. pl. 10, figs. 1 & 2.

† Dr. Lucae speaks of it in the Orang, and represents it (*loc. cit.* pp. 304 & 305, and pl. 3. fig. 8). It is figured by Professor Vrolik in Todd's Cyclopædia of Anat. and Phys. vol. iv. p. 204, fig. 124, *i*. Mr. W. H. Flower also informs me that he observed its existence in the wrist of an adult male Orang at the Museum of Leyden.

‡ *Loc. cit.* p. 305.

§ *E.g.* No. 5179 A in the Osteological Collection of the College of Surgeons.

disappears * (Plate XIV. fig. 3). In Simia it is sometimes as marked as in the Gorilla; sometimes, however, it can hardly be said to exist at all.

In Hylobates there is no trace of any concavity, but a strongly convex and rounded tubercle receives the concave articular surface of the base of the metacarpal of the pollex †.

In the lower Simiidæ the concavity is again sometimes present, though it is always very slight, and occasionally in Semnopithecus there is no indication of it, though in Colobus I have found it decidedly present, but slight; in Dr. Lucae's specimen, however, it was absent ‡.

In Ateles the trapezium is large in spite of the rudimentary condition of the pollex; but there is no saddle, *i. e.* no concavity for the metacarpal §.

In the other Cebidæ the depression sometimes exists. I have observed a decided, though small saddle in Brachyurus and some specimens of Cebus; in others I could detect no trace of such a structure, nor have I found such in the other genera of Cebidæ I have been able to examine.

In the Lemuroidea the concavity is very slight, though it may generally be detected.

The trapezium is always so placed that the axis of the convexity of the saddle forms a marked angle with a line drawn across the articulations of the four outer metacarpal bones with the proximal row of carpals ‖ (Plate XIV. figs. 6 & 7).

In Man and the Gorilla this angle is very open, but in the Chimpanzee and sometimes in lower Simiidæ it is smaller, the trapezium being, as it were, somewhat more pressed inwards, at its radial end, towards the middle of the palm. It is never, however, so inclined inwards as is the axis of the cylinder of the entocuneiforme of the pes, though the resemblance is considerable in the Chimpanzee, which thus differs from the inferior forms, as well as from Man.

In the American Anthropoidea the trapezium is well set out; and this, no doubt, contributes to produce that very feeble opposition and palmad flexion of the pollex which have been noticed to exist in them.

*Trapezoides.*—This bone is generally pyramidal in shape, the apex being towards the palm. It is more pointed at its palmar end in the lower Simiidæ than in Man, and still more so in Lemur.

It is very small in Tarsius ¶, but of ordinary relative size in Arctocebus and Perodicticus, in spite of the rudimentary condition of the index.

* *E. g.* the mounted manus, No. 744, in the same collection. The absence of a saddle in this species is noticed by Professor Huxley: see 'Medical Times,' 1864, vol. i. p. 428.

† Noticed by Dr. Lucae, *loc. cit.* p. 305, and Tab. 4. fig. 8.

‡ *Loc. cit.* p. 311.

§ Dr. Lucae speaks of the convex articular surface which, in Ateles, is received into the concavity of the metacarpal (*loc. cit.* p. 311).

‖ The "digital angulation" of Professor Huxley. See 'Medical Times,' vol. i. p. 177.

¶ Burmeister, *loc. cit.* Tab. 2. fig. 5, *g*.

In Man and the Simiidæ the proximal angle of its radial side is produced,—not so in other forms.

In all but Man, Troglodytes and the Indrisinæ, it articulates with the intermedium.

*Magnum.*—This bone is not generally the largest of the carpals*. As seen in a carpus with the bones articulated together, it appears much less than the unciforme in Hylobates, as also generally in the lower Anthropoidea, and always in the Lemuroidea.

It has throughout much the same shape as in Man; but its distal articular surface is often more concave, as are also its lateral margins.

It projects distally beyond the trapezoides in Man, the Simiidæ (though *very* slightly so in Hylobates), the Cebidæ (except Ateles and Lagothrix), Hapale, Lemur, the Nycticebinæ, Tarsius, and Cheiromys.

It may or may not articulate with the fourth metacarpal. Thus in the lower Simiidæ there is a distinct articular surface for the latter, but not in Lemur.

*Unciforme.*—In all the Primates this bone has a shape very similar to that which it presents in Man, but, as has been already remarked, it often predominates in size over other carpals.

The palmar process is enormously long in Hylobates, and I have found it† very large in the Nycticebinæ, large also in Indris, the Simiinæ (especially Simia), and Man. In him and in the Nycticebinæ this process projects much palmad, in other forms less palmad and more distad. In some, as in Macacus and Lemur (at least sometimes), it is so small as to form merely, as it were, the palmar lip of the distal articular concavity.

As has been said, an extra bone exists in Perodicticus, namely, an ossification of the anterior ligament of the carpus between the processes of the trapezium and unciforme. This small bone, which has been described and figured by Van Campen‡, is subtriangular in shape, and joining, as it does, the unciform process on one side, and the tuberosity of the trapezium on the other, it causes the flexor tendons to pass through a complete bony ring (Plate XIV. fig. 5).

### Metacarpus.

The greatest absolute length of this segment is exhibited by the third metacarpal of Simia.

The length of this part of the skeleton, as estimated by a comparison of the third metacarpal with that of the whole manus, is greatest in Simia and Troglodytes, where the length of the former is almost two-fifths of that of the latter. In the rest of the Order it varies between this dimension and that of Brachyurus and Loris, in which genera it is very little more than a quarter, except in Arctocebus, where it is even somewhat less.

* Speaking of Cercopithecus sabæus, De Blainville remarks that the unciforme is larger than the magnum (*l. c.* p. 16).

† Yet De Blainville found it little marked (*loc. cit.* Lemur, p. 15).

‡ In the periodical before referred to.

The metacarpus always much exceeds the carpus in length. It does so most, perhaps, in the Chimpanzee and Indris, and probably least in the Nycticebinæ.

The proportion of this segment to the spine is greatest in Hylobates, where it is sometimes nearly one-fifth, and then in Tarsius, the Chimpanzee, Cheiromys, and Ateles. It appears to be least in the Nycticebinæ.

*The four outer Metacarpals.*—These metacarpals are always more or less enlarged at each end. The proximal ends are never much wider transversely than the distal ones, and very rarely so at all. It is the case, however, in the Nycticebinæ, and at least sometimes in Lemur, and slightly in Cynocephalus. On the other hand, the distal ends often greatly exceed the proximal ends in breadth, as is the case in the Simiinæ and Ateles.

The proximal articular surfaces are more or less concave in Man and Troglodytes. In Simia and Hylobates those of the fourth and fifth metacarpals become decidedly convex, and in the lower Anthropoidea that of the third becomes convex also, and all four become so in the Lemuroidea.

The proximal surfaces of these metacarpals are in most Primates nearly at right angles with the long axes of their shafts; but in Hylobates a line joining these surfaces inclines distally as it proceeds ulnad from the index, and this inclination exists slightly in Troglodytes, sometimes in Lemur, and a trace of it is to be seen in Man.

The antero-posterior diameters of the heads (*i. e.* from dorsum to palm) are never greatly in excess of the transverse ones, except in Indris, Lagothrix, and Ateles, though slightly so in Simia and Hylobates. Generally the two diameters are about equal.

The shafts always broaden downwards (*i. e.* distad) decidedly. They are always flatter on the dorsum than are the metatarsals, and never so laterally compressed.

Antero-posterior planes extending vertically through the metacarpals from the dorsal, to the most prominent parts of the palmar surfaces, have their palmar edges in the fourth and fifth metacarpals, inclined towards the middle of the palm. Often the same can be said of such a plane traversing metacarpal of the index.

The shaft of the fifth metacarpal is never much flattened on its palmar surface.

The palmar surfaces of the metacarpals are more concave disto-proximally than those of the metatarsals in the same individual, yet scarcely so, perhaps, in Simia and Hylobates.

The metacarpals always diverge more or less distally; least so, perhaps, in Hylobates, Simia, and the Nycticebinæ, though (except in the last-mentioned subfamily) always more so than do their homotypes of the pes.

The heads are never bent ulnad at their extremities, but continue pretty much in the same direction as the shafts.

The distal articular surfaces are in all formed nearly as in Man, but are shortest dorsally in him.

In Man and the Simiinæ they are much larger than the homotypal parts of the pes. In the lower forms they are but slightly so.

*First Metacarpal.*—This metacarpal attains its greatest absolute length in the Orang. As compared with the spine, it is longest in Tarsius, where it is more than one-tenth of

the length of the latter—which it is also, sometimes, in Hylobates. It has always more than half that proportion (*i. e.* than one-twentieth), except in Chrysothrix, Loris, Nyctipithecus, Arctocebus, Lemur, Perodicticus, and Colobus. In the last it is to the spine only as about 3·5 to 100.

There is never a very large process on the palmar side of its proximal end, as there is so often in the corresponding part of the hallux.

The proximal articular surface is sometimes concavo-convex, as in Man, the Gorilla, sometimes in the lower Simiidæ and Cebidæ, and in the Nycticebinæ. But there is individual variation in this respect; and often this double curvature is scarcely to be detected, *e. g.* in the Chimpanzee, Simia, Lemur, and Ateles. In Hylobates there is a concave surface only, which unites with the articular ball of the trapezium.

The shaft of the bone generally broadens distally, but sometimes (*e. g.* in some of the lower Simiidæ) it tapers, yet never so much so as does the shaft of the hallux of the same species.

The distal end is rarely broader than the proximal one, generally it is of almost the same width, but sometimes narrower. It is probably never so much broader in the forms below Man as it is in him.

The angle formed by the transverse axis of the head with another similarly traversing the heads of the other metacarpals, always more or less nearly approaches a right angle, except in the Cebidæ and Hapale, where it is more obtuse, yet not so much so as is the homotypal angle of the pes of Man.

This metacarpal is never the longest one of the manus in any species, and it is the shortest one in all except the Nycticebinæ (where it exceeds in length the second metacarpal, and sometimes the fifth also) and Tarsius, where it slightly exceeds the fifth, but not the second one *.

*Second Metacarpal.*—This is sometimes the absolutely shortest metacarpal found in the whole order; namely, in Arctocebus. It is the longest of all in the same manus in Man, sometimes in Troglodytes and Simia, in Hylobates, and in the Cynopithecinæ.

It is the shortest one in the Nycticebinæ. It is shorter than the three metacarpals external to it in Indris, Cheiromys, and, of course, in the Nycticebinæ.

Its proportion to the metacarpal of the pollex is greatest in the Chimpanzee, where it is more than twice and a half its length, then in the Semnopithecinæ and the Gorilla. In all the rest it is longer than the metacarpal of the pollex, except in the Nycticebinæ, where, in Arctocebus, it is scarcely more than three-fourths its length.

The metacarpal of the index projects furthest (distad) of any in the same manus, in Man, sometimes in Troglodytes, in Hylobates, and sometimes in the lower Simiidæ.

It projects distad less than do the three metacarpals external to it in some, *e. g.* in Ateles, Pithecia, and sometimes Hapale, the Lemuridæ, and Cheiromys.

In Indris the shaft is much curved, with the concavity radiad.

* See Burmeister and Blanchard, *loc. cit.*

The proximal end always articulates more or less with the trapezium and magnum.

The proximal articular surface is sometimes strongly concave transversely, as in most Anthropoidea; sometimes it has a very strong projecting ridge, as in Loris, or the surface may be even, as in Lemur. In Indris it sends a remarkable lateral process to the radial side of the magnum.

*Third Metacarpal.*—This metacarpal is the one which attains the greatest absolute length in the whole order, which it does in Simia. Its proportion in length to the whole manus is greatest in Simia, Troglodytes, and Cynocephalus, where it is nearly two-fifths the length of that segment. It is always more than one quarter, except in Arctocebus, where it is a little less. It is the longest metacarpal of all in the same manus in many; namely, sometimes in Troglodytes and Simia, in almost if not all Cebidæ (except Pithecia), in Hapale, and in all the Lemuroidea, except Indris (Indrisinæ ?), above all in Cheiromys *. It is never the shortest metacarpal of any manus. It projects most distad of any in the same manus in Simia, sometimes in the lower Simiidæ, in Lagothrix, Cebus, Nyctipithecus, Chrysothrix, Hapale, the Nycticebinæ, Tarsius, and Cheiromys.

There is an angular projection at the radial side of the proximal end of the dorsum, in Man, and to a much less extent in the Simiinæ. I have not observed this in lower forms.

The proximal articular surface is concavo-convex in Man and the Simiinæ. In lower forms it is generally more or less strongly and exclusively convex; rarely it is decidedly concave, as I have found it in Ateles.

*Fourth Metacarpal.*—This is the longest one of the manus in Pithecia and Indris, but it is never the shortest one.

It projects slightly the most distad of all in Ateles, Pithecia, Indris, Lemur, and Perodicticus. The proximal articular surface is generally strongly convex antero-posteriorly, as in the lower Simiidæ and Cebidæ. It is less convex in Lemur, very slightly so in Indris. In Man and Troglodytes it is concavo-convex, and more or less, though sometimes *very* slightly, so in the other Simiinæ and in Ateles.

*Fifth Metacarpal.*—This metacarpal is never the longest of all in the same manus, except sometimes (as rarely in the lower Simiidæ) when its backwardly projecting process is included in the measurement.

It is the shortest of all except the pollex in Man, the Simiidæ, most Cebidæ, Hapale, and sometimes in Lemur. It is shortest of all, including the pollex, in Tarsius.

It never projects more distad than the other metacarpals, but it does so least, excluding the pollex, in Man, the Simiidæ, Lagothrix, Cebus, and lower Cebidæ. It does so least of all (pollex included) in Tarsius.

The proximal articular surface is always more or less strongly convex.

A process sometimes extends backwards from its proximal end, outside the carpus, like that of the homotypal bone of the pes. This, however, is large only in the lower

* In Cheiromys alone is it almost double the length of the second metacarpal, and nearly one-third longer than the fourth, being at the same time very much more slender than any of the other metacarpals.

and lowest Simiidæ; but even there it is never so large as the corresponding process of the pes in the same individual.

### Phalanges.

In almost all species the metacarpals all support phalanges, the only exceptions being in Colobus and Ateles *.

In all other genera the pollex has two phalanges, and the three outermost digits have always three each. The index has also always three phalanges, except in Perodicticus and Arctocebus, where there are only two.

The *proximal phalanx of the pollex* is absolutely longest in Man, the Chimpanzee, the Gorilla, and the Orang.

It is always shorter than the first metacarpal, except in Nyctipithecus, Perodicticus, and Cheiromys; in the last, indeed, it greatly exceeds it. There is very little difference, however, in Chrysothrix, Hapale, Lemur, Galago, Loris, and Arctocebus. On the other hand, in Colobus it is only one-third of the length of the metacarpal, and in Semnopithecus, Hylobates, and Simia there is also a great difference, though it is always more than half the length of the metacarpal.

The *second phalanx* is always shorter than the first; it is less even than half its length in Semnopithecus, sometimes in Macacus, in Nyctipithecus, Chrysothrix, Indris, Lemur, Galago, and Loris.

It is always flattened at its distal part from dorsum to palm, except in Hapale, in which genus it is laterally compressed, curved and pointed at the end.

The *phalanges of the other digits* are of very similar form throughout the order, and, as in Man, are convex transversely on the dorsum and flattened on the palmar side of each.

The ultimate phalanges are always flattened from dorsum to palm, except in Hapale, where they are laterally compressed, curved and pointed to support the similar-shaped claws of that genus. In Cheiromys they are much attenuated.

The *proximal phalanx of the third digit* is as long as, or longer than any other phalanx of the four outer digits in the Anthropoidea and Tarsius. That of the fourth digit is the longest in the Nycticebinæ and Cheiromys, but the predominance in length of the second phalanx of the fourth digit over the second phalanx of the third digit, which occurs in Cheiromys, is quite peculiar to that genus †.

The relative length of the phalanges may be estimated by selecting those of the third digit for comparison.

Thus the proximal phalanx is always much more than half the length of the third

* Dr. Lucae has found one phalanx in the pollex of Colobus, but none in that of Ateles. He concludes, however, from the form of the distal end of the metacarpal in the latter genus, that a phalanx has existed and been lost. Professor Huxley says that there is usually a small and nodular phalanx in Ateles (Medical Times, 1864, vol. i. p. 93).

† See Owen, Trans. Zool. Soc. vol. v. pl. 21. fig. 17.

metacarpal. It almost equals it in length in Lagothrix, Cebus, Pithecia, Chrysothrix, Lemur, Arctocebus, and sometimes in Hapale *. It slightly exceeds it in Brachyurus, Nyctipithecus, Galago, Loris, and Perodicticus, and very largely so in Tarsius and Cheiromys.

Its proportion to the manus is greatest in the two last-mentioned genera (about as 34 to 100), in all the rest it is more than a quarter, except in Ateles, Cynocephalus, Cercopithecus, Troglodytes, Man, and last of all, Arctocebus, where it is as 21·5 to 100.

It is at its greatest absolute length in Simia.

The *second phalanx* is at its greatest absolute length in Simia and the Chimpanzee.

It is always more than half the length of the proximal phalanx, except in Perodicticus and Cheiromys, where it is a little less. It is never, however, nearly so long as the first phalanx.

The *third phalanx* of the third digit is, like the other phalanges, absolutely longest in Simia.

It is always shorter than the second phalanx, and is less than half its length in the Chimpanzee, Orang, Hylobates, Semnopithecinæ, Macacus, Pithecia, Nyctipithecus, Chrysothrix, Indris, Lemur, Loris, and Cheiromys. In Tarsius alone is it less than one-third of the length of the second phalanx.

The phalanges always shorten successively, except that the second phalanx of the fourth digit is longer than the proximal phalanx of the index in Galago and the Nycticebinæ (especially, of course, Perodicticus and Arctocebus), and that it is longer than the proximal phalanx of the fifth digit also in Cheiromys.

### Digits without their Metacarpals.

The *pollex* thus measured is absolutely longest in Man when of average size.

As compared with the whole length of the manus, it is greatest in Arctocebus (about as 35 to 100), then in Hapale, Chrysothrix, and Man. In all it is more than one-fifth the length of the manus, except in the Simiinæ and Semnopithecinæ, Nyctipithecus, and Galago. It is less than a twentieth in Colobus †.

The pollex is never the longest digit of the manus, but, except in Perodicticus and Arctocebus, it is always the shortest one.

The *index* is never the longest digit of the manus, but in the two last-mentioned genera it is the shortest one. It is decidedly the shortest, except the pollex in the Lemuridæ and Cheiromys. It projects furthest distad in none; it does so least of the

* According to Dr. Lucae, the first phalanx is equal to the metacarpal in length in Hapale and Galago (*loc. cit.* p. 320).

† Dr. Lucae, *loc. cit.* p. 318, says that the pollex without its metacarpal does not by a good deal attain in the tailed Apes the proportionate length which it reaches in the Simiinæ. I have found all the lower Simiidæ to exceed all the Simiinæ in this respect except T. niger, and Dr. Lucae, in his Table B., gives for the proportion to the manus at 100, 22 & 24 to Cynocephalus, while to H. Leuciscus (which has the highest proportion of his Simiinæ) he only assigns 21·7 (see pp. 307 & 317).

four outer digits in all the Lemuroidea, except Tarsius. In Arctocebus and Perodicticus the index is extraordinarily short, consists only of two phalanges, and is absolutely and relatively the shortest digit of both manus and pes. It is also as short as, or shorter than the metacarpal which supports it, which in all other forms it exceeds in length.

As compared with the whole manus, the index is longest in Chrysothrix, Tarsius, Lagothrix, Mycetes, Lemur, and sometimes Hylobates, where it exceeds half the length of the former. Excluding the Nycticebinæ, it only falls below two-fifths in some of the lower Simiidæ.

The *third digit* is absolutely longest in Simia; as compared with the length of the whole manus, it is longest in Tarsius and Chrysothrix, where it exceeds three-fifths the length of that segment, and then in Lagothrix and Mycetes.

It exceeds two-fifths in all, but very slightly so in Arctocebus, in which it is shortest.

It is the longest digit of the manus in Man and the Simiidæ, Nyctipithecus, Chrysothrix, and Tarsius. In the Cebinæ the fourth equals it.

It is never the shortest nor ever the one which projects least distad. It projects furthest distad of the digits of the manus in the Anthropoidea, except Pithecia, and in Tarsius.

The *fourth digit* is the longest one of the manus, and also projects furthest distad in Pithecia, and in all the Lemuroidea except Tarsius.

It about equals the third digit in length in many of the Cebidæ and in Hapale.

It is never the shortest, even excluding the pollex. The fourth digit projects further distad than does the index, in Troglodytes, Simia, the Cebinæ, and Lemuroidea. The projection of the two is about equal in Hylobates and the lower Simiidæ.

The fourth digit is almost always somewhat longer than the second.

The *fifth digit* is never the longest or most distally projecting one of the manus.

It is the shortest one, except the pollex, in Man and the Simiidæ, and it is about equal to the index in the Cebidæ and Tarsius. It projects least of the four outer digits in the Anthropoidea and in Tarsius.

The proportion borne by the longest digit, without its metacarpal, to the longest metacarpal, is greatest in Cheiromys and Tarsius, where the first is more than twice and a half the length of the second. Then in the Nycticebinæ, where it is considerably more than twice as long. In the rest it varies between this proportion and once and a quarter, except sometimes in Cynocephalus, where the longest digit may scarcely exceed the longest metacarpal by more than one-fifth of the length of the latter.

### Digits with their Metacarpals.

Thus estimated the *pollex* is absolutely longest in Man, when of average size, and then in the Orang and Gorilla.

Its proportion to the spine is greatest in Tarsius, namely more than one-quarter; then in Cheiromys, and sometimes in Hylobates, where it equals one-fifth. In the rest

it varies between this and a tenth, except in Cercopithecus, Semnopithecus, Ateles, and Colobus, being in the last genus less than one-twentieth.

Its length, compared with that of the entire manus, is greatest in Arctocebus, and then in Chrysothrix, in both of which it considerably exceeds one-half. It does so slightly in Man, Hapale, and also in Loris. In the rest it varies between this proportion and two-fifths, except in the Simiinæ *, Cheiromys, and Semnopithecus, and finally, in Colobus and Ateles, in which two last genera it is less than one-fifth.

The pollex, when extended beside the index, exceeds it greatly in Arctocebus and Perodicticus †. It reaches nearly to the distal end of the second phalanx in Loris, and to the distal end (or very near it) of the proximal phalanx in the Cebidæ, and mostly in Hapale. It reaches considerably beyond the middle of that phalanx in Man, Tarsius, and Cheiromys, beyond the middle in Indris, and to its middle in Lemur and Galago. Occasionally in the lowest Simiidæ it nearly reaches the middle of the proximal phalanx, but generally does not extend so far. It goes but very little beyond the proximal end of that phalanx in the Gorilla and Hylobates, and still less beyond it in Semnopithecus. In the Chimpanzee it barely attains the distal end of the metacarpal of the index, while in Simia it decidedly falls short of its end. In Ateles, *i. e.* in the specimens examined (without a phalanx), it reaches more than halfway down the metacarpal of the index, but in Colobus (with a phalanx) it does not attain its middle.

The pollex, when compared with the longest digit of the manus, is at its maximum of relative length in Cheiromys (in spite of the great length of its longest digit), being almost quite three-fourths the length of that digit. Arctocebus and Hapale, Loris, and Man follow, its proportion in all four being more than three-fifths. The rest vary between this and Hylobates (where it is sometimes only as 33·4 to 100), except Colobus and Ateles, in which the proportion is only one-fifth or even less.

The *index*, compared with the spine, is longest in Hylobates and Tarsius, where it approaches one-half the length of the latter. Then in Simia, Cheiromys, and Ateles, where it is considerably more than one-third. In the rest it varies between this and a fifth, except in Cercopithecus, the Cebidæ below Mycetes, Lemur, Galago, and the Nycticebinæ, being least in Perodicticus and Arctocebus, where it scarcely exceeds one-twentieth.

The *longest digit*, whether third or fourth, compared with the length of the spine, is greatest in Tarsius, Cheiromys, and sometimes Hylobates, where it exceeds one-half. The rest vary between this and one-fifth, except Cercopithecus, Chrysothrix, Lemur, Loris, and Arctocebus, being least in the last-mentioned, where the proportion is as about 16 to 100.

* Dr. Lucæ found the pollex with its metacarpal to exceed that of Man in Macacus gelada, that of the Orang to be less than that of the Chimpanzee, and that of Hylobates to be greater than that of any other of the Simiinæ (*loc. cit.* pp. 307 & 317).

† In Perodicticus the pollex reaches somewhat beyond the middle of the proximal phalanx of the third digit; in Arctocebus it attains the ultimate phalanx of the third digit.

### The Pelvic Limb.

The entire length of the pelvic limb, measured from the summit of the femur to the distal end of the longest digit, is absolutely greatest in Man, and then in the Gorilla, Orang, and Chimpanzee successively.

The absolute length of the leg, without the pes, is again longest in Man and the Gorilla; but in the Chimpanzee it is longer than in the Orang.

The entire length of the pelvic limb, compared with that of the spine, is greatest by far in Tarsius, the former being nearly twice and a half as long as the latter. Then follow Hylobates and Ateles, in which I find the pelvic limb to be more than once and three fifths the length of the spine. Man and Galago follow, the proportion being in them a little less than one and a half to one. Cheiromys and some of the Cebidæ and Hapale succeed, and in all, the limb is at least one-tenth longer than the spine, except in Lemur, Perodicticus, and Arctocebus. It is less than the spine in Lemur, and still less in Perodicticus, while in Arctocebus it is least of all—about 85·2 to 100.

The proportion borne by the limb, without the pes, to the spine is greatest in Tarsius, being more than once and a half the length of the latter, then Hylobates, once and a quarter, or a little more. In Man the proportion is as about 117 to 100. The limb is a trifle longer than the spine in Ateles, and scarcely shorter than it in Galago, Lagothrix, and Indris; in all the others it is above seven-tenths its length, except in Perodicticus and Arctocebus, in which it is as 65 to 100.

The proportion borne by the entire pelvic limb to the entire pectoral one is far greatest in Galago, where the first is considerably more than once and a half the length of the second. In Indris it is as about 144·5 to 100, in Man and Nyctipithecus as about 135. In the rest it is less, but still the pelvic limb is longer than the pectoral one, except in Ateles* and the Simiinæ, in which it is shorter†, being sometimes only as 75·2 to 100.

The length of the pelvic limb minus the pes, compared with that of the pectoral one without the manus, is greatest in Galago and Indris, the first being in each more than once and a half the length of the second. Man follows (145 to 100), then Callithrix and Nyctipithecus (about 137 to 100), and all the other forms have the pelvic limb the longer, except in the Chimpanzee, Ateles, the Gorilla, Hylobates, and Simia, the proportion in the last being only as about 73·4 to 100.

### Os innominatum.

Throughout the order this bone consists of parts and processes homologous with those existing in Man; yet in him it assumes a form and proportions strikingly different from those existing in any other Primate.

* Professor Huxley notices the greater length of the pectoral limb in Ateles. See 'Medical Times,' 1864, vol. ii. p. 93.

† According to Dr. 'Lucae (*loc. cit.* p. 280) the proportion decreases in the following order:—Man, the Chimpanzee, Hylobates, the Gorilla, the Orang.

In absolute size this bone attains its maximum in the Gorilla*; it is next longest in the Chimpanzee, then in the Orang, afterwards in Man, and then in the largest Cynocephali; and there is even less difference in its length between the last and Man than there is between Man and the Gorilla.

The expansion of the ilium, as measured by the length of its crest, is again greatest in the Gorilla, and then in Man, who in this respect exceeds both the Chimpanzee and Orang. The Siamang follows these, and close upon it the Mandrill.

The part corresponding with the anterior margin of Man is longer than in him in all the largest Simiidæ, and sometimes in Ateles.

The pubic symphysis is actually longest in some Cynocephali, then in Troglodytes, Simia, and Man.

The conjugate diameter of the pelvis† is far greatest in the Gorilla, then in the Chimpanzee, Man, and the Orang.

Its transverse diameter is also greatest in the Gorilla.

The ilio-pubic angle, or that angle formed by the iliac part of the ilio-pectineal line with the anterior (in Man superior) margin of the so-called "horizontal" ramus of the pubis, varies from 88°, or even somewhat less‡, in Loris to 180 in Man.

That ilio-ischial angle formed by the superior part of the ilio-pectineal line with the superior (in Man posterior) margin of the ischium, is about 110° or 113° in Man; but in Apes and Lemuroids it varies from 140° to more than 180°, as also in some Cebidæ.

The angle formed by the same part of the ischium with the superior (in Man posterior) margin of the ilium is in Man about 140°, in Hylobates about 147°; in the rest it varies between this and 180°, which it attains in Troglodytes and Loris, but in Cynocephalus and some Cebidæ it is yet greater.

The extreme length of the os innominatum, when compared with that of the spine, is greatest in the Gorilla, where the former is decidedly more than one-half of the latter. It is also rather more than half in the Chimpanzee. In the Orang the proportions are as 45·8 to 100; in Hylobates, Ateles, Tarsius, Cynocephalus, and Lagothrix from 43·5 to 36·7; in Man and Cebus 32; the rest vary between this and 25, except Nyctipithecus, Callithrix, and Arctocebus, in which this bone is a trifle less than a quarter of the length of the spine.

The length of the inferior (in Man anterior) margin of the ilium between the spinous processes, compared with the length of the spine, is greatest in Tarsius and the Chimpanzee, where it is over one-fifth; the rest vary between this and Man, in whom it is considerably less than one-tenth.

Comparing the same margin with the total length of the os innominatum, the pro-

* Professor Owen remarks that it "would fit a human giant ten feet in height." Trans. Zool. Soc. vol. v. p. 12.

† Measured from the caudal end of the ventral surface of the first sacral vertebra to the symphysis pubis.

‡ Mr. John Wood, in his admirable article on the Pelvis in Todd's 'Cyclopædia of Anatomy and Physiology,' vol. v., gives 75° as the ilio-pubic angle of Loris. He calls attention to the great peculiarity in Man of the ilio-pubic angle being represented by a straight line.

portion is greatest in Tarsius, and the Lemuridæ other than the Indrisinæ. It is least in the Gorilla and Man.

The breadth of the ilium (as measured by a straight line drawn between its anterior —in Man superior—spinous processes), compared with the length of the spinal column, is greatest in the Gorilla, where it is more than three-tenths of the length of the latter. In the Orang and Man it is a little more than a fifth, and a trifle less in the Chimpanzee. In Hylobates it is about three-twentieths, and in the rest of the order it is between that and one-twentieth, except in Lemur and some of the Nycticebinæ, in which it is rather less.

The proportion borne by a line following the curve of the crest of the ilium to a straight one joining its anterior (in Man superior) spinous processes is greatest in Man, Hylobates, the higher Cebidæ, and the Chimpanzee and Gorilla. It is least in Galago, the Nycticebinæ, and Tarsius.

The length of a line extending from the ilio-pectineal eminence to the nearest point of the tuberosity of the ischium, as compared with that of the spinal column, does not vary much. It is greatest in the Gorilla, viz. as 21·8 to 100; then in the Chimpanzee, 17·7; in Man and Simia, 16; the rest vary between this and the proportion one-tenth, except Chrysothrix and the Nycticebinæ, where it is less.

The antero-posterior (in Man vertical) diameter of the acetabulum, as compared with the length of the spinal column, is greatest in the Orang, where it is as 8 to 100, then in Man and the Gorilla. It is smallest (under 3·5 to 100) in Arctocebus, Chrysothrix, and Hapale.

When the same dimension of the acetabulum is compared with the length of the os innominatum, the proportion appears to be greatest in Man, then in the Orang, and least in Hapale and some of the lower Simiidæ.

The length of the symphysis pubis, compared with that of the spine, appears subject to great variation in the same species; but it is greatest in the Simiidæ*, where it sometimes exceeds one-tenth; in the rest it ranges between this and one-twentieth, except in Man, Galago, Nycticebus, and Lemur, where it is a little less, and least of all in Arctocebus, Loris, and Perodicticus, where it is sometimes only as 1·7 to 100.

The length of the os innominatum, as compared with the greatest transverse diameter of the pelvis, is greatest in Loris, viz. as 442·4 to 100, then in the Pitheciinæ, 340·2 to 100; the rest vary between this and 211, except in Man, in whom, at least sometimes, it is a little under 2 to 1.

The breadth of the brim of the true pelvis, as compared with its conjugate diameter, is greatest in Man, where alone the former is in excess†, viz. as 105·5 to 100. Some-

* Professor Huxley remarks of Hylobates, "The subpubic arch, distinct in all the other great Apes, has almost disappeared, the symphysis pubis being inordinately long" (Med. Times, 1864, vol. i. p. 618). And of the lowest Simiidæ he says, "The symphysis is exceedingly long, the subpubic arch being very much reduced" (*loc. cit.* p. 672).

† Not always so, however, Professor Huxley has observed. See 'Medical Times,' 1864, vol. i. p. 344. See also Mr. John Wood's article on the Pelvis in Todd's 'Cyclopædia,' vol. v. pp. 150 & 151.

times in Cynocephalus the proportion is as 97 to 100, and it varies between this and 55 to 100, except in Arctocebus and Nycticebus, where the breadth is but very little above half the length, and in Loris, where it is even less than half.

The length of the os innominatum, compared with that of the scapula (measured from the anterior end, or summit of the glenoid surface to the posterior vertebral angle), is greatest in the Chimpanzee and some other Simiidæ, and is least in Man, the Orang and lower Cebidæ, Galago, Tarsius, Nycticebus, and Hapale.

The crest of the ilium undergoes a great change in form and proportion as we pass from one end to the other of the ordinal series.

Its length (measured along its curves) never equals half that of the os innominatum, except in Man and the Simiinæ; and, indeed, in Hylobates this is only sometimes the case. In Man alone does it nearly equal (and sometimes exceed) the length of the entire bone. It is at its minimum in Galago and the Nycticebinæ.

The breadth of the ilium, as measured by a straight line joining the points corresponding with the superior spinous processes of Man, exceeds half the total length of the os innominatum only in Man, the Gorilla, and the Orang, being respectively, to the latter dimension at 100, about as 69·8, 50·8, and 50·7 respectively.

The crest of the ilium is in some few forms much arched in a direction corresponding with that which is upwards in Man. This is especially the case in Hylobates, Mycetes, Ateles, and Lagothrix, and in a less degree generally in Man, Troglodytes, and Simia. In the other Anthropoidea it is but little so arched, the curve of the crest of the ilium being to a straight line joining its extremities as 126·4 to 100 on an average. In Indris and Lemur it is sometimes very slightly more arched; but in Galago, the Nycticebinæ, and Tarsius it is almost straight.

The lateral (in Man horizontal) curvature of the crest of the ilium presents a strongly-marked sigmoid flexure in none but Man, though in Troglodytes, and sometimes in Simia, there is a slight trace of such sigmoid curvature.

In the rest of the order the margin is all but, or quite straight, as in the Nycticebinæ; or there is but one lateral curve concave outwards, and this curvature is carried to its maximum in Ateles and Lagothrix (Plate XIII. fig. 1).

The crest of the ilium is generally thin, but more or less thickened at the points corresponding with the superior spinous processes of Man; in him alone, however, is it thickened at a point some distance behind the anterior superior spinous process, a thickened tract extending thence downwards to the acetabulum. In the lower Simiidæ (which have short iliac crests compared to those of Simiinæ) these crests are generally thicker relatively than in the latter. In the Cebidæ they are somewhat thinner, but the thickening at the ends is much marked in most Lemuroidea, though in the Nycticebinæ the very short crests are almost uniformly, as well as considerably, thickened.

The ventral (or anterior) margin of the ilium in Man is very short, but it is generally elongated and straight, or very slightly concave. It is, however, sometimes strongly concave in Troglodytes and Simia, still more so in Indris (Plate XIII. fig. 2), and

almost always much so in Lemur. It developes a slight prominence sometimes in Hylobates and the lowest Simiidæ, which prominence becomes much marked in Mycetes, Callithrix, Chrysothrix, and Hapale.

In certain Lemuroidea a peculiar condition obtains, in that the part answering to the ventral (or anterior) margin of the Anthropoidea runs obliquely backwards and upwards (in Man it would be downwards and backwards) over what is the external surface of the ilium so as to form an oblique ridge outside that bone, and reaching to the anterior (in Man superior) margin of the acetabulum, or even to a point above (behind) that cavity. This is the case in Galago, the Nycticebinæ (Plate XIII. fig. 3), and Tarsius, and an approach to the same condition is exhibited by Cheiromys.

There is a distinct superior anterior spinous process of the ilium in Man; but in no other primate is it so distinctly developed, but is represented only by the thickened end* of the crest of the ilium. In Indris and Lemur this is much produced in a direction which in Man would be forwards (Plate XIII. fig. 2).

The process of the ilium answering to the inferior anterior spinous one of Man attains its maximum of development in Indris, where it has a quite peculiar form (Plate XIII. fig. 2, *sp.*). Though sometimes indistinguishable, it is generally more or less marked throughout the order, especially in the Lemuroidea, except Tarsius and the Nycticebinæ, where it is minute or absent. Of all the Anthropoidea it is most marked in Man; in the Simiidæ it is little prominent, except in Cynocephalus, and in Ateles alone of the Cebidæ. In the latter family it often, as is also the case in Hapale, appears to be fused with the prominence which in them, as has been said, projects from the ventral margin of the ilium.

There is a distinct superior posterior spinous process in Man, but I have not found such in any other Primate, it being represented, in all the rest of the order, only by the more or less thickened upper (posterior) end of the crest of the ilium.

The part answering to the inferior posterior spinous process of Man is very rarely so sharp and distinct as in him†, and never so approximated as in him to the spinous process in front of (in Man above) it, except in Loris, the proportion borne by the distance between the processes to the length of the os innominatum taken at 100 being in Loris only 15·7 and in Man 17·7, while in the Gorilla it is 32·6, in the Chimpanzee 23·9, in the Orang 27·9; while in all the rest it is above 25, except in Tarsius, in which it is 23·9, and Nycticebus, where it is only 21·2.

The external surface of the ilium is generally more or less concave, and concave only. In Man, Troglodytes, and Simia, however, it is more or less convex, but in Man only is there that extensive anterior convexity and posterior concavity which determines the beautiful sigmoid curvature of the crest. The gluteal lines I have found distinctly marked only in Man.

* This is very thick sometimes in Simia, as in the specimen No. 3 c in the British Museum.

† I have found this process sometimes sharply marked in Cynocephalus, Ateles, Cebus, the Nyctipithecinæ Hapale, Indris, Lemur, Galago, and Tarsius.

This surface is most concave in Hylobates, the lowest Simiidæ, the Cebinæ, and Mycetes, less so in the other forms, and least of all in the Nycticebinæ, Tarsius, and Cheiromys, where there is either no external concavity at all, or only a very slight one near the crest of the ilium.

In the Nycticebinæ, Galago, and Tarsius, the part homologous with the outer surface of the ilium of the Anthropoidea becomes *exceedingly* small, as it forms only that part of the actual outer surface which is on the dorsal side of the oblique ridge, extending to the acetabulum, before mentioned (Plate XIII. fig. 3, *m*).

The internal surface of the ilium is generally narrow and flat, or only slightly concave. In none besides Man is it very wide, very concave, and directed entirely inwards. It is strongly concave in the Gorilla, however, and there is a very slight concavity in the Simiinæ generally, and sometimes in Cynocephalus, the Cebinæ, Mycetes, Hapale, and Indris. In the Nycticebinæ and Tarsius this surface is actually convex, by reason of the prominent ilio-pectineal line; but in Lemur, Galago, and Cheiromys the projecting spinous process near the acetabulum produces a concavity between that process and the ilio-pectineal line.

In the lower Cebidæ and Hapale the iliac fossa is extremely narrow.

The wall of the true pelvis, formed by the ischium, is generally elongated and narrow, but in Man and the Nycticebinæ (especially Loris) it is broad and short.

The auricular surface generally extends nearly to the crest of the ilium, and is especially high in the Cebidæ, the Nycticebinæ, and Tarsius. It is more distant from it in Man, Troglodytes, and Simia, and very much so in Lemur and Indris.

The ilio-pectineal line generally abuts against the first sacral vertebra, but sometimes against the second (as in Hylobates, Pithecia, and Chrysothrix), or between the two (as in some Cebidæ). In certain forms a ridge continues on as far as the crest of the ilium, ending near the part answering to the anterior superior spinous process of Man. This ridge is very marked in Indris and Lemur; and in Galago, the Nycticebinæ, and Tarsius it forms, as has been said, the actual ventral margin of the ilium (Plate XIII. fig. 3).

In Cebus the end of this ridge projects as a distinct and prominent process from the ventral end of the crest of the ilium, and a similar development is more or less marked in the lower Cebidæ and in Hapale.

The ileo-pectineal eminence is moderately marked in Man, as a rounded prominence. In Troglodytes it is sometimes absent, sometimes present as a distinct process, though more distant from the acetabulum, and nearer the symphysis, than in Man. In the Orang it is very large *, but in Hylobates there is only a slight ileo-pectineal prominence like that of Man. In the other Simiidæ it is not marked, except rarely in Cynocephalus.

In Mycetes there is sometimes a remarkable process, but it is not constant†, and in

* This is perhaps rather the *spine of the pubis* than an ilio-pectineal eminence.

† Present in the specimen in the Museum of the Royal College of Surgeons. That at the British Museum presents merely a trace of it.

the lower Cebidæ (*e. g.* Pithecia, Nyctipithecus) this eminence resembles that of Man. In Indris there is a certain thickening near the inner end of the acetabulum *; but in Galago a distinct small process is developed near to, but separated from, the spine of the pubis. In the Nycticebinæ a sharp-edged, ridge-like prominence appears to answer to both the last-mentioned processes united. The ileo-pectineal eminence is absent, or *very* slightly marked in Lemur, but it is rather marked in Cheiromys †.

The spine of the pubis is a less constant process. It is generally well developed in Man, but is indistinguishable in Troglodytes and Simia‡. On the other hand, it is immense in the Siamang, and large in almost all Hylobates. In the lower Simiidæ it is generally absent, and but very rarely much developed §, and appears to be absent in the lower forms of the order, except that in Indris there is sometimes a minute process (quite close to the symphysis), and in Galago a distinct projection like a second ilio-pectineal eminence.

The so-called horizontal ramus of the pubis has its anterior (in Man superior) surface very narrow, forming a sharp ridge, except in the Simiinæ and Man. Of all Apes it is broadest and most flattened in the Gorilla, but never in that species it is so much so as is generally the case in Homo.

The body of the pubis is relatively longer and more antero-posteriorly extended in the Simiidæ and some Cebidæ than in Man and most Lemuroidea; but in Loris it is at its maximum of development in the whole Order.

The subpubic groove, which is generally so marked a feature in the human os innominatum, is very rarely present in any other form. It is distinct, however, in the Gorilla, Orang, and Siamang, and is slightly marked in other species of Hylobates. I have only observed it besides in Mycetes and Lagothrix.

The ascending ramus of the ischium is very slender in many Lemuroidea, especially in Loris; it is broader in Man and the Cebidæ; but in the Simiidæ, especially in the Gorilla, Hylobates, and Cynocephalus, it becomes exceedingly broad, concave externally, and with an everted posterior (inferior of Man) margin.

The tuberosity of the ischium is always a marked and more or less rugose enlargement of the bone; but in the Simiidæ below Simia it is flattened and very much developed, and so much everted that sometimes (in Cynocephalus) its transverse exceeds its antero-posterior diameter. In Hylobates it is continued inwards almost to the symphysis pubis. In Troglodytes and Simia it is much larger than in Man, but not flattened; in the Cebidæ and Lemuroidea it is small and more or less rounded, but in none, except some of the Nycticebinæ, is it prolonged upwards near to the ace-

* De Blainville, 'Ostéographie,' Lemur, p. 11, speaks of a large ileo-pectineal spine in Lemur; but from what he says of Indris (p. 22), he evidently means the process corresponding to the anterior inferior spinous process of Man.

† Owen, Trans. Zool. Soc., vol. v. p. 53, and pl. 21, figs. 19 & 20.

‡ Unless what has been spoken of as an *ilio-pectineal eminence* be really the spine of the pubis,

§ As in the specimen No. 4720 in the Osteological Collection of the Royal College of Surgeons.

tabulum and spine of the ischium as in Man, Loris especially resembling the human structure in this. Of all the other Anthropoidea, Lagothrix perhaps makes the nearest approach to Man and Loris as to the tuberosity of the ischium (Plate XIII. fig. 1).

The spine of the ischium is generally very small yet distinct. In European Man it presents a development much greater than that existing in any other Primate, though sometimes the Orang rivals certain of the inferior races of mankind in this respect. It is never so sharp a process, however, as it always is in Man. In Mycetes and Nyctipithecus the spine of the ischium is hardly distinguishable.

The great sciatic notch is never very deep and concave, except in Man. Of all besides, it is most concave in the Gorilla, Orang, and Cynocephalus. It is rather strongly so also in Indris.

The lesser sciatic notch is generally represented by a margin which is so slightly concave as to be almost or quite straight, or even, as sometimes in Cynocephalus*, slightly convex; though in the lower Simiidæ a concavity is often occasioned by the eversion of the tuberosity. The projection of the spine of the ischium produces in Man a deep notch such as exists in no other Primate.

The acetabulum presents no very marked differences, but it is at its maximum of relative as well as absolute size in Man, Troglodytes, and Simia. It is largest and deepest, especially at the dorsal and towards the ventral side, in Man. In some of the Cebidæ (*e. g.* Ateles, Lagothrix, Mycetes, Pithecia, and Callithrix) it is very shallow, and it is so besides in Indris. In all species it is deepest at the part corresponding with the upper wall of Man.

The cotyloid notch and the excavation continuous with it are constantly present throughout the order, even in Simia† (where there is no *ligamentum teres*), though very small and narrow in that genus. In those skeletons of the Gorilla in which I have seen no trace of a depression for the round ligament on the head of the femur, the inner surface of the acetabulum is as usual ‡, or is but little less marked §. The notch is narrow in Ateles, but in Nycticebus it is sometimes relatively enormous ‖.

The general contour of the outer margin of the ischium, when the pelvis is viewed in front, is almost always more or less strongly concave. It is most so in the Gorilla and lowest Simiidæ, but very little so in Ateles, less in Lagothrix, and still less in the Nycticebinæ; in Loris and Nycticebus, as in Man, being positively convex from the prolongation upwards of the tuberosity.

In all the Anthropoidea, except the Simiinæ ¶, about two-thirds of the acetabulum are visible when the outer surface of the ilium is looked at, but in the Simiinæ it is only

* *E. g.* No. 4719 in the Osteological Collection of the Royal College of Surgeons.

† Its presence in the Orang has been noticed by Mr. John Wood (Todd's Cyclopædia, vol. v. p. 153).

‡ See No. 5179 A. College of Surgeons Museum.

§ See No. 5179 B. College of Surgeons Museum.

‖ See Nycticebus javanicus in British Museum.

¶ This condition in Troglodytes is noticed by Professor Owen (Trans. Zool. Soc. vol. v. p. 14).

seen in profile. In the Lemuroidea more of the acetabulum is visible than even in any of the Anthropoidea, if the whole of the actual outer surface of the ilium be in view.

The number of vertebræ with which the ilium articulates varies from one to four. Two is the usual number; but in Man, Troglodytes, Hylobates, Cynocephalus, Ateles, and Lemur four sometimes so unite.

The obturator foramen offers no very definite characters, but varies greatly from individual to individual. It is of great relative size, however, in Loris and Nycticebus.

The pubic symphysis forms an angle with the spinal column, open towards the head, not only in Man, but also in the Siamang, where it is about 43°. The pubis also appears sometimes to form a similar but smaller angle (about 25° or 30°) in Cynocephalus.

The brim of the pelvis is generally broadest between the acetabula. Sometimes in the Cebidæ it is so below those cavities, but only in Man, and not always in him is the outline of the brim heart-shaped.

The breadth of the true pelvis, as compared with the length of the spinal column, is greatest in Man and the Simiinæ (from 19·6 to 15 as compared with 100). In the rest the proportion is above 8 to 100, except in Pithecia, 7·5, and Loris, in which it is smallest, namely, only as 5·7 to 100.

The inferior outlet of the pelvis in Man is very small as compared with other Primates, from the relatively forward position of the sacrum *. Its height is in greatest excess in proportion to its breadth in the Nycticebinæ, especially in Loris.

### Femur.

Throughout the order the femur has a great general resemblance to that of Man. As regards absolute size, its length is considerably greater in Man than in even the largest of the Apes; but both in the transverse and antero-posterior diameters of the shaft near its middle, as well as in the width between the supracondyloid prominences, the Gorilla exceeds him.

The length of the femur, as compared with that of the spine, is far greatest in Tarsius, namely, as 81·9 to 100. The proportion is next greatest in Hylobates, about 67·8; then in Man, 64·9; Ateles, 61·4; and the Gorilla, 54·0. The other forms are between the last-mentioned proportion and that of 40 to 100, except Lemur and Hapale, which are a little less, and Arctocebus and Perodicticus, in which it is under 34·0 to 100.

The proportion of the length of the femur to that of the humerus is again far greatest in Tarsius, the first being more than double the second. Indris follows, and then Galago, in both of which, especially the former, the length of the femur is considerably more than once and a half that of the humerus. In some Semnopithecinæ and in Lemur it is but little less than as one and a half to one, and in Man about as 138·0 to 100. In all the rest it varies between the last-mentioned proportion and that of Loris (113·3 to 100), except in the Simiinæ, in all of which the femur is shorter than the humerus, and most so in the Orang.

* Wood, *loc. cit.* p. 152.

The proportion borne by the transverse diameter of the femur to its length is far greatest in the Gorilla, where it is more than a tenth, and but little less in the Chimpanzee and Orang. In the rest it is as much as one-twentieth, except sometimes in Hylobates, and in Tarsius.

The shaft of the femur is very often almost completely straight, as in Hylobates generally, and in most Cebidæ and Lemuroidea. It is decidedly curved, with the concavity backwards, in Man, Troglodytes, the lower Simiidæ, and sometimes in Hapale.

It is slightly curved, with the concavity forwards, in the Nycticebinæ, and sometimes in Lemur.

In all the Anthropoidea, except Hylobates, a straight line cannot be drawn from the most prominent point of the great (peroneal) trochanter to that of the condyles without cutting or meeting the front surface of the shaft; but in all the Lemuroidea this can easily be done.

The lateral expansion of the shaft downwards takes place gradually in the Gorilla and generally in Hylobates, in Mycetes (Plate XIII. fig. 4), the Pitheciinæ, Nyctipithecus, Callithrix, sometimes in Lemur and in Loris. It takes place suddenly in Man, the Chimpanzee, and mostly so in the lower Simiidæ; but it does so to a marked degree in Indris, Galago, Arctocebus, Perodicticus, andTarsius.

The shaft is especially angular in Man, the *linea aspera* being so prominent in none others as in him. Nevertheless the shaft is decidedly angular in Cynocephalus, and sometimes in Lemur. The *linea aspera* is also very distinct sometimes in Hylobates* and the lower Simiidæ as a longitudinal median groove bounded by two raised lips; these are very distinct also in Ateles and Mycetes.

The shaft is sometimes much compressed antero-posteriorly in the Gorilla and Orang, also in Mycetes and the Pitheciinæ. In Tarsius it is laterally compressed.

In the other genera it is more or less completely cylindrical.

The ridges, which in Man proceed from the *linea aspera* to the condyles, are rarely much marked in other species. That going to the inner condyle, which is moderate but distinct in Man, Cynocephalus, and Mycetes, is very faint or absent in all others. The branch going to the external condyle, which in Man is very prominent, is so in no other Primate, but almost or quite disappears, except in Troglodytes and Cynocephalus.

The neck of the femur is especially long and well defined in Man and the Simiinæ, but least so of these in the Gorilla. It is particularly short in Hapale and the Lemuroidea, especially in Indris, Galago, the Nycticebinæ, and Tarsius.

The great (peroneal) trochanter is generally pointed at its upper end, but in Man and the Simiinæ, Mycetes and Perodicticus, I have found it truncated. It is smaller in Ateles than in most other Anthropoidea, but it is particularly small in Galago and the Nycticebinæ. Its extremity often projects forwards, especially in Hylobates, Cebus, Hapale, Lemur, Perodicticus, and Tarsius. It sometimes rises higher than the summit of the head

* *E. g.* No. 5026 in the Museum of the Royal College of Surgeons.

of the femur in the Gorilla, and generally does so to a slight extent in the lower Simiidæ, though sometimes it is not quite so high as that summit. It rises considerably above it in Indris and Lemur. It does not reach it by a considerable interval in Man, the Orang, sometimes in Hylobates, and in Ateles.

It generally projects outwards beyond the general external margin of the shaft of the femur, but it does not do so in the Gorilla, Ateles, and Lagothrix, nor in Arctocebus, nor, sometimes, in Hylobates and Loris.

The external margin of the peroneal trochanter more or less blends with a marked gluteal ridge in Man and the lower Simiidæ and most Cebidæ. In Hapalè this ridge is very prominent, as also in Lemur, Galago, and Tarsius, in the three last developing a third trochanter.

In all the Lemuroidea, except sometimes in the Nycticebinæ, there is at least a trace of a third trochanter, and such a process is even rarely present in Hylobates *.

In Troglodytes and Simia there is often a marked concavity at this part.

The trochanteric fossa, which is rather shallow in Man, is particularly so in the Gorilla and Perodicticus. In the other forms it is deep, and in the Anthropoidea, is generally deeper relatively in the other genera of the suborder than in Homo. In the Lemuroidea it is small, especially in Galago, Arctocebus, Perodicticus, and Tarsius.

The lesser (tibial) trochanter is at its minimum of relative size in Man and the Simiinæ, except that sometimes in Hylobates it becomes very prominent †. In Hapale it is larger, relatively, than in any other of the Anthropoidea. In the Lemuroidea it is always very large, even sometimes exceeding in extent the peroneal trochanter. This is the case in the Nycticebinæ ‡, especially in Perodicticus and Arctocebus, where it is a large plate-like process, and attains the maximum of relative size in the whole order.

In the Anthropoidea this process is always at a greater distance from the head of the bone than in the Lemuroidea, and it is most approximated to it of all in Loris.

I have found only the anterior intertrochanteric line strongly marked in Man and the lowest Simiidæ, but it is faintly indicated sometimes in Chrysothrix, Indris, Lemur, Galago, Loris, and Tarsius.

The posterior intertrochanteric line is most prominent in the lower Simiidæ, then in Man and the Orang, and then in the other Simiinæ.

In Hapale the posterior surface of the femur between the trochanters is wide and flat (Pl. XIII. fig. 5), presenting an appearance existing in no other genus of the Anthropoidea, but very like that of all the Lemuroidea, where this large flat or concave surface serves for the extensive insertion of the *quadratus femoris* muscle.

The head of the femur is of a remarkably large relative size in the Orang, and it is also large in Indris. Sometimes, instead of being rounded, it is peculiarly compressed

* *E. g.* No. 5026 in the Museum of the Royal College of Surgeons.

† *E. g.* Nos. 5027 and 5027 A in the same museum.

‡ De Blainville remarks its great size (*l. c.* p. 16).

transversely. This is the case in the Nycticebinæ, especially in Nycticebus, and somewhat so in Tarsius.

The head is much inclined forwards in the Siamang, and in Indris also, though to a less extent; sometimes it is much so in Cynocephalus and in Man.

The pit for the insertion of the *ligamentum teres* is always present except in the Orang (in which it is almost constantly absent)*, and sometimes in the Gorilla †. It is larger and deeper relatively in the lower Simiidæ than in Man, and it is very large in Ateles. On the other hand, it is small in Indris and Lemur; and there is but a faint indication of it in Perodicticus, though it is large in Arctocebus and enormous in Nycticebus.

The condyles are prolonged backwards about equally in Man, and are nearly equal in size, and in most forms the outer one is but little smaller or less prolonged backwards than the inner one. In the Simiinæ, however (especially the Gorilla and Hylobates), as also in Ateles, Lagothrix, Indris, Arctocebus, Perodicticus, Cheiromys, and sometimes in Lemur, the internal condyle projects considerably further backwards than does the external one.

Supracondyloid prominences are more or less strongly and sharply marked in Man and the Simiinæ ‡. They are less so (except perhaps in some of the higher and larger Cebidæ) in the other Anthropoidea, and in the Lemuroidea; it is only in the Nycticebinæ that they become rather prominent and pointed.

In Tarsius the femur is exceptionally narrow at this part.

The intercondyloid space behind is especially wide in the Simiinæ and Pitheciinæ, and rather much so in Loris and Perodicticus. It is sometimes very shallow, as in Ateles.

The rotular surface is generally moderately concave from side to side, and is especially shallow in the Simiinæ and Nycticebinæ. It becomes deeper in Man and in most Anthropoidea, but in the Lemuroidea this deepening is carried much further, especially in Tarsius. The parts of this surface supported by the two condyles respectively are almost always pretty nearly of the same size; in Man alone the part supported by the external condyle has a great predominance over the other. In the Lemuroidea, other than the Nycticebinæ, however, the external margin of the rotular depression projects much more than does the internal one, especially in Indris.

The depression serving for the origin of the *plantaris* muscle is, as far as I have been able to observe, deepest in the Chimpanzee and sometimes in Hylobates. In the other Anthropoidea it is only slightly marked, and in the Nycticebinæ is altogether absent §.

* I find in the skeleton of an Orang, No. 3 *i* in the Osteological Collection of the British Museum, that each femur exhibits a small but distinct impression on its head, in the place occupied in other genera by the pit for the round ligament. See Trans. Zool. Soc. vol. vi. pl. xl. fig. 7 *i*.

† *E. g.* in the femora of the skeletons Nos. 5179 A and 5179 B in the Museum of the Royal College of Surgeons.

‡ In Man the inner one is the larger, in Troglodytes the outer one. See Owen, Trans Zool. Soc. pp. 14–18, and pl. 7. figs. 1, 4, 6.

§ The muscle itself being absent (Proc. Zool. Soc. 1865, p. 251).

The pit for the tendon of the *popliteus* is generally marked, and is deep in Troglodytes, Simia, Cynocephalus, and Man; also in Ateles, Mycetes, Cebus, and Hapale. It is very deep and large in the Nycticebinæ, and appears to attain its relative maximum in Nycticebus.

The depression for the internal lateral ligament seems less marked in Man than in the other Anthropoidea. Of the Lemuroidea I have found it very marked in Perodicticus and deep in Lemur.

The angle formed by the neck of the femur with its shaft varies from about 155° (Simia) to 128° (the Gorilla) or 125° (Indris).

The angle formed by the shaft of the femur with a horizontal surface on which both condyles are made to rest, varies from about 103° in Man to about 90° in the Chimpanzee. This angle measures the descent of the inner condyle beyond the outer one, which is greatest in Man, though very considerable in others, as, *e. g.*, sometimes in Cynocephalus, and especially Ateles *.

## Tibia.

In the whole of the Primates the tibia is an elongated bone, considerably enlarged at its proximal end, and less so at its distal extremity.

Except in the genus Tarsius, it never anchyloses with the fibula.

There is generally a distinct tubercle giving attachment to the ligament of the patella, and the external (peroneal) surface of the bone is almost always more or less excavated for the reception of the *tibialis anticus* muscle.

The posterior surface of the lower end of the bone has generally two distinct grooves, one for the passage of the tendons of the *tibialis posticus* and *flexor longus digitorum* muscles, the other for that of the *flexor longus hallucis*.

The tibia has the greatest absolute length in Man, to whom the Gorilla in this respect succeeds, but the breadth between the tuberosities is greatest in that Ape, Man being only second.

The length of the tibia (measured to the extremity of the malleolus), compared with that of the spine, varies from more than four-fifths, as in Tarsius, to scarcely more than three-tenths, as in Perodicticus and Arctocebus. In most, however, it is between two-fifths and one-half the length of the spine.

Its length is generally a little less than that of the femur, but it sometimes slightly exceeds it. In Cynocephalus, Troglodytes, and Nycticebus it is decidedly shorter, being to the femur in length as less than 85 to 100; but only in Man does the proportion fall so low as 80·5 to 100.

The length of the tibia, as compared with that of the humerus, is greatest in Tarsius, where it is more than twice as long, and then in Indris and Galago, where it is more than, or almost as much as, once and a half as long. In all, the femur's length exceeds that of the humerus, except in Mycetes, Ateles, Lagothrix, and the Simiinæ, being

* See the specimen No. 4708 in the Osteological Collection of the Royal College of Surgeons.

least of all in the Orang, where the length of the femur is less than seven-tenths of that of the humerus.

The proportion in length borne by the tibia to the radius is greatest, again, in Tarsius, viz. as 163·8 to 100. In Hapale it is as about 158·6, in Man about 150·5 to 100, and in Callithrix as 149·1. In all the rest the length of the former bone is in excess, or the two are equal, except sometimes in Cynocephalus, and in Ateles and the Simiinæ.

The breadth between the tuberosities, compared with the extreme length of the tibia, varies from about 28·5 to 100, as in the Gorilla, to only 9·8 to 100, as in Tarsius.

The antero-posterior diameter of the shaft also, compared with the length of the bone, varies from about 14·6 to 100 in the Gorilla, to about 6 to 100 in Loris.

The tibia is most laterally compressed in Tarsius, most cylindrical in Loris. It is most massive in Troglodytes and Simia.

The tubercle of the tibia is more distinctly prominent in Man than in other Primates. It is situated higher up, as regards the rest of the bone, in him than in any other of the Anthropoidea. In Indris it is as high up as in Man, and in Tarsius it is still higher.

The smooth surface above the rough projection of the tubercle is larger in the Simiinæ than in Man.

The tuberosities project out considerably on each side, except in Tarsius; and in most Primates the peroneal one projects outwards more strongly than it does in Man. A process is sometimes developed above the surface for the *tibialis anticus*, and projects sharply outwards. This is well seen in the Lemuroidea, except Tarsius, and is visible also in Mycetes. The amount of projection of the inner tuberosity varies but little, except that in Tarsius it is *very* slight.

The articular facets for the condyles of the femur rarely occupy the summit of the tibia so completely as in Man.

The outer facet is always decidedly convex antero-posteriorly, except in Ateles, Lagothrix, Indris, and Man, where it is flat or slightly concave antero-posteriorly.

The inner facet is almost always concave antero-posteriorly as well as transversely, but it is almost quite flat in Indris, while sometimes in the Lemuroidea (*e. g.* Galago and Perodicticus) its posterior part inclines strongly downwards.

The spine is always of moderate height, much as in Man, but is longest relatively perhaps in Indris.

The peroneal surface of the shaft is often much excavated for the *tibialis anticus*, and most so in Lemur.

The crest of the tibia is sometimes very prominent, as in Tarsius, Man, Lemur, and Indris. It is generally much sharper, however, in Man than in any other Primate.

The shaft of the tibia may be straight or variously curved. It is straight, or almost so, in Man, Lagothrix, Pithecia, Indris, and more or less so in Ateles and the Orang. It is considerably curved, convex forwards, in the Gorilla, the lower Simiidæ, and Lemur.

It is rather convex outwards in the Nycticebinæ. Sometimes there is a sigmoid vertical curvature, as in Nyctipithecus, Hapale, Indris, Galago, and Tarsius.

The ridge for the *popliteus* is very rarely distinguishable in any Anthropoidea except Man. In the Lemuroidea there is generally a marked vertical ridge at the upper part of the posterior surface of the tibia. This appears to attain its maximum in Arctocebus and Perodicticus.

The ridge for the interosseus membrane, which is so strongly marked in Man, is not distinct in the Simiinæ or higher Cebidæ, but it is more so in other Simiidæ and Cebidæ, and in Hapale. Sometimes in Lemur it is strongly marked, but not in any of the Nycticebinæ.

The malleolus is generally well developed, but sometimes, as in the Orang, very short. It is long in Cynocephalus, Lemur, Galago, and Cheiromys, and in some (the Nycticebinæ) it is much pointed, incurved, and antero-posteriorly compressed, with its articular surface very convex. Moreover, it seems rather to spring from the front than from the inner side of the shaft of the tibia, as is very well seen in Perodicticus.

The articular surface of the malleolus is sometimes nearly at right angles with the inferior surface of the shaft of the tibia, as in Man, the Chimpanzee, and the lower Anthropoidea; sometimes it forms an obtuse angle with that surface, as in the Gorilla, and still more in the Orang.

A groove for the tendon of the *tibialis posticus* marks the back of the malleolus; this attains its maximum of enclosure and relative depth in the Nycticebinæ, where the portion of bone which separates it from the (also strongly, though less marked) groove for the *flexor longus hallucis* has the appearance of a prominent process.

The distal articular surface of the shaft of the tibia is horizontal transversely in Man, Ateles, and Lagothrix. In the Simiidæ and lower Cebidæ the outer portion rises so that the articular surface slopes upwards and peronead; and this is still more the case in the Lemuroidea.

As regards the anterior and posterior margins of this articular surface, they descend in general about equally; but in some Cebidæ (*e. g.* Callithrix) and Lemuroidea (*e. g.* Lemur, Galago, Tarsius) the anterior border descends a little further than does the posterior one. On the other hand, the posterior margin descends considerably more than does the anterior one in Man, and might be supposed to do so in the Nycticebinæ, on account of the projection in the latter of the process of bone separating the grooves for the flexor tendons.

This inferior articular surface is generally subquadrate with a median antero-posteriorly directed prominence. In the Lemuroidea it tends to approach a triangular form, and the prominence in the Nycticebinæ (*e. g.* Loris) becomes very large.

The pit for the insertion of the tendon of the *semi-membranosus* is generally distinct, but often slight, as generally in Lemuroidea, though in Arctocebus it is very strongly marked. It is only in Indris that I have observed a tubercle projecting downwards immediately beneath it.

No constant characters appear to exist as to the medullary foramina, which are one or two in number, near the middle or upper part of the back of the shaft, on the peroneal side of the bone. The artery always enters from above downwards.

### The Patella.

This bone offers few marked or constant noteworthy characters.

It is generally oval, but is rounder and relatively thicker in the Gorilla than in Man; it is very small and round in the Orang.

It is longer and narrower in Mycetes than in most other Anthropoidea, but it attains its maximum of relative length in Indris, where it tapers downwards, and is so bent that the upper and lower halves of its outer (anterior) surface form together an angle which sometimes approaches 90°.

It is long also in Lemur and Cheiromys*, but it is small in the Nycticebinæ and Tarsius.

### Fibula.

This bone is always distinct from the tibia, except in Tarsius, where its lower half anchyloses with the tibia, which thus appears to furnish both the malleoli.

Its length varies with that of the tibia; and it is always very much more slender than that bone, especially in Man, Ateles, Hylobates, Indris, and Microrhynchus †.

The fibula is generally nearly straight, but curves slightly in one direction or in another. In Man it is very decidedly concave forwards, and a similar curvature, though less marked, exists in the lower Simiidæ, Pithecia, and Loris. I have observed it convex forwards in the Orang, Ateles, Mycetes, Indris, Lemur, and Galago, and convex outwards in Hylobates, Chrysothrix, Indris, and Lemur. But there is, I believe, but little constancy in this character.

The outer side of the head of the fibula may be convex, flat, or slightly or deeply concave, and the articular surface for the tuberosity of the tibia may also be flat or slightly or strongly concave. The head of the fibula is much expanded in the Nycticebinæ, and articulates with the tibia by an antero-posteriorly elongated groove.

The malleolus is generally much produced outwards, and projects about as much as, or rather less than, the tibial malleolus, except in Man, in whom alone the external (or peroneal) one is much deeper than the internal malleolus.

The under surface of the malleolus has generally a more or less marked fossa, but the presence and size of this are very irregular and inconstant. The malleolus is often grooved behind for the tendons of the *peronei* muscles, especially in Nycticebus.

The lower articular surface for the tibia varies but slightly in extent.

The fossæ, which more or less excavate the surface of the fibula, and the ridges which divide them, are in no Primate developed to such a degree as they generally are in Man. Yet Simia, the Gorilla, and Cynocephalus approach him rather nearly in this respect.

* Owen, Trans. Zool. Soc. vol. v. pl. 19.

† Proc. Zool. Soc. 1866, p. 165.

Often, in small species especially, there are no distinctly marked fossæ, but very generally there is a depression, on the tibial side towards the summit, which sometimes (as in Galago and Perodicticus) extends far down.

Very generally there is an anterior ridge, and often a posterior or external one also.

### Pes.

The absolute length of this segment is greatest in the Orang and Gorilla, then in Man, and afterwards in the Chimpanzee. In Indris and some other lower Simiidæ it is absolutely longer than in Hylobates, except the Siamang.

The proportion borne by the whole length of the pes to that of the spine is far greatest in Tarsius, where the first is more than four-fifths of the latter. In Cheiromys, Ateles, Simia, and Galago, the length of the pes is more than half that of the spine. All the rest exceed the proportion borne by Man (which is about 35·4 to 100), except Lemur and the Nycticebinæ. In Cynocephalus, however, the proportion is almost the same as in Man.

The length of the pes, as compared with that of the rest of the pelvic limb, is greatest in Simia, Cheiromys, and Tarsius, where the first is decidedly more than half the second. In Galago, Hapale, and Nyctipithecus it is about half; in the rest it is between this and two-fifths, except in Hylobates, sometimes in Cynocephalus, and in Man.

The proportion borne by it to the tibia is greatest in Simia, where it is more than one-fifth longer than the latter. It approaches this proportion in Cheiromys, and the pes is considerably longer than the tibia in Galago and Tarsius also. In all the rest the pes is more than four-fifths of the length of that bone, except in Hylobates, the Nycticebinæ, and Man.

The length of the pes, compared with that of the manus, is far greatest in Chrysothrix and Galago; the rest are intermediate between the latter genus and Ateles (where the proportion is as about 113·6 to 100), except the Chimpanzee, Cheiromys, and, last and least, Hylobates. In these alone, and not always in the Chimpanzee, is the pes shorter than the manus.

### Tarsus.

The absolute length of the tarsus of Man exceeds that of every other Primate, though that of the Gorilla approaches his very nearly.

Its length in proportion to the spine is far greatest in Tarsius, where it almost equals two-fifths of the length of the latter. In Galago it is nearly one quarter; then follow Cheiromys, Man, and the Gorilla, where it is more than three-twentieths. The rest vary between this and one-twentieth (which Indris, Lemur, and Loris scarcely exceed), except Arctocebus.

The length of the tarsus, as compared with that of the entire pes, is greatest in Galago, and then in Man and Tarsius, in all of which the first is between one-half and two-fifths

of the latter*. In the rest the proportion is less, but is still above 3 to 10, except in Mycetes, Hapale, Hylobates, Indris, Ateles, and Simia.

The length of the tarsus, as compared with that of the carpus, is far greatest in Tarsius and Galago. It is least in Simia, Arctocebus, and Hylobates.

The tarsus, besides sesamoids, always consists of seven bones only, except that, according to Van Campen†, an extra ossicle is developed in the transverse ligament enclosing the flexor tendons.

The tarsal bones almost always form an arch, both antero-posteriorly and transversely, but only in Man is the former so extended that the distal ends of the inner metatarsals form the anterior point of support. It must be remembered, however, that in him this is only the case as regards the tibial, or inner side of the foot. The fifth metatarsal is applied to the ground at its proximal end; and thus Man, like the lower Primates, puts the outer part of the tarsus and metatarsus to the ground‡.

The transverse arch is very marked in all Anthropoidea, it is less so in some of the other suborders.

*Os Calcis.*—The calcaneum is absolutely longest in the Gorilla §, but it is nearly as long in Man.

Its length in proportion to that of the spine is far greatest in Tarsius, namely, as 36·4 to 100; then in Galago, where it is just under one-fifth of the length of the spinal column; and then in the Gorilla and Man, where it is a little more than one-tenth. In the rest it varies between this proportion and one-twentieth, which is about that of Perodicticus and Arctocebus.

The tuberosity at its extremity is generally produced upwards or downwards, or both. It is produced both upwards and downwards in the Chimpanzee, Orang, Ateles, Lagothrix, and Mycetes, and more or less so in Arctocebus and Perodicticus. It is produced downwards only in the Gorilla and Loris, upwards only in the lower Simiidæ and Cebidæ, Hapale, Indris, and Lemur.

It (the tuberosity) is broadest at its plantar end in Man, and sometimes in the Gorilla; generally it is so at its middle, as in Simia and Ateles, or towards its upper end, as in the lower Simiidæ. In Hylobates it is sometimes as broad below as above.

In Man and the Gorilla the tuberosity is convex behind; it is concave in the Chimpanzee and in Hylobates; and in most of the forms below that genus it is vertically grooved behind. This is not the case, however, in the Nycticebinæ.

* Dr. Lucae estimates the tarsus by measuring it in front of the articular surface for the tibia, while I employ its extreme length from the tuberosity of the os calcis to the distal margin of the ecto-cuneiforme. Hence there are necessarily discrepancies between the results obtained by us.

† See 'Verhandelingen der Koninklijke Akademie van Wetenschappen.' Zevende Deel, 1859, p. 21, and plate 1. fig. 8*.

‡ Professor Huxley has called attention to this fact in his lectures at the Government School of Mines; and Mr. Henry Hancock, in his lectures on the anatomy and surgery of the foot, remarks, "The external margin, in standing, rests for the most part on the ground" (Lancet for June 1866, vol. xxiii. p. 618).

§ Professor Owen remarks that it is longer than in Man (Comp. Anat. of Vertebrates, vol. ii. p. 550).

In none is the long axis of the heel, or are the peroneal and tibial surfaces of the os calcis so vertical as they are in Man, but the bone is generally twisted, so that the *sustentaculum tali* forms a more or less acute angle with the long axis of the tuberosity; in Man, however, it is a rectangle. This twisting of the os calcis is very slight in the Orang and Pithecia, and not great in Ateles and Hylobates. It is more marked in the lower Simiidæ, and considerably more so still in Troglodytes. Amongst the Lemuroidea it is less so in Tarsius, Cheiromys, and Indris than in others, but it reaches its maximum in the Nycticebinæ, where, in Perodicticus, the *sustentaculum tali* is almost, if not quite parallel with the long axis of the tuberosity.

Concomitantly with this intwisting, the part answering to the inner face of the human calcaneum generally becomes more concave (though scarcely if at all more so in the Gorilla than in Man), reaching its maximum in Loris, where the tuberosity bends round and meets the posterior margin of the astragalus.

Again, a narrowing of the part which answers to the plantar surface of Man, also accompanies this intwisting. This part, indeed, becomes reduced to a narrow ridge by the approximation below of the inner and outer surfaces of the os calcis, and even in the Gorilla it is considerably narrower than in Man *.

The antero-posterior concavity of this plantar surface is very great in Troglodytes, being in both species greater than in Man † (Plate XIII. fig. 6), as also in the Nycticebinæ. On the other hand, in the lower Simiidæ and Cebidæ, Hapale, Indris, and Lemur, this surface is generally almost or quite level antero-posteriorly.

The length of the tuberosity behind the posterior margin of the posterior articular surface for the astragalus, is much longer than that surface in the Gorilla (Plate XIII. fig. 6), often so in the lower Simiidæ, and sometimes in Cebus, very slightly so in Perodicticus, and perhaps also in Tarsius ‡.

In Man the tuberosity about equals, or rather exceeds the same posterior articular surface, but never (except perhaps in some Negroes) equals the predominance attained in the Gorilla, where the part behind the posterior surface for the astragalus exceeds in length all the bone anterior to the hinder border of that posterior articular surface, and in this respect the Gorilla may be said to have the longest heel of any Primate (Plate XIII. fig. 6).

In the Galago the part behind this posterior surface for the astragalus about equals in length the antero-posterior dimension of that surface. In forms other than those before mentioned, it falls short of it; in Simia it is only half of it, and in Loris even much less than that.

The length of the heel behind the posterior articular surface for the astragalus

* But in the Gorilla, as Professor Huxley observes, "the calcaneum retains its narrowness and the single tubercle" (Medical Times, vol. i. p. 537). The two plantar tubercles of the plantar surface of the os calcis are only found distinct in Man.

† Its greater concavity is noticed by Professor Owen (Comp. Anat. of Vertebrates, vol. ii. p. 550).

‡ See Burmeister's 'Tarsius,' pl. 1.

exceeds that of the part of the os calcis altogether in front of that posterior articular surface, slightly in the Chimpanzee, greatly in Man; and in the Gorilla the first is more than double that of the second. The two parts are about equal in length in Perodicticus; but in all the rest of the order the first falls short of the second, especially in Indris, and immensely so, of course, in Galago and Tarsius.

The length of that part of the os calcis which is in front of the posterior articular surface for the astragalus falls very short of the antero-posterior diameter of that surface (less than half) in the Gorilla. It also falls short of it, though not to such an extent, in Man, the Chimpanzee, Simia, and Hylobates.

Of the other forms it about equals it in the Nycticebinæ; in the rest it exceeds it, greatly so in Indris, and immensely so in Galago and Tarsius.

The outer surface of the os calcis has generally one or two peroneal tubercles, but in the Gorilla *, and sometimes in the Chimpanzee, a very deep groove passes antero-posteriorly above one of them (Plate XIII. fig. 6).

The articular surfaces for the astragalus are generally more nearly equal in size in other Primates than in Man, and in the Lemuroidea the anterior one is often the larger. The posterior one is less convex, and the two are divided by a relatively wider groove in the lower Anthropoidea than in Man and the Gorilla.

The surface for the cuboides is generally much wider than the posterior articular surface for the astragalus; it is not so, however, in the Gorilla and Man, in which forms also it is less concave than in the others. It is very concave in the Nycticebinæ, Galago, and Tarsius.

*Astragalus.*—The head of the astragalus is generally united to the body of the bone by a tolerably long neck. This is very short, however, in Man, slightly more so in the Chimpanzee; and the bone has the minimum of length to breadth in the Gorilla † (Plate XIII. fig. 7). In other forms it is more elongated than in Man, and in the Orang it is exceedingly long ‡.

The upper surface is always more or less convex antero-posteriorly, and concave transversely. This convexity is generally more marked than in Man, but decidedly less so (than in him) in the Orang, and still less in Ateles and the Gorilla. This upper surface is almost always broader behind than in front; but the difference is very small in Man, and still less in Ateles, Simia, Hylobates, Lemur, and Loris.

When in Man, the astragalus is articulated with the os calcis, and the bones are placed in their natural position, with the long axis of the tuberosity of the os calcis vertical, then the upper surface is almost quite horizontal, and the lateral surfaces for the malleoli are vertical.

This condition is not so perfectly attained in any other form. In all other Primates,

* Noticed by Professor Owen (Comp. Anat. of Vetebrates, vol. ii. p. 550).

† Professor Owen remarks that it is broader in proportion to its length than in Man (Comp. Anat. of Vertebrates, vol. ii. p. 550).

‡ De Blainville speaks of its elongation in Cheirogaleus Milii as remarkable (*loc. cit.* Lemur, i. 12).

when the bones are naturally united, as when the tibia is vertical, then the long axis of the tuberosity of the os calcis inclines from below upwards and peronead, the outer surface of that bone tending towards the ground.

Now, with this bending downwards and inwards of the outer part of the os calcis, a concomitant upward development of the peroneal side of the astragalus often takes place, causing the surface for the outer malleolus to form an acute angle with the upper part of the astragalus. This is the case in Troglodytes, Simia, the lower Simiidæ, Cebus, and most of the inferior Cebidæ, and in Hapale and Tarsius. This angle, however, which is almost a right angle in Man, is nearly so in Hylobates and Pithecia, while in Ateles and Lagothrix it is obtuse, as also in the Lemuridæ, especially the Nycticebinæ, where it is so much so that the peroneal surface becomes not far from horizontal.

The angle formed with the top of the astragalus by the surface for the tibial malleolus is generally more or less obtuse, and most so in the Gorilla*, where it is almost on one plane with the upper surface (Plate XIII. fig. 7). In Man this is almost a right angle, and nearly so in Indris.

The peroneal surface generally looks more or less backwards, but not so in Man and Ateles, and scarcely so in Hylobates, Lagothrix, Tarsius, and Cheiromys.

The tibial malleolar surface is not generally so much smaller than the peroneal one, as in Man and the Gorilla; but, on the other hand, in the Nycticebinæ the predominance of the outer one is yet greater. In Ateles the equality of the two surfaces is remarkable.

When the bone is altogether detached and placed on a horizontal surface, the peroneal border of the upper surface, in Man and Ateles, is slightly below the tibial one, and this is still more the case in the Nycticebinæ, and sometimes in Lemur. In the Chimpanzee and Hylobates the peroneal border, when the bone is so placed, is slightly higher than the tibial one, and very much so in the Gorilla and the lower Simiidæ and Cebidæ.

The head of the astragalus is sometimes much compressed; this is the case in Ateles, but the compression is at its maximum in Loris.

The groove for the tendon of the *flexor longus hallucis* is sometimes marked off by a sharp process from that for the *flexor longus digitorum*. It is more or less so in Ateles, Lagothrix, Lemur, and Galago, but most so in the Nycticebinæ, where in Loris the tendon of the *flexor hallucis* is made to pass through almost a bony foramen by the large development of this process and the simultaneous intwisting of the tuberosity of the os calcis.

Of the inferior articular surfaces the anterior one is relatively smallest (compared with the posterior one) in Man and the Gorilla. It is rather larger in the Chimpanzee and Hylobates, still more so in the lower Simiidæ, and largest, relatively, in the Nycticebinæ, especially in Loris.

* Noticed by Professor Owen (Comp. Anat. of Vertebrates, vol. ii. p. 550).

The posterior inferior articular surface is always concave.

The anterior inferior articular surface is flat or more or less concave in Man and the Gorilla, generally it is slightly convex, and sometimes, as in Ateles and Loris, strongly so.

*Naviculare.*—This bone is always short disto-proximally in the Anthropoidea, longer in Indris and Microcebus *, but enormously long in Galago and Tarsius, especially the latter.

As compared with the os calcis, its length is also greatest in Galago and Tarsius, and its proportion in Microrhynchus greatly exceeds that in Indris or that in Lemur; in the Anthropoidea also it is relatively very short.

Its anterior and posterior faces are in Man nearly vertical and parallel. In all the other Anthropoidea the posterior face slopes more or less obliquely downwards, so that it looks somewhat upwards. In Lemur the two surfaces diverge as they descend from the dorsum, and they appear to do so generally in the other Lemuroidea, except in the Nycticebinæ, where they are again about parallel and nearly vertical.

The tuberosity of the naviculare is sometimes very large; it is so, and remarkably produced backwards, in Hylobates. It also extends much backwards in Ateles, Mycetes, and Cebus, but downwards in Lagothrix. This process in Man is generally † quite small.

The surfaces for the reception of the cuneiform bones are generally more convex and concave than in Man, but the convexity attains its relative maximum in Loris (Plate XIV. fig. 10), where two strongly projecting tubercles support the ento- and meso-cuneiform bones.

The naviculare almost always articulates distinctly with the cuboides; sometimes, however, only very slightly so.

*Ento-cuneiforme.*—The prevailing form of the internal cuneiform bone is antero-posteriorly short above, but longer towards the sole, *i. e.* its vertical extent is considerably greater at its distal than at its proximal end (Plate XIV. figs. 12 & 13).

Man, the Gorilla, and Orang differ from all other Primates in the more complete equality of the antero-posterior dimensions above and below, and of the vertical extent in front and behind (Plate XIV. fig. 11).

It is short antero-posteriorly as compared with its height in the Lemuroidea, especially in Indris, and most of all in the Nycticebinæ.

The outer surface is but slightly concave in Man, and some others, as Troglodytes and Ateles. It is more or less markedly concave in the lower Simiidæ.

The surface for the hallux has its long axis directed from the dorsum of the ento-cuneiforme towards the sole, and, except in Man, is always strongly convex.

* Proc. Zool. Soc. 1864, p. 624, fig. 1.

† In the skeleton of a giant, No. 5905 B, in the Museum of the Royal College of Surgeons, the tuberosity is very much produced, but not antero-posteriorly expanded. It is also rather produced in the skeleton of O'Byrne in the same Museum. Mr. Henry Hancock (lectures before referred to, 'Lancet' for June 16, 1866) calls attention to these instances.

The articular surface in Man looks more straight forwards than in the other forms, in which latter it is directed more tibiad, a condition which is very marked in Troglodytes.

The long axis of this articular surface always forms a more or less acute angle with a line drawn across the articulations of the four outer metatarsal bones where they join the proximal row of tarsals (Plate XIV. fig. 8).

In the lower Simiidæ this angle is quite as acute as in Man (Plate XIV. fig. 9), or even more so; and the same is the case in the Lemuroidea, but in Troglodytes the angle is a little more open, though nothing nearly so much so as is the homotypal angle in the human hand, nor even equalling that of the manus of the same species. The other Simiinæ * resemble Man and the lower Simiidæ in this respect.

The articular surface is sometimes (as in Hylobates, the lower Simiidæ, and to a certain extent in Man) notched on its peroneal side, but there is no concavity of the surface, making it a saddle joint, in any of the Anthropoidea, though I have observed a very slight depression towards the lower end of the cylinder in the Chimpanzee, Cebus, Mycetes, and Hapale. In the Lemuroidea, however, there is a true and decided saddle joint (Plate XIV. fig. 13), though the concavity is very slight in Indris, Galago, and Perodicticus, and all but obsolete in Nycticebus Javanicus.

The articular surface for the second metatarsal is almost always closely approximated to the surface destined for the hallux; in Simia, however, the two are widely separated†.

A strong tubercle or ridge sometimes projects from the middle of the inferior margin of the tibial surface, as in Ateles, Perodicticus, and Lemur.

*Meso-cuneiforme.*—This bone is sometimes very much vertically extended, as compared with its other dimensions, as in Man, Troglodytes, and the Nycticebinæ. Generally, perhaps, it is, as in Macacus, about as long as high. In Lemur the antero-posterior extent sometimes predominates.

The postero-inferior angle is sometimes produced into a sort of rounded head, as in the lower Simiidæ, and to a slight extent in the Nycticebinæ. In others, as Man, Troglodytes, Ateles, Lemur, this is not the case.

In some of the Lemuroidea (certainly in Lemur and Loris, and probably in Indris and Galago) it joins the cuboides beneath the ecto-cuneiforme. This is never the case in the Anthropoidea.

*Ecto-cuneiforme.*—The external cuneiform bone is sometimes much longer vertically than antero-posteriorly, as in Troglodytes and Simia. It is very slightly so in Hylobates. The two dimensions are about equal in Macacus, Ateles, Lagothrix, Mycetes, and Loris. In Man, the Pithecinæ, Nyctipithecus, and Chrysothrix and others, it is slightly longer than high, and twice as long as high in Lemur‡.

* Dr. Lucae remarks the difference between the Gorilla and Orang in this respect (*loc. cit.* p. 304. Tab. 3. fig. 6 *a*, *b*).

† Represented in Dr. Lucae's plate 3. figs. 5 & 6, and noticed by Professor Huxley (see 'Medical Times' for 1864, vol. i. p. 565).

‡ Well represented in Fischer's 'Anatomie der Maki,' Tab. 15. c. 9.

The proximal surface is sometimes convex, as in Macacus, Lemur, and others; sometimes, as in Man, it is nearly flat. The tibial surface has generally two facets, at its distal end, for the second metatarsal, as in Macacus, Lemur, and others. Sometimes there is only one such, as in Troglodytes and Ateles. The peroneal surface has generally two facets for the cuboides, as in Macacus, Lemur, Loris, and Troglodytes. In Ateles there is only one, which is at its upper anterior angle. In Man there is also only one, but it is posterior in position.

The vertical diameter of its distal articular surface is sometimes in excess, as in Man. Sometimes it is the transverse one which is so, as in Lemur and Loris. Sometimes, as in Macacus, the posterior inferior angle of the bone is produced into a rounded head, though this is not so marked as is that of the meso-cuneiforme. Sometimes, on the other hand, the posterior surface slopes rapidly downwards and forwards, as in Lemur and Hylobates.

The bone projects distally in Man considerably more than the cuboides or meso-cuneiforme do, and sometimes it does so in Ateles. It projects distally beyond the meso-cuneiforme in the Chimpanzee, Hylobates, Semnopithecus, Macacus, the Pitheciinæ, the Nyctipithecinæ, Cebus, Hapale, Indris, Lemur, Loris, and Galago. Much so in Tarsius, slightly so in Cheiromys. Very slightly or not at all so in the Gorilla, and not at all in Simia and Ateles. Sometimes it projects distally beyond the cuboides, but not beyond the meso-cuneiforme, as in Lagothrix and Mycetes.

*Cuboides.*—The length of the cuboides, as compared with that of the os calcis, is greatest in Hylobates, where it sometimes attains one-half. In the rest it varies between this and Galago, where it scarcely exceeds a quarter, and is least of all in Tarsius, where it is less than one-tenth *.

The line of junction of this bone with the os calcis is generally anterior to that of the astragalus and naviculare. It is exceedingly so, of course, in Tarsius and Galago, and it is markedly so in all the Lemuroidea besides, though least so in the Nycticebinæ, especially in Perodicticus. It is also decidedly anterior in Nyctipithecus, Chrysothrix, and Pithecia. In other forms it is generally slightly so, except in Lagothrix and Mycetes, where the two lines of junction form but one, Ateles, where the junction of the naviculare with the astragalus may be anterior, and Man and Troglodytes, where the latter condition generally, if not always obtains.

The distal articular surface is sometimes almost flat or only slightly concave, as in Man; sometimes decidedly concave but concave only, as in Lemur, Loris, and the Cebidæ; sometimes concave above and decidedly convex below, as in the lower Simiidæ.

The posterior surface offers an inferior projection (generally rather, or quite on the tibial side of the bone), which varies in size with the corresponding concavity of the os calcis, being very prominent in Loris and Galago †.

* Owing, of course, to the abnormal length of the os calcis in Galago and Tarsius.

† See the woodcut and description of this joint in Galago Senegalensis, given by Dr. Lucae (*loc. cit.* p. 314).

The plantar surface always offers a ridge, bounding posteriorly a groove for the tendon of the *peroneus longus*.

As has been before said, according to Van Campen * there is in the Potto (Perodicticus) an extra bone situated in the transverse ligament enclosing the flexor tendons, and near the ento-cuneiforme. He has figured it below the detached tarsal bones, and it is marked +.

It is noteworthy that in the species possessing the peculiar ossicle, already described, in the manus, this homotypal exceptional structure should also be developed in the pes!

### Metatarsus.

This segment attains its greatest absolute length in the second metatarsal bone of Simia.

The metatarsus, as estimated by a comparison of the length of the second metatarsal with that of the whole pes, is greatest in Hylobates †, where the first is about one-third of the second. In most it is above a quarter, but in the Nycticebinæ and Cheiromys it is between this and a fifth, while in Galago and Tarsius the proportion is still less. The proportion borne by the metatarsus to the pes is exceeded by that borne by the tarsus to the same, in none so much as in Man, except Tarsius and Galago, where the latter proportion is still greater. In the Gorilla, however, the excess comes very near to that existing in Man.

The metatarsus exceeds the tarsus in length in Simia, Hylobates, the Semnopithecinæ, Ateles, Pithecia, Chrysothrix, Hapale, and Indris. In the others the tarsus equals or exceeds the metatarsus, and largely exceeds it in Man and the Gorilla, and still more in Tarsius and Galago, where it is much more than twice the length of the metatarsus.

The proportion borne by the metatarsus to the spine is greatest in Tarsius, but it is very large also in Hapale and Cheiromys.

*The four Outer Metatarsals.*—These metatarsals are always more or less enlarged at each end. Almost always the proximal ends are wider transversely than are the heads (*i. e.* the distal ends) of these metatarsals. The disproportion in this respect is greatest in Man, though the lower Simiidæ approximate to him. In Simia, however, the heads are scarcely narrower than the proximal ends, and sometimes in the Nycticebinæ those of the third and fourth metatarsals are absolutely broader.

The proximal surfaces of these metatarsals, except that of the fifth metatarsal, are sometimes nearly at right angles to the long axes of their shafts, as is the case in the Simiinæ ‡, especially in Simia.

* Verhandelingen der Koninklijke Akademie van Wetenschappen. Zevende Deel, 1859, p. 21, & pl. 1. fig. 8†.

† From Dr. Lucae's measurements it appears that this proportion is sometimes greater in Simia than in Hylobates. Also that in Cynocephalus and Macacus the metatarsus is sometimes more than one-third of the length of the pes, and as 38·5 to 100 (see *loc. cit.* p. 317).

‡ See Dr. Lucae's figures, pl. 3. figs. 1, 2, 5, 10.

On the other hand, in Man *, the lower Simiidæ †, the Cebidæ, Hapale, and Lemur they are not so, but their bases are as it were bevelled off, so that a line drawn across the articulations of these outer metatarsal bones with the tarsus inclines proximally as it proceeds peronead from the index.

The head of each of these metatarsals has its vertical diameter always greatly in excess of its transverse one, but this excess is carried to its maximum, perhaps, in the Simiinæ (especially Hylobates) and Man.

The shafts (if taken from a short distance beyond the proximal ends to a similar distance from the heads) never broaden distally, but decidedly taper in the lower Simiidæ, and still more so in Man. In Troglodytes they taper slightly, and very slightly in the other forms, except in Simia, Hapale, and the Lemuroidea, where they cannot be said to do so at all.

The shafts are much laterally compressed in Man, the Simiinæ, and Ateles; in the rest they are more or less rounded.

Antero-posteriorly directed planes passing from the middle of the dorsum, of each of these metatarsals, to the most prominent parts of their plantar surfaces, never *con*verge below, in the Anthropoidea ‡, to the middle, or to the fourth metatarsal, but more generally *di*verge from the former. Such a plane in the second metatarsal (that of the index) generally inclines downwards and tibiad; and in the fourth and fifth metatarsals downwards and peronead. In Man this latter inclination is extreme, the fifth metatarsal being more flattened inferiorly in him than in any other Primate, though it is decidedly somewhat flattened below in many, *e. g.* in Hylobates and Ateles. In Lemur, Galago, and the Nycticebinæ this flattening is not at all marked, and in them perhaps even a certain convergence of these planes towards the middle metatarsal may be noted.

The under surfaces of the metatarsals are slightly, but never more than slightly, concave from before backwards; this is more marked, perhaps, in Troglodytes and Lemur than in others.

In Man, Hylobates, and the lowest Simiidæ these metatarsals are very nearly parallel; in the rest they diverge but very slightly from behind forwards, most so in the Lemuroidea.

The ends of the heads of these metatarsals are often inclined more or less strongly peronead, as compared with the long axes of their shafts. This is very marked in the lower Simiidæ and the Cebidæ, but little so in the Lemuroidea, and not at all, or only very slightly so, in Simia, Troglodytes, and Man.

These distal articular surfaces do not, in Man, bend downwards towards their ends, but continue almost on a level with the dorsum of the shafts; they are also limited

* Described by Professor Huxley in his 'Hunterian Lectures' (see 'Medical Times' for 1864, vol. i. p. 177).

† The obliquity of the metatarsals in Cynocephalus is well represented by Dr. Johan Georg Ile, in his 'Monographie der Schnenrollen.' Zweiter Abschnitt, Erste Abtheilung, fig. 2.

‡ De Blainville says of the metatarsals of Cercopithecus sabæus, they are arched, "le second en dedans et les trois autres en dehors" (*loc. cit.* p. 19).

posteriorly by a deeper transverse groove in him than in other forms. It must be admitted, however, that the Mandrill approaches Man very nearly in this.

As to the length of the metatarsals, compared with that of the metacarpals of the same individuals, and estimated by a comparison of those of the third digits of the manus and pes, the metatarsals are in excess in all Primates except the Simiinæ and Cheiromys; and such would also be the case in the last-mentioned genus but for its peculiarly elongated middle metacarpal. In Galago, Perodicticus, Arctocebus, and Tarsius the two segments are of very nearly the same length.

The breadth of the metatarsals, as compared with that of the metacarpals of the same individual, is always less * in Man and the Simiinæ, both absolutely and relatively to the length of the bones; in the latter respect it is also less in all the other genera of the order, except, perhaps, some of the Nycticebinæ and Tarsius.

*First Metatarsal.*—This bone attains its greatest absolute length in the Gorilla, if we except gigantic human forms.

As compared with the spine, this bone is longest in Tarsius, and then in Hylobates and Cheiromys, but it is as much as one-tenth the length of the spine in Indris and Ateles; in the rest it is less, and shortest in Hapale, Perodicticus, and Arctocebus, where it is little more than one-twentieth.

It is longer than the first metacarpal in every species except Simia, but in Perodicticus the excess is exceedingly small.

The proximal end has always an enlargement, for the tendon of the *peroneus longus* on the index side of its plantar surface. This process becomes extreme in size in the Lemuroidea, where it often comes in contact with the tibial side of the second metatarsal. There is a mere rudiment of this process in some, *e. g.* Ateles.

The proximal end also presents an articular surface elongated in the direction of the long axis of the corresponding articular surface of the ento-cuneiforme. This surface is sometimes slightly concave, as in Man, generally strongly so, as in most other forms; and in the Lemuroidea it is also somewhat convex in a direction at right angles to that of the concavity, corresponding to the saddle-shape of the articular surface of the ento-cuneiforme in them.

The long axis of this proximal articular surface of the first metatarsal very rarely (only in Man) runs almost vertically from the dorsal towards the plantar margin of the proximal end of the bone (Plate XIV. fig. 14). In the Simiinæ it runs obliquely from the peroneal side downwards and tibiad to the plantar and tibial angle of the proximal surface (Plate XIV. fig. 15); in all the lower forms it runs more transversely between the dorsal margin, which is turned more or less tibiad, and the peronead turned plantar margin of the same proximal surface (Plate XIV. figs. 16–18).

It is as if the metatarsal of Man had been removed, softened, and then, after being turned so that the dorsum looks tibiad as well as upwards, reapplied to the convex ento-

* Speaking of C. sabæus, De Blainville says, "Les os du métatarse sont surtout bien plus longs que ceux du métacarpe, un tiers en sus; ils sont en outre généralement plus grêles" (*loc. cit.* p. 19).

cuneiforme, and thus stamped with an oblique depression, or, if turned still further, with a transverse one.

The shaft of the bone is generally nearly straight, but in the Simiinæ it has a twist, which seems to disappear, or almost so, with the acquisition of a nearly transversely extended concavity in the articular surface. Thus in the Simiinæ the distal end of the bone can assume a position which would be impossible without this twist.

This distal end of the bone is rarely so large as the proximal one, and never predominates over it, as is the case with the homotypal parts in the pollex. The angle formed by an axis piercing it from side to side transversely, with another similarly traversing the heads of the other metatarsals *, is very rarely (only in Man) so extremely obtuse as to approach 180°.

In the Simiinæ, as well as in the lower forms, this angle is much more acute, approximating to 90°.

This metatarsal is the longest one of the whole pes in Galago and Arctocebus. It is shorter than the other metatarsals in all the other forms except Loris, Perodicticus, and Tarsius; though in Indris it is very little so.

*Second Metatarsal.*—This is sometimes the absolutely longest metatarsal of all, as has been said, but in a certain form (Arctocebus) it is the absolutely shortest of the order.

It is the longest of all in the same pes in Man, and (if we exclude the backwardly extending process of the base of the fifth metatarsal) also in the Simiinæ and Lemur, and at least sometimes in Ateles and Lagothrix.

It is the shortest metatarsal in Arctocebus, Perodicticus, and Tarsius; and it is the shortest except that of the hallux in the Semnopithecinæ, Pithecia, Nyctipithecinæ, Hapale, Indris, and Cheiromys.

It projects more forwards (*i. e.* distad) than the three metatarsals external to it in Man, the Simiinæ, Ateles, and Lagothrix.

It projects less than those do in Pithecia, Chrysothrix, Hapale, Indris, Tarsius, and Cheiromys.

It is longer than the second metacarpal in all except the Simiinæ, Tarsius, and Cheiromys.

The proximal end has an articular surface, which is flat, as in Man, or concavo-convex, as in the other Anthropoidea, or convex only, as in the Lemuroidea.

Sometimes (rarely), as *e. g.* in Lemur and Loris, it meets the proximal end of the fourth metatarsal beneath the ecto-cuneiforme.

The proximal end extends further back than the base of the third metatarsal. In Man, Troglodytes (only slightly so in the Gorilla), Hylobates, the lower Simiidæ, sometimes in Cebus, in Pithecia, Nyctipithecus, Hapale, in the Lemuroidea generally, and greatly so in Lemur.

*Third Metatarsal.*—This is absolutely longest in Simia.

* The plantar angulation of Professor Huxley.

It is the longest of all in the same pes in the lower Simiidæ, sometimes in Cebus, in Mycetes, Perodicticus, and Cheiromys.

It is never the shortest of the pes, but it is longer than the third metacarpal in all except the Simiinæ and Cheiromys, though it is almost the same length in Galago and Tarsius.

It projects most forwards, of the four outer metatarsals, in the lower Simiidæ, Cebus, Mycetes, the Nyctipithecinæ, and Lemuroidea.

The proximal articular surface is flat in Man, and more or less convex in the other forms.

*Fourth Metatarsal.*—This is the longest one of the pes in Pithecia, and the Nyctipithecinæ, Hapale, Indris, and Tarsius.

It is the shortest metatarsal in Loris and Galago.

It projects the most forwards of all the metatarsals in Pithecia and Hapale.

It is longer than the fourth metacarpal in all except the Simiinæ; but the length of these two bones is nearly the same in Galago and Arctocebus.

The proximal end is almost always strongly convex, except in Man.

*Fifth Metatarsal.*—When the backwardly-projecting process of its base is included, this metatarsal is the longest of the pes in the Gorilla, many Cebidæ, Cheiromys, and sometimes in Man.

Without including that process, it is the shortest of the four outer metatarsals in Man, the Simiinæ, Ateles, Lagothrix, Lemur, and Loris.

Except that of the hallux, it projects less forwards than any other of the metatarsals in Man, the Simiidæ, the Cebinæ, Mycetes, Nyctipithecus, Lemur, Galago, and Loris.

It is longer than the fifth metacarpal in all except the Chimpanzee, Simia, and Hylobates; but it is very little so in Arctocebus and Perodicticus, while in Brachyurus, Nyctipithecus, Chrysothrix, and Hapale it is more than double the length of the fifth metacarpal.

Its proximal end is in general strongly convex. A process projects backwards from the outside of the proximal end of the fifth metatarsal. This is at its maximum in Man and the Gorilla. It is smaller relatively, as well as absolutely, in most others, especially in the Lemuroidea.

### Phalanges.

The hallux has always two phalanges, except, as is well known, in Simia, where there is often but one.

Each of the other digits has three distinct phalanges, except in Man, where generally the ultimate and penultimate phalanges of the fifth digit become anchylosed together.

The *proximal phalanx of the hallux* is absolutely greatest in Man, but the Gorilla approaches him *very* closely * in this respect.

* Professor Owen has found it to equal that of Man (Comp. Anat. of Vertebrates, vol. ii. p. 551).

It is decidedly longer than its homotype of the manus in Man, Troglodytes, the long-tailed Simiidæ, and Indris, being in Colobus nearly three times as long. The two segments are about equal in Cynocephalus, in the Cebidæ (except, of course, Ateles), in Lemur, Galago, Loris, Arctocebus, and Tarsius. On the other hand, it is decidedly shorter in Simia, Hylobates, Hapale, Perodicticus, and Cheiromys.

It is always shorter than the first metatarsal, but at the same time is more than half its length, except sometimes in Simia and Hylobates and the lower Simiidæ, especially Semnopithecus and Cynocephalus.

The *second phalanx* is more than half the length of the first, except in the Semnopithecinæ, sometimes in Macacus, in Nyctipithecus, Hapale, and Tarsius, in which last it is very short, as compared with the proximal phalanx.

It is always flattened from above downwards at its distal part.

The *phalanges of the other digits* are, except in Man, very similar to those of the manus, but, as in him, they are narrower transversely than their respective homotypes. This difference is less marked below the Simiidæ, yet always exists except in Tarsius, and sometimes, perhaps, in the Nycticebinæ.

They are convex above and flattened or concave below; but in Man the shafts are so short that the inferior flattening is inconspicuous.

The distal ends of the several joints are formed nearly as in Man, except as regards the distal ends of the ultimate phalanges of some.

These ultimate phalanges are always flattened distally, more or less, as in Man, except in Hapale, where they are laterally compressed, curved, and pointed, like those of the manus; in the index of the Lemuridæ, which is elongated and pointed at its end; in Cheiromys, where they are "subcompressed and acutely pointed"*; as also in Tarsius.

The *proximal phalanx* of either the second or the third digit is the longest phalanx of all the four outer digits of the same pes in the Anthropoidea. In the Lemuroidea that of the fourth digit is the longest.

The length of the phalanges may be perhaps best estimated by selecting those of the third digit.

The *proximal phlanx* of this digit is much less than half the length of the third metatarsal in Man; it is about half the length of the latter in the Simiidæ below the Simiinæ, and in Hapale.

It is a little longer than the third metatarsal in Galago and some of the Nycticebinæ.

No proximal phalanx of any of the four outer digits exceeds its supporting metatarsal in the Anthropoidea; but in Galago, the Nycticebinæ, Tarsius, and Cheiromys, that of the fourth digit exceeds in length the fourth metatarsal; and in Galago, Loris, and Perodicticus† the same may be said of the third digit.

The proximal phalanx of the third digit greatly falls short of the length of its homo-

* Owen, Trans. Zool. Soc. vol. v. p. 54. † See Van Campen's figure, *loc. cit.*

type in Man; it falls less short in Troglodytes, Tarsius, and Cheiromys, and less still in Simia and Hylobates.

Often the two are nearly equal, and sometimes that of the pes is a little in excess. In Loris and Arctocebus it is considerably so.

The *second phalanx* is always more than half the length of the proximal phalanx, except in Tarsius, where it is a little less.

As compared with its homotype, it is less than half only in Man and Tarsius, but it is very little more than half in the Chimpanzee, and sometimes in Hylobates. It is still considerably shorter than its homotype in the Gorilla, Simia, Galago, and Cheiromys; in the rest there is but little difference, and the phalanx of the pes is sometimes a little the longer. In Indris, Loris, and Perodicticus it is decidedly so.

The *third phalanx*, as compared with the second one, is longest in Man, where it sometimes equals, and indeed even exceeds, the latter in length. It is often less than half the length of the second phalanx in other forms.

It is much shorter than its homotype, in Man and in all the Simiinæ. In the other forms there is no great difference, that of the pes being sometimes a little the shorter, sometimes, as in the Nycticebinæ, a little the longer.

The phalanges of the four outer digits shorten successively, except in Tarsius and sometimes in Galago, where the second phalanx of the fourth digit is as long as the proximal one of the index, and in the Nycticebinæ, where the latter is actually shorter.

### Digits without their Metatarsals.

The *hallux* thus measured is absolutely longest in Man.

As compared with the length of the pes, it is longest by much in Arctocebus, and then in the Chimpanzee and Man, Indris, and Loris, in all of which its length equals, or is but very little less than a quarter of that of the pes.

Not counting Ateles, it exceeds its homotype most in Colobus. It falls slightly short of it in Lagothrix, Mycetes, Hapale, and Hylobates; more considerably so in Cheiromys, and most so in Simia. In all the rest it equals or slightly exceeds it.

It is the longest digit of the pes only in Man, and only sometimes in him.

It is the shortest one in all except Man, but only in the Semnopithecinæ, Lagothrix, Hapale, and Cheiromys is it so small as only to equal half the length of the fifth digit, except in Simia, where it is sometimes less than a quarter of the length of that digit.

The *index* is the longest of the pes only in Man, and only sometimes in him.

It is never the shortest digit, but it is the shortest one, except the hallux, sometimes in Hapale, and in all the Lemuroidea. In Man alone it projects the most. In Perodicticus and Arctocebus it is unusually short, being very little longer than the hallux in the latter genus.

The index is always shorter than the third digit, except in Man *.

As compared with the whole pes, the index is longest in Simia and Cheiromys (nearly two-fifths of the length of the latter), and scarcely less in Lagothrix and Mycetes. In the rest the proportion is less, being least but one in Man, and least of all in Tarsius, where it is less than a quarter of the length of the pes.

The *third digit*, as compared with the whole length of the pes, is longest in Loris, where it is more than half of the length of the latter, then in Cheiromys and Indris; and in all the rest it is much above a quarter, except in Tarsius, in which it is only slightly so, and Man, where it is less than a fifth.

As compared to its homotype, it is much shorter in Man, Simia, and Tarsius also, but to a less degree in Cheiromys, the Simiinæ, and others.

It is never the shortest digit of the pes, but it is the longest one in the Simiidæ, Cebinæ, and Mycetes. It projects most of the digits of the pes in the Simiidæ and most Cebidæ.

The *fourth digit* is the longest of the pes in Pithecia, sometimes in Nyctipithecus, and always in the Lemuroidea.

It is never the shortest one, even without the hallux. It projects the most of the digits of the pes in Pithecia, sometimes in Nyctipithecus, in Hapale, and in all the Lemuroidea.

The *fifth digit* is the longest one of the pes in none; it is the shortest one in Man, but in him only.

It is the shortest one, except the hallux, in the Simiidæ and Cebidæ, and it is about as short as the index in Hapale. The fifth digit projects more than the index in Pithecia, Hapale, and the Lemuroidea; not so in other forms.

The length of the longest digit, compared with that of the tarsus, is greatest in Loris, Arctocebus, Simia, and Indris (more than 160 to 100); it is least in Tarsius, Galago, and Man, in the last being only about half.

The proportion borne by the longest digit to the longest metatarsal is greatest in Arctocebus and Perodicticus, where the first is near being twice and a half the length of the second. In Galago and Lemur it is also more than twice its length. In the rest it is more than once and a fifth as long, except in Troglodytes, the lower Simiidæ, Chrysothrix, Hapale, and Man. In Man and some of the lower Simiidæ it is shorter than the metatarsal.

The relation between the proportion borne by the longest digit to the longest metatarsal, and that borne by the longest digit of the manus to the longest metacarpal, is so far uniform that, except in Perodicticus †, the first proportion is always smaller than the second. The difference between the two proportions, however, is almost nil in Simia,

* Dr. Lucae says that the index is longer than the third digit in Troglodytes (*loc. cit.* pp. 306, 307, and 320); also that in Cynocephalus mormon, the second and third digits are equal (*loc. cit.* p. 317). I am inclined to think that this variation may be owing to an error in mounting the specimens.

† As Perodicticus was the only exception I found, I was inclined to suspect that the specimen in the British

small in Indris, Arctocebus, and Loris. In most, however, it is smaller than in Man; but he is exceeded in this respect by Mycetes, Tarsius, Chrysothrix, Cheiromys, Nyctipithecus, and Hapale, in which the difference between the proportions becomes successively greater, the excess of the longest digit of the manus over the longest metacarpal being in the last-mentioned genus more than four-fifths greater than that of the longest digit of the pes over the longest metatarsal.

### Digits with their Metatarsals.

The *hallux* thus estimated is absolutely longest in Man.

Its proportion to the spine is far greatest in Tarsius (more than a quarter), and then in Cheiromys and Hylobates, Indris, and Ateles, in all of which it is more than one-fifth the length of the spinal column. In Colobus and Hapale, on the other hand, it is but little more than one-tenth.

Its length, compared with that of the entire pes, is greatest in Arctocebus, Loris, and Indris, where it is more than one-half the length of the latter—a proportion it nearly attains in Man, and sometimes in Hylobates, while the Chimpanzee follows closely. In all the rest it is more than 33 to 100, except in Hapale and the Semnopithecinæ, where it is a little less, and Simia, where it is scarcely more than a quarter.

The hallux, when brought beside the index digit, attains to its extremity in Arctocebus; sometimes beyond its extremity in Man: to the middle of the distal phalanx, or rather beyond it, in the other Nycticebinæ, Galago, Tarsius, and sometimes in Man: to the middle or near the distal end of the second phalanx of the index in Lemur and Indris: to the proximal end of the second phalanx in the Chimpanzee and Cheiromys: to the distal end of the proximal phalanx in the Gorilla, sometimes Hylobates, Cynocephalus, Pithecia, and Nyctipithecus: to the middle, or nearly so, of the proximal phalanx in Hylobates (sometimes), the Cebinæ, Mycetes, Chrysothrix, and some lower Simiidæ: to a little beyond the base of the proximal phalanx in the Semnopithecinæ and Hapale: not nearly to the distal end of the metatarsal of the index in Simia.

The extent to which the hallux extends with regard to the index of the pes, when compared with the extent to which the pollex projects forwards beside the index of the manus in the same individual, is as follows:—

Almost always the hallux projects further than the pollex (omitting Arctocebus and Perodicticus).

The reverse condition, however, obtains largely in Hapale, in a less degree in some Cebidæ, *e. g.* Lagothrix, Mycetes, and also in Simia.

In most of the Cebidæ the relative extension is about equal; but in Man and Tarsius

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Museum had been wrongly articulated; but in Van Campen's memoir, before referred to, his plate represents the longest metacarpal as about equal to the longest metatarsal in length, while the longest digit of the pes decidedly exceeds that of the manus.

the difference is very great, and in this these two extreme forms agree together, and differ from all others.

The production of both, taken together, is greater in Loris than in Man and Tarsius, because, though the hallux projects a little less, the pollex projects so much more.

The combined projection is greatest of all in Arctocebus and Perodicticus, from the small development of the indices both of the pes and of the manus.

The hallux, when compared with the longest digit of the pes, is at its maximum in Man. Then follow Arctocebus, the Chimpanzee, and Indris, where the proportion is as about 7 to 10. In all the rest the proportion is greater than one-half, except in the Semnopithecinæ and in Simia.

The length of the hallux, as compared with that of the pollex, is, of course, far greatest in Ateles and Colobus, where the first is more than twice and a half the length of the second.

It always considerably exceeds the pollex in length, except in Hylobates, Tarsius, and Cheiromys, where it does so but little, and in Hapale and Simia, where it is absolutely less, the proportion in the last-named genus being as about 79·1 to 100.

The *index digit*, as compared with the spine, is longest in Simia, where it is nearly two-fifths of the length of the latter. It is only slightly less in Ateles, and but little so in Cheiromys and Tarsius. In all the rest it is less than in the last, but more than one-fifth of the length of the spine, except in Loris, Man, and Lemur, where it is a little less, and Perodicticus and Arctocebus, where it is little more than one-tenth.

The index of the pes is more than twice the length of its homotype in the manus in Arctocebus, and in Perodicticus it is more than once and a half as long.

It is longer than the index of the manus in all except Cheiromys, the Simiinæ, and Tarsius.

The *longest digit* of the pes (whether the third or the fourth), as compared with the longest one of the manus, is far greatest in Loris, where the first is nearly once and a half of the length of the second; but in all, except Man, the Simiinæ, Ateles, Tarsius, and Cheiromys, that of the pes is the longer. Of these last-mentioned genera the proportion borne by the digit of the pes is greatest in Ateles and Simia, least in Hylobates, where sometimes it is only as about 67·2 to 100.

### Hand and Foot.

Dr. Lucae, after terminating his description of the variations of structure noticed by him in the extremities of the Primates, observes that a more minute examination of the pes of apes shows it to agree more with the human hand than with any other mammalian extremity, that its resemblances to the human foot are superficial, and that the use of the name Quadrumana is thus fully justified.

His words are, "Denn nicht nur eine genauere anatomische Untersuchung weist nach, dass die s. g. 'hintere Hand' sowohl anatomisch als auch physiologisch weit mehr

Uebereinstimmung mit der 'menschlichen Hand' als mit irgend einer terminaler Abtheilung der Extremitäten in der ganzen Säugethierreihe besitzt, und dass in der That nur mehr oberflächliche Formähnlichkeiten mit dem menschlichen Fusse vorkommen.—Die Ordnung der Quadrumanen ist daher eine vollkommen berechtigte" *.

The result of my examination, on the contrary, convinces me that the so-called "hinder hand," as well anatomically as physiologically, far more agrees with the human foot than with the human hand, and that it agrees with the latter only in more superficial points †. Also that the old term Primates is far preferable ‡ to the name Quadrumana, which is not applicable exclusively to Apes and Lemuroids, whatever definition be accepted of the term "hand."

If we accept as our definition of the word "foot," "*an extremity in which the hallux forms the fulcrum in standing or walking*" §, then Man alone has a pair of feet; and if at the same time we define the hand as an unguiculate extremity more or less prehensile, with four or five complete digits, the innermost of which may or may not be opposable, then unquestionably Apes and Lemuroids have no feet, but four hands, and no one *using such definitions* could be justly blamed for speaking of those animals as quadrumanous, though the epithet should then be extended to others which are very different.

But Dr. Lucae, without any such preliminary qualification, states broadly that both anatomically and physiologically the posterior extremity of Apes far more nearly resembles the human hand than the human foot.

He does so on the following grounds ‖:—

1. The absence of the tarso-metatarsal arch in the foot of Apes, the inclined upper surface of the astragalus, and the support of the body by the anterior row of tarsal bones, the first and fifth metatarsals, and the toes, but not by the heel.

2. The short tarsus, no longer exceeding the metatarsus and toes; the greater rotation in the tarsal joint, and the hinge-joint formed by the metatarsals with the tarsus.

3. That the antero-posterior, dorso-plantar sections, "*sagittalen Durchschnitten*," are not parallel, but approximate to each other towards the sole. That all five metatarsals are not united together at their heads by ligaments, but only four of them, the fifth being free; also the form of these heads, which are but seldom, as in Man, provided with "*entwickelte Hemmungflächen*."

4. That the digits are long and mostly longer than the metatarsals; that the first toe is shorter than the second; that the second, however, is smaller than the third or even than the fourth.

* *Loc. cit.* p. 323.

† As justly observed by Professor Huxley (Man's Place in Nature, p. 91).

‡ It is not on anatomical grounds, however, that I would base my preference for the term Primates.

§ "The great toe, which forms the fulcrum in standing or walking, is perhaps the most characteristic peculiarity in the human structure; it is that modification which differentiates the foot from the hand, and gives the character to his order (*Bimana*)."—Owen on the Anatomy of Vertebrates, vol. ii. p. 553.

‖ Lucae, *loc. cit.* p. 321–323.

5. That not a dorsal, but a plantar flexion predominates in the tarso-metatarsal joint, and also * in the metatarso-phalangeal one.

6. Finally, on account of the mode of articulation of the hallux with the ento-cuneiforme.

Now, with regard to the plantar arch, it is, indeed, true that there is a certain difference between Man and Apes, owing to the hallux in the latter not being used as the fulcrum; but the tarsal bones, apart from the metatarsals, form in all the Anthropoidea an arch much as in Man, while in him, as in Apes, the fifth metatarsal takes no part in the arch, but is applied to the ground, as has been before noticed. The difference in this respect is small, indeed, between Man and the Gorilla as compared with that existing between the latter and other forms of the order, such as Tarsius, while the carpus of Man presents nothing at all resembling the antero-posterior arch of the tarsus of Apes.

The inclination of the upper surface of the astragalus very generally exists and has been described above, but it is difficult to see how this is any approximation to Man's hand.

As to the application of the heel to the ground, the difference is not between Man and the higher Apes, but between these and lower forms.

The shortness of the tarsus, as compared with the metatarsus, will not serve; for the proportion borne by the tarsus of the Gorilla to the metatarsus is overwhelmingly more like the proportion of the human foot than that of the homotypal parts of the human hand, the total length of the tarsus much exceeding that of the longest metatarsal, while in Galago and Tarsius the excess in length of the tarsus over the metatarsus is very far greater even than in the human structure. It is true that in none does the tarsus attain so great a length, as compared with the digits (whether with or without their metatarsals), as in Man, yet even in this respect the pes of the Gorilla and others far more nearly resembles the human foot than the human hand.

The rotation of the tarsal joint is certainly more extensive in Apes than in Man; but the shape of the joint closely resembles its homologue in Man's foot, and widely differs from his intercarpal articulation.

The convexity of the proximal articular surfaces of the metatarsals in the lower Apes does produce a sort of hinge-joint; but inasmuch as they *are* convex, they depart more from the structure of the proximal ends of the human metacarpals (some of which are more or less strongly *concave*) than from the flat proximal ends of the human metatarsals, while the highest Apes scarcely differ from Man in this respect.

As to the "sagittalen Durchschnitten," I must avow that I have been unable to find any indication of the plantar convergence of such in any Anthropoidea. There are differences indeed from Man's foot,—a lesser flattening beneath of the outermost metatarsals, and often a peroneal bending of their distal ends,—but no approximation to the human hand.

The absence of a ligamentous connexion between the heads of the first and second

* P. 322, line 13 from the bottom.

metatarsals might be neglected in considering the *osteology* of the limbs; but it may be remarked that this absence is a necessary condition of the strongly prehensile action of the hallux; and that the hallux *has* such action in the Primates below Man is admitted by all. The difference presented amongst Apes as to the extent of connexion by soft structures, of continuous digits is not less remarkable.

As to the form of the heads of the metatarsals, the transverse grooves on the dorsum, and the projecting tubercles beneath, are but little less marked in some Cynocephali than in Man; and in such the pes far more resembles, in this respect also, the human foot than the human hand.

In the length of the digits, as compared with the metatarsals, in the predominance of the third digit in so many forms, and in the greater plantar flexion of the tarso-metatarsal and metacarpo-phalangeal joints, the pes of Apes does rather resemble the hand of Man than his foot; but the elongation of the digits in the pes of Apes is a point conceded by all disputants.

The convexity of the distal articular surface of the ento-cuneiforme is again a point of resemblance to the hand of Man; but, as has been before said in describing that part, the angle formed by the long axis of that surface with a line traversing the distal surface of the other tarsals more resembles that of the human foot than the homotypal angle of Man's hand (Plate XIV. figs. 6–9); and in general form and proportion the ento-cuneiforme of the Gorilla is overwhelmingly more like its human homologue than it is like the trapezium of Man (Plate XIV. fig. 11).

But, in addition to these points, it should be borne in mind that the pes of the rest of the Primates resembles the foot of Man, in that—

1. Except in the Chimpanzee, Cheiromys, and Hylobates, it always exceeds in length the manus of the same individual.

2. Consequently with the same exceptions, the proportion borne by the pes to the spine exceeds that borne by the manus.

3. The proportion borne by the length of the tarsus to that of the spine always greatly exceeds that of the carpus.

4. The whole of the tarsal bones, in number, form, proportion, and connexions, resemble the human ones infinitely more than they do the carpals of Man.

5. The tarsus directly joins both the long bones of the middle segment of the limb, not only one, as in the human hand.

6. The articulation with the leg, however oblique, is on the type of the human foot, and not on that of the human hand and arm.

7. Very generally the ecto-cuneiforme projects distally considerably beyond the meso-cuneiforme.

8. The cuboides has a transverse ridge and no process like the unciforme, and it has a more or less sharply-marked prominence behind.

9. The tarsus sometimes exceeds the metatarsus in length.

10. If the line joining the bases of the metatarsals forms an angle with the long

axis of the pes (through their proximal ends being bevelled off), it inclines outwards and backwards, as in the human foot.

11. The shafts of the four outer metatarsals do not become broader distally, but almost, if not quite, always taper somewhat from near the base to near the head of each.

12. Planes antero-posteriorly directed and drawn from the middle of the dorsum of each metatarsal to the most prominent part of its plantar surface do *not* converge at least in any Anthropoidea.

13. Sometimes strongly marked transverse dorsal grooves limit proximally the articular surfaces of the distal ends of the metatarsals.

14. The third metatarsal (except in Simia and Cheiromys) always exceeds in length the third metacarpal of the same species.

15. Except perhaps in some Lemuroids, the metatarsals are more slender than are the metacarpals in the same individual.

16. There is a prominence at the proximal end of the plantar surface of the innermost metatarsal.

17. There is a large process projecting backwards in some from the proximal end of the fifth metatarsal.

18. The length of the hallux with its metatarsal always exceeds the pollex with its metacarpal, except in Simia, Hylobates, Hapale, Tarsius, and Cheiromys.

19. The hallux extends further, in relation to the index of the pes, than does the pollex in relation to the index of the manus, in the great majority of forms.

20. Except in Perodicticus, the proportion of the longest digit of the pes to the longest metatarsal is always less than that borne by the longest digit of the manus to the longest metacarpal.

21. The phalanges of the pes are generally more slender than are the homotypal ones of the manus in the same individual.

The pes of Apes and Lemuroids differs from the foot of Man and resembles his hand, in that—

1. The proportion borne by the pes to the rest of the pelvic limb almost always exceeds that borne by the manus to the rest of the pectoral one.

2. The proportion borne by the pes to the tibia is generally greater than that borne by the manus to the radius, reversing the conditions existing in Man.

3. The innermost digit is supported on a strongly convex surface.

4. The innermost digit diverges from the others, and the transverse axis of its head forms an angle which approaches 90°, with a line joining the heads of the other metatarsals.

5. The metatarsus in many exceeds the tarsus in length.

6. The phalanges, and therefore the four outer digits, are of such length as compared with their metatarsals and with the hallux.

7. Neither the first nor the second digit is ever the longest one of the pes.

8. There is such an amount of plantar flexion on the joints.

9. The dorsal grooves limiting the articular heads proximally are generally less marked.

The manus in Apes and Lemuroids agrees with the hand of Man and differs from his foot, in that—

1. The length of the manus is almost always less than that of the pes in the same individual.

2. The length of the manus, compared with that of the spine, is almost always less than that borne by the pes to the spine in the same individual.

3. The length of the carpus, as compared with that of the spine, manus, and digits, is smaller than that of the tarsus, as compared with the spine, pes, and digits, in the same individual.

4. The form, arrangement, and connexions of the bones are similar.

5. At least two bones of the manus articulate with the long bones of the limb.

6. There is a convex cylinder supporting the innermost digit, and its long axis forms an obtuse angle with a line joining the proximal ends of the metacarpals.

7. The cuboides has an unciform process and no transverse groove.

8. The line joining the proximal ends of the metacarpals never inclines outwards and backwards.

9. The metacarpals expand distally.

10. The antero-posteriorly directed planes, traversing the metacarpals from the dorsum to the palm, converge palmad.

11. The metacarpals are broader than the metatarsals of the same individual.

12. The pollex with its metacarpal is almost always shorter than the hallux with its metatarsal, in the same individual.

13. The pollex generally extends less far forwards with relation to the index of the manus, than does the hallux with relation to the index of the pes.

14. Except in Perodicticus, the proportion of the longest digit to the longest metacarpal always exceeds that of the longest digit of the pes to the longest metatarsal.

15. The phalanges of the manus are broader than their homotypes of the pes in the same individual.

16. The angle formed by the transverse diameter of the head of the pollex with a line connecting the heads of the other metacarpals is similar to the homologous angle in Homo.

17. Neither the first nor the second digit is the longest one.

18. The lengths of the phalanges, and hence of the digits, are similar.

The manus of Apes and Lemuroids differs from the hand of Man and resembles his foot, in that—

1. The proportion, as to length, borne by the manus to the rest of the pectoral limb almost always falls short of that borne by the pes to the rest of the pelvic one, reversing the conditions in Man.

2. The proportion borne by the manus to the radius is generally less than that borne by the pes to the tibia.

3. There is a strong dorsal flexion of the metacarpo-phalangeal joints.

4. The fifth metacarpal has sometimes a well-developed process extending backwards from the outside of its proximal end.

5. Generally there is no saddle-shaped surface to support the innermost metacarpal.

6. Almost always the carpus is directly connected with both the lower long bones of the limb.

A consideration of all the points above enumerated can, I think, leave little doubt on an unprejudiced mind that, as regards the form and relative size of the bones, their juxtaposition, connexions, and modes of union,—in other words, as far as osteological *anatomy* goes *, the posterior extremity of Apes much more resembles the foot of Man than it does his hand; while at the same time the manus of Apes differs widely from Man's foot, and closely resembles his hand.

The prolongation of the controversy, the last word of which, till now, has come from Dr. Lucae, is, I think, owing to the dispute being one rather about words than about material objects; and it is perhaps well further to consider the meanings given to the terms "hand" and "foot" respectively.

The popular use of the word "foot" shows that its connotation is "support." We speak of the foot of Man, the fore and hind foot of a horse, the foot of a wineglass or of a mountain; and in this sense the term is applicable both to the fore and hind extremities of most Apes and Lemuroids, which are thus, as they are often called, "Quadrupeds." If, neglecting common usage, we frame a special definition, then, as has been seen, one can readily be devised applicable exclusively to the lower extremities of Man.

As to the word "hand," the signification given to it by popular use is vague enough; but precise definitions of the term have been framed by Cuvier, Isidore Geoffroy St. Hilaire, and others; it remains to see if one has been devised which will justify the application of the term "quadrumanous" to all Primates besides Man, and to them exclusively.

Cuvier's definition, "*le pouce libre et opposable aux autres doigts, qui sont longs et flexibles*," which has been accepted by so many, cannot be applied to the anterior extremities of Colobus, Ateles, and Hapale, and scarcely, indeed, to any of the Cebidæ †.

* For an excellent summary of the myological resemblances and differences of the extremities, which lead to the same result, see the report of Professor Huxley's Hunterian Lectures in the 'Medical Times' for 1864, vol. i. p. 457. See also the article by Ludwig Fick in Müller's 'Archiv,' 1857, p. 435.

† The imperfect opposition of the thumb in the Cebidæ was first, I believe, pointed out by Don Felix d'Azara in his 'Essais sur l'Histoire Naturelle des Quadrupèdes de la Province du Paraguay,' 1801, vol. ii. pp. 213, 233, & 244; also by Geoffroy, 'Dictionnaire Classique d'Hist Nat.' t. xv. 1829. Again, and independently, by Mr. Ogilby in the Penny Cyclopædia, vol. i. p. 442; and again by the latter gentleman in a paper published in the Proceedings of the Zoological Society for 1836, p. 25. Mr. W. Martin notices the same point, 'Nat. Hist. of Man and Monkeys,' 1840, p. 341. More recently, this incomplete opposability has been noticed by Professor Huxley in his 'Evidence as to Man's Place in Nature,' 1863, p. 93; and in his Hunterian Lectures reported in the 'Medical Times' for 1864, vol. ii. p. 93. The only partial opposition of the pollex in Hapale and Cebus is mentioned by Professor Owen, 'Comp. Anat. of Vertebrates,' vol. ii. p. 543; and Trans. Zool. Soc. vol. v. p. 274. Also by Vrolik, Todd's Cyclop. vol. iv. p. 213.

If we extend this definition so as to include those forms and to be applicable at the same time to the pes of all Primates except Man, and therefore call every prehensile extremity with four or five unguiculate digits, with or without an opposable innermost one, "a hand," the same term must then be applied to the pes of the Bat and of the Parrot; nor could it be consistently refused even to the extremity of the Sloth; while some Marsupials, and even the Chameleon, might successfully lay claim to the epithet "quadrumanous."

M. Isidore Geoffroy St. Hilaire gave a definition of the word in some respects better: "*Toute extrémité pourvue de doigts allongés profondément divisés, très-mobiles très-flexibles et par conséquent susceptibles de saisir entre eux et la paume les objets placés à leur portée*" *. But even this is not exclusively applicable to Apes and Lemuroids, especially if the pedal digits of the Gorilla and Siamang are to be spoken of as "*profondément divisés.*"

But it is not only on account of form and structure that the same term cannot be applied with propriety to the hand of Man and the pes of Apes; for the careful consideration of the *function* of the parts shows more difference between them than is often supposed, as well as a greater agreement between the pelvic extremity of Man and that of the other genera of the order. All admit that the hand of Man is almost exclusively prehensile, his foot almost exclusively locomotive; and it is commonly asserted that in Apes and Lemuroids the pes resembles the hand of Man in function far more than it does his foot. I believe, however, that this is not the case; for, in the first place, the foot of Man is not quite destitute of prehensile action, as Ludwig Fick † has noticed. In his excellent article on the hand and foot, that author truly observes that in locomotion, especially on an uneven surface, there is a certain abduction and adduction of the digits in the human foot. Professor Huxley has also called attention to its occasional grasping action ‡. In the second place, though this prehension is very much more developed in all the other Primates than in Man, yet in them this prehension is like that which exists rudimentarily in the human foot and not that of the hand of Man. It is a prehension *subsidiary to locomotion*, and a modification of the action of the pes in harmony with the form of the most frequent supporting surfaces (the boughs and twigs of trees), not a true assumption of the function of a hand which is still preserved by the anterior extremity. This view, that the prehension of the pes is a locomotive and not a manual prehension, is confirmed by some observations kindly communicated to me by Mr. A. D. Bartlett, Superintendent of the Gardens of the Zoological Society, to whom we are indebted for so much interesting information respecting the habits of animals. He informs me that he is confident that Apes and Lemurs do not use the pes as a hand, that is, for conveying food to the mouth, &c., unless the anterior extremities are already occupied; and this is the more remarkable, because he has

* Archives du Muséum d'Histoire Naturelle, 1839, p. 17.

† "Hand und Fuss," Müller's Archiv, 1857, p. 456.

‡ Man's Place in Nature, p. 86.

observed the spider-monkeys successfully employ their long prehensile tails to obtain an object otherwise out of their reach. He has also noticed that the flying fox, Pteropus (to the pes of which no one has applied, as far as I am aware, the term "hand"), will hold its food in, and eat from, its pelvic extremity.

As to the constant elevation of the heel above the ground spoken of by Dr. LUCAE, it has been already observed that, at least in the higher forms (Simiinæ), this is not the case, especially, perhaps, in the Gibbons, where, in terrestrial progression, the pectoral extremities are raised entirely from the ground.

Thus, physiologically as well as anatomically, the same term may certainly be applied to the pelvic extremities of both Man and Apes.

Some, however, while denying that the term "hand" is applicable to the pes of Apes, go yet further and refuse to apply it even to the manus of those animals. BURDACH observes * that the term "hand" applies truly neither to the anterior nor to the posterior extremity of Apes; and just as the word "foot" may be so defined as to apply exclusively to the pes of Man, so, no doubt, it might be possible to frame such a definition of the word "hand" as that it should be applicable only to the human manus.

Every one knows that the hand of Man possesses a perfection of structure such as exists in the extremity of no other animal; but this perfection consists in a number of minute points and delicate distinctions; and in descending the order Primates we are led by small steps from this highly finished structure to the comparatively imperfect manus of Ateles or Hapale. Indeed the difference is small, both anatomically and physiologically, between Man and the highest Apes, as compared with that existing between the latter and lower forms; and it is with perfect justice that Professor HUXLEY remarks †, in speaking of the manus of the Marmoset, "There can be no doubt but that the hand is more different from that of the Gorilla than the Gorilla's hand is from Man's." If, therefore, the same term is to be applied to the manus of all Apes and Lemuroids, it is difficult to see how the hand of Man can reasonably be excluded.

Thus, then, anatomically the pes of Apes agrees in a far greater number of points with the foot of Man than with his hand, and similarly the Simian manus resembles his hand and differs from his foot. At the same time there is a similar physiological resemblance, as the manus throughout remains *the* prehensile organ, while the predominant function of the pes is constantly *locomotion*. Although, therefore, to avoid ambiguity, it would be well in scientific treatises to avoid entirely these disputed designations, and to employ instead well-defined and unmistakeable homological terms, such as "pes" and "manus;" yet, if the former *are* used, the conclusion appears to me irresistible, that of Apes and Lemuroids (as well as of Man) it must be said that each and all they are severally provided with "TWO HANDS AND A PAIR OF FEET."

* Beiträge zur vergleichenden Anatomie der Affen, last page.

† Man's Place in Nature, p. 93.

## DIMENSIONS AND PROPORTIONS.

The skeletons which have been measured for comparison are the following:—For Man, the skeleton No. 5569 in the Museum of the Royal College of Surgeons; for the Gorilla, No. 5178; Chimpanzee, No. 5082; Orang, No. 5050; Hylobates, Nos. 5026 and 5027; Colobus, No. 5008 A; and Semnopithecus, No. 5504,—all in the same collection. For Cercopithecus, a skeleton in my own collection; for Macacus, No. 4991 in the Museum of the Royal College of Surgeons; Cynocephalus, Nos. 4719 and 4720; Ateles, No. 4687; Lagothrix, No. A 4718 a; Cebus, No. 4671; Mycetes, No. 4718 b; and Pithecia, No. A 4670,—all in the same collection. For Brachyurus, No. 806 b; and for Callithrix, No. 969 a,—both in the Osteological Collection of the British Museum. For Chrysothrix, No. 4667, in the College of Surgeons Museum; for Nyctipithecus, No. 4665 A; Hapale, No. 4664 A; Indris, No. 4631; Lemur, No. 4661 A; Loris, No. 4633; Nycticebus, No. 4634 A; and Arctocebus, No. A 4632 a,—all in the same collection. For Perodicticus I have used the skeleton No. 743 c in the British Museum; and for Galago, No. 68 d, and Tarsius, No. 318 b, both in the same collection. Finally, for Cheiromys I have employed the skeleton in the Museum of the Royal College of Surgeons.

In estimating proportions, I have in general only employed one specimen of each genus; and therefore, as there is considerable individual variation, the proportions here given are offered merely as approximations to the true standard of each genus.

An average *, drawn from the comparison of a considerable number of specimens in each case, would have been more satisfactory; but, in the first place, materials for such an estimate are not as yet accessible, and in the second, even were they so, the expenditure of time would have been out of proportion to the result. I venture to think, therefore, that it may be left to such succeeding observers as may confine themselves to special groups, to rectify the results here given.

Following the happy idea started by Professor Huxley †, I have taken as my main standard of comparison (in estimating proportions) the vertebral column, estimating it by measuring it along its inferior (in Man anterior) curvature from the anterior (in Man upper) end of the atlas to the posterior (in Man lower) end of the sacrum.

The other dimensions given in the following Tables have been estimated as follows:—

The entire pectoral is measured from the summit of the head of the humerus to the distal end of the longest digit, whichever that may be.

* Such as is given by Mr. George Busk in his admirable paper "On the Cranial and Dental Characters of the existing Species of *Hyæna*," Journal of the Linnean Society, vol. ix. p. 59, 1866.

† Man's Place in Nature, p. 71. Dr. Lucae has not, unfortunately, pursued this plan, but in Man and the higher Apes he has estimated the spinal column by measuring from the atlas to the end of the coccyx, while in the lower forms he has measured to the end of the last caudal vertebra provided with a complete neural arch (*loc. cit.* p. 285). This divergence of mode necessitates a certain discrepancy between my results and those of Dr. Lucae, nevertheless a considerable correspondence exists between them.

The pectoral limb, minus the manus, is measured from the same point above to the anterior margin of the distal articular surface of the fore-arm.

The length of the scapula is estimated by a line drawn from the anterior (in Man superior) margin of the glenoid surface to the posterior (in Man inferior) vertebral angle.

The Humerus is measured from the summit of its head to the distal end of the ulnar (or inner) margin of the trochlea.

The Radius is measured from its head to the end of the styloid process.

The Ulna, from the end of the olecranon to that of the styloid process.

The Manus is measured from the distal margin of the radius to the extremity of the longest digit.

The length of the Carpus is estimated by a line drawn from the summit of the semilunare to the distal end of the magnum.

The length of the phalanges of the pollex, hallux, and third digits are given, as seen in skeletons, with the bones articulated together.

The Pelvic limb is measured from the summit of the head of the femur to the distal end of the longest digit, the pes being articulated, and the posterior part of the tarsus, of course, not counted.

The same, minus the pes, to the margin of the inferior surface of the shaft of the tibia.

The length of the os innominatum has been estimated by a line extending from the highest point of the crest of the ilium to the lowest one of the tuberosity of the ischium.

The conjugate diameter of the pelvis is measured from the anterior end of the symphysis pubis to the posterior (in Man inferior) margin of the first sacral vertebra.

Its transverse diameter is measured (wherever the brim of the true pelvis appears widest) in a line at right angles to the long axis of the trunk.

Its oblique diameter is estimated by a line extending from the ilio-pectineal eminence to the summit of the sacro-iliac synchondrosis of the opposite side.

The ilio-ischial angle No. I. is that formed by the superior (in Man posterior) margin of the ischium with the ilio-pectineal line.

The ilio-ischial angle No. II. is the one made by the same with the upper (in Man posterior) margin of the ilium.

The length of the femur is taken by measuring from its highest to its lowest extremity.

The tibia is measured to the lower end of the internal malleolus.

The length of the pes is taken from the distance between the end of the tuberosity of the os calcis and that of the longest digit.

That of the tarsus, from the posterior end of the os calcis to the distal margin of the ecto-cuneiform *.

* Dr. Lucae measures this segment only from the front of the articular surface for the tibia; hence there must necessarily be discrepancies between his estimates and mine.

LENGTH AND PROPORTION OF PECTORAL LIMB WITH AND WITHOUT THE MANUS.

| | Length from atlas to caudal end of sacrum. | Length of entire pectoral limb. | Pectoral limb —manus. | Spine : 100 :: entire pectoral limb : | Spine : 100 :: pectoral limb —manus : |
|---|---|---|---|---|---|
| | inches. | inches. | inches. | | |
| Man | 28·5 | 30·50 | 23·00 | 107·3 | 80·7 |
| T. Gorilla | 27·0 | 40·75 | 31·00 | 150·9 | 114·8 |
| T. niger | 22·0 | 31·25 | 22·00 | 142·0 | 100·0 |
| Simia | 21·5 | 36·70 | 26·70 | 170·6 | 124·1 |
| Hylobates, 27 | 12·3 | 25·00 | 19·10 | 203·2 | 155·2 |
| Hylobates, 26 | 10·9 | 24·20 | 18·20 | 222·0 | 166·9 |
| Colobus | 18·7 | 17·15 | 12·15 | 91·7 | 64·9 |
| Semnopithecus | 16·0 | 16·50 | 12·20 | 103·1 | 76·2 |
| Cercopithecus | 12·5 | 11·90 | 9·00 | 95·2 | 72·0 |
| Macacus | 12·8 | 14·80 | 11·20 | 115·6 | 87·5 |
| Cynocephalus, 19 | 21·3 | 25·85 | 19·75 | 121·3 | 92·7 |
| Cynocephalus, 20 | 20·6 | 22·10 | 16·70 | 107·2 | 81·0 |
| Ateles | 12·7 | 22·20 | 16·20 | 174·8 | 127·5 |
| Lagothrix | 13·2 | 16·80 | 12·50 | 127·2 | 94·6 |
| Cebus | 10·3 | 11·30 | 8·30 | 109·7 | 80·5 |
| Mycetes | 14·7 | 15·45 | 11·10 | 105·1 | 75·5 |
| Pithecia | 8·5 | 8·60 | 6·20 | 101·1 | 72·9 |
| Brachyurus | 9·7 | 10·40 | 7·25 | 107·2 | 74·7 |
| Nyctipithecus | 8·0 | 6·90 | 4·70 | 86·2 | 58·7 |
| Callithrix | 10·5 | ? | 6·40 | ? | 60·9 |
| Chrysothrix | 8·7 | 7·20 | 5·40 | 82·7 | 62·0 |
| Hapale | 6·0 | 5·00 | 3·50 | 83·3 | 58·3 |
| Indris | 18·0 | 16·55 | 11·25 | 91·9 | 62·5 |
| Lemur | 14·8 | 11·30 | 8·10 | 76·3 | 54·7 |
| Galago | 5·1 | 4·40 | 2·89 | 86·2 | 56·6 |
| Loris | 5·7 | 5·85 | 4·65 | 102·6 | 81·5 |
| Nycticebus | 6·7 | ? | ? | ? | ? |
| Perodicticus | 10·2 | 8·15 | 5·75 | 79·9 | 56·3 |
| Arctocebus | 6·9 | 5·20 | 3·90 | 75·3 | 56·5 |
| Tarsius | 3·1 | 5·80 | 4·00 | 187·1 | 129·0 |
| Cheiromys | 7·4 | 9·50 | 5·25 | 128·3 | 70·9 |

DIMENSIONS OF SCAPULA.

| Scapula of | Length from anterior end of glenoid surface to posterior vertebral angle. | Length of axillary margin. | Vertebral margin following curves. | Vertebral margin behind spine. | Vertebral margin measured by a straight line. | From margin of glenoid surface to vertebral end of spine. | Length of glenoid surface. | Breadth of glenoid surface. |
|---|---|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. |
| Man | 6·5 | 5·3 | 7·4 | 4·10 | 6·6 | 4·3 | 1·5 | 1·1 |
| T. Gorilla | 9·6 | 8·5 | 11·9 | 6·2 | 10·0 | 6·7 | 1·9 | 1·3 |
| T. niger | 6·6 | 5·9 | 6·0 | 3·3 | 6·4 | 5·0 | 1·3 | ·9 |
| Simia | 7·0 | 6·4 | 5·6 | 4·5 | 5·6 | 3·9 | 1·6 | 1·0 |
| Hylobates | 3·2 | 3·1 | 2·6 | 1·15 | 2·5 | 2·4 | .. | .. |
| Hylobates | 3·05 | 2·8 | .. | ·95 | 2·0 | 2·4 | .. | .. |
| Colobus | 3·7 | 3·3 | 3·7 | 2·1 | 3·0 | 2·7 | ·7 | ·4 |
| Semnopithecus | 3·2 | 2·6 | 3·2 | 1·8 | 2·4 | 2·6 | .. | .. |
| Cercopithecus | 2·9 | 2·5 | 2·1 | 1·45 | 1·9 | 2·4 | 4·8 | 3·4 |
| Macacus | 3·4 | 3·0 | 2·5 | 1·6 | 2·0 | 2·9 | ·8 | ·6 |
| Cynocephalus | 5·2 | 4·4 | 4·3 | 2·7 | 3·5 | 4·5 | .. | .. |
| Cynocephalus | 4·6 | 3·8 | 4·5 | 3·0 | 3·7 | 4·2 | .. | .. |
| Ateles | 3·3 | 3·0 | 3·1 | 1·6 | 2·8 | 2·1 | ·6 | ·45 |
| Lagothrix | 3·0 | 2·7 | 2·7 | 1·5 | 2·4 | 2·4 | ·6 | ·4 |
| Cebus | 2·4 | 2·2 | 2·1 | 1·2 | 1·6 | 1·9 | ·5 | ·3 |
| Mycetes | 3·4 | 3·0 | 3·1 | 1·8 | 2·9 | 2·4 | ·6 | ·4 |
| Pithecia | 1·6 | 1·4 | .. | 1·0 | 1·3 | 1·2 | ·3 | ·2 |
| Brachyurus | 2·0 | 1·8 | .. | 1·2 | 1·4 | 1·5 | ·4 | ·23 |
| Nyctipithecus | 1·35 | 1·2 | .. | ·65 | ·8 | 1·0 | .. | .. |
| Callithrix | 1·85 | 1·7 | .. | 1·1 | 1·4 | 1·4 | .. | .. |
| Chrysothrix | 1·7 | 1·5 | .. | ·9 | 1·1 | 1·3 | ·35 | ·20 |
| Hapale | 1·25 | 1·1 | ·90 | ·65 | ·8 | ·9 | .. | .. |
| Indris | 3·1 | 2·8 | 2·3 | 1·4 | 2·0 | 2·2 | ·6 | ·3 |
| Lemur | 2·5 | 2·3 | 1·2 | ·8 | 1·2 | 2·1 | ·55 | ·3 |
| Galago | 1·1 | 1·0 | .. | ·35 | ·5 | ·9 | .. | .. |
| Loris | 1·0 | ·9 | .. | ·55 | ·7 | ·8 | .. | .. |
| Nycticebus | 1·5 | 1·2 | 1·5 | ·95 | 1·2 | 1·1 | ·4 | ·2 |
| Perodicticus | 1·6 | 1·2 | 1·8 | 1·15 | 1·5 | 1·2 | ·45 | ·23 |
| Arctocebus | 1·1 | ·9 | 1·05 | ·6 | ·95 | ·85 | ·25 | ·15 |
| Tarsius | ·85 | ·80 | .. | ·3 | ·35 | ·75 | .. | .. |
| Cheiromys | 1·4 | 1·3 | ·80 | ·6 | ·78 | 1·2 | ·3 | ·2 |

DIMENSIONS AND PROPORTIONS OF SCAPULA.

| Scapula of | Length of anterior margin. | Spine : 100 :: length of scapula : ? | Axillary margin : 100 :: vertebral margin : | Axillary margin : 100 :: anterior margin : | Posterior vertebral angle. | Angle of glenoid surface with spine. | Angle of glenoid surface with axillary margin. | Angle of spine with vertebral margin. | Angle of spine with axillary margin. |
|---|---|---|---|---|---|---|---|---|---|
| | inches. | | | | ° | ° | ° | ° | ° |
| Man | 3·4 | 22·8 | 124·5 | 64·1 | 40 | 80 | 135 | 95 | 55 |
| T. Gorilla | 4·3 | 35·5 | 117·6 | 50·5 | 34 | 90 | 120 | 110 | 30 |
| T. niger | 2·4 | 30·0 | 108·4 | 40·6 | 22 | 95 | 125 | 125<br>135 | 20<br>24 |
| Simia | 3·2 | 31·4 | 87·5 | 50·0 | 35 | 80 | 110<br>118 | 104 | 41 |
| Hylobates | 1·75 | 26·0 | (86·8) | .. | 30<br>35 | 96 | 105 | 125 | 15 |
| Hylobates | 1·6 | 27·9 | 71·4 | 57·1 | 25 | 92 | 93 | 115 | 12 |
| Colobus | 2·48 | 19·8 | 90·9 | .. | 50 | 95 | 132 | 100 | 40 |
| Semnopithecus | 1·95 | .. | .. | .. | 50 | 102 | 145 | 80<br>85 | 32<br>45 |
| Cercopithecus | 2·18 | 23·2 | 76·0 | 87·2 | 50 | 97 | 130 | 97 | 34 |
| Macacus | 2·7 | 21·0 | 66·6 | 90·0 | 60 | 95 | 135<br>140 | 90 | 30<br>37 |
| Cynocephalus | 4·3 | .. | .. | .. | 63 | 95 | .. | .. | .. |
| Cynocephalus | 3·7 | 24·4 | 92·3 | 107·6 | 75 | 93 | 130 | 74<br>86 | 38 |
| Ateles | 1·3 | 25·9 | 93·1 | 48·2 | 30 | 95 | 110 | 127 | 20<br>25 |
| Lagothrix | 1·9 | 22·7 | 88·8 | 70·3 | 50 | 94 | 130 | 95 | 35 |
| Cebus | 1·5 | 23·3 | 72·7 | 68·1 | 45 | 85<br>95 | 130 | 103 | 30 |
| Mycetes | 2·0 | 23·1 | 96·6 | 66·6 | 48 | 85<br>95 | 123<br>115 | 95 | 40 |
| Pithecia | 1·0 | 18·8 | 92·8 | 71·4 | 45 | 80 | 120 | 95 | 45 |
| Brachyurus | 1·3 | 20·6 | 77·7 | 72·2 | 48 | 80<br>90 | 125 | 100 | 37 |
| Nyctipithecus | ·9 | 16·8 | 66·6 | 75·0 | 42 | 95 | 130 | 112 | 28 |
| Callithrix | 1·2 | 17·6 | 82·3 | 70·5 | 40 | 94 | 135 | 110 | 35 |
| Chrysothrix | 1·2 | 19·5 | 73·3 | 80·0 | 47 | 93 | 130 | 90 | 34 |
| Hapale | ·8 | 20·8 | 72·7 | 72·7 | 45 | 97 | 130 | 100 | 25 |
| Indris | 1·8 | 17·2 | 71·4 | 64·2 | 45 | 110 | 130 | 115 | 28 |
| Lemur | 1·9 | 16·9 | 52·1 | 82·6 | 48 | 110 | 126 | 120 | 20 |
| Galago | ·8 | 21·5 | 50·0 | 80·0 | 50 | 105 | 128 | 120 | 17 |
| Loris | ·7 | 17·5 | 77·7 | 77·7 | 49 | 92 | 130 | 112 | 37 |
| Nycticebus | ·9 | 22·3 | 100·0 | 75·0 | 43 | 90 | 130 | 110 | 42 |
| Perodicticus | 1·1 | 15·6 | 125·0 | 91·6 | 55 | 98 | 145 | 95 | 42 |
| Arctocebus | ·7 | 15·9 | 105·5 | 77·7 | 35 | 110 | 142 | 110 | 30 |
| Tarsius | ·75 | 27·4 | 43·7 | 87·5 | 65 | 97 | 120 | 95 | 24 |
| Cheiromys | 1·1 | 18·9 | 60·0 | 84·6 | 65 | 112 | 132 | 95 | 20 |

DIMENSIONS AND PROPORTIONS OF CLAVICLE.

| Clavicle of | Length following curves. | Length measured by a straight line. | Breadth at middle. | Spine : 100 :: length of clavicle : | Length of scapula : 100 :: that of clavicle : | Length of clavicle : 100 :: its breadth : |
|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | | | |
| Man | 6·8 | 6·0 | ·50 | 21·0 | 92·3 | 8·3 |
| T. Gorilla | 6·0 | 5·9 | ·60 | 21·8 | 61·4 | 10·1 |
| T. niger | 5·0 | 4·9 | ·50 | 22·2 | 74·2 | 10·2 |
| Simia | 6·8 | 6·5 | ·50 | 28·0 | 95·9 | 7·3 |
| Hylobates | 3·6 | 3·58 | .. | 29·1 | 111·8 | .. |
| Hylobates | 3·52 | 3·50 | ·25 | 32·1 | .. | .. |
| Colobus | 2·55 | 2·4 | ·20 | 12·8 | 64·8 | 8·3 |
| Semnopithecus | 2·55 | 2·4 | ·25 | .. | .. | .. |
| Cercopithecus | 2·0 | 1·83 | ·15 | 14·6 | 63·1 | 8·1 |
| Macacus | 2·4 | 2·2 | ·20 | 15·6 | 58·8 | 10·0 |
| Cynocephalus | 3·3 | 3·0 | ·30 | 14·0 | 57·6 | 10·5 |
| Cynocephalus | 3·0 | 2·7 | ·30 | .. | .. | 11·1 |
| Ateles | 2·65 | 2·45 | ·20 | 19·2 | 74·2 | 8·1 |
| Lagothrix | 2·7 | 2·6 | ·20 | 19·6 | 86·6 | 7·6 |
| Cebus | 1·8 | 1·5 | ·12 | 14·5 | 62·5 | 8·0 |
| Mycetes | 2·8 | 2·7 | ·12 | 18·3 | 79·4 | 4·4 |
| Pithecia | 1·3 | 1·2 | ·10 | 14·1 | 75·0 | 8·3 |
| Brachyurus | 1·5 | 1·4 | ·13 | 14·4 | 70·0 | 9·2 |
| Nyctipithecus | 1·1 | ·9 | ·09 | 11·2 | 60·6 | 10·0 |
| Callithrix | 1·4 | 1·25 | ·09 | 11·9 | 67·5 | 7·2 |
| Chrysothrix | 1·25 | 1·1 | ·08 | 12·6 | 64·7 | 7·2 |
| Hapale | ·85 | ·7 | ·08 | 11·6 | 56·0 | .. |
| Indris | 2·25 | 2·10 | ·15 | 11·6 | 67·7 | 7·1 |
| Lemur | 1·5 | 1·45 | ·15 | 9·7 | 58·0 | 10·3 |
| Galago | ·83 | ·74 | ·06 | 14·2 | 67·2 | 8·1 |
| Loris | 1·0 | ·8 | ·06 | 14·0 | 80·0 | 8·1 |
| Nycticebus | 1·15 | 1·05 | ·09 | 15·6 | 70·0 | 8·5 |
| Perodicticus | 1·4 | 1·3 | ·13 | 12·7 | 81·2 | 10·0 |
| Arctocebus | ·89 | ·80 | ·09 | 11·5 | 72·7 | .. |
| Tarsius | ·65 | ·45 | ·05 | 14·5 | 52·9 | 10·0 |
| Cheiromys | 1·2 | 1·1 | ·10 | 14·8 | 78·5 | 9·0 |

DIMENSIONS AND PROPORTIONS OF HUMERUS.

| Humerus of | Length from summit of head to bottom of inner margin of trochlea. | Breadth of shaft at its middle. | Breadth of both tuberosities. | Breadth between the condyles. | Spine : 100 :: length of humerus : | Length of scapula : 100 :: that of humerus : | Length of humerus : 100 :: breadth of its shaft : | Length of humerus : 100 :: width of its tuberosities : | Length of humerus : 100 :: breadth between its condyles : |
|---|---|---|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | inches. | | | | | |
| Man | 13·4 | ·85 | 1·8 | 2·55 | 47·0 | 206·1 | 6·3 | 13·4 | 19·0 |
| Gorilla | 17·4 | 1·25 | 2·2 | 3·55 | 64·4 | 181·2 | 7·1 | 12·6 | 20·4 |
| T. niger | 11·7 | ·9 | 1·65 | 2·7 | 53·1 | 177·2 | 7·6 | 14·1 | 23·0 |
| Simia | 13·8 | ·9 | 1·8 | 2·65 | 64·1 | 197·1 | 6·5 | 13·0 | 19·2 |
| Hylobates | 9·1 | ·35 | ·70 | 1·0 | 73·9 | 284·3 | 3·8 | 7·6 | 10·9 |
| Hylobates | 9·0 | .. | ·65 | .. | 82·5 | 295·0 | .. | .. | .. |
| Colobus | 6·25 | ·43 | ·92 | 1·07 | 33·4 | 169·9 | 6·8 | 14·7 | 17·1 |
| Semnopithecus | 5·75 | ·39 | ·83 | 1·12 | 35·9 | 179·6 | 6·7 | 14·4 | 19·4 |
| Cercopithecus | 4·52 | ·37 | ·65 | ·69 | 36·1 | 207·3 | 8·1 | 14·3 | 15·2 |
| Macacus | 5·75 | ·40 | ·90 | 1·2 | 44·9 | 169·1 | 6·9 | 15·6 | 20·8 |
| Cynocephalus | 9·2 | ·78 | 1·35 | 1·88 | 43·1 | 176·9 | 8·4 | 14·6 | 20·4 |
| Cynocephalus | 8·3 | ·72 | 1·35 | 1·88 | 40·2 | 180·4 | 8·6 | 16·2 | 22·6 |
| Ateles | 7·7 | ·35 | ·75 | 1·07 | 60·6 | 233·3 | 4·5 | 8·9 | 13·8 |
| Lagothrix | 6·65 | ·40 | ·79 | 1·05 | 52·3 | 221·6 | 6·0 | 11·8 | 15·7 |
| Cebus | 4·35 | ·29 | ·57 | ·82 | 42·2 | 181·2 | 6·6 | 13·1 | 18·8 |
| Mycetes | 5·8 | ·41 | ·85 | 1·15 | 39·6 | 170·5 | 7·0 | 14·6 | 19·8 |
| Pithecia | 3·30 | ·19 | ·44 | ·65 | 38·8 | 206·2 | 5·7 | 13·3 | 19·6 |
| Brachyurus | 4·15 | ·23 | ·54 | ·70 | 42·7 | 207·5 | 5·5 | 13·0 | 16·8 |
| Nyctipithecus | 2·51 | ·16 | ·32 | ·48 | 31·3 | 185·9 | 6·3 | 12·7 | 19·1 |
| Callithrix | 3·5 | ·20 | ·43 | ·61 | 33·3 | 189·1 | 5·7 | 12·2 | 17·4 |
| Chrysothrix | 2·8 | ·20 | ·39 | ·51 | 32·1 | 164·7 | 7·0 | 13·9 | 18·2 |
| Hapale | 1·8 | ·13 | ·28 | ·38 | 30·0 | 144·0 | 7·2 | 15·5 | 21·1 |
| Indris | 5·15 | ·32 | ·70 | 1·14 | 28·6 | 166·1 | 6·2 | 13·5 | 22·1 |
| Lemur | 4·05 | ·30 | ·62 | ·93 | 27·3 | 162·0 | 7·4 | 15·3 | 22·9 |
| Galago | 1·62 | ·11 | ·25 | ·40 | 31·7 | 147·2 | 6·7 | 15·4 | 24·6 |
| Loris | 2·25 | ·12 | ·28 | ·31 | 39·4 | 225·0 | 5·3 | 12·4 | 13·7 |
| Nycticebus | 2·45 | ·18 | ·36 | ·52 | 36·5 | 163·3 | 7·3 | 14·2 | 21·2 |
| Perodicticus | 2·90 | ·25 | ·50 | ·73 | 28·4 | 181·2 | 8·6 | 17·2 | 25·1 |
| Arctocebus | 2·05 | ·12 | ·28 | ·38 | 29·7 | 186·3 | 5·8 | 13·6 | 18·5 |
| Tarsius | 1·2 | ·10 | ·21 | ·31 | 38·7 | 141·1 | 8·3 | 17·5 | 25·8 |
| Cheiromys | 2·65 | ·23 | ·55 | ·78 | 35·8 | 189·2 | 8·6 | 20·7 | 29·4 |

DIMENSIONS AND PROPORTIONS OF RADIUS AND ULNA.

| Radius and ulna of | Length of radius. | Transverse diameter of its distal end. | Length of ulna. | Spine : 100 :: length of radius : | Length of humerus : 100 :: that of radius : |
|---|---|---|---|---|---|
| | inches. | inches. | inches. | | |
| Man | 9·90 | 1·40 | 10·85 | 34·7 | 73·8 |
| T. Gorilla | 14·20 | 1·80 | 15·00 | 52·5 | 81·6 |
| T. niger | 10·90 | 1·31 | 11·60 | 49·5 | 93·1 |
| Simia | 13·65 | 1·54 | 14·25 | 63·4 | 98·9 |
| Hylobates | 10·13 | ·61 | 10·30 | 82·3 | 111·3 |
| Hylobates | 9·55 | ·60 | 9·75 | 87·6 | 106·1 |
| Colobus | 5·68 | ·63 | 6·60 | 30·3 | 90·8 |
| Semnopithecus | 6·25 | ·64 | 7·00 | 39·0 | 108·6 |
| Cercopithecus | 4·4 | ·45 | 5·00 | 35·2 | 97·3 |
| Macacus | 5·50 | ·62 | 6·25 | 42·9 | 95·6 |
| Cynocephalus | 10·18 | ·95 | 11·00 | 47·7 | 110·6 |
| Cynocephalus | 8·35 | ·99 | 9·25 | 40·5 | 100·6 |
| Ateles | 8·00 | ·60 | 8·70 | 62·9 | 103·8 * |
| Lagothrix | 5·90 | ·53 | 6·70 | 44·6 | 88·7 |
| Cebus | 3·68 | ·41 | 4·20 | 35·7 | 84·5 |
| Mycetes | 5·40 | ·54 | 6·10 | 36·7 | 93·1 |
| Pithecia | 2·80 | ·31 | 3·20 | 32·9 | 84·8<br>95·2 |
| Brachyurus | 3·15 | ·39 | 3·73 | 32·4 | 75·9 |
| Nyctipithecus | 2·25 | ·23 | 2·50 | 28·1 | 89·6 |
| Callithrix | 2·95 | ·31 | 3·30 | 28·0 | 84·2 |
| Chrysothrix | 2·60 | ·29 | 3·00 | 29·8 | 92·8 |
| Hapale | 1·50 | ·16 | 1·87 | 25·0 | 83·3 |
| Indris | 6·00 | ·52 | 6·80 | 33·3 | 116·5 |
| Lemur | 3·71 | ·40 | 4·60 | 25·0 | 91·6 |
| Galago | 1·67 | ·15 | 1·90 | 32·7 | 103·0 |
| Loris | 2·48 | ·17 | 2·64 | 43·5 | 110·2 |
| Nycticebus | 2·30 | ·23 | 2·60 | 34·3 | 93·8 |
| Perodicticus | 3·10 | ·36 | 3·50 | 30·3 | 106·8 |
| Arctocebus | 2·10 | ·17 | 2·23 | 30·4 | 102·4 |
| Tarsius | 1·55 | ·14 | 1·72 | 50·0 | 129·1 |
| Cheiromys | 2·50 | ·35 | 3·00 | 33·7 | 94·3 |

* Sometimes in Ateles the radius is shorter than the humerus.

DIMENSIONS AND PROPORTIONS OF MANUS.

| Manus of | Length of manus. | Spine : 100 :: manus : | Rest of pectoral limb : 100 :: manus : | Radius : 100 :: manus : | Length of carpus. | Spine : 100 :: length of carpus : | Length of carpus : 100 :: its breadth : |
|---|---|---|---|---|---|---|---|
| | inches. | | | | inches. | | |
| Man | 7·50 | 26·3 | 32·6 | 75·7 | 1·4 | 4·5 | 161·5 |
| T. Gorilla | 9·75 | 36·1 | 31·4 | 68·6 | 1·5 | 5·5 | 166·6 |
| T. niger | 9·25 | 42·0 | 42·0 | 84·8 | 1·25 | 5·6 | 160·0 |
| Simia | 10·00 | 46·5 | 37·4 | 73·2 | 1·42 | 6·6 | 147·8 |
| Hylobates | 5·90 | 47·9 | 30·8 | 58·2 | .. | .. | .. |
| Hylobates | 6·00 | 55·0 | 32·9 | 62·8 | ·73 | 6·6 | 82·1 |
| Colobus | 5·00 | 26·7 | 41·1 | 88·6 | ·65 | 3·4 | 138·4 |
| Semnopithecus | 4·30 | 26·8 | 35·2 | 68·8 | ·80 | 5·0 | 113·7 |
| Cercopithecus | 2·90 | 23·2 | 32·2 | 65·9 | ·53 | 4·2 | .. |
| Macacus | 3·60 | 28·1 | 32·1 | 65·4 | ·60 | 4·6 | 166·6 |
| Cynocephalus | 6·10 | 28·6 | 30·8 | 59·9 | 1·08 | 5·07 | 150·9 |
| Cynocephalus | 5·40 | 26·2 | 32·3 | 64.6 | ·93 | 4·5 | 154·8 |
| Ateles | 6·00 | 47·2 | 37·0 | 75·0 | ·65 | 5·1 | 107·6 |
| Lagothrix | 4·30 | 32·5 | 34·4 | 72·8 | ·59 | 4·4 | 144·0 |
| Cebus | 3·00 | 29·1 | 36·1 | 81·5 | ·43 | 4·1 | 125·5 |
| Mycetes | 4·35 | 29·5 | 39·1 | 80·5 | ·65 | 4·4 | 135·3 |
| Pithecia | 2·40 | 28·2 | 38·7 | 85·7 | ·45 | 5·2 | 113·3 |
| Brachyurus | 3·15 | 32·4 | 43·4 | 100·0 | ·45 | 4·6 | 144·4 |
| Nyctipithecus | 2·20 | 27·5 | 46·8 | 97·7 | ·28 | 3·5 | 142·8 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. |
| Chrysothrix | 1·80 | 20·6 | 33·3 | 69·2 | ·34 | 3·9 | 126·4 |
| Hapale | 1·50 | 25·0 | 42·8 | 100·0 | ·21 | 3·5 | 157·1 |
| Indris | 5·30 | 29·4 | 47·1 | 88·3 | ·58 | 3·2 | 134·5 |
| Lemur | 3·20 | 21·6 | 39·5 | 86·2 | ·57 | 3·8 | 128·0 |
| Galago | 1·51 | 29·6 | 46·6 | 90·4 | ·17 | 3·4 | 142·8 |
| Loris | 1·20 | 21·0 | 25·8 | 48·3 | ·18 | 3·1 | 144·4 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | 2·40 | 23·6 | 41·7 | 77·4 | ·44 | 4·3 | 96·6 |
| Arctocebus | 1·30 | 18·8 | 33·3 | 63·7 | ·25 | 3·6 | 96·0 |
| Tarsius | 1·80 | 58·1 | 45·0 | 116·1 | .. | .. | .. |
| Cheiromys | 4·25 | 57·4 | 80·9 | 170·0 | ·50 | 6·7 | 114·0 |

DIMENSIONS OF MANUS.

| Manus of | Length of first metacarpal. | Length of second metacarpal. | Length of third metacarpal. | Length of fourth metacarpal. | Length of fifth metacarpal. | Length of first phalanx of pollex. | Length of second phalanx of pollex. | First phalanx of third digit. | Second phalanx of third digit. | Third phalanx of third digit. | Length of index with metacarpal. | Length of longest digit with metacarpal. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. |
| | | | | | | | | | | | | 3rd. |
| Man | 1·74 | 2·70 | 2·60 | 2·30 | 1·97 | 1·23 | ·97 | 1·80 | 1·24 | ·83 | 6·0 | 6·47 |
| T. Gorilla | 1·82 | 3·75 | 3·72 | 3·48 | 3·30 | 1·10 | ·75 | 2·29 | 1·60 | ·82 | 7·65 | 8·43 |
| T. niger | 1·30 | 3·38 | 3·45 | 3·23 | 2·98 | 1·11 | ·80 | 2·25 | 1·74 | ·83 | 7·25 | 8·27 |
| Simia | 2·00 | 3·81 | 3·93 | 3·67 | 3·37 | 1·10 | ·60 | 2·80 | 1·74 | ·85 | 8·35 | 9·32 |
| Hylobates | 1·11 | 2·26 | 2·20 | 1·98 | 1·77 | ·70 | ·36 | 1·66 | 1·13 | ·50 | 5·18 | 5·49 |
| Hylobates | 1·20 | 2·23 | 2·09 | 1·85 | 1·62 | ·78 | ·42 | 1·72 | 1·18 | ·52 | 5·37 | 5·51 |
| Colobus | ·67 | 1·62 | 1·70 | 1·63 | 1·61 | ·22 | .. | 1·31 | ·99 | ·43 | 3·80 | 4·43 |
| Semnopithecus | ·81 | 1·67 | 1·76 | 1·70 | 1·69 | ·49 | ·17 | 1·18 | ·73 | ·32 | 3·30 | 3·99 |
| Cercopithecus | ·63 | 1·03 | ·96 | ·92 | ·87 | ·35 | ·20 | ·70 | ·45 | ·27 | 2·20 | 2·38 |
| Macacus | ·90 | 1·40 | 1·39 | 1·33 | 1·29 | ·50 | ·24 | ·96 | ·63 | ·30 | 3·10 | 3·28 |
| Cynocephalus | 1·60 | 2·32 | 2·30 | 2·30 | 2·36 | ·90 | ·50 | 1·44 | ·90 | ·48 | 4·77 | 5·12 |
| Cynocephalus | 1·40 | 2·06 | 2·00 | 1·97 | 2·00 | ·80 | ·42 | 1·35 | ·85 | ·52 | 4·40 | 4·72 |
| Ateles | ·91 | 1·84 | 1·92 | 1·84 | 1·80 | ? | .. | 1·48 | 1·03 | ·52 | 4·55 | 4·95 |
| Lagothrix | ·81 | 1·11 | 1·26 | 1·25 | 1·12 | ·73 | ·47 | 1·23 | ·82 | ·46 | 3·40 | 3·77 |
| Cebus | ·65 | ·84 | ·90 | ·86 | ·72 | ·51 | ·33 | ·82 | ·53 | ·35 | 2·31 | 2·60 |
| Mycetes | ·87 | 1·20 | 1·23 | 1·19 | 1·16 | ·73 | ·49 | 1·28 | ·81 | ·42 | 3·40 | 3·74 |
| | | | | | | | | | | | | 4th. |
| Pithecia | ·50 | ·64 | ·70 | ·71 | ·63 | ·37 | ·24 | ·70 | ·44 | ·21 | 1·69 | 2·09 |
| Brachyurus | ·60 | ·72 | ·81 | ·81 | ·70 | ·50 | .. | ·83 | ·58 | .. | .. | .. |
| | | | | | | | | | | | | 3rd. |
| Nyctipithecus | ·37 | ·55 | ·58 | ·56 | ·48 | ·38 | ·17 | ·60 | ·42 | ·15 | 1·55 | 1·75 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 1·27 |
| Chrysothrix | ·43 | ·57 | ·58 | ·56 | ·50 | ·40 | ·17 | ·54 | ·39 | ·18 | 1·57 | 1·69 |
| Hapale | ·31 | ·41 | ·45 | ·43 | ·35 | ·29 | ·19 | ·39 | ·27 | ·18 | 1·18 | 1·29 |
| | | | | | | | | | | | | 4th. |
| Indris | 1·10 | 1·67 | 1·87 | 1·85 | 1·84 | ·98 | ·42 | 1·60 | ·91 | ·40 | 3·80 | 4·82 |
| Lemur | ·63 | ·97 | ·98 | ·91 | ·88 | ·61 | ·24 | ·96 | ·61 | ·23 | 2·58 | 2·80 |
| Galago | ·29 | ·34 | ·42 | ·39 | ·34 | ·26 | ·11 | ·45 | ·39 | ? | ·89 | ? |
| Loris | ·27 | ·23 | ·31 | ·30 | ·25 | ·23 | ·11 | ·34 | ·21 | ·10 | ·72 | 1·00 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | ·41 | ·40 | ·65 | ·63 | ·61 | ·40 | ·28 | ·69 | ·31 | ·20 | ·80 | 2·05 |
| Arctocebus | ·30 | ·23 | ·32 | ·32 | ·30 | ·26 | ·20 | ·28 | ·15 | ·10 | ·44 | 1·11 |
| | | | | | | | | | | | | 3rd. |
| Tarsius | ·37 | ·44 | ·50 | ·43 | ·36 | ·28 | ·19 | ·62 | ·45 | ·13 | 1·44 | 1·70 |
| | | | | | | | | | | | | 4th. |
| Cheiromys | ·43 | ·61 | 1·12 | ·80 | ·66 | ·62 | ·49 | 1·46 | ·68 | ·24 | 2·72 | 3·77 |

PROPORTIONS OF MANUS.

| Manus of | Spine : 100 :: pollex with metacarpal : | Spine : 100 :: longest digit with metacarpal : | Spine : 100 :: index with metacarpal : | Spine : 100 :: metacarpal of pollex : | Metacarpal of pollex : 100 :: metacarpal of index : | Metacarpal of third digit : 100 :: its proximal phalanx : | Longest digit : 100 :: pollex with metacarpal : | Manus : 100 :: third metacarpal : | Manus : 100 :: pollex with metacarpal : | Manus : 100 :: first phalanx of third digit : | Manus : 100 :: pollex without its metacarpal : | Manus : 100 :: third digit without its metacarpal : | Longest metacarpal : 100 :: longest digit without metacarpal : |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Man | 13·8 | 22·7 | 21·0 | 6·1 | 155·1 | 69·2 | 60·8 | 34·6 | 52·5 | 24·0 | 29·3 | 51·6 | 143·3 |
| T. Gorilla | 13·5 | 31·2 | 28·3 | 6·7 | 206·0 | 61·5 | 43·5 | 39·1 | 37·6 | 23·4 | 18·9 | 48·3 | 125·6 |
| T. niger | 14·5 | 37·5 | 32·9 | 5·9 | 260·0 | 65·2 | 38·8 | 38·3 | 35·6 | 25·0 | 21·2 | 53·5 | 139·7 |
| Simia | 17·2 | 43·3 | 38·8 | 9·3 | 190·5 | 71·2 | 39·6 | 39·3 | 37·0 | 28·0 | 17·0 | 53·9 | 137·1 |
| Hylobates | 17·6 | 44·6 | 42·1 | 9·0 | 203·6 | 75·4 | 33·4 | 35·4 | 35·0 | 26·7 | 17·0 | 53·0 | 145·5 |
| Hylobates | 20·0 | 50·5 | 49·2 | 11·0 | 185·8 | 82·2 | 36·8 | 32·6 | 37·5 | 26·8 | 18·7 | 53·4 | 153·3 |
| Colobus | 4·7 | 23·1 | 20·3 | 3·5 | 241·7 | 77·0 | 20·0 | 34·0 | 17·8 | 26·2 | 4·4 | 54·6 | 160·5 |
| Semnopithecus | 9·1 | 24·9 | 20·6 | 5·0 | 206·1 | 65·1 | 36·8 | .. | 34·1 | 26·9 | 17·2 | 51·3 | 126·7 |
| Cercopithecus | 9·4 | 19·2 | 17·6 | 5·0 | 155·5 | 72·9 | 48·9 | 33·1 | 40·6 | 24·2 | 25·8 | 48·9 | 137·8 |
| Macacus | 12·8 | 25·6 | 24·2 | 7·8 | 165·5 | 69·0 | 50·0 | 38·6 | 45·5 | 26·6 | 20·5 | 52·5 | 135·0 |
| Cynocephalus | 14·0 | 24·0 | 22·3 | 7·5 | 145·0 | 62·6 | 58·5 | 37·7 | 49·1 | 23·6 | 22·9 | 46·2 | 121·5 |
| Cynocephalus | 12·7 | 22·7 | 21·3 | 6·7 | 147·1 | 66·0 | 54·6 | 37·0 | 48·5 | 24·4 | 22·5 | 49·8 | 132·0 |
| Ateles | 7·1 | 38·9 | 35·8 | 7·1 | 202·1 | 77·0 | 18·3 | 32·0 | 15·1 | 24·6 | .. | 50·5 | 157·1 |
| Lagothrix | 15·2 | 28·5 | 25·7 | 6·1 | 137·0 | 97·6 | 53·3 | 29·3 | 46·7 | 28·6 | 27·9 | 58·3 | 199·2 |
| Cebus | 14·4 | 25·2 | 22·4 | 6·3 | 129·2 | 91·1 | 57·3 | 30·0 | 49·6 | 27·3 | 28·0 | 56·6 | 188·8 |
| Mycetes | 14·2 | 25·4 | 23·1 | 5·9 | 137·9 | 104·0 | 55·8 | 29·0 | 48·6 | 29·7 | 28·3 | 58·3 | 204·0 |
| Pithecia | 13·0 | 24·5 | 19·8 | 5·8 | 128·0 | 102·8 | 53·1 | 29·5 | 46·2 | 30·4 | 25·4 | 57·5 | 200·0 |
| Brachyurus | .. | .. | .. | 6·1 | 120·0 | 102·4 | .. | 25·7 | .. | 26·3 | .. | .. | .. |
| Nyctipithecus | 11·5 | 21·8 | 19·3 | 4·6 | 148·6 | 103·4 | 52·5 | 26·4 | 41·8 | 27·2 | 24·1 | 53·1 | 201·7 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Chrysothrix | 11·4 | 19·4 | 18·0 | 4·9 | 132·5 | 93·1 | 59·1 | 32·2 | 55·5 | 30·0 | 31·6 | 61·6 | 191·3 |
| Hapale | 13·1 | 21·5 | 19·6 | 5·1 | 132·2 | 86·6 | 61·2 | 30·0 | 52·6 | 26·0 | 32·0 | 56·0 | 186·6 |
| Indris | 13·8 | 27·6 | 21·1 | 6·1 | 151·8 | 85·5 | 51·8 | 36·4 | 47·1 | 30·1 | 26·4 | 54·9 | 158·8 |
| Lemur | 10·0 | 18·9 | 17·4 | 4·2 | 153·9 | 97·9 | 52·8 | 30·6 | 44·6 | 30·0 | 26·5 | 56·2 | 192·8 |
| Galago | 12·9 | .. | 17·4 | 5·6 | 117·2 | 107·1 | .. | 27·8 | 43·7 | 29·8 | 24·5 | .. | 250·0 |
| Loris | 10·7 | 17·5 | 12·6 | 4·7 | 85·1 | 109·6 | 61·0 | 25·8 | 50·8 | 28·3 | 28·3 | 54·1 | 225·8 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | 10·6 | 20·1 | 7·8 | 4·0 | 97·5 | 106·1 | 53·1 | 27·0 | 45·4 | 28·7 | 28·3 | 50·0 | 218·4 |
| Arctocebus | 11·0 | 16·0 | 6·3 | 4·3 | 76·6 | 87·5 | 68·4 | 24·6 | 58·4 | 21·5 | 35·3 | 40·7 | 246·8 |
| Tarsius | 27·0 | 54·8 | 46·4 | 11·9 | 118·9 | 124·0 | 49·4 | 27·7 | 46·6 | 34·4 | 26·1 | 66·6 | 254·0 |
| Cheiromys | 20·8 | 50·9 | 36·7 | 5·8 | 141·8 | 130·3 | 74·2 | 26·3 | 36·2 | 34·3 | 26·1 | 56·0 | 265·1 |

LENGTH AND PROPORTIONS OF PELVIC LIMB WITH AND WITHOUT THE PES.

| Pelvic limb of | Length of entire pelvic limb. | Pelvic limb —pes. | Spine : 100 :: entire pelvic limb: | Spine : 100 :: pelvic limb—pes : | Entire pectoral limb : 100 :: entire pelvic limb: | Pectoral limb—manus : 100 :: pelvic limb—pes : |
|---|---|---|---|---|---|---|
| | inches. | inches. | | | | |
| Man | 41·20 | 33·40 | 144·5 | 117·1 | 135·0 | 145·2 |
| T. Gorilla | 34·82 | 26·50 | 128·9 | 98·1 | 85·4 | 85·4 |
| T. niger | 28·00 | 21·00 | 127·2 | 95·4 | 90·3 | 95·4 |
| Simia | 30·25 | 19·60 | 140·6 | 91·1 | 82·4 | 73·4 |
| Hylobates | 19·95 | 15·45 | 162·1 | 125·6 | 78·8 | 80·8 |
| Hylobates | 18·50 | 13·80 | 169·7 | 126·6 | 75·2 | 75·8 |
| Colobus | 22·15 | 15·70 | 118·4 | 83·9 | 129·1 | 129·2 |
| Semnopithecus | 21·80 | 15·93 | .. | 85·1 | 132·1 | 130·5 |
| Cercopithecus | 14·60 | 10·70 | 116·8 | 85·6 | 122·6 | 118·8 |
| Macacus | 17·71 | 12·91 | 138·3 | 100·8 | 119·6 | 115·2 |
| Cynocephalus | 26·90 | 20·30 | 126·3 | 95·3 | 104·0 | 102·7 |
| Cynocephalus | 24·10 | 18·35 | 116·9 | 89·0 | 109·0 | 109·8 |
| Ateles | 20·81 | 14·90 | 163·8 | 101·5 | 93·7* | 91·9 |
| Lagothrix | 17·70 | 12·90 | 134·0 | 97·7 | 105·3 | 103·2 |
| Cebus | 13·58 | 9·96 | 131·8 | 96·6 | 120·1 | 120·0 |
| Myeetes | 16·50 | 11·77 | 112·2 | 80·0 | 106·7 | 106·0 |
| Pithecia | 11·15 | 7·90 | 131·1 | 92·9 | 129·6 | 127·4 |
| Brachyurus | 13·10 | 9·28 | 135·0 | 95·6 | 125·9 | 128·0 |
| Nyctipithecus | 9·33 | 6·43 | 116·6 | 80·3 | 135·2 | 136·8 |
| Callithrix | .. | 8·81 | .. | 83·9 | .. | 137·6 |
| Chrysothrix | 9·70 | 6·78 | 111·4 | 79·3 | 134·7 | 127·7 |
| Hapale | 6·65 | 4·63 | 110·8 | 77·1 | 133·0 | 132·2 |
| Indris! | 23·93 | 17·40 | 132·9 | 96·6 | 144·5 | 154·6 |
| Lemur | 14·32 | 10·88 | 96·7 | 73·5 | 126·7 | 134·3 |
| Galago | 7·38 | 5·05 | 144·7 | 99·0 | 167·7 | 174·7 |
| Loris | 6·63 | 5·03 | 116·3 | 88·2 | 113·3 | 108·1 |
| Nyeticebus | .. | 5·42 | .. | 80·8 | .. | .. |
| Perodicticus | 9·17 | 6·72 | 89·9 | 65·8 | .. | 116·8 |
| Arctocebus | 5·88 | 4·51 | 85·2 | 65·3 | 113·0 | 115·6 |
| Tarsius | 7·57 | 5·08 | 244·1 | 163·8 | 130·5 | 127·0 |
| Cheiromys | 10·47 | 6·99 | 140·5 | 94·4 | 110·2 | 133·1 |

* Sometimes, as in No. 4690 in the Museum of the Royal College of Surgeons, the entire pelvic limb is slightly longer than the entire pectoral one.

DIMENSIONS OF OS INNOMINATUM.

| Os innominatum of | Length from summit of crest of ilium to bottom of tuberosity of ischium. | Crest of ilium following curves. | Crest of ilium measured by a straight line. | From crest of ilium to spine of ischium. | From spine of ischium to nearest point of its tuberosity. | From ilio-pectineal eminence to tuberosity of ischium. | Length of symphysis. | Antero-posterior diameter of acetabulum. | Conjugate diameter of pelvis. | Oblique diameter of pelvis. | Transverse diameter of pelvis. | Ilio-pubic angle. | Ilio-ischial angle I.* | Ilio-ischial angle II. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | inches. | | | |
| Man | 9·30 | 9·20 | 6·50 | 3·80 | 1·45 | 4·60 | 1·40 | 2·19 | 4·50 | 4·80 | 4·75 | 180° | 113° | 140° |
| T. Gorilla | 14·75 | 11·60 | 8·50 | 8·80 | 3·1 | 5·90 | 2·00 | 2·11 | 7·00 | 6·00 | 5·30 | 130 | 150 | 180 |
| T. niger | 11·50 | 6·00 | 4·25 | 6·95 | 1·80 | 3·90 | 1·80 | 1·49 | 5·40 | 5·00 | 4·00 | 120 | 172 | 180 |
| Simia | 9·85 | 6·50 | 5·00 | 5·50 | 1·80 | 3·48 | 1·80 | 1·73 | 4·50 | 4·50 | 3·71 | 125 | 153 | 173 |
| Hylobates | 4·75 | 2·75 | 1·83 | 3·10 | ·60 | 1·62 | 1·20 | ·78 | 3·00 | 2·75 | 2·25 | 110 | 140 | .. |
| Hylobates | 4·75 | 2·40 | 1·63 | 3·25 | ·60 | 1·78 | 1·25 | ·70 | 2·95 | 2·50 | 1·64 | 130 | 147 | 147 |
| Colobus | 5·50 | 1·82 | 1·45 | 3·45 | ·40 | 2·00 | 1·66 | ·70 | 2·00 | 2·20 | 1·93 | 102 | 140 | 152 |
| Semnopithecus | 5·46 | 1·75 | 1·26 | 3·63 | ·53 | 2·10 | 1·76 | ·84 | 2·12 | 2·12 | 1·57 | 110 | 160 | 162 |
| Cercopithecus | 4·36 | 1·25 | 1·02 | 2·78 | ·68 | 1·60 | ·94 | ·51 | .. | .. | .. | 110 | 162 | 170 |
| Macacus | 5·40 | 1·70 | 1·33 | 3·13 | ·84 | 2·02 | 1·35 | ·68 | 2·12 | 2·01 | 1·80 | 108 | 164 | 178 |
| Cynocephalus | 7·87 | 2·75 | 2·20 | 4·75 | 1·25 | 2·75 | 2·15 | 1·00 | 2·72 | 3·12 | 2·64 | 110 | 160 | 165 |
| Cynocephalus | 7·80 | 3·00 | 2·50 | 4·70 | 1·40 | 3·10 | 2·40 | 1·16 | 3·00 | 2·92 | 2·50 | 110 | 162 | 185 |
| Ateles | 4·93 | 2·20 | 1·54 | 2·93 | 1·00 | 1·68 | ·90 | ·70 | 2·93 | 2·65 | 1·95 | 125 | 175 | 167<br>180 |
| Lagothrix | 4·85 | 1·75 | 1·09 | 3·35 | ·53 | 1·75 | ·93 | ·70 | 2·82 | 2·43 | 1·70 | 113 | 170 | 185 |
| Cebus | 3·31 | 1·00 | ·73 | 2·36 | ·60 | 1·40 | ·80 | ·43 | 1·58 | 1·50 | 1·28 | 107 | 187 | 192 |
| Mycetes | 4·50 | 1·55 | 1·04 | 3·08 | ·60 | 1·73 | ·90 | ·69 | 2·60 | 2·52 | 1·84 | 115 | 175 | 178 |
| Pithecia | 2·20 | ·65 | ·50 | 1·60 | ·36 | ·90 | ·60 | ·44 | 1·00 | 1·05 | ·64 | 125 | 180 | 195 |
| Brachyurus | 2·79 | ·85 | ·70 | 1·93 | ·64 | 1·28 | ·60 | ·50 | 1·47 | 1·35 | ·82 | 125 | 180 | 207 |
| Nyctipithecus | 1·97 | ·44 | ·34 | 1·41 | ·35 | ·90 | ·39 | ·32 | ·86 | ·91 | ·66 | 115 | 184 | 187 |
| Callithrix | 2·61 | ·70 | ·56 | 1·84 | ·40 | 1·10 | ·60 | ·42 | 1·10 | 1·09 | ·91 | 130 | 160 | 175 |
| Chrysothrix | 2·25 | ·70 | ·55 | 1·60 | ·28 | ·84 | ·51 | ·29 | ·88 | 1·03 | ·86 | 125 | 160 | 180 |
| Hapale | 1·50 | ·41 | ·34 | 1·09 | ·21 | ·65 | ·49 | ·19 | ·60 | ·72 | ·52 | 115 | 160 | 165 |
| Indris | 4·52 | 2·00 | 1·55 | 3·25 | ·60 | 1·88 | 1·06 | ·81 | 1·88 | 1·80 | 1·57 | 135 | 137 | 165 |
| Lemur | 3·70 | ·80 | ·62 | 2·64 | ·47 | 1·54 | ·63 | ·61 | 1·50 | 1·60 | 1·45 | 120 | 165 | 165 |
| Galago | 1·52 | ·30 | ·29 | 1·19 | ·19 | ·60 | ·24 | ·22 | ·90 | ·74 | ·50 | 115 | 166 | 162 |
| Loris | 1·46 | ·31 | ·30 | 1·10 | ·08 | ·50 | ·13 | ·23 | ·74 | ·71 | ·33 | 88 | 170 | 180 |
| Nycticebus | 2·02 | ·40 | ·33 | 1·54 | ·17 | ·79 | ·31 | ·30 | 1·35 | 1·11 | ·71 | 108 | 174 | 170 |
| Perodicticus | 2·65 | ·46 | ·45 | 2·03 | ·19 | 1·01 | ·18 | ·46 | 1·69 | 1·34 | 1·09 | 119 | 166 | 172 |
| Arctocebus | 1·67 | ·32 | ·30 | 1·29 | ·11 | ·54 | ·18 | ·24 | 1·15 | ·98 | ·61 | 117 | 168 | 160 |
| Tarsius | 1·17 | ·24 | ·24 | ·93 | ·14 | ·43 | ·19 | ·17 | ·70 | ·68 | ·48 | 98 | 172 | 166 |
| Cheiromys | 2·30 | ·51 | ·41 | 1·49 | ·48 | 1·09 | ·46 | ·46 | ·94 | ·89 | ·70 | 125 | 165 | 164 |

* I. = angle formed by ischium with ilio-pectineal line. II. angle formed by ischium with upper (in Man posterior) margin of ilium.

PROPORTIONS OF OS INNOMINATUM.

| Os innominatum of | Spine : 100 :: extreme length of os innominatum : | Spine : 100 :: crest of ilium measured by a straight line : | Spine : 100 :: length from ilio-pectineal eminence to tuberosity of ischium : | Length of os innominatum : 100 :: antero-posterior diameter of acetabulum : | Spine : 100 :: distance between inferior (in Man anterior) spinous processes : | Spine : 100 : length of symphysis pubis : | Breadth of pelvis : 100 :: length of os innominatum : | Conjugate diameter of pelvis : 100 :: its breadth : | Length of femur : 100 :: that of os innominatum : |
|---|---|---|---|---|---|---|---|---|---|
| Man | 32·6 | 22·8 | 16·1 | 23·5 | 7·0 | 4·8 | 195·7 | 105·5 | 50·2 |
| T. Gorilla | 54·6 | 31·4 | 21·8 | 14·3 | 16·6 | 7·4 | 278·3 | 75·5 | 101·0 |
| T. niger | 52·2 | 19·3 | 17·7 | 12·9 | 22·7 | 8·1 | 287·5 | 74·0 | 100·0 |
| Simia | 45·8 | 23·2 | 16·1 | 17·5 | 16·0 | 8·3 | 265·4 | 82·0 | 96·5 |
| Hylobates | 38·6 | 14·8 | 13·1 | 16·4 | 16·2 | 9·7 | 211·1 | 75·0 | 56·8 |
| Hylobates | 43·5 | 14·9 | 16·3 | 14·7 | 16·5 | 11·4 | 289·6 | 55·5 | 64·1 |
| Colobus | 29·4 | 7·7 | 10·6 | 12·7 | 13·1 | 8·8 | 284·9 | 96·5 | 67·9 |
| Semnopithecus | 34·1 | 7·8 | 13·1 | 15·3 | 16·5 | 11·0 | 347·7 | 74·0 | 65·0 |
| Cercopithecus | 34·8 | 8·0 | 12·8 | 11·6 | .. | 7·4 | .... | .. | 81·1 |
| Macacus | 42·1 | 10·3 | 15·7 | 12·5 | 18·9 | 10·5 | 300·0 | 84·9 | 81·0 |
| Cynocephalus | 36·9 | 10·1 | 12·9 | 12·7 | 19·7 | 10·0 | 298·1 | 97·0 | 71·5 |
| Cynocephalus | 37·8 | 12·1 | 15·0 | 14·8 | 18·2 | 11·1 | 312·0 | 83·3 | 78·0 |
| Ateles | 38·8 | 12·1 | 13·2 | 14·1 | 17·7 | 7·0 | 252·8 | 66·5 | 63·2 |
| Lagothrix | 36·7 | 8·2 | 13·7 | 14·4 | 16·4 | 7·0 | 285·2 | 60·2 | 71·3 |
| Cebus | 32·1 | 7·0 | 13·5 | 12·9 | 16·5 | 7·7 | 258·5 | 81·0 | 64·2 |
| Mycetes | 30·6 | 7·0 | 11·7 | 15·3 | 14·1 | 6·1 | 244·5 | 70·7 | 71·7 |
| Pithecia | 25·8 | 5·8 | 10·8 | 20·0 | 11·7 | 7·0 | 343·7 | 64·0 | 53·6<br>55·6 |
| Brachyurus | 28·7 | 7·2 | 13·1 | 17·9 | 13·5 | 6·1 | 340·2 | 55·7 | 58·1 |
| Nyctipithecus | 24·6 | 4·2 | 11·2 | 16·2 | 10·6 | 4·8 | 298·4 | 76·7 | 60·6 |
| Callithrix | 24·8 | 5·3 | 10·4 | 16·0 | 10·6 | 5·7 | 286·8 | 82·7 | 59·1 |
| Chrysothrix | 25·8 | 6·3 | 9·6 | 12·9 | 12·8 | 5·8 | 261·6 | 97·7 | 64·6 |
| Hapale | 25·0 | 5·6 | 10·8 | 12·6 | 10·1 | 8·1 | 288·4 | 86·6 | 66·6 |
| Indris | 25·1 | 8·6 | 10·4 | 17·9 | 11·3 | 5·8 | 287·9 | 83·5 | 48·0 |
| Lemur | 25·0 | 4·1 | 10·4 | 16·4 | 13·8 | 4·2 | 258·7 | 95·3 | 63·7 |
| Galago | 29·8 | 5·6 | 11·7 | 14·4 | 17·2 | 4·7 | 304·0 | 55·5 | 56·7 |
| Loris | 25·6 | 5·2 | 8·7 | 15·7 | 14·9 | 2·2 | 442·4 | 44·5 | 57·2 |
| Nycticebus | 30·1 | 4·7 | 11·7 | 14·8 | 15·9 | 4·6 | 274·6 | 52·5 | 70·3 |
| Perodicticus | 25·9 | 4·4 | 9·9 | 17·3 | 13·7 | 1·7 | 243·1 | 64·0 | 77·9 |
| Arctocebus | 24·2 | 4·3 | 7·8 | 14·3 | 14·0 | 2·5 | 273·7 | 53·0 | 71·3 |
| Tarsius | 37·7 | 7·7 | 13·8 | 14·5 | 22·9 | 6·1 | 243·7 | 68·5 | 46·0 |
| Cheiromys | 31·0 | 5·5 | 14·7 | 20·0 | 13·1 | 6·2 | 328·5 | 74·4 | 67·3 |

DIMENSIONS OF FEMUR.

| Femur of | Length of femur. | Transverse diameter at its middle. | Antero-posterior diameter at the same part. | Extreme width at the condyles. | Angle formed by neck with shaft. | Angle formed by shaft with horizon. |
|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | inches. | ° | ° |
| Man | 18·50 | 1·05 | 1·13 | 3·15 | 135 | 103 |
| T. Gorilla | 14·60 | 1·57 | 1·26 | 3·56 | 128 | 96 |
| T. niger | 11·50 | 1·10 | ·83 | 2·30 | 135 or 140 | 90 |
| Simia | 10·20 | ·90 | ·76 | 2·30 | 155 | 98 |
| Hylobates | 8·35 | ·38 | ·37 | 1·02 | 135 or 145 | 92 |
| Hylobates | 7·40 | ·37 | ·39 | ·93 | 130 | .. |
| Colobus | 8·10 | ·47 | ·49 | 1·14 | 135 | .. |
| Semnopithecus | 8·40 | ·42 | ·48 | 1·22 | 135 | 97 |
| Cercopithecus | 5·37 | ·37 | ·33 | ·74 | 135 | 93 |
| Macacus | 6·66 | ·40 | ·46 | 1·06 | 145 | 92 |
| Cynocephalus | 11·00 | ·70 | ·72 | 1·49 | 133 | .. |
| Cynocephalus | 10·00 | ·71 | ·74 | 1·61 | 130 | 98 |
| Ateles | 7·80 | ·45 | ·42 | 1·17 | 145 | 95 or 99 |
| Lagothrix | 6·80 | ·40 | ·42 | ·94 | 135 | 95 |
| Cebus | 5·15 | ·29 | ·30 | ·76 | 135 | 95 |
| Mycetes | 6·27 | ·47 | ·37 | 1·02 | 137 | 97 |
| Pithecia | 4·10 | ·22 | ·20 | ·64 | 145 | 95 |
| Brachyurus | 4·80 | ·24 | ·23 | ·72 | 140 | 91 |
| Nyctipithecus | 3·25 | ·17 | ·16 | ·49 | 140 | .. |
| Callithrix | 4·41 | ·22 | ·21 | ·60 | 140 | 93 |
| Chrysothrix | 3·48 | ·20 | ·20 | ·46 | 138 | .. |
| Hapale | 2·25 | ·14 | ·14 | ·34 | 130 | .. |
| Indris | 9·40 | ·47 | ·48 | 1·00 | 125 | 90 |
| Lemur | 5·80 | ·34 | ·36 | ·79 | 132 | 92 or 103 |
| Galago | 2·68 | ·14 | ·15 | ·30 | 145 | 90 |
| Loris | 2·55 | ·13 | ·12 | ·33 | 145 | 95 |
| Nycticebus | 2·87 | ·19 | ·19 | ·46 | 145 | 95 |
| Perodicticus | 3·40 | ·27 | ·27 | ·53 | 145 | 92 |
| Arctocebus | 2·34 | ·13 | ·14 | ·34 | 145 | 95 |
| Tarsius | 2·54 | ·10 | ·13 | ·22 | 135 | 94 |
| Cheiromys | 3·43 | ·23 | ·22 | ·63 | 140 | 95 |

PROPORTIONS OF FEMUR.

| Femur of | Spine : 100 : : length of femur : | Humerus : 100 : : length of femur : | Length of femur : 100 : : its breadth at condyles : | Length of femur : 100 : : breadth of its shaft : |
|---|---|---|---|---|
| Man ..... | 64·9 | 138·0 | 17·0 | 5·6 |
| T. Gorilla .... | 54·0 | 83·9 | 24·3 | 10·7 |
| T. niger ..... | 52·2 | 98·2 | 20·0 | 9·5 |
| Simia ..... | 47·4 | 73·9 | 22·5 | 8·8 |
| Hylobates .... | 67·8 | 91·7 | 12·2 | 4·5 |
| Hylobates .... | 67·8 | 82·2 | 12·5 | 5·0 |
| Colobus ..... | 43·3 | 129·6 | 14·0 | 5·8 |
| Semnopithecus . | 52·5 | 146·1 | 14·5 | 5·0 |
| Cercopithecus .. | 42·9 | 118·8 | 13·7 | 6·8 |
| Macacus..... | 52·0 | 115·8 | 15·9 | 6·9 |
| Cynocephalus .. | 51·6 | 119·5 | 13·5 | 6·3 |
| Cynocephalus .. | 48·5 | 120·4 | 16·1 | 7·1 |
| Ateles..... | 61·4 | 101·3 | 15·0 | 5·7 |
| Lagothrix .... | 51·5 | 102·2 | 13·8 | 5·8 |
| Cebus..... | 50·0 | 118·3 | 14·7 | 5·6 |
| Mycetes ..... | 42·6 | 109·0 | 16·2 | 7·4 |
| Pithecia ..... | 48·2 | 124·2 | 15·6 | 5·3 |
| Brachyurus .. | 49·4 | 115·6 | 15·0 | 5·0 |
| Nyctipithecus .. | 40·6 | 129·4 | 15·0 | 5·2 |
| Callithrix .... | 42·0 | 126·0 | 13·6 | 4·9 |
| Chrysothrix .. | 40·0 | 124·2 | 13·2 | 5·7 |
| Hapale ..... | 37·5 | 125·8 | 15·1 | 6·2 |
| Indris..... | 52·2 | 182·5 | 10·6 | 5·0 |
| Lemur ..... | 39·1 | 143·2 | 13·6 | 5·8 |
| Galago ..... | 52·5 | 165·4 | 11·1 | 5·2 |
| Loris ..... | 44·7 | 113·3 | 12·9 | 5·0 |
| Nycticebus .... | 42·8 | 117.1 | 16·0 | 6·6 |
| Perodicticus .. | 33·3 | 117·2 | 15·5 | .. |
| Arctocebus .... | 33·9 | 114·1 | 14·5 | 5·5 |
| Tarsius ..... | 81·9 | 211·6 | 8·6 | 3·9 |
| Cheiromys .... | 46·3 | 129·4 | 18·3 | 6·7 |

DIMENSIONS AND PROPORTIONS OF TIBIA.

| Tibia of | Length of tibia. | Greatest breadth between tuberosities. | Antero-posterior diameter of shaft. | Spine : 100 : : length of tibia : | Femur : 100 : : length of tibia : | Humerus : 100 : : length of tibia : | Radius : 100 : : length of tibia : | Length of tibia : 100 : : antero-posterior diameter of shaft : | Length of tibia : 100 : : width at tuberosities : |
|---|---|---|---|---|---|---|---|---|---|
| | inches. | inches. | inches. | | | | | | |
| Man ..... | 14·90 | 2·87 | 1·61 | 52·2 | 80·5 | 111·1 | 150·5 | 10·8 | 19·2 |
| T. Gorilla .... | 11·90 | 3·40 | 1·74 | 44·0 | 81·5 | 68·4 | 83·8 | 14·6 | 28·5 |
| T. niger ..... | 9·50 | 2·33 | 1·17 | 43·1 | 82·6 | 81·0 | 87·1 | 12·3 | 24·5 |
| Simia ..... | 9·40 | 2·30 | 1·16 | 43·7 | 92·1 | 68·1 | 68·8 | 12·3 | 24·4 |
| Hylobates .... | 7·10 | 1·02 | ·50 | 57·7 | 85·0 | 78·0 | 70·0 | 7·0 | 14·3 |
| Hylobates .... | 6·40 | ·93 | .50 | 58·7 | 86·4 | 71·1 | 67·0 | 7·8 | 14·5 |
| Colobus ..... | 7·60 | 1·17 | ·70 | 40·6 | 93·8 | 121·6 | 133·8 | 9·2 | 15·4 |
| Semnopithecus . | 7·53 | 1·23 | ·72 | 47·0 | 89·6 | 130·9 | 120·4 | 9·5 | 16·3 |
| Cercopithecus .. | 5·08 | ·71 | ·40 | 40·6 | 94·5 | 112·3 | 115·4 | 7·8 | 13·9 |
| Macacus ..... | 6·25 | 1·11 | ·53 | 48·8 | 93·8 | 108·6 | 113·6 | 8·4 | 17·7 |
| Cynocephalus .. | 9·30 | 1·57 | ·89 | 43·6 | 84·5 | 101·0 | 91·3 | 9·5 | 16·8 |
| Cynocephalus .. | 8·35 | 1·66 | ·91 | 40·5 | 83·5 | 100·6 | 100·0 | 10·8 | 19·8 |
| Ateles ..... | 7·10 | 1·12 | ·57 | 55·9 | 91·0 | 92·2 | 88·7 | 8·0 | 15·6 |
| Lagothrix .... | 6·10 | ·99 | ·52 | 46·2 | 89·7 | 91·6 | 103·3 | 8·5 | 16·2 |
| Cebus ..... | 4·81 | ·74 | ·44 | 46·7 | 93·3 | 110·5 | 130·7 | 9·1 | 15·3 |
| Mycetes ..... | 5·50 | ·99 | ·58 | 37·7 | 87·7 | 94·8 | 101·8 | 10·5 | 18·0 |
| Pithecia ..... | 3·80 | ·61 | ·27 | 44·7 | 92·6 / 94·6 | 115·1 | 135·7 | 7·1 | 16·0 |
| Brachyurus .... | 4·48 | ·74 | ·32 | 46·1 | 93·3 | 107·9 | 142·2 | 7·1 | 16·5 |
| Nyctipithecus .. | 3·18 | ·48 | ·28 | 39·7 | 97·8 | 126·6 | 141·3 | 8·8 | 15·0 |
| Callithrix .... | 4·40 | ·56 | ·31 | 41·9 | 99·7 | 125·7 | 149·1 | 7·0 | 12·7 |
| Chrysothrix .. | 3·30 | ·47 | ·29 | 37·9 | 94·8 | 117·8 | 126·9 | 8·7 | 14·2 |
| Hapale ..... | 2·38 | ·30 | ·20 | 39·6 | 105·7 | 132·2 | 158·6 | 8·4 | 12·6 |
| Indris ..... | 8·00 | 1·06 | ·65 | 44·4 | 85·1 | 155·3 | 133·3 | 8·1 | 13·2 |
| Lemur ..... | 5·08 | ·78 | ·50 | 34·0 | 87·5 | 125·4 | 136·9 | 9·8 | 15·3 |
| Galago ..... | 2·37 | ·41 | ·20 | 46·4 | 88·3 | 146·2 | 141·9 | 8·4 | 17·3 |
| Loris ..... | 2·48 | ·32 | ·15 | 43·5 | 97·2 | 110·2 | 100·0 | 6·0 | 12·9 |
| Nycticebus .... | 2·55 | ·47 | ·17 | 38·0 | 81·8 | 104·0 | 110·8 | 6·6 | 18·4 |
| Perodicticus .. | 3·32 | ·60 | ·27 | 32·5 | 97·6 | 114·4 | 107·0 | 8·1 | 18·0 |
| Arctocebus .... | 2·17 | ·30 | ·13 | 31·4 | 92·7 | 105·8 | 103·3 | 5·9 | 13·8 |
| Tarsius ..... | 2·54 | ·25 | ·18 | 81·9 | 100·0 | 211·6 | 163·8 | 7·0 | 9·8 |
| Cheiromys .... | 3·56 | ·59 | ·24 | 48·1 | 103·7 | 131·3 | 142·4 | 6·7 | 16·5 |

DIMENSIONS AND PROPORTIONS OF PES.

| Pes of | Length of pes. | Spine : 100 :: pes : | Rest of pelvic limb : 100 :: pes : | Tibia : 100 :: pes : | Manus : 100 :: pes : | Length of tarsus. | Spine : 100 :: tarsus : | Pes : 100 :: tarsus : | Length of tarsus : 100 :: its breadth : | Carpus : 100 :: tarsus : | Tarsus : 100 :: longest digit without metatarsal : |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | inches. | | | | | inches. | | | | | |
| Man | 10·10 | 35·4 | 30·2 | 67·7 | 134·6 | 4·70 | 16·4 | 46·5 | 48·0 | 335·6 | 50·0 |
| T. Gorilla | 11·20 | 41·4 | 42·2 | 94·1 | 114·9 | 4·45 | 16·4 | 39·7 | 56·1 | 296·6 | 81·3 |
| T. niger | 8·85 | 40·2 | 42·1 | 93·1 | * 95·6 / 101·8 | 3·20 | 14·5 | 36·1 | 62·5 | 256·0 | 94·6 |
| Simia | 11·50 | 53·4 | 58·6 | 122·3 | 115·0 | 3·07 | 14·2 | 26·6 | 56·9 | 216·1 | 163·8 |
| Hylobates | 5·00 | 40·3 | 32·3 | 70·4 | 84·7 | 1·40 | 11·3 | 28·0 | 50·0 | .. | 150·0 |
| Hylobates | 5·13 | 47·0 | 37·1 | 80·1 | 85·5 | 1·40 | 12·8 | 27·2 | 55·0 | 191·7 | 157·8 |
| Colobus | 7·17 | 38·3 | 45·6 | 94·3 | 143·4 | 2·19 | 11·7 | 30·4 | 43·1 | 336·9 | 125·5 |
| Semnopithecus | 6·84 | 42·7 | 42·9 | 90·8 | 159·0 | 2·30 | 14·3 | 33·6 | 42·6 | 287·5 | 97·3 |
| Cercopithecus | 4·62 | 36·9 | 43·1 | 90·9 | 159·3 | 1·53 | 12·2 | 33·1 | .. | 288·6 | 106·7 |
| Macacus | 6·00 | 46·8 | 46·4 | 96·0 | 166·6 | 1·91 | 14·9 | 31·8 | 50·7 | 318·3 | 113·6 |
| Cynocephalus | 8·10 | 37·7 | 39·9 | 86·0 | 131·1 | 2·67 | 12·5 | 32·9 | 53·5 | 247·2 | 102·6 |
| Cynocephalus | 7·40 | 35·9 | 40·3 | 88·6 | 137·0 | 2·61 | 12·6 | 35·2 | 60·5 | 280·6 | 90·4 |
| Ateles | 6·82 | 53·7 | 45·1 | 96·0 | 113·6 | 1·84 | 14·4 | 26·9 | 53·2 | 283·0 | 152·1 |
| Lagothrix | 5·70 | 43·1 | 44·1 | 93·4 | 132·5 | 1·80 | 13·6 | 31·5 | 41·1 | 305·0 | 134·8 |
| Cebus | 4·40 | 42·7 | 44·1 | 91·4 | 146·6 | 1·42 | 13·7 | 32·2 | 40·1 | 330·2 | 119·7 |
| Mycetes | 5·70 | 38·7 | 48·4 | 103·6 | 131·0 | 1·70 | 11·5 | 29·8 | 49·4 | 261·5 | 138·8 |
| Pithecia | 3·72 | 43·7 | 47·0 | 97·8 | 155·0 | 1·17 | 13·7 | 31·4 | 45·2 | 260·0 | 123·0 |
| Brachyurus | 4·30 | 44·3 | 46·3 | 95·9 | 136·5 | 1·40 | 14·4 | 32·5 | 42·1 | 311·1 | .. |
| Nyctipithecus | 3·23 | 40·3 | 50·2 | 101·5 | 146·8 | 1·01 | 12·6 | 31·2 | 36·0 | 360·7 | 122·7 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Chrysothrix | 3·20 | 36·7 | 47·1 | 96·9 | 177·7 | ·97 | 11·1 | 30·3 | 39·1 | 285·2 | 125·7 |
| Hapale | 2·35 | 39·1 | 50·7 | 98·7 | 156·6 | ·66 | 11·0 | 28·0 | 39·3 | 330·0 | 131·8 |
| Indris | 7·05 | 39·1 | 40·5 | 88·1 | 133·0 | 1·92 | 10·6 | 27·2 | 45·8 | 331·0 | 163·5 |
| Lemur | 4·50 | 30·4 | 41·3 | 88·5 | 140·6 | 1·60 | 10·8 | 35·5 | 40·0 | 280·7 | 111·2 |
| Galago | 2·57 | 50·3 | 50·8 | 108·4 | 170·1 | 1·24 | 24·3 | 48·2 | 16·9 | 723·5 | 76·6 |
| Loris | 1·82 | 31·9 | 36·1 | 73·3 | 151·6 | ·58 | 10·1 | 31·8 | 44·8 | 322·2 | 170·6 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Arctocebus | 1·64 | 23·7 | 36·3 | 75·5 | 126·1 | ·50 | 7·2 | 30·4 | 48·0 | 200·0 | 164·0 |
| Tarsius | 2·72 | 87·7 | 53·5 | 107·0 | 151·1 | 1·22 | 39·3 | 44·8 | 15·5 | .. | 80·3 |
| Cheiromys | 4·02 | 54·3 | 57·5 | 115·5 | 94·5 | 1·23 | 16·6 | 30·5 | 34·9 | 246·0 | 152·0 |

* In the skeleton No. 5083 A. the pes is a little longer than the manus, not so in Nos. 5082 & 5084.

DIMENSIONS OF PES.

| Pes of | Length of first metatarsal. | Length of second metatarsal. | Length of third metatarsal. | Length of fourth metatarsal. | Length of fifth metatarsal. | First phalanx of hallux. | Second phalanx of hallux. | First phalanx of third digit. | Second phalanx of third digit. | Third phalanx of third digit. | Length of index with its metatarsal. | Length of longest digit with its metatarsal. | Length of os calcis. | Dorsum of navicular. | Dorsum of cuboid. | Length of longest digit without metatarsal. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. | inch. |
| Man | 2·44 | 3·00 | 2·82 | 2·70 | 2·95 | 1·50 | 1·03 | 1·00 | ·48 | ·47 | 5·35 | 5·35 (2nd) | 3·17 | ·62 | 1·27 | 2·35 |
| T. Gorilla | 2·46 | 3·20 | 3·18 | 3·11 | 3·40* | 1·32 | ·85 | 1·77 | 1·20 | ·70 | 6·50 | 6·80 (3rd) | 3·26 | ·50 | ·93 | 3·62 |
| T. niger | 2·08 | 2·71 | 2·62 | 2·35 | 2·63 | 1·20 | ·90 | 1·48 | ·95 | ·60 | 5·60 | 5·65 | 2·40 | ·40 | ·80 | 3·03 |
| Simia | 1·95 | 3·70 | 3·60 | 3·30 | 3·18 | ·98 | ? | 2·63 | 1·51 | ·80 | 8·25 | 8·63 | 2·10 | ·46 | ·80 | 5·03 |
| Hylobates | 1·33 | 1·72 | 1·59 | 1·49 | 1·45 | ·62 | ·37 | 1·09 | ·68 | ·33 | 3·53 | 3·69 | ·94 | ·23 | ·47 | 2·10 |
| Hylobates | 1·36 | 1·67 | 1·57 | 1·47 | 1·43 | ·68 | ·49 | 1·11 | ·70 | ·40 | 3·53 | 3·78 | ·89 | ·20 | ·40 | 2·21 |
| Colobus | 1·34 | 2·25 | 2·44 | 2·40 | 2·34 | ·62 | ·30 | 1·30 | ·95 | ·50 | 4·32 | 5·19 | 1·44 | ·31 | ·48 | 2·75 |
| Semnopithecus | 1·34 | 2·20 | 2·28 | 2·32 | 2·42 | ·55 | ·24 | 1·18 | ·75 | ·31 | 4·12 | 4·52 | 1·60 | ·31 | ·54 | 2·24 |
| Cercopithecus | ·93 | 1·39 | 1·49 | 1·44 | 1·30 | ·45 | ·29 | ·86 | ·46 | ·31 | 2·70 | 3·12 | 1·03 | ·22 | ·38 | 1·63 |
| Macacus | 1·29 | 1·79 | 1·89 | 1·86 | 1·85 | ·65 | ·30 | 1·07 | ·70 | ·40 | 3·55 | 4·06 | 1·32 | ·27 | ·49 | 2·17 |
| Cynocephalus | 2·13 | 2·56 | 2·67 | 2·60 | 2·67 | ·94 | ·53 | 1·54 | ·88 | ·41 | 5·13 | 5·41 | 1·84 | ·37 | ·72 | 2·74 |
| Cynocephalus | 1·80 | 2·30 | 2·38 | 2·28 | 2·27 | ·81 | ·50 | 1·19 | ·82 | ·35 | 4·69 | 4·74 | 1·83 | ·32 | ·64 | 2·36 |
| Ateles | 1·29 | 2·15 | 2·03 | 2·00 | 2·04 | ·80 | ·46 | 1·40 | ·90 | ·50 | 4·69 | 4·83 | 1·43 | ·33 | ·49 | 2·80 |
| Lagothrix | 1·13 | 1·62 | 1·62 | 1·61 | 1·60 | ·73 | ·40 | 1·19 | ·80 | ·42 | 3·80 | 4·03 | 1·25 | ·28 | ·36 | 2·41 |
| Cebus | ·90 | 1·23 | 1·32 | 1·29 | 1·28 | ·50 | ·34 | ·88 | ·54 | ·28 | 2·86 | 3·02 | ·99 | ·15 | ·37 | 1·70 |
| Mycetes | 1·21 | 1·60 | 1·71 | 1·70 | 1·58 | ·72 | ·43 | 1·12 | ·79 | ·45 | 3·80 | 4·07 | 1·21 | ·25 | ·43 | 2·36 |
| Pithecia | ·75 | 1·09 | 1·18 | 1·20 | 1·23 | ·42 | ·28 | ·71 | ·50 | ·25 | 2·30 | (4th) 2·64 | ·83 | ·20 | ·23 | 1·44 |
| Brachyurus | ·79 | 1·29 | 1·34 | 1·39 | 1·50 | ·54 | .. | ·82 | ·58 | .. | .. | .. | ·94 | ·23 | ·35 | .. |
| Nyctipithecus | ·60 | 1·00 | 1·00 | 1·01 | 1·01 | ·40 | ·20 | ·61 | ·43 | ·20 | 2·09 | (3rd) 2·24 | ·65 | ·20 | ·27 | 1·24 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | ·85 | .. | .. | |
| Cryosothrix | ·63 | 1·03 | 1·03 | 1·05 | 1·09 | ·39 | ·23 | ·58 | ·41 | ·23 | 2·12 | 2·25 | ·68 | ·22 | ·26 | 1·22 |
| Hapale | ·41 | ·72 | ·82 | ·83 | ·84 | ·24 | ·11 | ·42 | ·28 | ·17 | 1·52 | 1·70 | ·47 | ·12 | ·19 | ·87 |
| Indris | 1·95 | 2·00 | 2·04 | 2·07 | 2·02 | 1·15 | ·60 | 1·71 | 1·05 | ·44 | .. | (4th) 5·21 | 1·37 | ·31 | ·53 | 3·14 |
| Lemur | 1·06 | 1·25 | 1·21 | 1·14 | 1·15 | ·61 | ·32 | ·94 | ·62 | ·22 | 2·81 | 2·92 | 1·00 | ·24 | ·46 | 1·78 |
| Galago | ·45 | ·42 | ·43 | ·40 | ·42 | ·30 | ·17 | ·46 | ·27 | ·10 | 1·13 | 1·35 | ·97 | ·69 | ·26 | ·95 |
| Loris | ·47 | ·45 | ·47 | ·43 | ·44 | ·27 | ·18 | ·51 | ·34 | ·14 | 1·11 | 1·42 | ·36 | ·12 | ·17 | ·99 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | ·60 | ·58 | ·68 | ·66 | ·65 | ·45 | ·31 | ·71 | ·41 | ·24 | 1·40 | 2·24 | ·57 | ·21 | ·26 | 1·58 |
| Arctocebus | ·41 | ·33 | ·35 | ·34 | ·33 | ·29 | ·21 | ·34 | ·18 | ·13 | ·90 | 1·16 | ·34 | ·09 | ·14 | ·82 |
| Tarsius | ·46 | ·41 | ·52 | ·54 | ·50 | ·31 | ·13 | ·42 | ·20 | ·11 | ·95 | 1·52 | 1·13 | ·83 | ·10 | ·98 |
| Cheiromys | ·87 | ·91 | ·98 | ·99 | 1·00 | ·48 | ·30 | ·87 | ·60 | ·40 | 2·50 | 2·86 | ·85 | ·19 | ·34 | 1·87 |

* Including the backwardly extending process, otherwise the fifth metatarsal is shorter than the fourth.

PROPORTIONS OF PES.

| Pes of | Spine : 100 :: hallux with metatarsal : | Spine : 100 :: longest digit with metatarsal : | Spine : 100 :: index with metatarsal : | Spine : 100 :: metatarsal of hallux : | Longest digit : 100 :: hallux with metatarsal : | Pes : 100 :: second metatarsal : | Pes : 100 :: hallux with metatarsal : | Pes : 100 :: hallux without metatarsal : | Pes : 100 :: third digit without metatarsal : | Longest digit of manus with metacarpal : 100 :: that of pes with metatarsal : | Longest metatarsal : 100 :: longest digit without metatarsal : | Pollex : 100 :: hallux (with anterior. with metatarsal) : | Index of manus : 100 :: that of pes : | Spine : 100 :: os calcis : | Os calcis : 100 :: cuboides : |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Man | 17·4 | 18·7 | 18·7 | 8·5 | 92·8 | 29·7 | 49·2 | 25·0 | 19·3 | 82·6 | 78·3 | 126·1 | 89·1 | 11·1 | 40·0 |
| T. Gorilla | 17·1 | 25·1 | 24·0 | 9·1 | 68·0 | 28·5 | 41·3 | 19·3 | 32·7 | 80·6 | 113·1 | 126·1 | 84·9 | 12·0 | 28·5 |
| T. niger | 18·2 | 25·5 | 25·4 | 9·4 | 73·9 | 30·6 | 47·2 | 25·1 | 34·2 | 68·3 | 111·8 | 130·2 | 77·2 | 10·9 | 33·3 |
| Simia | 13·6 | 40·1 | 38·3 | 9·0 | 33·9 | 32·1 | 25·4 | 8·5 | 42·9 | 92·5 | 135·9 | 79·1 | 98·8 | 9·7 | 38·1 |
| Hylobates | 18·8 | 30·0 | 28·6 | 10·8 | 62·8 | 34·4 | 46·2 | 19·8 | 42·0 | 67·2 | 122·0 | 106·9 | 68·1 | 7·6 | 50·0 |
| Hylobates | 23·2 | 34·6 | 32·3 | 12·4 | 66·9 | 32·5 | 49·3 | 22·8 | 43·0 | 68·6 | 132·3 | 105·4 | 65·7 | 8·1 | 44·9 |
| Colobus | 12·0 | 27·7 | 23·1 | 7·1 | 43·5 | 31·3 | 31·5 | 12·5 | 38·3 | 117·3 | 112·7 | 253·9 | 113·6 | 7·7 | 33·3 |
| Semnopithecus | 13·3 | 28·2 | 25·7 | 8·3 | 47·1 | 32·1 | 31·1 | 11·5 | 32·7 | 113·2 | 96·5 | 144·8 | 124·8 | 10·0 | 33·7 |
| Cercopithecus | 13·3 | 24·9 | 21·6 | 7·4 | 53·5 | 30·0 | 36·1 | .. | 35·7 | 131·0 | 109·3 | 141·5 | 122·0 | 8·1 | 36·8 |
| Macacus | 18·1 | 31·7 | 27·7 | 10·0 | 57·1 | 29·8 | 38·8 | 15·8 | 36·1 | 123·7 | 114·8 | 141·4 | 114·5 | 10·3 | 37·1 |
| Cynocephalus | 16·9 | 25·4 | 24·0 | 10·0 | 66·5 | 31·6 | 44·4 | 18·1 | 33·7 | 105·6 | 102·4 | 120·0 | 107·5 | 8·6 | 39·1 |
| Cynocephalus | 15·1 | 23·0 | 22·2 | 8·7 | 65·6 | 31·0 | 42·0 | 17·7 | 31·8 | 101·0 | 99·1 | 118·7 | 106·5 | 8·8 | 34·9 |
| Ateles | 20·0 | 38·0 | 36·9 | 10·1 | 52·7 | 31·5 | 37·3 | 18·4 | 41·0 | 97·5 | 130·2 | 280·2 | 103·0 | 11·2 | 34·2 |
| Lagothrix | 17·1 | 30·5 | 28·7 | 8·5 | 56·0 | 28·4 | 39·6 | 19·8 | 42·2 | 106·8 | 148·7 | 112·4 | 111·7 | 9·4 | 28·8 |
| Cebus | 16·8 | 29·3 | 27·6 | 8·7 | 57·6 | 27·9 | 39·2 | 19·0 | 38·6 | 116·1 | 128·7 | 116·7 | 123·8 | 9·6 | 37·3 |
| Mycetes | 16·0 | 27·6 | 25·8 | 8·2 | 57·9 | 28·0 | 41·4 | 20·1 | 41·4 | 108·8 | 138·0 | 112·9 | 111·7 | 8·2 | 35·5 |
| Pithecia | 17·0 | 31·0 | 27·0 | 8·8 | 54·9 | 29·3 | 38·9 | 18·8 | 38·7 | 126·3 | 120·0 | 130·6 | 136·0 | 9·7 | 25·3 |
| Brachyurus | .. | .. | .. | 8·1 | .. | 30·0 | .. | .. | .. | .. | .. | .. | .. | 9·6 | 37·2 |
| Nyctipithecus | 15·0 | 28·0 | 26·1 | 7·5 | 53·5 | 30·9 | 37·1 | 18·5 | 37·7 | 128·0 | 122·7 | 132·6 | 134·8 | 8·1 | 41·5 |
| Callithrix | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 8·1 | .. |
| Chrysothrix | 14·3 | 25·8 | 24·3 | 7·2 | 55·1 | 32·1 | 39·1 | 19·3 | 38·1 | 133·1 | 116·1 | 125·0 | 135·0 | 7·8 | 38·2 |
| Hapale | 12·6 | 28·3 | 25·3 | 6·8 | 44·7 | 30·6 | 32·3 | 14·8 | 37·0 | 132·5 | 106·0 | 96·2 | 128·8 | 7·8 | 40·4 |
| Indris | 20·5 | 28·9 | .. | 10·8 | 71·0 | 28·3 | 52·4 | 24·8 | 45·3 | 108·0 | 151·6 | 148·0 | .. | 7·6 | 38·6 |
| Lemur | 13·4 | 19·7 | 18·9 | 7·1 | 68·1 | 27·7 | 44·2 | 20·6 | 39·5 | 104·2 | 142·4 | 134·4 | 108·9 | 6·7 | 46·0 |
| Galago | 18·0 | 26·4 | 22·1 | 8·8 | 68·1 | 16·3 | 35·7 | 18·2 | 32·2 | .. | 220·9 | 139·3 | 126·9 | 19·0 | 26·8 |
| Loris | 16·1 | 24·9 | 19·4 | 8·2 | 64·7 | 24·7 | 50·5 | 24·7 | 54·3 | 142·0 | 210·6 | 150·8 | 154·1 | 6·3 | 47·2 |
| Nycticebus | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Perodicticus | 13·3 | 21·9 | 13·7 | 5·8 | 60·7 | .. | .. | .. | .. | 109·2 | 232·3 | .. | 166·6 | 5·5 | 45·6 |
| Arctocebus | 13·1 | 16·8 | 13·0 | 5·9 | 78·4 | 20·1 | 55·4 | 30·4 | 39·6 | 104·5 | 234·2 | 119·7 | 204·5 | 4·9 | 41·1 |
| Tarsius | 29·0 | 49·0 | 30·6 | 14·8 | 59·2 | 15·0 | 33·1 | 16·1 | 26·8 | 89·4 | 181·4 | 107·1 | 65·9 | 36·4 | 8·8 |
| Cheiromys | 22·2 | 38·6 | 33·7 | 11·7 | 57·6 | 22·6 | 41·0 | 19·4 | 46·5 | 75·8 | 188·8 | 107·1 | 91·9 | 11·4 | 40·0 |

## EXCEPTIONAL FORMS.

Having now enumerated the principal modifications in the form, size, and proportions of the several segments and bones entering into the composition of the appendicular skeleton, it is desirable to consider the more remarkable points of structure presented by some of the most specially modified and peculiar forms of the order, such as Man, the Orang, Hapale, Indris, Loris, Tarsius, and Cheiromys.

MAN.

Although the arm and hand of Man are exceeded in absolute length by the pectoral limb of Troglodytes and Simia, yet his leg, both with and without the foot, presents an absolute length of limb such as exists in no other member of the Order.

The length of the whole pectoral limb, compared with that of the spine, is remarkably short when contrasted with the proportions existing in the highest Apes; but some of the lower Simiidæ and Cebidæ resemble Man closely in this respect, while in others of them, as also in all the Lemuridæ, it (the whole pectoral limb) has a less relative length (often much less) than in him—Arctocebus falling almost as much below Man, in this respect, as the Chimpanzee exceeds him.

The proportion borne by the arm without the hand to the spine presents us with nearly similar conditions.

The length of the whole pelvic limb, when compared with that of the spine, is considerably greater than in the majority of the Order; nevertheless this relative size is approached in Simia and Cheiromys, equalled in Galago, surpassed in Hylobates * and Ateles, and very greatly so in Tarsius, in which it much more exceeds the next greatest proportion than that of Man exceeds that of the smallest of the Order.

The length of the leg without the foot, compared with that of the arm without the hand, is still more exceptional, far exceeding, as it does, that of any other Primates except Indris and Galago; yet in those genera it is considerably greater still.

The proportion borne by the leg without the foot to the spine is much greater than in any other of the Primates, except Hylobates and Tarsius, where it is still greater; in the last, indeed, exceeding the proportion in Man more than that exceeds the proportion existing in any other forms save only Perodicticus and Arctocebus.

The *scapula* is surpassed in absolute size by that of the Gorilla, and is nearly equalled by that of the Chimpanzee and that of the Orang.

If its axillary margin be taken as the standard of comparison, then the vertebral margin is longer in Man than in any other Primate, except perhaps Perodicticus, and much longer than in any one except the Gorilla; while the anterior margin is considerably shorter than in any others of the Primates, except the Simiinæ, Ateles, Indris, and sometimes, perhaps, Mycetes.

The posterior vertebral angle is more acute than in most species of the Order, and greatly more so than in some; nevertheless it is not so acute as in the Simiinæ, Ateles, and Arctocebus.

The anterior vertebral angle is sharp and marked † to a degree rarely, if at all, met with in the Order besides, except in Troglodytes.

* Professor HUXLEY truly observes that, thus compared, Hylobates is as much longer in the legs than Man, as Man, is longer in the legs than the Gorilla.—Man's Place in Nature, p. 72.

† Not so, however, in the male and female Boschisman, Nos. 5357 & 5357A. in the Museum of the Royal College of Surgeons.

The angle formed by the spine of the scapula with the axillary margin is probably more obtuse than in any other Primate.

The breadth of the glenoid surface, compared with its length, is greater than in almost any other of the Order; nevertheless it is approached by that of the Gorilla, and exceeded, sometimes at least, by that of Ateles.

The proportion of the supraspinous fossa to the infraspinous one is smaller than in the great bulk of the Order; nevertheless it is larger than in the Pitheciinæ, Nycticebus tardigradus, and Tarsius.

The anterior margin is scarcely ever convex, and the suprascapular notch is well marked, thus differing from the Simiidæ and Lemuroidea; but then in many of the Cebidæ it is much more defined than in Man.

The surface for the *teres major* is more strongly marked than in the Simiinæ, or than in Indris, Loris, and Nycticebus, but it is not so much so as in others, *e. g.* the lower Simiidæ, Cebus, and Chrysothrix.

At the vertebral end of the spine there is a flat triangular surface, absent or less marked in most, but present in Mycetes, Loris, and Arctocebus.

The root of the spine approaches the glenoid surface more nearly than in the Gorilla and Hylobates, but yet not so closely as in the lower Simiidæ and Cebidæ.

Unlike the bulk of the Primates, the infraspinous fossa, close to the glenoid surface, is wider than the supraspinous one; but then Perodicticus, Pithecia, Loris, and Nycticebus resemble Man in this respect.

The spine differs from that of most Primates in not being grooved below (in all but Man behind) at its base; but then it is not so either in the Simiinæ, Ateles, Indris, or Loris.

The coracoid process is largely developed, and projects more away from the glenoid surface than in any Simiidæ; in many of the Lemuroidea, however, it is much the same as in Man (Pl. XII. fig. 2).

The acromion is so produced that (the long axis of the glenoid surface being vertical) it would meet, or nearly so, a plane bisecting the glenoid surface vertically and produced upwards; and it rises at least as high as does the coracoid. In these points Man differs from the bulk of the Primates, including the lower Simiidæ, but agrees with the Simiinæ and Lemur.

The ridge for the trapezoid ligament is less marked than in most Primates, but more so than in others, as, *e. g.*, Indris, Lemur, Loris, and Cheiromys.

The *Clavicle*.—The length of this bone, compared with that of the spine, is greater than in any others, except the Simiinæ, but it falls short of the extreme proportion of Hylobates about as much as it exceeds the very small one of Nyctipithecus.

Its length, as compared with that of the scapula, is greater than in any other Primates, except Simia and Hylobates.

The subacromial surface is almost always convex, thus differing from nearly all the rest of the order; but the Simiinæ and Nycticebinæ are more or less similar to Man in this respect.

The sigmoid curvature of the bone is more marked than in the great majority of forms; but some of the Cebidæ and Nycticebinæ are similar to him.

The *humerus* is longer, as compared with the spine, than in the great bulk of the order; but its relative length is exceeded in the Simiinæ, Ateles, and Lagothrix—the Gorilla and Orang surpassing Man in this respect almost as much as he exceeds Perodicticus.

The articular head looks much more inwards and less backwards than in any other Primate. The tuberosities do not quite rise to the summit of the head, thus differing from some; but then in Lagothrix, Ateles, Simia, and Hylobates they do not rise as high as they do in Man.

The bicipital groove extends about one-third down the bone, and is more marked than it generally is in the Simiinæ, but much less so than it is in the lowest Simiidæ.

The supinator ridge is moderate in size, and much less developed than in the lower Simiidæ and most Lemuroidea; on the other hand, it is more developed than in Hylobates.

The external condyle is a moderate process distinct from the capitellum, and, unlike its position in most Primates, looks forwards and backwards, as in the Simiinæ, Ateles, and Indris.

The projection of the ulnar ridge of the trochlea is again intermediate between other forms as to its extent. The same may be said of the distinctness of its radial ridge and the depth of the olecranal fossa; but the musculo-spiral groove is more marked than in any other Primate.

*Radius.*—In absolute length the radius of Man is not only exceeded by that of the whole of the Simiinæ (except the smallest Gibbons), but by that of the largest of the Cynocephali also. The diameters of its two extremities, however, are greater than in any other Primates, except Troglodytes and Simia.

Its proportionate length to the spine is smaller than in almost all the Simiidæ. It exceeds that, however, existing in the lower Cebidæ, in Hapale, and in all the Lemuroidea except Loris and Tarsius.

Its proportion to the humerus is characteristic, being generally less than in any other Primate, and only approached by the Gorilla, Brachyurus, and Hapale.

Its thickness, in relation to its length, is also extreme.

The shaft is moderately curved, more so than in many Primates, but less so than in some others, *e. g.* the Gorilla, Cebus, Indris.

The bicipital tubercle is as marked as in any of the Order.

The ulnar margin is sharp, as in the lowest Simiidæ, not rounded as in many others, *e. g.* Troglodytes.

The ridge for the origin of the *flexor sublimis digitorum* is marked, thus differing from all but the lowest Simiidæ and some Lemuroidea, *e. g.* Indris, Arctocebus.

The excavation for the origin of the *flexor longus pollicis* is more marked than in most forms, as is also the rough surface for the insertion of the *pronator teres*.

The excavation for the origin of the *extensores pollicis* is again more marked than in

most, though it is very much so sometimes in some, *e. g.* the larger Cynopithecinæ, Chrysothrix, and Hapale.

The radial margin is unusually sharp, yet not so much so as it is in the Cynopithecinæ, and sometimes in Cebus, Lemur, and Galago.

The ulnar angle of the surface for the scapho-lunar articulation is slightly produced, but not so much so as in some, as, *e. g.*, the Gorilla, while in very many species it is not at all so.

The styloid process is exceptionally long, rivalled, however, by that in some Simiinæ.

The grooves for the extensor tendons are unusually distinct; and that for the *extensor secundi internodii pollicis** I have only found distinct in the Chimpanzee and Orang amongst Apes.

*The ulna* is separated by a wider interval from the shaft of the radius in Man than in most Primates, but not so much so relatively as in some, *e. g.* as in Troglodytes, Cebus, Indris.

The greater sigmoid cavity is exceptionally broad, differing in this from all others except Troglodytes and Simia.

The lesser sigmoid cavity looks outwards and not forwards, in which it differs from almost all Primates, and is most nearly approached by Troglodytes, Simia, and the Nycticebinæ.

The olecranon is very broad, indeed at its maximum in breadth compared with length. It is most closely approached, however, in this respect by the Simiinæ. It is not much excavated at its apex, as is so often the case (*i. e.* much excavated) in the Order.

The anterior surface of the bone is distinctly marked off from its inner (ulnar) side, by which it differs from the great bulk of the order, by Troglodytes and, to a less degree, by Simia only resembling it.

The fossa for the *extensores pollicis* is marked in a way existing in no other of the Simiidæ except Simia and Hylobates. However, the genera Chrysothrix, Hapale, Lemur, Galago, Nycticebus, and *especially* Arctocebus, resemble Man in this fossa being distinct. In the last of these it is more marked than that for the *flexor profundus digitorum*.

The surface for the *supinator brevis* is deep and broad, as only in Troglodytes besides, being narrower in the other Simiidæ.

The ridge for the attachment of the *pronator quadratus* is less marked than in many of the Order, but more so than in others.

The head of the ulna is large and rounded, and bears a proportion to the styloid process larger than that existing in any other of the Primates except the Simiinæ.

The styloid process is moderate, not so long as in many (especially Ateles and the Nycticebinæ), but longer than some (*e. g.* Gorilla and Orang).

*Manus.*—The whole manus of Man is exceeded in actual length by that of Troglodytes and Simia only. Its length, as compared with that of the spine, however, is less than in

* This is not constant in Man; at least no trace of it exists in the Boschisman and the Australian (No. 5184) in the Museum of the Royal College of Surgeons.

the bulk of the Order, but not so small as in the lower Simiidæ, and much greater than in some of the lowest Cebidæ, Lemur, and the Nycticebinæ*.

Its length, as compared with that of the arm, is nearer the lower limit of the Order, as it is exceeded by all except the Gorilla, sometimes Hylobates, some of the lower Simiinæ, and Loris; and that of Cheiromys exceeds Man's more than his exceeds the least of the Order, while the difference between the last and that of Cheiromys is much exceeded by the difference between the proportions of Cheiromys and Tarsius.

Its length compared with the radius much exceeds that in Loris, but is almost as much exceeded by that in Hapale, which is again surpassed to a much greater degree by that in Cheiromys.

The number of carpal bones is very exceptional, still (as we have seen) there are but eight, not only in Troglodytes, but also in the widely different Indrisinæ.

The small relative size of the pisiforme distinguishes the manus of Man from almost all Primates, but Simia and the Nycticebinæ resemble him in this. Simia and Troglodytes agree with Man in the absence of direct connexion between the cuneiforme and ulna.

The trapezium has a more concave surface for the pollex than in almost any other Primate; but in the Gorilla and in Simia there is sometimes almost, if not quite, as deep a concavity.

The os magnum predominates over the other carpals more in Man than in other Primates, and the unciforme has its process directed more forwards (palmad) and less towards the digits; but in the latter respect Man is closely resembled by the Nycticebinæ.

The metacarpals (in a skeleton of ordinary size) are exceeded in actual length only by those of Simia and Troglodytes, except the fourth and fifth ones of the Siamang.

The length of the third metacarpal, compared with that of the entire manus, is less than in most Simiidæ, but it exceeds that of the same bone in all Primates below that family except Indris.

The length of the metacarpus compared with that of the spine (as estimated by the same metacarpal) is very much less than in some, *e. g.* Hylobates; much greater than in others, *e. g.* the Nycticebinæ.

The proximal ends of some of the metacarpals have more concave articular surfaces than in most genera.

The distal articular surfaces are somewhat less developed dorsally than in other forms.

The first metacarpal, as compared with the spine, is considerably longer than in some, *e. g.* Perodicticus, but is still more exceeded by others, *e. g.* Hylobates and Tarsius.

In the proportion borne by the pollex, index, and third digits (including their metacarpals) to the spine, as also in the length of the metacarpal of the index as compared with that of the pollex, Man holds an intermediate position in the Order.

As regards the proportion of the pollex to the longest digit (metacarpals included),

* *I. e.* as far as can be judged from the limited comparisons which have been made for this paper. I have had no opportunity of examining Nycticebus; and Perodicticus may sometimes, of course, be longer in the manus than in the specimens examined.

that of Man exceeds the other Primates, except Hapale, Loris, Arctocebus, and Cheiromys.

The absolute length of the pollex, both with and without its metacarpal, is absolutely greater in Man than in any other Primate that I have measured*.

The proportion borne by the pollex, without its metacarpal, to the entire manus exceeds that in all others except Chrysothrix, Hapale, and Arctocebus.

The length of the first phalanx of the third digit, as compared with that of the entire manus, is smaller in Man than in any others of the Order except the Gorilla, some of the lower Simiidæ, and Arctocebus,

The proportion borne by the same first phalanx to its metacarpal is less than that in any other Primates except Troglodytes and some of the Simiidæ other than the Simiinæ.

The length of the third digit, without its metacarpal, compared with that of the manus, is greater than in some and less than in others; and the human proportion is almost as much exceeded in Chrysothrix as it exceeds that in Arctocebus.

In the length of the first phalanx of the same digit, compared with that of the whole manus, as in the proportion borne by longest digit, without its metacarpal, to the longest metacarpal, Man exceeds some and is exceeded by others.

*Os innominatum.*—This is perhaps the most characteristic bone in the appendicular skeleton of Man.

In absolute size the human pelvis is vastly exceeded by that of the Gorilla†, and in length by that of the Chimpanzee and that of the Orang.

The human pelvis is also distinguished by the almost constant‡ excess of its transverse over its conjugate diameter, and by the absence of an ilio-pubic angle §.

The absolute length of a straight line joining the superior spinous processes of the ilium, and the proportion of the same to the spine, are greater than in any other Primate except the Gorilla. The latter species exceeds to a less degree when the crest of the ilium is measured along its curves.

The ilio-ischial angles are less than in any other Primate, but the length of a line drawn from the ilio-pectineal eminence to the nearest point of the tuberosity of the ilium is greater, as compared with that of the spinal column, than in any other of the Primates, except Troglodytes and Simia.

The vertical diameter of the acetabulum, compared with the length of the os innominatum, is greater than in all the rest of the Order.

* Dr. Lucae, however, found the pollex of the Chimpanzee (both with and without the metacarpal) slightly longer than that of the European woman.—*Loc. cit.* p. 306.

† Trans. Zool. Soc. vol. v. p. 12.

‡ As Professor Huxley has pointed out (Medical Times, 1864, vol. i. p. 344), the transverse diameter is less than the conjugate one in the female Boschisman's skeleton, No. 5357A. in the Museum of the College of Surgeons. In Nos. 5257 and 5300 in the same collection the two are about equal. Mr. John Wood gives an instance of the same predominance of the antero-posterior diameter in a male negro, and quotes instances from Professor Weber.—Todd's Cyclopædia, vol. v. pp. 150 & 151.

§ John Wood, *loc. cit.* p. 152.

The distance between the superior posterior spinous process of the ilium and the tuberosity of the ischium, as compared with the length of a line joining the inferior anterior spinous process with the symphysis pubis, is least in Man of all Primates.

The same is the case with the proportion of the length of the os innominatum to that of the transverse diameter of the brim of the pelvis, and with the distance between the inferior posterior spinous process and the spine of the ischium, when compared with that between the inferior anterior spinous process and the pubic symphysis.

The length of the os innominatum, as compared with that of the femur, is less in Man than in any other Primate except Indris and Tarsius.

The crest of the ilium, measured along its curves, almost or quite equals, sometimes even exceeds, the extreme length of the os innominatum—a condition existing in Man alone of all the Order. Moreover, the crest has a degree of sigmoid curvature, and is generally thickened above the acetabulum in a way existing in no other Primate; and the superior anterior spinous process is rather more distinct than in any other genus of the Order.

The inferior anterior spinous process is developed to a greater extent than in any other of the Anthropoidea, but it is exceeded by that of some Lemuroidea.

The posterior spinous processes are more sharp and distinct than in other Primates, and are nearer together, in comparison with the length of the os innominatum, than in any other except Loris.

The outer surface of the ilium is at the same time convex anteriorly and concave posteriorly to a degree existing in no other Primate; moreover, the gluteal lines are much marked, and the iliac fossa is very wide, very concave, and looks mainly inwards —conditions peculiar to Man.

The spine of the pubis is more marked than in most forms, and the superior surface of the so-called horizontal ramus of the pubis is generally broadened, and the subpubic groove in most cases marked to a degree existing in no other Primate *.

The pubic symphysis is shorter relatively than in any other of the Anthropoidea.

The shortness of the body of the ischium, the smallness and non-eversion of the tuberosities of the ischium, and their prolongation backwards and upwards to very near the spine of the ischium, are characters almost peculiar to Man, but not quite so, because they exist in the Nycticebinæ, especially in Loris. Man, however, is the only Primate in which these characters coexist with a broadly expanded ilium.

The heart-shaped brim of the pelvis, so general in Man, exists in no other member of the Order.

The spine of the ischium is generally developed in Man in a way absolutely peculiar to him †, as also is the great concavity of the sciatic notches.

* In skeleton No. 5184 in College of Surgeons, the ramus is very narrow; and in 5357A. the subpubic groove is so faint as to be hardly distinguishable.

† In the skeleton of a male and female of Boschisman race in the College of Surgeon's Museum, the spine is very small, as Mr. John Wood has remarked (*loc. cit.* p. 149). Indeed, in the female it is scarcely larger, though more pointed, than in the Orang.

When the outer surface of the blade of the ilium is looked at, about two-thirds of the acetabulum are visible, thus differing from the Simiinæ, but agreeing with the bulk of the order.

*Femur.*—The femur attains an absolute length much exceeding that of any other Primate; nevertheless, the transverse diameter of the shaft is absolutely greater in the Gorilla.

The angle formed by the neck with the shaft is greater than exists in some other Primates, but is less than in many. The angle formed by the shaft with a horizontal surface, on which both condyles are made to rest, deviates more from a right angle than in any other Primate, though in this there is greater difference between inferior forms.

The proportion of the femur to the spine is greater than in any other genus of the Primates except Hylobates and Tarsius, in which it is still larger. It is, however, very nearly approached in Ateles; and Tarsius exceeds Man, in this proportion, much more than Man exceeds the Gorilla.

The length of the femur, compared with that of the humerus, is much greater than in any other of the Anthropoidea, and very much greater than in the Simiinæ; nevertheless, I find it (thus compared) exceeded in all the Lemuroidea I have examined except the Nycticebinæ and Cheiromys—Tarsius exceeding Man much more than he exceeds even Simia.

Its length, compared with that of the os innominatum, is greater than in any other of the Anthropoidea; nevertheless, it is exceeded in Indris and Tarsius.

The angularity of the shaft and the prominence of the linea aspera are greater than in any other Primate, as also, most probably, the prolongation of the latter to the outer condyle.

The trochanteric fossa is rather more shallow than in most, but not so much so as in the Gorilla and Perodicticus; and in the remaining characters of the femur Man occupies an intermediate position.

The tibial trochanter is as small as, or smaller relatively, than in any other Primate, and the intertrochanteric line in front (as in the largest Cynocephali) is very distinct. The two condyles are pretty equally developed as to projection backwards (differing thus from the Simiinæ and others), but the inner condyle descends peculiarly. Still in this the difference between Man and certain Apes * is less than that between forms of the Order inferior to him.

The rotular surface differs from that of all other Primates in the great predominance of that part of it which is supported by the external condyle. The transverse concavity of the rotular surface is greater than in the Simiinæ† and Nycticebinæ, but it is exceeded in the Lemuroidea other than the last-named subfamily.

* See, *e. g.*, the femur of Ateles (No. 4708 in the Museum of the Royal College of Surgeons), where the descent of the inner condyle is very marked indeed.

† Owen, Trans. Zool. Soc. vol. v. p. 16, as regards Troglodytes.

*Tibia*.—This bone in Man is again absolutely longer than in any other Primate, and absolutely broader at its proximal end than in all but the Gorilla.

Its length, as compared with that of the spine, is (like that of the femur) greater than in any other of the Primates except Hylobates and Tarsius. As compared with that of the humerus, however, it is exceeded by the Semnopithecinæ, some of the lower Cebidæ, and by the Lemuroidea other than the Nycticebinæ. Its length exceeds that of the radius more in Man than in any other Primates except Hapale and Tarsius.

Its length, as compared with that of the femur, is less in Man than in any other Primate, though he is very closely approached by Troglodytes, Nycticebus, and Cynocephalus.

The tubercle of the tibia is at its maximum of distinctness in Man, and is placed higher up than in other Anthropoidea.

The articular surfaces for the condyles of the femur more completely occupy the upper surface of the tibia than in any other Primate; and in Man the outer of the two articular facets is generally more or less strongly concave antero-posteriorly.

The exceeding sharpness of the crest is absolutely peculiar to Man.

The ridge for the popliteus is more developed in him than in any other of the Anthropoidea, and in the fact that the posterior border of the articular surface of the astragalus descends further down than does the anterior margin of that surface, he differs from every other Primate.

In the *Fibula* the peroneal malleolus is not produced out into a strong process as in other Anthropoidea, but it descends much further down than does the tibial one, by which character Man differs from all the rest of the Order. The fibula of Man is excavated and ridged in a degree existing very rarely, if ever, in other Primates.

*Pes*.—The absolute length of this segment of the skeleton of Man is exceeded only by that of the same part in the Gorilla and Orang.

Its length, in proportion to that of the spine, is exceeded by that of all other Primates except Lemur, and (as far as I have been able to ascertain) the Nycticebinæ, and perhaps also some of the lower Simiidæ and Cebidæ. Man, however, more exceeds Arctocebus in this proportion than he is exceeded by the Gorilla.

Its length, as compared with that of the pelvic limb minus the pes, is less than in any other Primate, Hylobates and the Nycticebinæ, however, approaching him rather nearly in this respect.

The same is the case as regards the proportion borne by the pes to the tibia.

The antero-posterior plantar arch is, as has been before said, extensive and peculiar (from the fact of the hallux forming the fulcrum in standing and walking), the tuberosity of the calcis and the distal ends of the inner metatarsals resting on the ground; but, as has been pointed out already, even in Man the outer side of the tarsus and metatarsus is applied, in standing, to the supporting surface, while in many other Primates the inner side of the tarsus and metatarsus is more raised from the ground than in him. So that the distinction between Man and Apes, in this respect, is much less than is often supposed.

The tarsus is longer, in proportion to the spine, than in any other Primates except Cheiromys, Galago, and Tarsius, that of each of the two last exceeding Man's in relative length far more than his exceeds that of Arctocebus, which is the relatively shortest of the Order.

The length of the tarsus, compared with that of the whole pes, is greater in Man than in any other Primate except Galago, though it is very nearly equalled by that of Tarsius.

The os calcis appears to be longer, in proportion to the spine, than in any other Primate except the Gorilla and Ateles, Cheiromys, Galago, and Tarsius. That of Man is twice the relative length of that of the relatively shortest; but that of Tarsius is three times the relative length of that of Man.

The tuberosity of the os calcis is at its maximum breadth inferiorly, where it has two tubercles, and is thus peculiar.

The cuboides is absolutely longer than in any other Primate; and the distal articular surface of the entocuneiforme is strikingly and characteristically flatter than in any other species of the Order.

The absolute length of the hallux, both with and without its metatarsal, is greater than in any other Primate. It also differs from that of all other Primates in not being directed outwards at an angle to the other metatarsals.

The superior surface of the astragalus is almost perfectly horizontal.

The first phalanx of the hallux is slightly, and the second one considerably longer, absolutely, than in any other Primate.

The proportion borne by the hallux, with its metatarsal, to the pollex and also to the spine, as also that of its metatarsal to the latter, are all intermediate in the order; but the proportion of the hallux, with its metatarsal, to the longest digit of the foot, is greater in Man than in any other Primate.

The proportion of the hallux, with its metatarsal, to the whole pes is greater than in any other of the Primates except Indris, the Nycticebinæ, and sometimes Hylobates.

Without its metatarsal, its length, when compared with that of the pes, is decidedly exceeded by that in Arctocebus and perhaps Perodicticus, and slightly by that in the Chimpanzee, but by no other, though it is very nearly equalled by that in Loris and Indris.

The length of the third digit, without its metatarsal, compared with that of the pes, is far less in Man than in any other Primate; and the proportion of the longest digit, with its metatarsal, to the spine is less than in any other Primate except Arctocebus.

The first or the second digit is the longest of the pes, a condition existing in no other Primate.

The phalanges are shorter, as compared with the metatarsals, than in any other Primate, and are narrow and rounded inferiorly in a way found in no other.

The index is sometimes shorter than the hallux (compared without their metatarsals), a condition existing only in Man in the whole Order, though nearly approached in Arctocebus and Perodicticus.

The proportion of the longest digit of the foot to the longest one of the hand (metatarsal and metacarpal included) is less in Man than in any other of the Primates except the Chimpanzee, Hylobates, and Cheiromys.

The excess in length of the third digit of the manus over the third digit of the pes (without the metacarpal and metatarsal) is far greater in Man than in any other Primate except the Orang.

The distal ends of the metatarsals are small, and the vertical diameter predominates more over the transverse one than in the other Primates.

The successive shortening of the three divisions of the pes (tarsus, metatarsus, and digits) exists in Man alone of all Primates; also the great flattening of the plantar surface of the fifth metatarsal.

The proportion of the index to the spine is less than in any others of the Order except Perodicticus and Arctocebus.

The proportion borne by the second metatarsal to the pes is intermediate; that of the index of the foot to the index of the hand is less than in any other Primates except Troglodytes, Hylobates, and Tarsius.

That of the longest digit, without its metatarsal, to the longest metatarsal is much less than in any other Primate whatever; that of the same to the tarsus is also less than in any; but the proportion is approached by Galago, Tarsius, and the Gorilla.

The extent to which the hallux reaches with regard to the index is greater than in almost all, but is exceeded by that in Perodicticus and Arctocebus.

Thus the characters absolutely peculiar to Man, as compared with all the rest of the Primates, are—

1. The very inward aspect of the head of the humerus.
2. The radius being less than three-fourths the length of the humerus *.
3. The small relative distance between the anterior spinous processes of the ilium.
4. The large size of the acetabulum compared with the length of the os innominatum.
5. The small length of the os innominatum compared with the breadth of the pelvis.
6. The small distance between the inferior posterior spinous process of the ilium and the spine of the ischium, compared with that between the anterior inferior spinous process of the ilium and the symphysis pubis.
7. The fact that the length of the crest of the ilium (measured along its curves) about equals that of the os innominatum.
8. The strongly concavo-convex outer surface of the ilium, with marked gluteal lines.
9. The great concavity and inward direction of the iliac fossa.
10. The strongly marked concavity of the sciatic notches, and sharply projecting spine of the ischium.
11. The coexistence of small tuberosities, prolonged up nearly to the ischial spines, with a broad ilium.
12. The absolute length of the femur.

* Though, as far as I have seen, Man is the only Primate ever so conditioned, yet he does not appear to be invariably so. See above, note ‡, p. 311.

13. The angularity of the shaft of the femur, and strong projection of the linea aspera.

14. The predominance of that part of the rotular surface which is supported by the peroneal condyle.

15. The absolute length of the tibia.

16. The sharpness of the crest of the tibia.

17. The descent of the posterior border of the distal articular surface of the tibia (for the astragalus) below its anterior border.

18. The much greater descent of the peroneal than of the tibial malleolus.

19. The shortness of the pes compared with the length of the pelvic limb minus the pes, and compared with that of the tibia.

20. The great breadth of the lowest part of the tuberosity of the os calcis, and the presence of two tubercles on its plantar surface.

21. The flattened surface (for the hallux) of the entocuneiform bone.

22. The fact that either the first or the second digit is the longest and most projecting one of the pes.

23. The absolute size of the hallux, both with and without its metatarsal, and especially of its second phalanx.

24. The *very* slight outward direction of the first metatarsal.

25. The very obtuse angle (plantar one) formed by the transverse axis of the head of the first metatarsal, with a line passing transversely through the heads of the other metatarsals.

26. The great proportion borne by the hallux to the longest digit.

27. The small proportion borne by the four outer digits to the whole pes and to the metatarsals.

28. The very slight flattening of the plantar surfaces of the phalanges.

29. The narrowness and elevation of the distal ends of the four outer metatarsals.

30. The great flattening beneath of the outermost metatarsal.

31. The successive decrease in length of the tarsus, metatarsus, and digits.

32. The form and construction of the antero-posterior plantar arch.

### Simia.

The Orang-outan, when compared with all the other Primates, presents the following notable conditions:—

The proportion borne by the pectoral limb to the spine is greater than in any other genera of the order except Tarsius and Hylobates.

The proportion of the length of the radius to that of the spine is greater than in any other except Hylobates.

The length of the index, with its metacarpal, compared with the spine, is greater than in any except Tarsius and Hylobates.

The length of the metacarpal of the pollex is greater, in proportion to that of the spine, than in any others except Hylobates and Tarsius.

The spine of the ischium is more largely developed than in any other Primate except Man.

The length of the pes is greater, in proportion to that of the spine, than in any except Ateles, Cheiromys, and Tarsius.

The proportion borne by the longest digit of the pes to the spine is greater than in any other Primate except Tarsius.

The length of the longest digit, without its metatarsal, compared with that of the tarsus, is greater than in any of the order except the Nycticebinæ and perhaps Indris.

In addition to these more or less exceptional conditions, the Orang differs from every other Primate without exception in:—

1. The great absolute length of the pectoral limb minus the manus.
2. The small length of the pelvic limb minus the pes, compared with that of the pectoral limb minus the manus.
3. The great absolute length of the manus.
4. The great absolute length of the third digit of the manus, both with and without its metacarpal.
5. The great absolute length of the metacarpal of the pollex.
6. The great difference between the length of the pollex and that of the index.
7. The large diameter of the acetabulum compared with the length of the spine.
8. The small proportion borne by the femur to the humerus.
9. The very obtuse angle formed by the neck of the femur with its shaft.
10. The all but constant* absence of a pit for the ligamentum teres, on the head of the femur.
11. The shortness of the tibia compared with the humerus.
12. The length of the pes compared with that of the rest of the pelvic limb.
13. The length of the pes compared with that of the tibia.
14. The absolute length of the three middle metatarsals.
15. The absolute length of the longest digit with its metatarsal.
16. The very small proportion borne by the length of the hallux to that of the longest digit of the pes.
17. The occasional absence of the second digit of the hallux.
18. The great length of the index, with its metatarsal, compared with that of the spine.
19. The small length of the hallux (both with and without its metatarsal) compared with that of the whole pes.
20. The great length of the second digit, without its metatarsal, compared with that of the whole pes.
21. The very similar length of the indices of the pes and manus, both with and still more without, the metatarsal and metacarpal.
22. The shortness of the tarsus compared with the length of the pes.

* It is *occasionally* absent in the Gorilla, as has already been mentioned; but in the Orang I have only found it present in one out of fourteen skeletons examined, namely in the specimen No. 3 *i* in the Osteological Collection of the British Museum. For this and other details concerning the appendicular skeleton of Simia, see Trans. Zool. Soc. vol. vi. p. 165, and pls. 25–43, especially pl. 40. fig. 7 *i*.

### Hapalæ.

This genus has the pectoral limb, without the manus, shorter in proportion to the spine than it is in any other of the Anthropoidea.

The same remark applies to the pelvic limb minus the pes; and even with the latter it is shorter, as compared with the spine, than in any of the Primates, except Lemur and some of the Nycticebinæ.

The limbs are the shortest and smallest found in the Anthropoidea.

The proportion of the length of the clavicle to that of the scapula is less than in any other of the Primates except Tarsius.

The length of the humerus, compared with that of the spine, is less than in any except Indris, Lemur, Perodicticus, and Arctocebus.

The length of the manus about equals that of the radius, a proportion greater than in any of the Lemuridæ or other of the Anthropoidea except Brachyurus.

The proportion of the pollex to the longest digit is greater than in any other of the Anthropoidea (unless perhaps sometimes in Man) or Lemuroidea, except Arctocebus, Cheiromys, and perhaps Loris. The proportion of the pollex (without its metacarpal) to the whole length of the manus is greater than any other Primate that I have measured except Arctocebus.

The length of the femur, compared with that of the spine, is less than in any of the order except Perodicticus and Arctocebus.

The femur differs from that of the other Anthropoidea by its very short neck and the wide flat space between the trochanters behind (Plate XIII. fig. 5), thus approaching the Lemuroidea.

The proportion borne by the length of the tibia to that of the humerus is about as in Cheiromys, and greater than in any except Indris, Galago, and Tarsius.

The length of the pes, compared with that of the rest of the pelvic limb, is greater than in any except Galago, Tarsius, Cheiromys, Simia, and perhaps Nyctipithecus.

The proportion borne by the hallux to the spine is less than in any other of the Primates except Colobus.

That of its metatarsal to the spine is less than in any except Perodicticus and Arctocebus.

The length of the hallux, compared with that of the pollex, is less than in any other genus except Simia.

The excess, in length, of the longest digit of the pes over that of the manus is greater than in any other of the Primates measured by me except Chrysothrix and Loris*.

The length of the tarsus, compared with that of the pes, is less than in any genera of the order except Indris, Hylobates, Ateles, and Simia.

The length of the longest digit (without its metatarsal), compared with that of the longest metatarsal, is less than in any except Man and some of the lower Simiidæ.

* I have not been able to ascertain the proportion existing in Callithrix, Brachyurus, and Nycticebus.

Hapale differs from all other Primates in—

1. The small length of the radius compared with that of the spine.
2. The small length of the os innominatum compared with that of the scapula.
3. The degree to which the tibia sometimes exceeds the femur in length.
4. The laterally compressed ultimate phalanges of all the digits except the hallux.

### Indris.

This remarkable Lemuroid has the whole hind limb, when compared with the whole fore limb, longer than in any other Primate * except Galago; and this is still more marked when the pelvic limb minus the pes is compared with the pectoral one minus the manus †.

The proportion borne by the length of the humerus to that of the spine is less than in any other of the Primates except Perodicticus and Lemur.

The length of the radius is greater, as compared with that of the humerus, than in any other Primate except Tarsius. That of the manus is to the rest of the pectoral limb greater than in any other except Cheiromys.

The length of the os innominatum, as compared with that of the femur, is less than in any other Primate except Tarsius.

That of the femur, compared with that of the humerus, is greater than in any other except Tarsius.

The same is the case as regards the tibia.

The proportion borne by the hallux to the spine is greater than in any other of the Primates except Tarsius, Cheiromys, sometimes Hylobates, and perhaps Ateles. The same is the case as regards its metatarsal.

The length of the hallux, as compared with that of the longest digit of the pes, is greater than in any Primate except the Chimpanzee, Arctocebus, and Man.

The proportion of the hallux to the pollex is greater than in any other Primate (in which the pollex has two phalanges) except Loris.

The length of the hallux (with its metatarsal), compared with that of the pes, is greater than in any other Primate except perhaps Arctocebus.

The proportion borne by the third digit (without its metatarsal) to the pes is greater than in any other Primates except Loris and Cheiromys; but Hylobates comes very near.

Indris differs from all other Primates in—

1. The dorsal portion of the groove near the axillary margin of the scapula.
2. The great relative size of the posterior inferior (in Man inferior anterior) spinous process of the ilium.

* Excluding, of course, Propithecus and other forms, the skeletons of which I have had no opportunity of examining.

† The great difference as to length between the arm and the leg in this genus is noticed in 'Man's Place in Nature,' pp. 72 & 73.

3. The degree to which the head of the femur is bent forwards.

4. The presence of a tubercle, projecting downwards on the tibia, beneath the pit for the semi-membranosus.

### Loris.

In this genus the humerus is longer, in proportion to the spine, than in any other Lemuroid, and longer, in proportion to the scapula, than in any others of the Primates except Ateles and Hylobates, which two genera alone have the shaft narrower, in proportion to its length, than in Loris.

The radius exceeds the humerus more than in any others except Tarsius, the Indrisinæ, and sometimes Cynocephalus and Hylobates.

The manus, as compared with the spine, is shorter than in any other Primates except Chrysothrix and Arctocebus.

The proportion borne by the longest digit to the spine is less than in any other Primate except Arctocebus.

The proportion of the index to the spine is less than in any except Perodicticus and Arctocebus*; as also is that of the metacarpal of the index to that of the pollex.

The length of the pollex, as compared with that of the longest digit, is greater than in any others except Hapale, Arctocebus, Cheiromys, and perhaps Man.

The length of the third metacarpal, compared with that of the whole manus, is less than in any others except Brachyurus and Arctocebus.

The pubic symphysis, as compared with the length of the spine, is shorter than in any other Primate except Perodicticus.

The tuberosity of the ischium is more prolonged upwards and forwards towards the spine of the ischium (Plate XIII. fig. 3) than in any other Primate except Man.

The proportion of the length of the femur to that of the humerus is less than in any other Primates (other than the Simiinæ) except Ateles, Lagothrix, and Mycetes.

The length of the pes in proportion to that of the rest of the hind limb and of the tibia is less than in any other Primate except Man and Hylobates. The same in proportion to that of the spine is less than in any other Primate except Lemur and Arctocebus.

The os calcis is shorter, compared with the spine, than in any other genera except Arctocebus and Perodicticus; but the cuboid is longer, as compared with the os calcis, than in any other Primate except sometimes Hylobates.

The proportion borne by the hallux to the pollex is greater than in any other Primates except Colobus and Ateles.

The index of the pes more exceeds that of the manus in length than in any other genera of the order except Perodicticus and Arctocebus.

The length of the hallux, with its metatarsal, compared with that of the pes, is greater than in any other Primates except Indris and Arctocebus, and perhaps (?) Perodicticus.

Without its metatarsal, and so compared, it exceeds all except the same last-mentioned genera, the Chimpanzee, and Man.

* I have not been able to ascertain the proportion in Nycticebus.

Loris differs from every other Primate in—

1. The very cylindrical shape of the radius and ulna.
2. The shortness of the manus to the rest of the pectoral limb.
3. The shortness of the manus to the radius.
4. The exceeding smallness of the ilio-pubic angle.
5. The narrowness of the pelvis compared with its conjugate diameter.
6. The large relative size of the body of the pubis.
7. The extent to which the vertical diameter of the posterior outlet of the pelvis exceeds its transverse diameter.
8. The very small absolute length of the symphysis pubis.
9. The coexistence of small tuberosities (prolonged up nearly to the spines of the ischium) with a narrow ilium.
10. The small breadth of the true pelvis compared with the length of the os innominatum.
11. The closeness of the tibial trochanter to the head of the femur.
12. The extent to which the longest digit of the foot exceeds that of the hand.
13. The large extent to which the pollex exceeds a hallux with two phalanges.
14. The great length of the third digit (without its metatarsal) compared with that of the whole pes.
15. The length of the longest digit (without its metatarsal) compared with that of the tarsus.

### Tarsius.

This highly interesting form, which perhaps stands lowest in the order*, presents many peculiarities.

The length of the entire pectoral limb, when compared with that of the spine, is greater than in any other genus of the order except Hylobates; and the same is the case with regard to the pectoral limb minus the manus (in the specimens examined by me), though Ateles and Simia come very close to Tarsius in this proportion. The proportion borne by the length of the scapula to that of the spine is greater than in any except the Simiinæ.

Similarly compared, the radius exceeds that of all except the Simiinæ and Ateles, and the manus is longer than in any except perhaps Cheiromys; and the last named is, moreover, the only genus in which the manus is yet more in excess of the radius in length.

The length of the first phalanx of the third digit, compared with that of its metacarpal, is greater than in any other Primate except Cheiromys; and its length, as compared with that of the manus, is about the same as in that genus, and greater than in any other.

I have found the ilio-pubic angle smaller than in any other Primate except Loris.

The length of the os innominatum, compared with that of the spine, is greater than in any other Primate except the Simiinæ, Cynocephalus, and Ateles.

* It has been suggested to me by Dr. Peters that Tarsius is a lower form than Cheiromys.

The proportion borne by the pes to the rest of the pelvic limb is greater than in any except Cheiromys and Simia; that borne by it to the tibia is greater than in any except the two last-mentioned genera and Galago.

The longest digit of the pes is shorter, in proportion to that of the manus, than in any other Primates except Man, the Simiinæ, and Cheiromys.

The index of the pes is shorter, as compared with that of the manus, than in any other member of the order except sometimes Hylobates.

The proportion borne by the index of the pes to the spine is greater than in any other genera except Hylobates, Cheiromys, Ateles, and Simia.

The length of the tarsus, compared with that of the entire pes, is greater than in any except Man and Galago.

The length of the third digit, without its metatarsal, compared with that of the pes, is less than in any other Primate except Man.

Tarsius differs absolutely from all other Primates in—

1. The whole pelvic limb being more than double the length of the spine.
2. The great length, compared with that of the spine, of the whole pelvic limb minus the pes.
3. The small proportion borne by the vertebral border of the scapula to its axillary border.
4. The shortness of the clavicle compared with the scapula.
5. The shortness of the humerus compared with the same.
6. The great length of the radius and ulna compared with that of the humerus.
7. The great length of the pollex (with its metacarpal) compared with that of the spine.
8. The great length of the longest digit (with its metacarpal) compared with the same.
9. The great length of the metacarpal of the pollex to the same.
10. The great length of the third digit (without its metacarpal) to that of the whole manus.
11. The difference in length of the ultimate and penultimate phalanges.
12. The shortness of the os innominatum compared with the femur.
13. The distance between the inferior (in Man anterior) spinous processes of the ilium compared with the length of the spine.
14. The same, compared with the length of the os innominatum.
15. The very great length of the femur compared with that of the spine.
16. The femur being more than double the length of the humerus.
17. The slenderness of the shaft of the femur, *i. e.* its transverse diameter compared with its length.
18. The narrowness of the femur at the condyles compared with the length of the bone.
19. The small extent of the free upward projection of the peroneal (great) trochanter compared with the same.
20. The great length of the tibia compared with that of the spine.

21. The great length of the same (more than double) compared with that of the humerus.

22. The great length of the same compared with that of the radius.

23. The narrowness of the proximal end of the tibia compared with the length of the bone.

24. The high position of the tubercle on the shaft of the tibia.

25. The anchylosis inferiorly of the tibia and fibula.

26. The great length of the whole pes compared with that of the spine.

27. The great length of the tarsus compared with the same.

28. The small breadth of the tarsus compared with its length.

29. The great length of the os calcis compared with that of the spine.

30. The very small length of the cuboid compared with that of the os calcis.

31. The great length of the hallux compared with that of the spine.

32. The great length of its metatarsal similarly compared.

33. The longest digit equalling very nearly half the length of the spine.

34. The shortness of the second metatarsal when compared with the length of the whole pes.

35. The shortness of the third digit (without its metatarsal) compared with the same.

### Cheiromys.

This aberrant form, the close affinity of which to the other Lemuroids has been lately placed beyond the possibility of dispute *, differs from all the rest of the order except Tarsius, in the length of the manus compared with that of the spine; and I find the same to be the case with respect to the pollex, though here it is sometimes very closely approached, if not equalled, by Hylobates.

The longest digit, with its metacarpal, is also longer, when compared with the spine, than in any except Tarsius.

The index is longer, compared with the spine, than in any other Primates except Tarsius, Simia, and Hylobates.

The first phalanx of the third digit is longer, in proportion to the length of the manus, than in any other Primate except Tarsius.

The length of the tibia, compared with that of the spine, is greater than in any others of the order except Man, Hylobates, Ateles, and Tarsius; its length, compared with that of the humerus, is only exceeded by Hapale, Galago, Indris, and Tarsius; and compared with that of the radius, by Man, Callithrix, Hapale, and Tarsius.

The proportion of the tibia to the femur is greater than in any other Primate except Hapale.

The length of the pes, compared with the rest of the pelvic limb, is greater than in any other Primate except Simia.

* See the very excellent memoir on Cheiromys by Professor Peters, 'Abhandlungen der Königl. Akademie der Wissenschaften zu Berlin,' 1865.

The same, compared with that of the spine, is only exceeded by Tarsius, unless it may be sometimes also by Simia.

Compared with that of the manus, it is less than in any except Hylobates.

Compared with the tibia, it is longer than in any except Simia.

The cuboid bone is longer, in proportion to the os calcis, than in any except Man, Hylobates, Nyctipithecus, Hapale, Lemur, and the Nycticebinæ.

The proportion of the hallux, with its metatarsal, to the spine is greater than in any other except Tarsius, and sometimes Hylobates.

The proportions of the longest digit, and of the metatarsal of the hallux to the spine, are greater; the first than in any genus of the order except Tarsius and Simia, and perhaps Ateles; the second than in any except Tarsius and Hylobates.

The proportion of the hallux to the pollex is less than in any others except Hylobates, Hapale, Simia, and Tarsius.

The length of the longest digit of the pes, compared with that of the manus, is less than in any except Hylobates and the Chimpanzee.

The third digit, without its metatarsal, when compared in length with the pes, I have only found exceeded by Loris*.

Cheiromys differs from absolutely every other Primate in—

1. The great length of the manus as compared with that of the rest of the pectoral limb.
2. The great breadth of the two ends of the humerus compared with the length of the bone.
3. The manus being more than once and a half the length of the radius.
4. The great predominance, in length, of the third metacarpal over the others.
5. The great excess, in length, of the first phalanx of the third digit over its metacarpal.
6. The length of the pollex as compared with that of the longest digit.
7. The slenderness of the third as compared with the other digits of the manus.
8. The length of the longest digit, without its metacarpal, compared with that of the longest metacarpal.

Thus Man is but one of several exceptional forms of the Primates; nor does it appear that the bony structure of his limbs presents more peculiarities of form and proportion than may be detected in that of Tarsius, if even so many.

Again, the differences in anatomical structure between the appendicular skeleton of Man and that of certain Apes is certainly less than that which exists between the same parts in other genera which are counted by followers of Cuvier as Quadrumana, and therefore cannot have an *ordinal* value.

As might have been anticipated, it is the os innominatum and foot which supply the great majority of the absolutely distinctive characters. But the pelvis of Man differs decidedly less from that of the Gorilla than does the latter from that of Loris;

* I have not been able to compare Brachyurus, Callithrix, Nycticebus, and Perodicticus.

and indeed the last differs from the pelvis of Indris as much perhaps as from that of Man, which it resembles as regards the tuberosities of the ischium.

So also the amount of difference in form and proportion between the parts forming the pes of Man and their homologues in the Gorilla is far less, as has been before abundantly demonstrated*, than that existing between the same parts in the Gorilla and Orang, *à fortiori*, than that which distinguishes the pes of the Gorilla from that of Tarsius!

Again, the hand of Cynocephalus is indeed like that of Man, when considered beside the manus of Ateles, Arctocebus, Tarsius, and Cheiromys!

Yet the differences which *do* exist between the appendicular skeleton of Man and that of all other Primates harmonize with his location in a distinct family.

This family is evidently one more closely allied to the Apes than to the Lemuroids; it is one belonging to the Anthropoidea, not to the Lemuroidea. Yet in certain points Man approximates to the latter group: thus the condition of the tuberosity of the ischium presented by him is most closely imitated by Loris; and in the small proportion borne by both the humerus and by the pelvis to the femur, Man resembles some or other of the Lemuroids. The same might be said as regards the length of the tarsus as compared with that of the spine; but this resemblance is only owing to the peculiar tarsal structure of Galago and Tarsius. In the small proportion borne by the index to the spine, however, and in the length of the hallux compared with the longest digit of the pes, Man is more nearly approached by some of the Lemuroids than by any of the Anthropoidea inferior to him. Man resembles some or other of the forms of his own suborder, however, in the absolute size of the limbs and of the several bones composing them; also in the well-marked anterior vertebral angle of the scapula and the sigmoid form of its vertebral margin; in the breadth of the glenoid cavity, the well-defined suprascapular notch, and the length of the clavicle as compared with that of the spine. Also in the inward direction of the head of the humerus he differs less from the highest Apes than from all the Lemuroidea; and he differs from the last and resembles the Anthropoidea in the great length of the humerus as compared with that of the spine, the moderate supinator ridge, the absence of the supracondyloid foramen, which is present in all Lemuroids except Arctocebus, but is, as we have seen, absent in the majority of the Anthropoidea.

Again, he approximates to some of the last-named group in the small proportion borne by the radius to the humerus, in the width of the greater sigmoid cavity of the ulna, and the indirect articulation of the latter with the carpus, in the small proportion borne by the first phalanx of the third digit to its metacarpal, in the width and internal concavity of the ilium, in the flattened horizontal ramus of the pubis, in the subpubic groove, and in the length of the femur compared with that of the spine, in which last Man is about equalled by Ateles†. Finally, he agrees with all the Anthropoidea, and differs from a greater or less number of the Lemuroidea in the ilio-pectineal

* Man's Place in Nature, p. 93.

† In Tarsius the femur is still longer relatively, but so much so as to diverge further from the human proportions by excess than does that of most Apes by defect.

line never forming the actual anterior margin of the ilium, in the small tibial trochanter, the rounded patella, the moderate concavity and elongation of the rotular surface.

On the whole, then, the family Hominidæ ranges itself side by side with the Simiidæ, Cebidæ, and Hapalidæ (*i. e.* judging from the appendicular skeleton only), though probably it is more distinct from them than they are from each other. But before considering the affinities of the various groups of Primates to each other, it is desirable to enumerate the more prominent characters which exist in the several divisions of the order. In endeavouring to collect such I have found considerable difficulty in obtaining characters to separate the two suborders; this has not been, however, on account of any great resemblance between them, but, as in the axial skeleton*, because of the great diversity of structure presented by the suborder Lemuroidea.

ANTHROPOIDEA.

Suprascapular notch often well defined, sometimes a foramen; humerus often without a supracondyloid foramen; os intermedium not interposed between the semilunare and the unciforme; pollex never reaching to the middle of the second phalanx of the index; index always with three phalanges; third digit almost always† projecting furthest, and being the longest; ilium often broad, its crest often much arched; ilio-pectineal line never forming the actual inferior (anterior) margin of the ilium; tuberosities of ischium often much expanded and flattened; shaft of femur often curved, convex forwards, a line drawn from the peroneal (great) trochanter to the condyles, almost always cutting its anterior surface; neck more or less elongated; tibial trochanter always less than the peroneal one; no third trochanter‡; pit for ligamentum teres not always present; patella more or less rounded; tibial malleolus never much incurved at its extremity; fibula always distinct from the tibia; distal articular surface of entocuneiforme never saddle-shaped; fourth digit of pes scarcely ever projecting most, or being the longest§; no proximal phalanx of any of the four outer digits exceeding in length its supporting metatarsal.

HOMINIDÆ.

The characters of Man have been given above, under the head of "Exceptional forms."

SIMIIDÆ.

Suprascapular notch not strongly marked; anterior vertebral angle mostly ill-defined; humerus without a supracondyloid foramen; pollex never reaching the middle of the proximal phalanx of the index; conjugate diameter of pelvis always exceeding the transverse; length of crest of ilium never nearly equalling that of the

* Proceedings of the Zool. Soc. 1865, p. 578.

† Except in Pithecia.

‡ Except sometimes in Hylobates, *e. g.* No. 5026 in the Museum of the Royal College of Surgeons.

§ Except, again, in Pithecia, and also sometimes in Nyctipithecus. It sometimes projects most also in Hapale.

os innominatum; crest without a marked sigmoid curvature; inferior (in Man anterior) spinous processes of ilium little marked; symphysis pubis much elongated; tuberosities of ischium large, everted, and always distant from the spine of the ischium; sciatic notches never very concave; brim of pelvis never heart-shaped; linea aspera only moderately marked; rotular surface supported but little more by the peroneal condyle than by the tibial one; surface of tibia for the peroneal condyle convex antero-posteriorly; crest of tibia never very sharp; posterior border of articular surface for astragalus not descending below its anterior border; peroneal malleolus strongly projecting outwards, but only descending about as much as the tibial one; hallux never equalling three-fourths the length of the longest digit of pes; tuberosity of os calcis, with its long axis, always bent inwards below; distal articular surface of entocuneiforme convex; metatarsal of hallux with no articular surface for the metatarsal of the index; third digit of pes extending further forwards; phalanges of pes long, broad, and flattened or concave beneath, much like those of the manus; either pollex or hallux with two phalanges, but not always both with two.

SIMIINÆ.

Proportion of whole pectoral limb to spine, estimated at 100, from about 142 to about 220; pelvic limb always shorter than pectoral one; angle formed by the vertebral and axillary margins of the scapula, from about 22° to about 35°; spine of scapula not always extending to the vertebral margin; anterior vertebral angle well defined; spine not grooved behind (below) at its base; surface for *teres major* little marked: acromion as high as coracoid or higher; coracoid long; anterior margin of scapula less than, or not much more than half the length of, the axillary one; clavicle more than one-fifth of the length of the spine; humerus more than half of the length of the same; deltoid surface of humerus not much marked; external condyle projecting outwards; styloid process of ulna not so large as the distal end of the shaft of that bone; manus more than one-third of the length of the spine; ilium more or less largely expanded; tuberosity of ischium only sometimes flattened inferiorly; only profile of acetabulum, and not its cavity, visible when outer surface of ilium is looked at; femur shorter than humerus; outer condyle decidedly smaller than the inner one.

TROGLODYTES.

Length of whole pectoral limb less, or but little more than once and a half the length of the spine; angle of axillary margin of scapula, and its glenoid surface, about 120° to 125°; no os intermedium; os pisiforme large; cuneiforme not articulating directly with the ulna; length of os innominatum more than half of that of the spine; os calcis more than one-tenth of the length of the spine.

T. GORILLA.

Proportion of supraspinous fossa of scapula very large as compared with infraspinous fossa; spine not reaching to the vertebral margin of scapula; posterior vertebral angle

about 34°; angle formed by spine with the axillary border about 30°; supra- and infra-spinous fossæ about equal in breadth at the glenoidal end of the spine; sternal horizontal curve of clavicle almost obsolete; subacromial space scarcely at all concave; humerus when looked at in front, with the ulnar tuberosity hiding the neck of the bone; ulnar ridge of trochlea projecting very little below the capitellum; manus less than three-fourths of the length of the radius; iliac fossa strongly concave; a distinct subpubic groove; shaft of femur expanding gradually downwards; great trochanter not projecting at all, peronead beyond the shaft of the femur; trochanteric fossa very shallow; sometimes no depression for the ligamentum teres; external condyle projecting back much less than the internal one; surface above the inner condyle not prominent; inner surface of malleolus not nearly at right angles with the distal articular surface of the tibia; length of hallux, without the metatarsal, less than one-fifth of that of the pes; tuberosity of calcaneum at its maximum of relative length, and produced only downwards (Plate XIII. fig. 6); astragalus at its minimum of length to breadth (Plate XIII. fig. 7); surface for tibial malleolus nearly on a level with the top of the astragalus; tarsus largely exceeding metatarsus in length; hallux reaching the distal end of the proximal phalanx of the index.

T. NIGER.

Proportion of supraspinous fossa of scapula only moderate as compared with infraspinous fossa; spine reaching quite or almost to vertebral margin, and sometimes ending nearer to the anterior than to the posterior end of the latter; posterior vertebral angle only about 22°; angle formed by spine with axillary margin about 20° or 24°; supraspinous fossa much wider than the infraspinous one at the glenoidal end of the spine; sternal horizontal curve of clavicle rather marked; subacromial space decidedly concave; ulnar tuberosity not hiding the neck of the humerus when the bone is looked at in front; ulnar ridge of the trochlea projecting much below capitellum; manus more than three-fourths of the length of the radius; iliac fossa *very* slightly concave, or not at all so; no subpubic groove; shaft of femur expanding suddenly at the condyles; great trochanter projecting slightly peronead beyond the shaft of the femur; trochanteric fossa very deep; pit for the ligamentum teres constantly present; condyles projecting backwards not very unequally; surface above the inner condyle prominent; inner surface of malleolus nearly at right angles with the distal articular surface of tibia; length of hallux, without the metatarsal, one-fifth of that of the pes, or more; tuberosity of calcaneum concave behind, and produced both upwards and downwards; surface for tibial malleolus forming a decided angle with upper surface of astragalus; hallux reaching a little beyond the proximal end of the second phalanx of the index.

SIMIA.

The main characters of this genus have been already given among "Exceptional forms."

HYLOBATES.

Pectoral limb about twice the length of the spine; entire pelvic limb scarcely more than three-fourths of the length of the pectoral one; angle of spine of scapula, with axillary margin, about 12° or 15°; angle of glenoid surface, with the same, about from 93° to 105°; proportion of supra- to infraspinous fossa very large; spine ending considerably nearer the posterior than the anterior end of the vertebral margin, which margin, however, it scarcely attains; axillary margin nearly straight; supraspinous fossa largely exceeding the infraspinous one at the glenoidal end of the spine; glenoid surface rounded and shallow; clavicle more than one-quarter the length of the spine; acromial horizontal curve of clavicle almost obsolete; humerus somewhere about three-fourths of the length of the spine, and approaching three times that of the scapula; head of humerus very globular; tuberosities very small, and much below the summit of the head of the humerus; radius more than four-fifths of the length of the spine; metacarpal of pollex about one-tenth of the length of the spine; an os intermedium; cuneiforme articulating directly with the ulna; metacarpals increasing markedly (in length and in extension distad) from the fifth to the second; pisiforme long but slender; trapezium with a rounded convex articular surface for metacarpal of pollex; ilio-pectineal eminence very large; subpubic groove generally distinct; symphysis pubis very long; tuberosities of ischium large and flattened beneath; shaft of femur very straight; trochanteric fossa deep; external condyle projecting back much less than the internal one; tibia more than half of the length of the spine; hallux reaching to the middle or more of the proximal phalanx of index; metatarsal of hallux more than one-tenth of the length of the spine.

SIMIIDÆ OTHER THAN THE SIMIINÆ.

Proportion of whole pectoral limb to spine, at 100, from about 91·7 to about 121·3; pelvic limb always longer than pectoral one; angle formed by the vertebral and axillary margins of the scapula from about 50° to about 75°; spine of scapula always reaching its vertebral margin; anterior vertebral angle not well defined; spine grooved behind at its base; surface for *teres major* much marked; acromion often not nearly so high as coracoid; coracoid very short and bent in towards glenoid surface (Plate XI. fig. 3); anterior margin of scapula much more than half of the length of the axillary margin (Plate XI. fig. 2); clavicle less than one-fifth of the length of the spine; humerus less than half of the length of the same; deltoid surface much marked (Plate XII. fig. 4); external condyle pressed forward and, as it were, flattened against capitellum; styloid process as large as, or larger than, the distal end of the shaft of the bone; manus less than one-third of the length of the spine; often a process extending backwards from the proximal end of the fifth metacarpal; an os intermedium; cuneiforme joining ulna directly; pisiforme large; crest of ilium short; no subpubic groove; tuberosity of ischium large and always flattened beneath; greater part of cavity of acetabulum visible when outside of ilium is looked at; femur longer than humerus; outer condyle of nearly the same size as the inner one; hallux always with two phalanges.

SEMNOPITHECINÆ.

Pollex sometimes only with one phalanx, and if with two, only reaching slightly beyond the base of the proximal phalanx of the index; hallux extending a little beyond the base of the proximal phalanx of the index of the pes.

CYNOPITHECINÆ.

Pollex always with two phalanges, and sometimes nearly reaching the middle of the proximal phalanx of the index; hallux nearly attaining the distal end of the proximal phalanx of the index of the pes.

There is great similarity in the limb-structure of all the Simiidæ other than the Simiinæ; and a series of gradual modifications leads from the form and proportions found in Semnopithecus to those existing in Cynocephalus.

CEBIDÆ AND HAPALIDÆ.

Pelvic limb always longer than the pectoral one, except sometimes in Ateles; posterior vertebral angle from about 30° to about 50°; anterior margin always more than half the axillary one, except in Ateles; spine ending in front of the middle of the vertebral margin of the scapula; suprascapular notch generally well defined, sometimes a foramen; supraspinous fossa much exceeding the infraspinous one at the glenoidal end of the scapula; clavicle with a sigmoid curvature; external condyle of humerus very small, and more or less confounded with capitellum; olecranal fossa shallow, imperforate; capitellum relatively very large; ulnar ridge of trochlea very little produced; styloid process of ulna very large; ridge for *pronator quadratus* often much marked; proportion of pollex, without metacarpal, to manus always greater than in the Simiidæ (except of course in Ateles); pollex attaining distal end of proximal phalanx of index (except of course in Ateles); an os intermedium present; cuneiforme articulating directly with ulna; only a very small process projecting back from the proximal end of the fifth metacarpal; ilium never much expanded; iliac fossa very narrow and never strongly concave; brim of pelvis never heart-shaped; conjugate diameter of pelvis always exceeding the transverse one; length of crest of ilium never nearly equalling that of the os innominatum, and without any sigmoid curvature; spinous process of ilium little marked; tuberosities of ischium never broad and flattened beneath, but distant from spine of ischium; sciatic notches very slightly concave; greater part of cavity of acetabulum visible when the outside of the ilium is looked at; trochanteric fossa deep; linea aspera little marked; no third trochanter; rotular surface but little more on the peroneal condyle than on the tibial one; surface of tibia for peroneal condyle more or less convex antero-posteriorly; crest of tibia never very sharp; posterior border of articular surface for astragalus not descending below its anterior border; peroneal malleolus strongly projecting outwards, but not descending further than the tibial one; hallux never much exceeding half the length of the longest digit, metatarsals included; tuberosity of os calcis with its long axis always bent inwards below; entocuneiforme with its distal

articular surface convex; metatarsal of hallux with no articular surface for metatarsal of index; third digit of pes almost always the one extending furthest forwards, if not the third, then the fourth; phalanges of pes long, broad, and flattened or concave beneath, much like those of the manus; hallux always with two phalanges.

CEBIDÆ.

Inner condyle of humerus generally produced downwards nearly to the level of the margin of the inner part of the trochlea; often a supracondyloid foramen; radius only one-fourth of the length of the spine; proportion of hallux, without its metatarsal, at 100, to pes, from about 18·4 to 20·1; hallux reaching from the middle to the distal end of the proximal phalanx of the index; ultimate phalanges never laterally compressed, sharply curved and pointed.

ATELES.

Pelvic limb sometimes shorter than the pectoral one; proportion of pectoral limb to spine greater than in any other Cebidæ, or than in any Lemuridæ or lower Simiidæ; proportion of pelvic limb to spine greater than in any other Anthropoidea except Hylobates; length of scapula to spine greater than in any other of the Anthropoidea except the Simiinæ; its anterior border less than half of the length of its axillary border; angle of glenoid surface, with axillary margin, about 110°; posterior vertebral angle about 30°; proportion of the supraspinous fossa to the infraspinous fossa very large; spine not quite reaching the vertebral margin; a suprascapular foramen; surface for *teres major* not projecting; acromion very long and narrow; clavicle nearly a fifth of the length of the spine; tubercle for trapezoid ligament very marked; humerus more than half of the length of the spine; tuberosities small and decidedly below the head of the humerus; no supracondyloid foramen; ulnar ridge of trochlea very small and not descending below the inner condyle; medullary foramen of humerus opening at the end of a long groove; styloid process of ulna enormous; medullary foramen of radius often directed distad; manus nearly half of the length of the spine; pollex often less than one-tenth of the length of the same; proportion of metacarpal of pollex to spine sometimes greater than in any other of the Cebidæ or Hapalidæ; pollex often with only one phalanx; pisiforme small; trapezium with no distal concavity; crest of ilium much arched, concave outwards; tuberosity of ischium very small; acetabulum very shallow; peroneal trochanter small, not so high as head of femur; peroneal trochanter not projecting peronead beyond shaft; pit for ligamentum teres very large; outer condyle much smaller than the inner one; pes exceeding half the length of the spine; pes shorter in proportion to the manus than in any other Primates except the Simiinæ and Cheiromys; hallux much more than twice and a half the length of the pollex.

LAGOTHRIX.

Pelvic limb slightly longer than pectoral one; pectoral limb more than once and a quarter the length of the spine; proportion of supra- to infraspinous fossa even

greater than in Ateles; anterior margin much more than half the length of the axillary one, and with a deep notch sometimes converted into a foramen; spine at its glenoidal end coming close to axillary border; clavicle nearly one-fifth of the length of the spine; tuberosities of humerus small and decidedly below its head; no supracondyloid foramen; pollex always with two phalanges; tuberosity of ischium approaching slightly nearer to the spine of the ischium than in Ateles (Plate XIII. fig. 1); peroneal trochanter larger than in that genus, and rising above the summit of the head; pit for ligamentum teres of moderate size; outer condyle much smaller than the inner one; hallux not reaching quite so far forwards, with regard to the index of the pes, as does the pollex with respect to that of the manus.

CEBUS.

Scapula much like that of the lower Simiidæ, but trapezoid ridge very prominent and sharply defining the suprascapular notch; spine transverse and grooved behind at its base; spine at its glenoidal end coming close to axillary margin; surface for *teres major* very prominent; clavicle much less than one-fifth of the length of the spine; sternal horizontal curve of clavicle less marked than in Lagothrix and Ateles; tuberosities of humerus almost rising to a level with the summit of its head; bicipital groove and deltoid surface very marked; a distinct supracondyloid foramen; radius and ulna much curved; styloid process of ulna long, but not like that of Ateles; pisiforme large; trapezium sometimes with a very slight distal concavity; tuberosity of ischium rounded; anterior inferior (superior anterior) spinous process of ilium disguised by the projection of the anterior (superior) end of the ilio-pectineal line; peroneal trochanter well developed, pointed, and projecting peronead beyond the shaft of the femur; condyles approaching, but not attaining the equality existing in those of the lower Simiidæ.

MYCETES.

Proportion of the supraspinous fossa to the infraspinous one greater than in any other of the Anthropoidea except the Gorilla; a peculiar process projecting forwards from the anterior margin of the suprascapular foramen (Plate XI. fig. 4 *x*); spine flattened at its upper (posterior) end; subscapular fossa crossed by very strong ridges; clavicle remarkably and exceptionally slender (Plate XII. fig. 3); tuberosities of humerus very much below its head; no supracondyloid foramen; inner condyle descending as low as the inner margin of the trochlea; olecranal fossa shallow, imperforate; ridge on ulna, for *pronator quadratus*, very strongly marked; pisiforme large, and expanded at its distal end; anterior inferior (superior anterior) spinous process of the ilium distinct from that process which is the prolongation forwards (upwards) of the ilio-pectineal line; ilio-pectineal eminence sometimes a very prominent process; a more or less marked subpubic groove; shaft of femur expanding very gradually downwards (Plate XIII. fig. 4), and much antero-posteriorly compressed at its lower half; linea aspera represented by a wide groove with very distinct lips; trochanteric fossa deep; peroneal trochanter

not rising to the level of the top of the head of femur, and projecting *very* slightly peronead beyond the shaft; ridge for *gluteus maximus* very strongly marked; outer condyle much smaller than the inner one; patella rather long and narrow; hallux reaching nearly to the middle of the proximal phalanx of the index.

NYCTIPITHECUS.

Anterior margin of scapula strongly convex, and suprascapular notch well defined, but never replaced by a foramen; a supracondyloid foramen generally * present; condyles of femur nearly equal; proportion of pes to whole hind limb and to tibia greater than in any other of the Anthropoidea except Simia and Hapale; fourth digit of pes sometimes longest and most projecting.

CALLITHRIX.

Scapula with no projection for the *teres major*; no suprascapular foramen, but the notch sharply defined; a supracondyloid foramen; length of radius to spine less than in any other of the Anthropoidea except Hapale; condyles of femur about equal in size and production backwards.

CHRYSOTHRIX.

Proportion of pectoral limb to spine smaller than in any other of the Anthropoidea; anterior margin of scapula convex, and equalling four-fifths of the length of its axillary one; no suprascapular foramen; spine at its glenoidal end coming remarkably close to the same margin; surface for *teres major* very marked; suprascapular notch not well defined; manus scarcely more than a fifth of the length of the spine; length of pollex and of longest digit to spine less than in the other Cebidæ; femur only two-fifths of the length of the spine, and therefore relatively shorter than in any other Anthropoidea except Hapale; condyles rather unequal; pes shorter in proportion to spine than in any other of the Anthropoidea except Man, and sometimes Cynocephalus.

PITHECIA.

Supraspinous fossa very small when compared with the infraspinous one (Plate XI. fig. 5); anterior vertebral angle sometimes distinctly marked; suprascapular notch very little defined; a supracondyloid foramen; internal condyle not descending to the level of the inner margin of the trochlea; fourth digit the longest, and projecting most in both the manus and the pes.

BRACHYURUS.

Anterior vertebral angle of scapula not distinctly marked; suprascapular notch very little defined; a supracondyloid foramen; internal condyle descending as low as the inner margin of the trochlea; spine of ischium rather prominent from the inclination downwards (forwards) of the tuberosity of the ischium.

* In N. villosus (No. 58 B. in the British Museum) there is none.

HAPALIDÆ.

The peculiarities of this family have been mentioned amongst the "Exceptional forms."

LEMUROIDEA.

Spine of scapula ending anteriorly (superiorly) to the middle of the vertebral margin of the bone; suprascapular notch ill defined; glenoid surface narrow; clavicle never so much as one-fifth of the length of the spine; humerus never half the length of the same; internal condyle not descending so low as the edge of the inner margin of the trochlea; almost always a supracondyloid foramen *; supinator ridge almost always large and prominent; styloid process of ulna always articulating directly with the cuneiforme; intermedium generally present, and sometimes interposed between the semilunare and the unciforme; pollex sometimes reaching beyond the middle of the second phalanx of the index, which latter has sometimes only two phalanges; pollex always with two phalanges; fourth digit of manus almost always † the longest; ilium never very broad, or its crest much arched; ilio-pectineal line often forming the actual inferior (anterior) margin of the ilium; tuberosities of ischium never flattened beneath; no subpubic groove; sciatic notches very slightly concave; brim of pelvis never heart-shaped; conjugate diameter of pelvis always exceeding the transverse; a line drawn from the most anterior part of the peroneal trochanter to that of the condyles passing quite in front of the shaft of the femur; linea aspera faintly marked or absent; neck of femur *very* short; tibial trochanter sometimes larger than the peroneal one; often a third trochanter; pit for ligamentum teres constant; rotular surface but little more supported by the peroneal condyle than by the tibial condyle; patella often much elongated; inner malleolus often much incurved and compressed; fibula not quite always distinct from the tibia; peroneal malleolus not descending below the level of the tibial one; anterior border of articular surface of tibia for astragalus descending a little further than the posterior border; crest of tibia never very sharp; hallux sometimes equalling three-fourths of the length of the longest digit; tuberosity of calcis always inclined more or less inwards inferiorly; distal articular surface of entocuneiforme convex, and mostly saddle-shaped; hallux always with two phalanges; fourth digit of pes always longest and most projecting; phalanges of pes resembling much those of manus; proximal phalanges sometimes longer than their supporting metatarsals.

LEMURIDÆ.

Pectoral limb never more than slightly exceeding the length of the spine; pelvic limb, minus the pes, never so long as the spine; surface of scapula for teres major very slightly marked; coracoid process long; radius never half the length of the spine; manus never nearly attaining that proportion; manus never longer than radius; pollex, with its metacarpal, never one-fifth of the length of the spine; longest digit never attaining that proportion; fibula never anchylosed to tibia; hallux reaching to the middle of the second phalanx of the index, or beyond it.

* Always except in Aretocebus. † Always except in Tarsius.

LEMURIDÆ OTHER THAN THE NYCTICEBINÆ.

Pectoral limb always shorter than spine; anterior margin of scapula convex; clavicle often with only one horizontal curvature; great tuberosity of humerus as high as its head; supracondyloid foramen large and constant; olecranal fossa imperforate; intermedium present or absent; tuberosities of ischium never approaching very near its spine; posterior inferior (inferior anterior) spinous process of ilium well developed (Plate XIII. fig. 2, *sp*); condyles of femur unequal; head of femur not compressed; rotular surface deep, its margins very unequal; patella elongated; tibial malleolus opening from the side of the tibia; groove for tibialis posticus not extraordinarily marked; tarsus sometimes much elongated; naviculare more or less considerably enlarged antero-posteriorly.

INDRIS.

For this see above, "Exceptional forms."

MICRORHYNCHUS*.

Humerus with a strong sigmoid curvature, also supinator ridge; olecranon very small; ulna not diverging from radius so much as in Indris; pisiforme very small; no os intermedium; proportion of metacarpal of index to that of pollex as 183·3 to 100, or greater than in any other Lemuroid; tibia to radius as about 149·1 to 100, or almost the proportion of Man; fibula exceedingly slender; length of cuboid to os calcis as about 37·6 to 100; of naviculare to the same, about 38·9; naviculare expanding downwards but little.

LEMUR.

Pectoral limb scarcely more than three-fourths of the length of the spine; angle of spine of scapula with vertebral margin less than in the other Lemuridæ measured; supraspinous fossa very large as compared with the infraspinous one; margin of spine of scapula slightly bent over the infraspinous fossa; spine approaching very near to the axillary margin and glenoid surface; supraspinous fossa slightly exceeding the infraspinous one at the glenoidal end of the spine of the scapula; acromion remarkably flattened and concave externally, with a strong metacromion-like projection over the infraspinous fossa; clavicle sometimes less than a tenth of the length of the spine; shaft of humerus much curved; great tuberosity rising above the head of the humerus; supinator ridge exceedingly marked; radius sometimes only a quarter of the length of the spine; ridge on ulna for pronator quadratus very large; an os intermedium; fifth digit longer than index; ilio-pectineal line not forming the actual inferior (anterior) margin of the ilium; peroneal trochanter rising more or less above the head of the femur, and projecting very peroneal beyond the shaft; trochanteric fossa small but deep; a third trochanter; peroneal condyle smaller than the tibial one; tarsus but little more than one-tenth of the length of

* For further details see Proc. Zool. Soc. 1866, p. 133. From the structure of the skull I have now no doubt but the *Propithecus diadema*, of Bennett, closely resembles *Indris brevicaudatus* in its appendicular skeleton. See Proc. Zool. Soc. 1867, p. 247.

the spine; cuboid nearly half the length of the os calcis; length of os calcis less than one-third of that of the tibia.

MICROCEBUS*.

Os calcis one-third of the length of the tibia.

GALAGO.

Entire pelvic limb more than once and a half the length of the entire pectoral one, both with and without manus and pes; angle of spine of scapula with its axillary margin less than in any other of the Primates measured, except Hylobates; supraspinous fossa nearly equal in size to the infraspinous one; spine of scapula approaching close to axillary margin and glenoid surface; clavicle considerably more than one-tenth of the length of the spine; shaft of humerus nearly straight; tuberosities not quite rising to the level of the top of the head of the bone; ilio-pectineal line forming the actual inferior (anterior) margin of the ilium; a small ilio-pectineal eminence; peroneal trochanter small, rising very slightly above the neck of the femur, but projecting much peronead beyond its shaft; a large third trochanter; trochanteric fossa very small; outer condyle extending back almost as much as the inner one; patella small, not much elongated; os calcis and naviculare very much elongated, the first being nearly one-fifth of the length of the spine; cuboides but very little more than one quarter of the length of the os calcis, which last is more than one-third of the length of the tibia; whole pes about half the length of the spine; pes more than once and a half the length of the manus; tarsus nearly one-fifth of the length of the spine.

NYCTICEBINÆ.

Pectoral limb sometimes longer than the spine; anterior vertebral angle well defined; spine ending not far from the posterior end of the first third of the vertebral margin, which margin it almost always reaches; infraspinous fossa more or less exceeding the supraspinous one near the glenoidal end of the spine (Plate XI. fig. 6); spine of the scapula not nearly approaching the axillary margin; no marked surface for the teres major; glenoid surface, as it were, twisted above (Plate XII. fig. 2); clavicle more than one-tenth of the length of the spine, with a more or less marked sigmoid curvature, and no acromial expansion; tuberosities of humerus always more or less below its head; supinator ridge slightly or strongly marked; supracondyloid foramen sometimes absent; olecranal fossa sometimes perforated; styloid process of ulna always exceedingly long, sometimes curved; os intermedium constant; pisiforme very small; sometimes an extra ossicle in transverse ligament of carpus; index digit sometimes at its minimum; tuberosities of ischium approaching near to the ischial spine (Plate XIII. fig. 3); posterior inferior (inferior anterior) spinous process absent or very minute; crest of ilium remarkably short; pelvis very narrow transversely; no pubic spine; ilio-pectineal line forming the actual inferior (anterior) margin of the ilium (Plate XIII. fig. 3); symphysis pubis

* For further details see Dr. Peters's 'Reise nach Mossambique,' p. 17.

generally *very* short; neck of femur all but obsolete; peroneal trochanter very small, sometimes smaller than the tibial one, which is always large; scarcely any distinct third trochanter; head of femur more or less compressed; rotular surface very shallow; tibia with a strongly marked process overhanging the fossa for the *tibialis anticus*; facet of tibia for outer condyle very convex antero-posteriorly; tibial malleolus pointed, curved, and compressed, and sometimes springing rather from the front than from the side of the tibia; groove for *flexor longus hallucis* very marked; groove for *flexor longus digitorum* separated from the others by a strong process; surface of tibia for astragalus with its long axis directed antero-posteriorly; patella small and rounded; os calcis less, or but little more, than one-twentieth of the length of the spine; tuberosity of os calcis small and much inflected; naviculare very short antero-posteriorly (Plate XIV. fig. 10); hallux reaching beyond the middle of the distal phalanx of the index.

### LORIS.

For the characters of this genus see above, "Exceptional forms."

### NYCTICEBUS.

Proportion of supra- to infraspinous fossa very small; anterior margin of scapula uniformly concave (Plate XI. fig. 6); a supracondyloid foramen; olecranal fossa imperforate; peroneal trochanter not rising so high as the summit of the head of the femur, which is extremely compressed; pit for ligamentum teres enormous; condyles projecting about equally backwards.

### PERODICTICUS.

Posterior vertebral angle of scapula very obtuse (Plate XII. fig. 1); angle of spine of scapula with axillary margin very open; proportion of supra- to infraspinous fossa large; anterior margin of scapula with a slight prominence in its middle, otherwise straight; anterior vertebral angle very well defined; spine of scapula approaching axillary margin and glenoid surface more closely than in Loris; shaft of humerus much curved; a remarkably deep pit for the insertion of the *infraspinatus*; supinator ridge very strongly marked; external condyle, as it were, much extending the distal articular surface of the humerus; a large supracondyloid foramen; olecranal fossa imperforate; internal condyle large, truncated; an extra ossicle beneath (*i. e.* palmad to) carpus (Plate XIV. fig. 5), and another beneath tarsus; pollex much exceeding index in length, and reaching to the middle of the third digit; index with only two phalanges; tuberosity of ischium rather more everted than in Loris, and not approaching spine of ischium so nearly; peroneal trochanter projecting strongly forwards, and a little peronead beyond the shaft of the femur; tibial trochanter a wide flat process, sometimes larger than the peroneal one; only a faint impression of the ligamentum teres on femur; inner condyle projecting back more than the outer one; peroneal malleolus very large and massive; hallux attaining to nearly the middle of the distal phalanx of the index.

ARCTOCEBUS.

Proportion of supra- to the infraspinous fossa very large; free edge of spine of scapula much flattened; anterior margin of scapula with a convexity as in Loris; great tuberosity of humerus not rising to the level of the summit of its head; internal condyle prominent, but no supracondyloid foramen; index with only two phalanges; pollex reaching altogether beyond index and to ultimate phalanx of third digit, which third digit is remarkably short; peroneal trochanter very small, and not projecting peronead beyond the shaft of the femur; trochanteric fossa very small; tibial trochanter a large plate-like process, exceeding the peroneal trochanter in size; no third trochanter; hallux reaching to the end of the index digit of the pes.

TARSIIDÆ & CHEIROMYIDÆ.

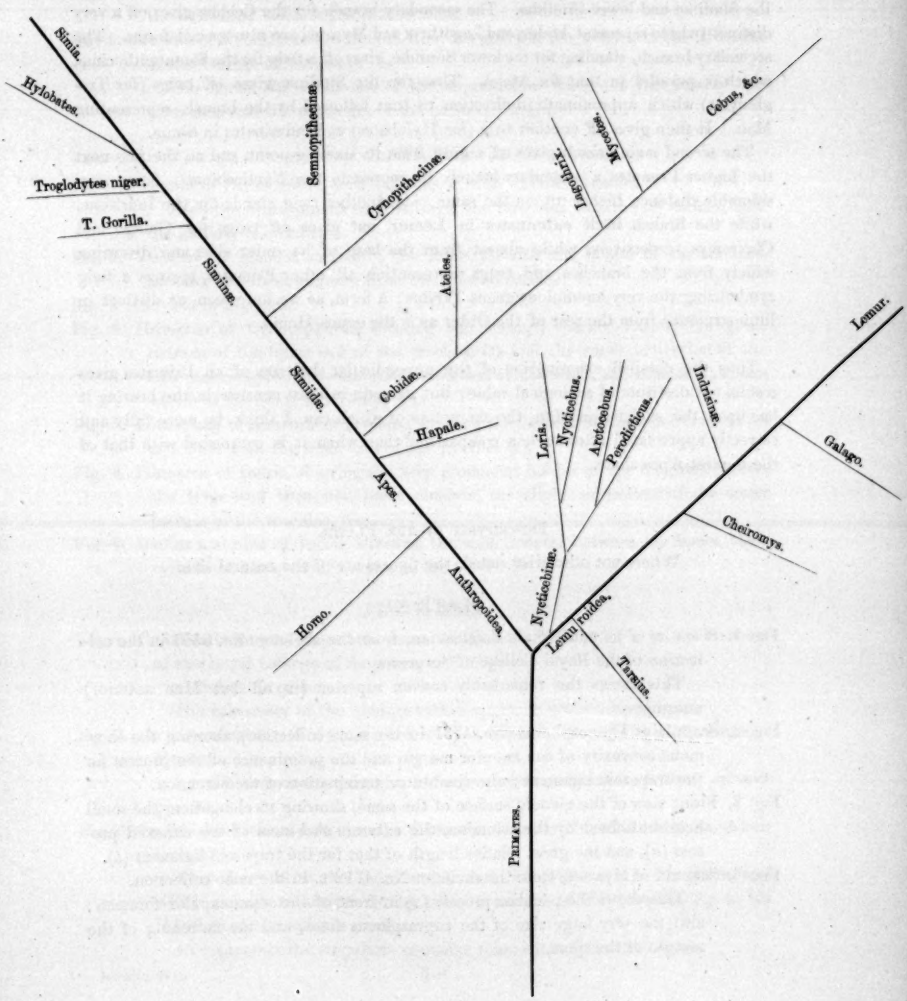
For the main peculiarities of these families see above, where they are the last two of the "Exceptional forms" given above.

To sum up the results of the foregoing observations, the Primates appear to present us (as regards their appendicular skeleton) with six principal types of structure, namely, (1) Homo, (2) Simia, (3) Cercopithecus, (4) Nycticebus, (5) Lemur, and (6) Tarsius. The first, however, has relations both with the third and fourth, some of the Nycticebinæ resembling Man more than all, or almost all, the other Primates in the proportion borne by the arm, without the manus, to the spine; in the proportion borne by the radius and ulna to the same; in the length of the pollex as compared with that of the longest digit; in the proportion borne by the tibia to the humerus and to the femur; in the length of the pes as compared with that of the tibia; in the marked anterior vertebral angle of the scapula; in the small supraspinous fossa; in the excess of the infraspinous fossa over the supraspinous one near the glenoid surface; in the short symphysis pubis; in the very peculiar tuberosity of the ischium. Moreover, one or more of the Nycticebinæ differ from the other Lemuroidea and approximate to Man in the greater or less degree of sigmoid curvature of the clavicle, and in the absence, in one genus, of the supracondyloid foramen in the humerus.

Besides the above six types, other forms show, as we have seen, more or less marked peculiarities; and perhaps the affinities between the various groups of the order (as regards the characters offered by their appendicular skeleton exclusively) may be fairly represented under the symbol of a tree. The trunk of such a tree divides into two main branches, for the Anthropoidea and Lemuroidea respectively.

The first main branch gives off a secondary one, which represents Man *, and then

* It should be borne in mind that this is only an attempt to express the degrees of resemblance existing amongst the appendicular skeletons of Primates, not the affinities indicated by their osteology generally, still less that evidenced by the totality of their organization. It is, in great part, the ossa innominata which cause Man to diverge so from the other Anthropoidea.


Simia.
Hylobates.
Troglodytes niger.
T. Gorilla.
Semnopithecinæ.
Cynopithecinæ.
Mycetes.
Nyctipitheci.
Cebus, &c.
Ateles.
Simiinæ.
Simiidæ.
Cebidæ.
Hapale.
Apes.
Homo.
Anthropoidea.
Loris.
Nycticebus.
Arctocebus.
Perodicticus.
Nycticebinæ.
Indrisinæ.
Lemur.
Galago.
Cheiromys.
Lemuroidea.
Tarsius.
Primates.

two other secondary branches for the American Apes. It then bifurcates to symbolize the Simiinæ and lower Simiidæ. The secondary branch for the Cebidæ gives off a very distinct twig to represent Ateles, and Lagothrix and Mycetes are also special forms. The secondary branch, standing for the lower Simiidæ, gives off a twig for the Semnopithecinæ, which is parallel to that for Ateles. That for the Simiinæ gives off twigs (for Troglodytes) which approximate in direction to that followed by the branch representing Man. It then gives off another twig (for Hylobates) and culminates in Simia.

The second main branch gives off, almost from its starting-point, and on the side next the higher Primates, a secondary branch to represent the Nycticebinæ. Some considerable distance higher up, on the same side, another twig stands for the Indrisinæ, while the branch itself culminates in Lemur, but gives off twigs for Galago and Cheiromys respectively, while almost from the base of its outer side, and diverging widely from the branches and twigs representing all other Primates, springs a twig symbolizing the very anomalous genus Tarsius; a form, as we have seen, as distinct in limb structure from the rest of the Order as is the genus Homo.

Thus the detailed examination of the appendicular skeleton of all Primates gives results not destitute of zoological value; but its main interest consists in the bearing it has upon the skeleton of Man, the characters of which can, I think, be more fully and correctly appreciated after such a comparison than when it is contrasted with that of the highest Apes alone.

## Description of the Plates.

Where not otherwise stated, the figures are of the natural size.

## PLATE XI.

Fig. 1. Scapula of an adult male Boschisman, from the skeleton No. 5357 in the collection of the Royal College of Surgeons.

This shows the remarkably convex superior (in all but Man anterior) margin.

Fig. 2. Scapula of Cynocephalus (No. 4731 in the same collection), showing the enormous convexity of the anterior margin and the prominence of the process for the trapezoid ligament; also the obtuse termination of the acromion.

Fig. 3. Front view of the glenoid surface of the same, showing its elongation, the small height attained by the acromion, the extreme shortness of the coracoid process (*a*), and the great relative length of that for the trapezoid ligament (*b*).

Fig. 4. Scapula of Mycetes, from the skeleton No. 4718 B. in the same collection.

This shows the peculiar process (*x*) in front of the suprascapular foramen; also the very large size of the supraspinous fossa, and the flattening of the margin of the spine.

Fig. 5. Scapula of Pithecia from the mounted specimen in the British Museum, showing the extremely small supraspinous fossa.

Fig. 6. Scapula of Nycticebus tardigradus, from the skeleton No. 4634 A. in the Museum of the College of Surgeons.

This shows the small size of the supraspinous fossa, the shortness of the acromion, and the concavity of the axillary margin.

## PLATE XII.

Fig. 1. Scapula of Perodicticus, from a skeleton in the British Museum.

Here the supraspinous fossa is large, the vertebral margin very convex, and the axillary one concave.

Fig. 2. View of the glenoid surface of the same, showing the length of the coracoid process, and its divergence from the glenoid surface.

Fig. 3. Clavicle of Mycetes.

Fig. 4. Humerus of Cynocephalus, showing the strongly marked ridges, the great descent of the inner end of the trochlea (*t*), and the small projection of the external condyle; also the great thickness of the shaft, the considerable elevation of the great tuberosity, &c.

Fig. 5. Lower half of humerus of Mycetes, showing the descent of the inner condyle (*c*) to a level with the inner end of the trochlea (*t*).

Fig. 6. Humerus of Indris, showing the very prominent border of the bicipital groove, the large and truncated inner condyle, the slight projection of the inner margin of the trochlea, &c.

Fig. 7. Radius and ulna of Indris, showing the wide interval between the bones, their cylindrical form, small olecranon, &c.

## PLATE XIII.

Fig. 1. Os innominatum of Lagothrix, from the skeleton No. A 4718 *a* in the Museum of the Royal College of Surgeons.

*p*. Subpubic groove.

The tuberosity of the ischium rather approaches the human form.

Fig. 2. Os innominatum of Indris, from the skeleton in the College of Surgeons Museum.

*s p*. The enormous posterior inferior (inferior anterior of Man) spinous process.

Fig. 3. Os innominatum of Loris gracilis, from the skeleton No. 4633 in the same collection.

This shows at *m* the ridge which runs obliquely outside the ilium, and which appears to answer to the anterior margin of Man; as also *s p* to his inferior anterior spinous process.

*t i* represents the singularly man-like tuberosity of the ischium.

Fig. 4. Femur of Mycetes (from the same skeleton as the scapula and clavicle were taken from), showing the very gradual increase downwards of the transverse diameter of the shaft, &c.

Fig. 5. Posterior surface of the upper part of the femur of Hapale, from the skeleton No. 4666 in the Museum of the College of Surgeons.

This shows the wide space between the trochanters behind.

Fig. 6. Outer surface of the os calcis of the Gorilla (5179 *a. q.* College of Surgeons), showing the deep groove outside it, and the very prolonged tuberosity, with the strongly marked inferior concavity of the bone.

Fig. 7. Astragalus of Gorilla (No. 5179 *a. r.* College of Surgeons).

This shows the great breadth and shortness of the bone, and the surface for the tibial malleolus (*t m*) almost in the same plane with the surface for the shaft of the tibia.

## PLATE XIV.

Fig. 1. Dorsum of the carpus of the left manus of Troglodytes niger, from the specimen No. 5083 A. in the Museum of the Royal College of Surgeons.

In this the scaphoides (*sc.*) is large, and sends a marked process over the os magnum (*mg.*).

Fig. 2. The same view of the same part in the Orang (No. 5076 in the same Collection).

Here the scaphoides (*sc.*) and the intermedium (*i*) seem together evidently to answer to the scaphoides of the Chimpanzee.

Fig. 3. Trapezium of Troglodytes niger from the mounted detached manus in the Museum of the College of Surgeons.

In this there is no trace of a concavity for the metacarpal.

Fig. 4. Trapezium of Nycticebus tardigradus, from a specimen in my own collection. (Four times the size of nature.)

Fig. 5. Palmar surface of right carpus of Perodicticus, from the specimen in the British Museum.

*Sc.* Scaphoides. *l.* Lunare. *c. u.* Cuneiforme. *pi.* Pisiforme. *u.* Process of unciforme. *tz.* Trapezium. *x* Extra ossicle placed between the unciform process and the very large process of the trapezium.

A bristle is represented passing through the arch formed by the extra ossicle and these last-mentioned processes. (Twice the size of nature.)

Fig. 6. Distal surfaces of the human carpal bones.

Fig. 7. The same view of those of Macacus.

Fig. 8. Distal surfaces of the human tarsal bones.

Fig. 9. The same view of those of Macacus.

These five figures exhibit the correspondence between the angles formed by the articular surfaces in the manus and pes.

Fig. 10. Naviculare of Loris gracilis, from a specimen in my own collection.
C 1 & C 2, the prominences for the cuneiform bones. (Four times the size of nature.)
Fig. 11. Entocuneiforme of Gorilla.
Fig. 12. Entocuneiforme of Cynocephalus. (Twice the size of nature.)
Fig. 13. Entocuneiforme of Lemur, showing the saddle-shaped distal surface. (Twice the size of nature.)
Fig. 14. Proximal articular surface of the metatarsal of the hallux of Man.
Fig. 15. The same of the Gorilla.
Fig. 16. The same of Cynocephalus. (Twice the size of nature.)
Fig. 17. The same of Macacus. (Twice the size of nature.)
Fig. 18. The same of Lemur. (Twice the size of nature.)

These last five figures show the change in direction of the concavity of the articular surface from vertical, in Man, to horizontal in Lemur, that of the Gorilla being oblique.

## XIV. *Observations on the Ovum of Osseous Fishes.*

*By* W. H. Ransom, *M.D.* *Communicated by Dr.* Sharpey, *Sec. R.S.*

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In November 1854 I had the honour of presenting to the Royal Society a short paper, containing the principal results of observations made on the ova of two species of *Gasterosteus*, and I then expressed an intention of furnishing a more detailed record of them at a future time. Since that date my experiments have been continued and extended, and I now purpose so to recount them, that physiologists interested in this department may be able to judge of the value of the results, and to repeat the observations.

It is intended in the following pages to consider, first, the unimpregnated ovum, with reference to its structure, its physical and chemical properties in the mature state, and during its development; and afterwards the impregnated egg, in reference to the mode in which fecundation is effected, the phenomena which follow it, and their modifying and essential conditions.

### The Unimpregnated Ovum.

*The Stickleback* (Gasterosteus leiurus and G. pungitius).

1. *Ripe deposited ova.*—The 3-spined and 10-spined sticklebacks may be taken together, as there are no important differences in the structure of their eggs. These observations, however, were for the most part made on the former, which are more easily obtained in numbers, and where the latter were employed it will be so stated.

The freshly deposited ova are held together by a colourless, transparent, viscid, mucoid matrix; they are few in number compared with those of many other fishes, and of large size, measuring about $\frac{1}{17}''$ on the average. To the naked eye they have a pale amber tint, and are semitransparent, spherical in form, but irregular, from mutual pressure (Plate XV. fig. 1). Each consists of an outer covering, the well-known dotted membrane or chorion, which I shall speak of as yelk-sac, understanding that term to mean that covering of the yelk which, being formed in the ovary, is placed next in contact with the yelk, but takes no part in cleavage; and a yelk-ball, divisible into formative and food-yelk, the former forming a complete cortical layer of granular matter, the latter, the chief mass, containing oil in large drops.

a. *The viscid layer* is a secretion from the oviduct. It resists for some time the action of water and prevents its imbibition in unimpregnated eggs, so that they remain flaccid after at least $2\frac{1}{2}$ hours immersion. It is then no longer distinguishable as a viscid layer, but it makes the eggs cohere firmly together, as if it had the property of setting in water.

This substance has an alkaline reaction on red litmus-paper, but does not change turmeric. A very weak solution of potash destroys its viscidity, and permits water to enter and distend the egg. A weak solution of acetic acid destroys its viscidity, and renders it opaque and flocculent. Boiling and nitric acid do not coagulate it.

b. *The yelk-sac* is a rather thick membrane, measuring $\frac{1}{515}''$. It surrounds the yelk-ball, and is, on its inner surface, in immediate contact with it, so long as no water has been imbibed. It is characterized by a tolerably regular fine dotting, the dots being arranged in lines which cross each other, so that lozenge-shaped spaces are left between them. At the folded margin corresponding radial lines appear, themselves resolvable into dots, as if the membrane consisted of concentrically arranged laminæ, each dotted so as to correspond with the next layer. The further details of this structure will be given in the section devoted to the development of the ovarian ova.

One part of the surface of these eggs is distinguished from the rest by a scattered group of stalked, cup-shaped processes, or buttons, which covers about one-fourth of the surface, and marks the germinal pole of the unimpregnated egg. The form of these buttons varies a great deal, and the size is by no means constant, but for the most part they a little exceed in length the thickness of the yelk-sac, and the form, when not changed mechanically, is shown in Plate XV. fig. 2. Further details of their structure are given in the section on development of ovarian ova.

In the centre of this group of buttons is the micropyle. It may be seen in various modes. I first noticed it while crushing an unimpregnated egg in June 1854, but its characters are best studied by removing the germinal segment, and examining it separately after washing away the contents. Sections may also be made by imbedding ova in strong warm jelly, and slicing the mass when cold, and in this way, some may be obtained which cut the micropyle vertically. It consists of a wide-mouthed, funnel-shaped pit, directed towards the centre of the egg, near its apex becoming more acute, and terminating in a short narrow tube with almost parallel sides (Plate XV. fig. 3). The inner end of this tube is apparently open, and however viewed, whether from within or without, whether in sections or in whole eggs, looks like a clear, pale-blue, oval or circular aperture, and measures $\frac{1}{1850}''$ (Plate XV. fig. 4). The fine dottings of the yelk-sac cease abruptly at the margins of this opening. Powers of $\times 100$ to $\times 200$ are well suited for the examination of these sections, but higher ones may be used without difficulty. To examine the relation of the micropyle to the egg as a whole, or to the yelk-ball, powers of $\times 50$ to $\times 100$ are more convenient.

Unimpregnated ova yield to moderate pressure without rupture, and when the germinal pole presents, the micropyle may be seen either full face, or at various inclinations; and if the position be suitable, the terminal opening is still visible as a clear bluish spot, although the whole of the yelk-ball is below it. When the germinal pole is in profile under suitable pressure, the funnel is seen projecting into that portion of the yelk-ball which corresponds to this pole (Plate XV. fig. 5), and which I have called the discus proligerus. In unimpregnated ova this relation is not quite so distinct as in those which

have been recently impregnated. If eggs are examined in a similar manner about four or five minutes after having been fertilized, the funnel is seen very clearly, half withdrawn from a corresponding pit in the centre of the discus proligerus (Plate XV. fig. 6). The ordinary mode of examining the surface of these eggs under a lens is not convenient for observing the micropyle, on account of the strong reflection from the viscid layer, which cannot be got rid of by drying or wiping.

The yelk-sac resists the action of water for a very long time, in so far as it is not decomposed or materially changed in its optical properties by it, and indeed it differs markedly from the contents of the egg by its greater stability. It is, however, changed by imbibition of water, from a soft easily lacerable membrane to a firm elastic one, each small section of which returns to its normal form very quickly after pressure. It is not much altered by spirit of wine; it is rendered clearer, slightly swollen, and its markings are made less distinct by dilute acetic acid and by dilute solutions of potash.

c. *The yelk-ball.*—A delicate, colourless, translucent, homogeneous membrane, which I call provisionally the inner sac, covers the whole surface of the yelk-ball within the yelk-sac. It is not so easily shown in the eggs which have not imbibed water, but with care may be seen to escape with its contents, by causing a sudden rupture with a large opening in the yelk-sac. By examining recently impregnated eggs, which, being elastic, allow the contents to escape with a jerk under suitable pressure, it may be better seen, as it escapes, thrown into distinct folds, contracting or collapsing as its contents pass out into the water around (Plate XV. fig. 7). I have reason to think that it is hardened by the action of water on its outer surface, its membranous characters being more distinct in eggs which have been some time impregnated. When free in water it shows a double contour at the folds or wrinkles only; for where it still contains in its pouches yelk-substance, the inner surface line is not well defined. Under pressure it seems capable of almost indefinite extension before it ruptures. It thus presents more the characters of a firmer layer or crust upon the surface of the thick fluid yelk-ball, than of a separable membrane. Perhaps the best view of it is got while it is yet within the yelk-sac, after partial escape of the yelk (Plate XV. fig. 8). I formerly spoke of this membrane as elastic, but I have now some doubt of this, as its tendency to shrink and collapse after rupture may be due merely to the escape of its fluid contents.

Dilute acetic acid and dilute solution of sal-ammoniac do not dissolve it; dilute hydrocyanic acid does.

*The formative yelk* is that portion of the yelk-ball which is afterwards directly transformed into the germ. It exists in the unimpregnated egg as a superficial layer completely surrounding the food-yelk, and is closely connected near the germinal pole with the soft, ill-defined internal surface of the inner sac, which, as it ultimately takes part in the cleavage, may to that extent be considered a part of the formative yelk. At the germinal pole it forms a thicker layer or disk, extending over about one-fourth to one-third of the surface of the yelk-ball, marked at its centre by a pit which receives the

micropyle, and at its margins passing imperceptibly into the thinner layer of similar material which extends over the rest of the yelk (Plate XV. fig. 5, and Diagram A).

Although this layer is found in the eggs of both species of *Gasterosteus* with identical structure and properties, it is best observed in those of *G. pungitius*.

It may be traced over the whole surface by examining the eggs under gentle pressure, either rolling over the same egg, or using at the same time a number of eggs in different positions, or by treating an egg with a weak solution of acetic acid, which so hardens the cortical layer that it cracks under pressure, and is then very distinct, even at the ventral pole, where it is thinnest. To show the thicker portion of it at the germinal pole, or discus proligerus, it may be viewed either full face under gentle pressure short of rupture, when it is seen as an opaque, yellowish halo around the micropyle; or in profile, when it is seen as in Plate XV. figs. 5, 6 & 8.

A power of $\times 50$ enables one familiar with the object to trace this layer very well, but to make out its constituent parts $\times 100$ is required, and then it is better to examine both before and during rupture, under suitably graduated pressure. By such means it may be made out to contain a number of droplets, sometimes irregular in form from mutual pressure, but usually round, and varying much in size from a diameter equal to the thickness of the yelk-sac down to an immeasurable granule, in which case the characters of a drop are lost. They have a yellowish colour, are placed near the surface of the egg next to the inner sac, and the larger ones are much more numerous in the germinal than in the ventral segment. They are imbedded in a mass of fine, yellowish, granular matter, which in part at least consists of very minute similar droplets, but principally of a substance having somewhat different reactions.

These elements are held together by a homogeneous matrix, which in the discus proligerus makes the mass semi-solid, and under some conditions may be seen drawn into threads, as if very viscid.

In consequence of the presence of the granular elements of the formative yelk in the cortical layer of the unimpregnated eggs, they are more opaque to the naked eye than impregnated ones.

Attached to the basal surface of the discus proligerus, in contact with the clear food-yelk, there is a small collection of dark oil-granules, distinct from the larger drops which float in the food-yelk.

The yellow droplets are characterized by their reactions with water, and indeed they are so unstable that I could find no neutral medium in which to examine them when the egg is ruptured. When seen *in situ* in unimpregnated, unruptured eggs, into which no water has been imbibed, in consequence of the defensive action of the viscid layer, their aspect is perfectly homogeneous and highly refractive, although less so than oil. On rupturing an egg in water, they exhibit vacuolation very rapidly, and undergo a very varied series of changes, during which they become pale and disappear, often presenting appearances like cells, with clear, lilac-tinted, vesicular nuclei in a mass of a deeper yellow colour, either granular or homogeneous. Similar changes occur, but less rapidly,

if eggs be ruptured with no other fluids than the adhering maternal secretions, such as the serum, or the alkaline viscid layer; and care being taken to note the position of the escaping currents, it seemed as if the acid food-yelks had a similar effect.

These changes, so common in protoplasmic matter, wherever met with, give rise to the suggestion that they may be due to the separation of two immiscible fluids, by the contact of a third, from a previous state of feeble combination. The lilac or faint blue tint of the vacuoles may be an optical effect of contrast, as I produced a similar appearance by shaking together water and yellow fat, but, as the depth of the lilac tint is not in the ratio of the depth of the yellow, I am not certain on this point. That I am justified in calling them droplets, is shown by the fact that they sometimes fuse together.

The granular basis, which, besides the yellow droplets in a granular condition, forms the formative yelk, is exhibited best by the action of water, which, while it causes the latter to disappear gradually while vacuolating, makes the former at once become darker and more distinct, and does not cause its ultimate disappearance.

The homogeneous matrix of the cortical layer contains also a peculiar form of albumen, which water precipitates in fine molecules, and which I shall have to speak of as albumen *b*.

The yellow droplets disappear also with vacuolation in a solution of sal-ammoniac, although less rapidly than in water. A weak solution of acetic acid acts on them somewhat similarly. A weak solution of potash rapidly dissolves them, and those of them which are in the granular condition are similarly acted on by these reagents.

The larger part of the granular substance of the cortical layer, rendered darker by the action of water, is not dissolved by sal-ammoniac or by dilute acetic acid. These agents cause a precipitate to appear, of dark granules of a coarse kind, and make the whole layer solid, by coagulation. A weak solution of potash also leaves a granular solid layer after dissolving the yellow droplets.

That portion of the granular matrix which is dissolved by sal-ammoniac, is finer than the substance precipitated by it, and if after this reaction a solution of acetic acid be added, a further precipitate of very fine granules is formed, which is due to the presence of albumen *b*. The finer granular deposit caused by the contact of water is easily dissolved by the solution of sal-ammoniac, not by acetic acid.

Thus it is possible to distinguish in the formative yelk (1) part of the substance of the inner sac, (2) the matter of the yellow droplets and the granules of the same material, (3) the granules darkened by the action of water and not dissolved by the alkaline chlorides, (4) the smaller oil-drops—all existing as separable substances already formed. In solution or otherwise not optically separable, (1) the albumen *b*, (2) the matter so largely precipitated by the sal-ammoniac.

No trace of a germinal vesicle or of its contents could be found after the most careful and repeated searching.

*The food-yelk* forms the chief mass of the egg. It is a thick fluid drop, covered by the formative yelk, colourless, transparent, and without visible contained particles, ex-

cept the oil-drops. The surface of the drop within the inner sac I had some reason to think a little less dense than the centre, as it ran rather more freely, but all parts flowed from a rupture like a very thick syrup.

The oil is collected into a group of large and small drops and granules, which moves freely through the fluid food-yelk, but not through its centre, to get to the uppermost segment when the egg is rolled; in so doing the drops often separate to unite again. Sometimes, especially in impregnated ova, they adhere to the germinal mass, and then cause it always to float uppermost.

The food-yelk is acid in reaction to blue litmus, to an extent which more than suffices to neutralize the alkalinity of the viscid layer, but its taste is astringent rather than acid. It is coagulated firmly by boiling, by nitric acid, or by spirit of wine. Water causes a fine molecular precipitate in it, soluble in alkaline chlorides and acetates. Dilute acetic acid precipitates freely very fine dark molecules in rapid vibratory movement, after the action of the chlorides, or previously, and then dilute nitric acid causes a still further molecular deposit of a darker aspect.

I tested for cellulose in the different parts of the egg without finding any.

2. *The ovarian ovum.*—The development of ova in ovario I have only attempted to trace in its later stages, *i. e.* after the first germs of the egg in a distinctly recognizable form have been laid. This has been done, mainly, with a view of throwing light upon the mode of growth of the parts of the ovum, and upon the ultimate fate of the germinal vesicle and its contents.

a. *The ovaries* appear completely formed, and containing their characteristic elements in very young fry, certainly in those not more than a month old and about $\frac{1}{2}''$ long. They are, however, better studied in the adult, in which they exist as complex folds of vascular connective tissue, not separable from the peritoneum, attached on either side of the bodies of the vertebræ, and projecting as leaflets, in which lie the ova contained in ovisacs which are lined with epithelial cells, but have no demonstrable basement membrane. There is no connexion, beyond that of simple contact, between the outer surface of the yelk-sac and the ovisac; certainly no peduncle. As the position of the micropyle can be easily and certainly determined in early ovarian ova by the buttons which surround it, the absence of such attachment connected with it is not difficult to prove, by watching eggs escape from the ovisacs under graduated pressure.

The ovisacs are supplied with blood-vessels running in one, two, or more directions, so that they are not pedunculated. Younger and more advanced eggs are met with together in all parts of the ovary without any definite arrangement.

The ovaries are enclosed in a sac, which is attached to their bases on either side, and anteriorly, but continuous with the sexual aperture behind the vent, and must be looked upon as the oviduct. Into its cavity the ova escape when ripe, and remain there for a short time before they are deposited; its walls are muscular, and its inner surface secretes a viscid substance which defends the eggs when deposited from the too rapid action of water, and which serves as a suitable medium for the spermatozooids to move in.

b. *Nearly ripe ova* will be conveniently described here before the earlier ovarian ova; and at the same time I shall attempt to trace the fate of the germinal vesicle and its contents. As the germinal spots have some resemblance to the yellow droplets of the formative yelk, and as both are so very unstable that they undergo visible changes in all ordinary media, a large number of observations had to be made with a view of determining their characteristic reactions.

Nearly ripe ovarian ova, having a diameter of $\frac{1}{13}''$ to $\frac{1}{18}''$, have the oil collected but imperfectly, and are not quite so yellow and clear-looking as ripe ova. The germinal vesicle can be seen *in situ* very well, by placing an egg without the previous contact of water on the slide, and using some pressure, the micropyle either presenting or in profile: the latter is the better mode. The vesicle is then constantly found excentrically placed in the egg, imbedded in the centre of the semi-solid discus proligerus, so that the apex of the micropyle comes nearly into contact with the centre of its surface directed to the germinal pole. At this period it appears lenticular when viewed in profile, and so closely connected with the substance of the discus proligerus, that on rupture of the egg by pressure, the vesicle carries with it, in escaping, a portion of the granular matter in which it lies.

In an egg which measured $\frac{1}{10}''$ the germinal vesicle had a diameter of $\frac{1}{104}''$. Viewed with a power of $\times 50$ the germinal spots are just visible, the other contents of the vesicle not at all, while it is *in situ*. To see the spots with higher powers, it is better to puncture the yelk-sac before applying pressure, so that the contents may escape with less violence. If an egg be ruptured by pressure in water, the germinal vesicle often escapes detection altogether; but pressure short of rupture, without water, shows the vesicle very well, and if it be then increased so as to produce rupture, the vesicle may be easily traced as it passes from its natural position to the aperture in the yelk-sac, where, however, it often breaks and is lost. In all these modes the contents of the vesicle are apt to be displaced, and are so variable in aspect, that no doubt can remain of their being changed somewhat, either by the media they are examined in, or by mechanical violence. When by pressure and rupture without water the vesicle escaped without being destroyed, it was much distorted at the opening in the yelk-sac, but as it lay in the fluid of the egg it appeared round. Examined with a power of $\times 200$ the germinal spots were numerous, loosely aggregated, rounded irregularly, of considerable refractive power, and without any distinct vacuolation in their substance, which had a homogeneous aspect and was almost solid looking (Plate XV. fig. 9). They measured on the average $\frac{1}{1000}''$. Besides the germinal spots, the vesicle was nearly filled with a very delicate molecular matrix. When to this slide water was added, the vesicle imbibed it, and became distended at first unequally, the molecular matrix was displaced, the molecules seemed to darken (although this might be an effect of the greater contrast of refractive index merely), and among them delicate vacuoles appeared (Plate XV. figs. 10 & 11). The wall of the vesicle resisted the action of water and showed considerable tenacity, admitting of free manipulation. It was not dissolved by a solution of sal-ammoniac, or by weak acetic acid, which shrivelled it.

When an egg is ruptured by pressure in water, and the germinal vesicle is seen as it escapes, it shows the contents more changed, and it is very apt to vanish quickly in the water, as if from some injury received during its expulsion. However, sometimes it may be examined in this mode, if no time be lost, and then the germinal spots are very irregular. At a rough estimate the number of spots is about 100 at this period. They are slowly dissolved by a solution of sal-ammoniac, the molecular matrix more quickly so. Dilute acetic acid, added after the action of the sal-ammoniac, causes a copious, very fine dark granular precipitate within the vesicle.

In one instance I found eggs measuring $\frac{1}{17}''$, the average size of ripe free ova, which contained the germinal vesicle with contents precisely as above described. In this case the oil was more concentrated, the eggs clearer-looking to the naked eye; and as a further proof of their being nearly ripe, the oviduct was furnished with a store of the viscid material ready to cover the ova as they burst the ovisacs. Not all the eggs, however, of the same batch have exactly the same dimensions, and still less have those of different individual parents when ripe and free. In one, at least, of these ovarian ova I ascertained that the germinal vesicle had disappeared.

In the nearly ripe ovarian ova, from about $\frac{1}{18}''$ to $\frac{1}{13}''$, in which the germinal vesicle is visible, there is a cortical layer of formative yelk, a thicker layer of it, or discus proligerus, at the germinal pole, essentially identical in structure and properties with the same parts in the deposited eggs. There are some slight differences however, the most distinctive being, that larger droplets, apparently identical with the matter forming the yellow droplets, but a little paler in colour, occupy a deeper plane in the cortex, chiefly of the germinal segment. These undergo similar changes of vacuolation, and have identical reactions with the yellow droplets. This vacuolation presented at the same time in various parts of the escaped formative yelk both lilac-tinted and colourless vacuoles, shown, by their inverting an image seen through them when beyond focus, to have less refractive power than the surrounding medium, and by their gradual growth and fusion, to be in reality drops of a limpid fluid. Between these limpid drops there appeared various kinds of granules and semisolid-looking refractive, yellow, crescent-shaped masses, partly or wholly surrounding the vacuoles, thus giving rise to appearances like young cells. Sometimes a number of minute vacuoles formed within a large homogeneous-looking yellow droplet, and thus a pseudo-granular corpuscle resulted. But all these appearances were fleeting, and the variations infinite, depending in some degree on the nature of the medium which had been used. Thus water caused the vacuolation most rapidly; the viscid secretion of the oviduct and the food-yelk, I have before said, excite it, although slowly; the same may be said of the scanty succus, or the blood or serum contained in the tissues of the ovary, or of weak solutions of glycerine or sal-ammoniac; this latter, by causing a precipitate in the matter of the formative yelk, complicates still further the forms which result from these changes. There is seen at the under surface of the discus proligerus, and forming a part of it, the little heap of fine dark oil-granules, distinct from the store of oil which floats in the food-yelk. The inner sac is distinctly to be made out in these ova.

In still more advanced eggs, *i. e.* those of full size, with concentrated, grouped oil-drops, although still within their ovisacs, no trace of germinal vesicle could be found on repeated and careful examination; so that its disappearance precedes the escape of the eggs into the oviduct. The most careful and prolonged examination of the substance of the discus proligerus, more particularly of that part of it which had so recently contained the germinal vesicle, failed to show any trace of the germinal spots, or any other change in the structure or reactions of the matter of the discus proligerus, after the germinal vesicle had vanished, which could be looked on as due to its disappearance. In face of this negative result, however, it may be mentioned that in cases where I watched the germinal vesicle escape through a rupture in the yelk-sac into the surrounding fluid, whether water was present or not, it often happened that the vesicle was ruptured and its contents escaped; and when this did occur, I frequently could not see the spots among the surrounding materials if, perchance, they were lost sight of for a few minutes. This was doubtless due, in part, to the fact that all the objects in the field were in a state of constant change; still, the changes which occur in the germinal spots are such as might distinguish them from the only visible elements of the formative yelk which can be compared with them, viz. the yellow droplets; for these latter vacuolate, fade, and disappear in water, while the former vacuolate and become darker in outline and do not disappear, at least for a very long time, in water. Perhaps the saline or other constituents of the yelk-ball may have a solvent action on the germinal vesicle or spots; but this point, which might be submitted to experiment, I had not an opportunity of determining.

The ripe or nearly ripe ovarian ova have their ovisacs decidedly thinner at that part which covers the germinal segment. These eggs, placed in water, speedily imbibe it, and become faintly opalescent. Vacuolation soon appears in the matter of the cortical layer, beginning at the germinal segment, and in ten minutes the yellow drops disappear, and a slight interval appears between the yelk-sac and the outer surface of the yelk-ball—a true breathing-chamber. In one or two cases, where some rupture or injury had taken place, a partial concentration of the formative yelk also took place, but this was exceptional.

c. *Earlier ovarian ova.*—An adult female fish, taken from the natural haunts in the month of June, after she has deposited her first batch of eggs, and in which a second is ripening, may be used conveniently to examine the developing ovarian ova in all their earlier stages. Three principal groups may then be made out with the naked eye; 1st, large, nearly ripe, semitransparent, yellowish ova, the oil grouped more or less; 2nd, medium-sized creamy tinted opaque ones, with oil scattered; 3rd, smallest, colourless or whitish, semitransparent ones without oil-drops. The two latter groups alone remain to be described, and I shall examine them chiefly with reference to their mode of growth.

There is no advantage gained in the study of the earliest distinguishable ova by taking young fry; for adults in the autumn, winter, or early spring contain the smallest visible ova as easily observable as in young fish of $\frac{1}{2}''$ in length, and not a month old. I may

mention here, incidentally, that young fry, taken from the streams in November, measure about 1″ to 1¼″, and their ova, which belong to Groups 2 and 3, reach about $\frac{1}{75}$″. Young fry in June of ½″ in length, contain ova not much exceeding $\frac{1}{300}$″, but some fry are met with in this month about 1″ in length, and in all respects as far advanced as are some of the later hatches met with in early spring, and which do not seem to have grown at all during the winter. In adult females in November, the largest eggs are about $\frac{1}{33}$″.

3rd Group.—The smallest certainly recognizable ova measured $\frac{1}{300}$″ (excluding now a single observation in which I met with what appeared to be still earlier ova without distinguishable yelk of any kind around the germinal vesicles), Plate XV. fig. 12. They are spherical, have a distinct ovisac lined with cells, a central, comparatively large germinal vesicle, a yelk of one kind only, which is solid, yellowish, refractive, homogeneous, semitransparent, and is not covered by a distinctly separable yelk-sac, but has a smooth defined border, probably indicating its first trace (Plate XV. fig. 13). As the eggs grow larger, the first change noted is a faintly granular aspect of the yelk, and, with certain methods of examination, an appearance as if a clear substance occupied the centre around the germinal vesicle.

The yelk-sac is separable in eggs measuring about $\frac{1}{300}$″, and may be seen in the fluids on the slide as a homogeneous-looking, collapsed sac. Eggs a little larger are less translucent, the yelk is more granular, the free yelk-sac is seen to be furnished with buttons, and has the dotted structure; these eggs measured $\frac{1}{140}$″, the germinal vesicle measuring $\frac{1}{333}$″. Later on, the cortical layer is seen to have the yellow droplets.

2nd Group.—The oil begins to appear in eggs about $\frac{1}{80}$″ in diameter, at first as scattered small granules, and the whole egg is then more opaque, not only from the presence of the oil, but partly from its larger size, partly from the more markedly granular structure of the cortical layer (Plate XV. fig. 14). These eggs pass into the above described first group, with gradual increase of the oil and grouping of the large drops.

In both of these groups the germinal vesicle is central and globular; nor could I succeed at any time in making out how it became transferred to its excentric position, and received its lenticular form. I saw also no discus proligerus, although in all the eggs of the second group a food-yelk exists and escapes on rupture, apart from the formative yelk or cortex, and probably also in some of the later stages of the third group.

To examine the relations of the parts of the eggs in both these groups, water is not a good medium, as it changes them too rapidly by imbibition; but its action on the ovisac is noteworthy. It distends the contained cells rapidly, and passes through the yelk-sac so as to act on the yelk without forming a breathing-chamber. A solution of chloride of sodium, 1 per cent., is a very good medium to be used,—it exhibits well the cells lining the ovisac, and causes very little distension; but a 1 per cent. solution of glycerine is perhaps better for taking measurements in, as it neither distends nor shrivels for some time, while it leaves the whole field very clear. A solution of the acetate of potash of the same strength is also a good medium. All these, after some minutes, permit one

to see in eggs of about $\frac{1}{100}''$, a clear halo around the germinal vesicle, bounded by a defined, but irregular granular outline. At present I hesitate to express any decided opinion as to whether this indicates a first separation of the yelk into two kinds.

A solution of chloride of sodium of 2 per cent. contracts or shrivels the tissue of the ovary, and makes the eggs, when they are above the very smallest size, opaque, by precipitating in them the matter of the cortex.

Solutions of acetate of potash of 5 per cent. or 2 per cent. cause the egg, with its yelk-sac, to shrink within the ovisac, which dilates; make the larger eggs of the third group opaque by precipitating the formative yelk, and leave the smallest homogeneous eggs clear, do not permit the clear halo to be seen, and make the yelk-sac paler.

Solution of glycerine, $2\frac{1}{2}$ per cent. is a very good medium, it leaves the field clear, all the objects well defined, but shrivels the egg a little in the ovisac.

Strong glycerine is quite unsuitable, it changes the appearance and form of the yelk-mass, and obscures the germinal vesicle at times completely; nor can the natural appearances be restored to specimens preserved in it by adding water. The ovarian stroma is also obscured in it.

*The yelk* requires to be examined in various media in order to make out its structural elements, and its separation into food and formative yelk.

Water, although in using it great care is required, on account of the rapidity with which it changes everything, is very useful. When used abundantly it causes a fine granular precipitate in the substance of the yelk, in the very smallest ova met with, due probably to the presence of albumen *b*, but does not cause visible vacuolation in the yelk. It also permits, in the larger eggs of group 3, and the smaller of group 2, the clear halo around the germinal vesicle to be seen; at the same time it causes a granular deposit in the cortical layer, and then gives rise to vacuolation; and if the eggs are at that stage that yellow droplets have appeared, they grow pale and disappear.

A 1 per cent. solution of acetate of potash slowly causes a precipitate in the cortex of eggs above the very smallest; very slightly also a turbidity of the smallest egg.

A solution of $1\frac{1}{4}$ per cent. of chloride of sodium, which does not alter the blood-disks of the same fish, also causes a dark precipitate in the cortex of eggs which have a distinction of yelks.

A 1 per cent. solution of glycerine is the most neutral agent, as far as regards the yelk-substance, but after some time the yelk of all ages becomes slightly granular in it.

In trying to determine at what stage of development the granular elements of the cortical layer appeared, it was necessary first to find a fluid medium which did not determine a precipitate. The maternal fluids may be used, but do not enable one to obtain a clear field.

5 and $2\frac{1}{2}$ per cent. solutions of acetate of potash precipitate the larger of these eggs strongly, the smaller less so, the smallest not at all, and their yelks escape in a solid form. The yellow droplets change very slowly in these solutions.

Hence it is safe to say, that the substance which is first seen around the germinal

vesicle is neither food-yelk nor formative yelk, but differs in structure and reactions from both in a very marked manner, although it contains probably a little albumen *b**. It may be called primitive yelk. That water and other fluids cause no visible vacuolation in its substance may possibly be due to the fact, that if it were delicate it might be obscured by the changes which are produced at the same time in the cells of the ovisac, and which at times are very confusiug.

The primitive yelk is very firm, and often escapes on cutting up a fragment of an ovary in 1 per cent. solution of chloride of sodium, or in a solution of glycerine 2 per cent., as a solid-looking, somewhat angular body, with its contained germinal vesicle (Plate XV. fig. 15). In water also it is solid, though much paler, when escaped, than in the other media. Although it is not like the perfect formative yelk in structure or properties, it may possibly be continued in a modified form, and exist in some proportion in the ripe ovum, as its reactions are rather negative than positive, and it might therefore easily escape detection.

To the primitive yelk, as the eggs grow, are added the other elements; first, of the formative yelk, and afterwards of the food-yelk, as the above-mentioned reactions prove. The precise time of the appearance of the food-yelk was not made out with certainty; it is probably some time before the oil-drops make their appearance, and possibly the clear halo around the germinal vesicle is the first optical expression of it. What its precise relation to the germinal vesicle and formative yelk may be at first, I could not determine. The solidity of the primitive yelk reminds one forcibly of the early condition of the yelk in the ovum of Birds.

Whether any inner sac exists in ova of the groups 3 and 2, I cannot say. I could only find it in group 1, *i. e.* nearly ripe ova; and one observation seemed to indicate that in group 3, at least, it is not present; for these eggs, when examined in saliva, show the yelk-sac distended, together with the ovisac as one membrane, and then the surface of the yelk is granular and irregular, not smoothly defined as it would be, were an inner sac present.

At no time did I observe any contractions of the protoplasm of immature ova: perhaps I did not use the requisite media; but the solid state in which it exists at first makes it difficult to conceive how such could occur.

*The germinal vesicle and its contents* also require that various reagents, of different degrees of concentration, shall be employed in their examination.

The first difficulty is to get to understand the natural aspect of such variable objects, and to appreciate duly the influence of the media used.

By cutting up a large piece of an ovary without any fluid, and selecting a small fragment for examination, the smaller eggs and the germinal vesicles may be studied, and the latter seen both *in ovo*, and free in the field; but the field is turbid, and the refractive index of the medium, which is a mixture of escaped yelk and serum, is too much like

* See page 451 for a description of this variety of albumen, which is probably a constant constituent of the yelk of vertebrata.

that of the objects to be examined, for good definition. The germinal vesicle appears to be filled with a homogeneous and colourless colloid material. It is true, however, that a finely molecular structure in it might in this way escape notice. The germinal spots are imbedded in this colloid matrix, on its surface only, so as to be in contact with the inner surface of the wall of the vesicle; unless, as often happens from mechanical causes and from imbibition, they have been displaced. They have a round form, homogeneous aspect, and a refractive power but little greater than that of the yelk-fluid (Plate XV. fig. 16).

A more convenient mode of seeing the natural condition of the germinal spots, is to use a very minute quantity of water in cutting up the fragment, and to examine rapidly before time is allowed for changes to take place. Then, the spots are seen round and homogeneous-looking, when within their vesicles in larger ova, water not having reached them by imbibition. But in the vesicles of the smallest eggs, or in those which lie free in the field, the spots when first seen are variously tailed and vacuolate, parietally placed in the vesicle, lenticular when seen edgeways, their outlines much darker, and harder, perhaps, in part, an effect of contrast. The colloid matrix is usually seen delicately shaded by a fine molecular deposit, but the conditions of the formation of this molecular deposit I could not feel quite sure of, except that water favours it.

More abundant and prolonged action of water is apt to displace the germinal spots, by distending the vesicle, but this it does irregularly, so as to make it appear in some measure a result of mechanical injury (Plate XV. fig. 17). The colloid matrix, after a time, becomes more granular, and this change may even obscure the germinal spots, when a weak solution of chloride of sodium dissolves the fine granules, without impairing the consistence of the colloid matrix. This was well seen in one instance, at a rupture in the wall of a large escaped vesicle, in which also the extreme toughness and strength of the vesicular wall was manifest. The action of water, however, on the spots and on the colloid matrix is not the same on free uninjured vesicles, as it is on those still within the egg, especially the larger ones; in which I found that the results were in great measure due to the influence exerted by the saline or other constituents of the yelk, which were carried into the vesicle by osmose. Thus, in larger eggs of group 3 and smaller ones of group 2, when long acted on by water, the germinal spots of contained vesicles are seen to get pale and disappear; at the same time the ovisac and yelk-sac show evidences of abundant endosmose, and there is also some granular deposit in the cortical layer of the yelk, but not such as will account for the obscuration of the spots, as the position of the germinal vesicle is well seen, marked by a clear area (Plate XV. figs. 13 & 14). Still later on, the contents of the germinal vesicle are seen as distinct granules in rapid tremor. These facts strongly suggest the notion that the germinal spots are soluble in some of the constituents of the yelk, and we may thus explain their disappearance in ripe ova.

It should be here mentioned that free germinal vesicles, being uninjured, remain in water for hours, without much visible distention or displacement of their contents, or disappearance of the germinal spots; and the same may be said of those contained in very

small eggs; and if the fragment has been prepared in an abundance of water at first, the free vesicles may show no molecular deposit in the colloid matrix after seven hours (Plate XV. fig. 18).

The germinal spots, after this prolonged action of water on the free vesicles, are not soluble in a 10 per cent. solution of acetate of potash, but the spots in the unbroken larger ova are; thus water must chemically change the substance of the spots or their surface, as before its action $2\frac{1}{2}$ per cent. of the same salt dissolved them.

A 1 per cent. solution of glycerine is an excellent medium for showing the germinal spots; they remain for ten minutes in it without showing changes of form or vacuolation even in the free vesicles. It does not precipitate the colloid matrix.

A $1\frac{1}{4}$ per cent. solution of chloride of sodium, which does not change blood-corpuscles, added to a fragment of ovary, prepared in the maternal fluids, made the colloid matrix which was not previously granular look brighter, and changed the germinal spots from round homogeneous-looking bodies to variously tailed and vacuolated forms (Plate XV. fig. 19). On then gradually increasing the strength of the solution to 5 per cent., it was observed in a ruptured free vesicle that the germinal spots, as they lay adhering to the colloid matrix near, and partly within the rupture, gradually became paler, coalesced, and fused into a large pale drop, with vacuolation in and around it. The stages of this change are seen in Plate XVI. fig. 20. Precisely similar changes were seen to occur in the spots of germinal vesicles while yet contained in the eggs. It seems probable, then, that we must look on the germinal spots as drops of a thick fluid, or at least not as solid bodies.

A solution of only $2\frac{1}{2}$ per cent. of chloride of sodium which crenates the blood-disks, similarly caused fusion of the germinal spots. In a solution of 1 per cent. only, the spots vacuolate and become tailed very slowly, and after an hour I found them again round—suggesting the possibility that they may have a power of changing their form analogous to that possessed by the protoplasm of white blood-corpuscles. This solution causes the red blood-disks of the same fish to become paler: it does not ultimately dissolve the spots, but like water changes them, so that they are no longer soluble in even a 10 per cent. solution of chloride of sodium.

The solutions of acetate of potash act very much like those of chloride of sodium.

A weak acetic acid solution does not dissolve the wall of the germinal vesicle or further distend it. It precipitates the colloid matrix, leaving the spots dark-bordered and distinct.

*The yelk-sac* also merits a minute investigation.

The precise period at which it is formed is difficult or impossible to determine. In the smallest eggs seen, those of $\frac{1}{100}''$, it is not separable, but is probably indicated by the smooth hard outline which the yelk shows on its surface, when a $2\frac{1}{2}$ per cent. solution of glycerine or of chloride of sodium is used, which contracts the yelk-ball with the yelk-sac, and leaves a space between it and the ovisac. A little later it is seen indicated by folds on the surface of the yelk, the result of the shrivelling which the solution

causes; and in eggs about $\frac{1}{100}''$ in diameter it may be separated, but I failed to show any structure in it, probably from not succeeding in mounting it for examination with the highest powers. It is known by its characteristic foldings and refractive index.

The best way to get out the yelk-sac is to cut up a fragment of the ovary with very fine scissors in all directions, after having macerated it well in water; then by removing the larger pieces several yelk-sacs are seen free from their ovisacs.

The yelk-sacs of eggs of about $\frac{1}{140}''$ have certainly a fine dotted structure, and are furnished with buttons and a micropyle. The buttons can be seen on still smaller eggs, in which I found no dotted structure. As the eggs grow the yelk-sac gets thicker, and its markings more distinct: at first it is flaccid at all times; but in eggs of the 2nd group, and larger, the segments, after imbibition of water, during which they seem to increase in thickness, become elastic, so that each segment springs into its shape again like a segment of an india-rubber ball. When the fine dottings can be observed, they have the same characters in all the stages of growth.

The dots have a similar aspect on both inner and outer surfaces of the sac, are arranged in tolerably regular diagonal, curved lines, alternating, so that they enclose lozenge-shaped spaces. With powers up to $\times 500$ they appear round, and even with the highest used, $\times 2600$ and $\times 3000$, they are but obscurely hexagonal. They are seen blackest when a plane rather deeper than the true surface is in focus, and then appear round. With very careful adjustments, and the true surface in focus, they have a polygonal or hexagonal, not very sharp outline, and seem like pits; the elevated ridges between which look like a very fine, rather irregular reticulation. At a folded edge they produce an appearance of radial striation, the striæ resolvable into dots, due to the laminated structure of the sac. This is best seen at a cut edge, especially after longer maceration. The cut edge reminds one of the edge of cut lace or perforated zinc, but whether this is due to an actual falling out of the matter which caused the appearance of dots, or is an illusion, I cannot say. The dots act, in focusing, like a substance of low refractive power, and I incline to the view that the appearance described is illusory. In the smallest-sized eggs of which I measured the dots, they were $\frac{1}{14000}''$ apart; these eggs were about $\frac{1}{100}''$ in diameter. In nearly ripe ova, on the point of quitting the ovisacs, they were $\frac{1}{11000}''$ apart, measured from centre to centre, each dot being about $\frac{1}{30000}''$ in diameter; these results were the mean of several measurements with a power of $\times 1000$. The dots are the same distance from each other on the inside as on the outside of the sac, and the radial lines are the same distance apart. All the measurements were made in the eggs of the 3-spined species.

The outer surface of the sac suffers sometimes, after long maceration in water, a peculiar change in its consistence, so that on its rupture by very strong pressure, the surface-layer yields like a soft, almost viscid substance, seen as a colourless, structureless film, stretched across the rent, with a power of $\times 250$ (Plate XVI. fig. 21). But with a power of $\times 3000$ it has an exquisitely delicate structure, like net, very regular and perfect, and

evidently identical with the dotted structure. It forms thin layers, of which several may be counted. The dottings are dark with light interspaces, or light with dark interspaces, according to the focus (Plate XV. figs. 22 & 23).

Besides these minute regular dots, in larger eggs of group 2 there is a darker kind of dot, which I will call the stellar dot. It is irregularly scattered over the inner surface only, and can only then be seen from the outside, when a lower objective than $\frac{1}{8}''$ is used, which penetrates sufficiently. They are larger and much blacker than the regular small dots, of a stellar form, are wider apart (on an average $\frac{1}{3000}''$), but vary much in this respect. Examined and measured with a power of $\times 930$, each has a diameter of about $\frac{1}{8000}''$. They are in sharpest focus and blackest when a plane rather deeper than the true inner surface is in focus, and with that surface in focus they look like stellar-shaped pits. On focusing they act like bodies of low refractive power. At the cut edge they may be seen to pass radially about two-thirds into the substance of the yelk-sac, gradually coming to a point and ceasing. They do not look like spaces at the cut edge, as do the fine regular dots.

The buttons may be well examined in unripe ova, especially those nearly ripe. They are attached to the outer surface of the yelk-sac by a bright, highly refractive point, from which radiate along their under surfaces to the periphery, little folds of the substance, which is clear, homogeneous, soft, and easily distorted, by contact, in consequence of its adhesiveness.

I counted the number of buttons on five small eggs of group 3, and on the average found 80 to each. The average number on each yelk-sac of nearly ripe ova, or group 1, is 207, a result of five countings. In ripe deposited ova I could not prepare the yelk-sac so as to count them, on account of the readiness with which they became detached, by adhering to external objects; but there is no reason to think the number increases after the stage of group 1. I think it probable that they are organs of adhesion, and serve to fix the egg.

d. *The method of staining tissues*, so strongly recommended by Dr. Beale, was tried with reference to its importance as a test of germinal matter, a term which I assume to be synonymous, or nearly so, with protoplasm.

If a fragment of ovary be digested in the carmine solution* for half an hour, and then washed with the acetic acid glycerine, it will be found to be irregularly and unequally dyed, this irregularity affecting the ovarian stroma, the yelk-sac, and the yelk. The tissue of the ovary is softened to an almost viscid consistence; whether an effect of the ammonia or of the acetic acid I did not stop to ascertain.

After twenty-four hours' digestion in the carmine solution the stroma is deeply dyed, but less so than the yelk of the youngest ova. Those parts of the ovarian tissue which are thickest, such as vessels, have the deepest tinge; the films of connective tissue show no colour, perhaps on account of their extreme tenuity and translucency.

In the fragment which had been digested for half an hour, the minutest eggs showed

* This was prepared according to Dr. Beale's formula.

the deepest tinge in their yelk-mass, especially those most exposed to the fluid. Those larger eggs which had a food-yelk showed less colour; what stain had taken place was limited to the formative yelk, but many of these yelks were not dyed at all; the food-yelk flowed out from a rupture as a colourless fluid, or changed chemically into a mass of clear, starch-like corpuscles. The whole primitive yelk, when dyed, was deformed, rendered opaque, vacuolating, and granular, a physical condition favourable for reflecting its colour, but unfavourable for exhibiting its true structure or characters. The germinal vesicle and spots were obscured or quite undistinguishable.

To test the action of this dye-fluid on the germinal vesicle and contents, I prepared a piece of ovary in a 1 per cent. solution of chloride of sodium, as a neutral solution; then bringing into view a free germinal vesicle, I gradually added the carmine solution; slowly the vesicle swelled out, the spots became pale and vanished, the vesicular wall seeming to shrivel up.

That this was due to the ammonia was shown afterwards by repeating the experiment, using a dilute solution of ammonia (about $1\frac{1}{2}$ per cent. of Liquor Ammoniæ); the germinal spots vanished as in the dye-solution, and the vesicular wall also, but an hour later.

Thus it may be said, I think, that whatever value we may attach to this process of dyeing tissues, we must not neglect the consideration of the changes which may be produced in these sensitive states of matter by the menstrua employed. If the strong, tough, and very distinctly solid wall of the vesicle may vanish, if the highly refractive, striking-looking spots may be rapidly dissolved by Beale's carmine solution, who can tell what changes it may produce in the almost equally unstable and sensitive substances which constitute the growing parts of tissues, and probably even the functionally active parts? It certainly seems necessary to supplement this method by others capable of determining the appearances presented in perfectly indifferent media.

The yelk-sac took the dye freely; considering its thinness and translucency perhaps as much so as the formative yelk; the dye was deepest in the thickest sacs. When an egg was crushed by strong pressure, so as to reduce the layer of yelk-substance to the same thickness as the yelk-sac, the colour was seen to be quite as deep in the buttons on this latter as in the formative yelk, but of a somewhat more yellowish red.

The dyeing is independent of any acid reaction of the substance dyed, as macerated yelk-sacs, which had become alkaline from decomposition, took the dye freely. This independence of the acid reaction is also further seen by the fact that the acid food-yelk does not take the colour at all.

The fine structure of the yelk-sac is rendered less distinct by this method, partly in consequence of the action of the glycerine, partly from the action of the acetic acid. Ammonia does not impair the distinctness of this structure, although it makes the sac very clear.

It will perhaps not be entirely out of place to introduce here the following

*Observations on the staining of the eggs of the Pike.*

Ripe ova put into water tinged with common alkaline magenta dye, after ninety-six hours show a rose colour in the yelk-sac, and a deeper tinge in the granular contents of the yelk, in those eggs only, which had a ruptured and collapsed inner sac, with alkaline contents. But in those with clear acid food-yelk and unruptured inner sac, there was no dye seen beyond the yelk-sac; while in those with partly diffused formative yelk, and partly emptied inner sac, the dye extended only to the diffuse granular matter. Thus the inner sac resists the passage of the solution, and the deeper parts of the egg are not stained until materially altered.

Fertile ova, ninety-six hours after impregnation, placed for twenty-four hours in a similar solution, continued to develope as in water; the yelk-sac alone took the dye, the embryonic tissues resisted it completely.

Unimpregnated eggs, put fresh from the parent fish into the same solution, soon become stained in the yelk-sac, and if then they be broken, the contents escape free from colour.

A weak ammoniacal solution of carmine acts just like that of magenta, and when tried on ovarian ova, I could not succeed in staining the yelk-matter of either kind by cutting them in halves, and leaving them in it, although the yelk-sac was easily dyed in the same time. The yelk-sac must, I suppose, be looked upon as "formed material," yet it takes the carmine dye even more quickly and almost, if not quite as deeply, as does the formative yelk. The food-yelk, with its portion of inner sac, must, I imagine, be looked upon as "germinal matter;" at least it is a protoplasm and contractile, yet it cannot be made to take the dye.

The substance of the primitive yelk after a time takes the dye strongly, and then, compared with the more delicate translucent tissue in which it lies, is a very prominent object; but it is important not to forget the effect of thickness and physical condition in influencing the apparent colour of objects.

The substance of the formative yelk appears only then to take the stain when it is no longer defended by that of the inner sac; so that it is changed in form and structure before it can be dyed.

I am therefore not disposed to consider staining a satisfactory test of germinal matter, for some "formed material" takes the colour quicker and some "germinal matter" is destroyed, while other is much changed by the dye fluid; and some cannot be stained if the food-yelk and its cortex be considered one of its varieties. So far as my own observations permit me to form an opinion on the constant characters of protoplasm, I should at present say that the tendency to vacuolate is the most trustworthy test; in other words, protoplasm is in such unstable equilibrium that its proximate elements easily separate by contact with most aqueous solutions.

e. *Remarks on the mode of growth of the yelk-sac and on the germinal vesicle.—The former* I am, notwithstanding its highly complex structure, disposed to consider as a cell-membrane. Whatever may be said as to the mode of its earliest formation, it cannot

be conceived to grow by apposition of layers added from the inside or outside, although its laminated structure might at first be supposed to afford some support to this view; for the increase in number, as the growth proceeds, of the buttons placed on the outer surface, and their early appearance, make it impossible to understand growth by either of these modes. It is, I think, certain that it grows interstitially, and the suggestion arises that the larger stellar dots may in some way be connected with this increase.

The fact that young ovarian ova, of $\frac{1}{100}''$ in diameter, when the dots are first measured show them to average about 24,000 to an inch linear, and ripe eggs five or six times their diameter have 11,000 dots to an inch, thus but little more than doubling their size or distance apart, proves that during growth the number of these structural elements of the yelk-sac must increase, as well as their size. This may be taken as an additional proof of interstitial growth.

There is no evidence of the conversion of the substance of the outer layer of the protoplasm, *i. e.* the cortex of the yelk, into yelk-sac, in the sense in which that gradual conversion is believed by Dr. BEALE to take place in cartilage; at least after arrival at that stage, and it is a very early one, at which the yelk-sac is separable: as then it always shows its inner and outer surfaces equally sharp, hard, and distinct.

There are no facts known to me to point out whether the pabulum for the growth of this membrane is derived directly from the currents passing inwards, or from the material elaborated in the egg and passing out of it, or from both sources indifferently.

The extreme delicacy of the film which covers the yelk at first, makes it impossible to say positively whether it appertains rather to the layer of cells lining the ovisac, or whether it is more closely adherent to the yelk; but I incline to the latter view.

*The germinal vesicle*, which, both from the facts here recorded, and from the analogy of the eggs of invertebrata, appears to be formed before the primitive yelk, may be supposed, from its disappearing when the egg is ripe, before fecundation, or the action of external agents, to preside over the origin and growth of the egg. However, the position in which it is last seen with respect to the micropyle, as strongly indicates that its remnants have some important relation to the act of fecundation.

BALBIANI* has recently, in a paper which received a prize from the French Academy, stated that the germ (with him the equivalent of the formative yelk) may be traced to a preexisting nucleated cell spontaneously arising on the surface of the food-yelk, which cell by endogenous development of cells at the expense of the "primordial protoplasm" (food-yelk) forms the future germ on its surface. He expressly extends this view to osseous fishes, although he repeats the error made by COSTE, that the germ (formative yelk) only appears after fecundation.

This description of the mode of origin and growth of the parts of the ovum, I feel justified in stating is not in accordance with observed facts.

* "Sur la Constitution du Germe dans l'œuf animal avant la fécondation," Comptes Rendus, 1864, t. lviii.

*The Trout* (Salmo fario).

In ripe ova expressed from the fish I found the micropyle, December 1854, by carefully turning the egg over before water was applied; then by drying the surface somewhat, it was visible with the naked eye, but more easily with a lens (Plate XVI. fig. 24). At first it corresponds to the centre of the discus proligerus or germinal pole, but after the egg has been in water a few minutes, even when not fertilized, water enters, and the formative yelk which is at first as in *Gasterosteus*, a complete cortical layer, concentrates, and is collected into a nipple-shaped heap at the germinal pole; and from having attached to it some oil, it always floats uppermost, the yelk-ball being free to move in the now distended yelk-sac; so that the correspondence between the micropyle and the centre of the discus proligerus ceases. The terminal opening measures $\frac{1}{6300}''$ across, and the funnel or pit at its mouth $\frac{1}{90}''$.

The formative yelk, the discus proligerus with a deep central pit to receive the micropyle, the clear food-yelk, and the group of oil-drops, are all essentially the same as in *Gasterosteus*.

These eggs, pressed from the parent into water, stick to the dish for a time, but if first left exposed to the air for a little while, do not. This was not explained. They formed a breathing-chamber by imbibition, but no active protoplasmic contractions were seen.

*The Salmon* (Salmo salar).

These ova were examined in January 1855. They have a micropyle precisely similar to that met with in the egg of the trout, and the general structure of the egg is the same. The yelk-sac is very well suited for examining the dotted structure, especially after prolonged immersion in water, in which it retains its structure for four months at least; the details of its structure are essentially the same as in *Gasterosteus*.

These eggs, like those of the trout, imbibe water when not impregnated. If uninjured, they remain in water without apparent change, at least forty-three days. If injured, the inner sac often ruptures, and then the yelk coagulates, by the action of the water. If kept for the same time in damp moss, they decompose, become fœtid and alkaline, and then, if crushed in water, do not coagulate, the salt of ammonia produced keeping the albumen *b* in a state of solution. The inner sac thus seems, when intact, to resist the passage of osmotic currents through it.

I tried to test this in the following manner:—

Eight clear ova which had been kept without change in water for twenty days were well shaken in an empty bottle for a minute or two; then distilled water was added, and all became opaque at once, showing no longer a breathing-chamber, the inner sac being ruptured. The water was found to be acid, and contained an organic substance, which, when incinerated, left an alkaline ash containing chlorides and phosphates.

*The chemical reactions of the yelk.*—In the three species of Salmonidæ which I have examined, the yelk reacts similarly, but it differs somewhat from that of most other osseous fishes, in having a larger proportion of a peculiar variety of albumen precipitable

by water. This substance, which I have spoken of as albumen *b* provisionally, I have thought worthy of a somewhat more detailed examination, because it appears to be present in some proportion in the yelk of all the osseous fishes which I have been able to procure, and it, or a closely allied substance, is a constituent of the yelk of frogs and birds.

Its ready precipitation by water suggests the notion that the inner sac may possibly be formed by a gradual hardening of the surface of the yelk, through a chemical action of the surrounding medium, which in the ovarian ova would be an exudation from the blood. Certainly the inner sac was noticed to become firmer and more distinct in eggs which had been long exposed to the action of water. Whether this property of albumen *b* has any part in the formation of cell-walls is an interesting speculation, but one to which these observations give no direct support. Be that as it may, a substance easily precipitable by water is, I believe, very widely met with in animal protoplasms, and the firmer limiting surfaces, which in the protoplasmic balls of rapidly growing structures are the only representatives of true cell-walls, may owe their formation to its precipitation.

a. *Fluid albumen b.*—The food-yelk of the salmon is a thick fluid albumen, entirely precipitable by water in excess, if free ova be used; if ripe ova *in ovario* be used, a small proportion of ordinary albumen remains dissolved in the supernatant water, and may be coagulated by boiling, and nitric acid; but this is derived from the vascular tissue of the ovary.

The characters of this albumen, when in solution, may be studied in the entire egg, or in crushed eggs treated with water, in too small a proportion to precipitate all the albumen.

It then is coagulated by boiling, nitric acid, hydrochloric acid, alcohol, and ether. Dilute acetic acid coagulates it, and stronger acetic acid redissolves the coagulum, after which water will not precipitate it, nor will carbonate of potash added to alkalinity. In the alkaline solution mineral acids do not cause a precipitate unless heat is applied, nor does boiling without a mineral acid coagulate it. The acetic acid solution is, however, precipitated by yellow prussiate of potash.

b. *The solid albumen b.*—In the solid state this albumen, obtained by precipitating it with water, is white, finely pulverulent, composed of immeasurably fine molecules; while moist, it gives an acid reaction, after the most prolonged washings, short of decomposition; part of it always passes through the filter on account of its fineness and the absence of dense flocculi.

A strong solution of *chloride of ammonium* dissolves it, and the solution is not reprecipitated by alcohol unless boiled, and only imperfectly coagulated by boiling alone. It is precipitated, on the addition of strong acetic acid, and also by water in excess, when the precipitate is soluble in phosphoric, nitric, hydrochloric, sulphuric, tartaric, and acetic acids, also in potash and ammonia. The nitric acid solution is not coagulated by boiling, and the potash solution only then coagulated on boiling when nitric acid is added hot.

A strong solution of *acetate of potash* acts much as does the above reagent. The solution may be boiled without much coagulation, unless acetic acid be added, when flocculi form.

*Chlorides of sodium* and *potassium* dissolve it, and the solution is reprecipitated by water; it then may be again dissolved by acetic acid, phosphoric acid, hydrochloric acid, and nitric acid, the acid solutions not being coagulated by boiling unless nitric acid be added while hot, and not being precipitated by water in excess. This solubility of the second precipitate by water, in nitric acid, is the more remarkable, as the first solution of the albumen *b* in chloride of potassium is precipitated by nitric acid.

*Ammonia* dissolves it, the solution is not precipitated by water in excess, and only very imperfectly coagulated by prolonged boiling, after which acetic acid causes a copious precipitate.

Weak solutions of *potash* and its *carbonate* do not dissolve it; they partially coagulate a clear egg.

The common *phosphate of soda* dissolves it, and if precipitated again by excess of water, and redissolved by phosphoric acid, the acid solution may be boiled without coagulating, unless ammonia be added while hot, or may be diluted freely with water, without clouding. If acetic acid be used instead of phosphoric, the solution may be boiled without coagulation, unless potash be added.

The *phosphate of ammonia* acts like the above; the precipitate from it by water may be dissolved in phosphoric acid, and the solution, which is faintly clouded by prussiate of potash, and more so by nitric acid, is not at all so by chloride of mercury or by boiling, unless ammonia in excess be added hot.

Weak *acetic acid* does not dissolve it; stronger does, and the solution is not precipitated by water used freely; the acid solution clouds a little on boiling.

Weak *phosphoric acid* dissolves it; the solution is not reprecipitated by water or by boiling.

A concentrated, acid-reacting, aqueous, solution of the *salts* and watery extractive of the egg dissolves it, the solution being precipitable by water in excess.

While a strong solution of *sugar*, in which the eggs float, causes the albumen *b* to precipitate if an egg be broken in it, a weaker solution of *glycerine* does not.

These somewhat complicated reactions do not precisely accord with any modification of albumen with which I am familiar. Perhaps it has the closest affinity with MYOSIN, recently discovered by KÜHNE* in the juice of muscle. Besides its precipitation by water and easy solubility in most alkaline salts, the most characteristic reactions are, that under certain conditions its acid solutions do not coagulate on boiling, even when the acid is the nitric, and that its solution in some neutral salts is not precipitable by alcohol. The ichthuline of VALENCIENNES and FREMY† may be the same substance, but I cannot feel sure.

The natural salts of the yelk probably hold this albumen in solution, at least it has been shown that they can do so; whether the acid of the yelk contributes thereto I could not make out.

* Untersuchungen über das Protoplasma und die Contractilität, 1864.

† "Recherches sur la Composition des œufs dans la série des Animaux," Journal de Pharmacie et de Chimie, t. xxv. and t. xxvi.

c. *The acid reaction.*—I sought in vain to isolate the acid of the yelk. In precipitating the albumen *b* by water, the supernatant fluid becomes acid, and the precipitate itself is so, until it commences to decompose.

The acid, whatever it may be, is insoluble in alcohol, soluble in water, and forms a non-crystallizable soluble salt with baryta, which remains in solution in the alkaline reacting fluid, after the phosphates and sulphates have fallen, and is not precipitated from it by carbonic acid, but is by ammonia; when dried and burned, it chars, and leaves an alkaline ash. It is, I believe, a compound into which phosphoric acid and an organic substance enter as constituents.

A substance much resembling leucin was found among the alcoholic extractives of the yelk.

*The Grayling* (Thymallus vulgaris).

The ripe ova resemble those of trout and salmon in all essential particulars.

*The Pike* (Esox lucius).

The ripe ova have a general structure, essentially the same as that of *Gasterosteus*; the inner sac is particularly easy to demonstrate, and may be separated, teazed with a needle, and mounted. The yelk-sac is covered externally by a thin layer quite structureless under a power of $\times 400$, the equivalent of the "eikapsel" in Perch. It is visible only in moderated light, at least with the lower powers, and then requires, in order to distinguish it from a diffraction effect, that note be taken of the flow of fluids, and the position of solid particles on its surface.

It is this layer which causes freshly expressed eggs to adhere to each other and to the dish when in water. They do not thus adhere in air. Yelk-sacs long digested in a weak ammoniacal solution of carmine become rather friable, but the homogeneous outer layer retains its plasticity, and on rupture by pressure may be seen stretched across the gaping fissures of the yelk-sac, tinged faintly with the dye, but quite structureless under a power of $\times 400$, and careful illumination.

The micropyle may be sought for in the same way as in trout eggs. The aperture at the apex is easily seen under a power of $\times 200$ in full face. When the pit is viewed in $\frac{3}{4}$ face with oblique light, it seems to have a trumpet-shaped tube, standing erect from the bottom of it (Plate XVI. fig. 25). This appearance is due to the fact that in a strong illumination the clear, colourless, outer layer is quite invisible, and the trumpet-shaped tube appearing to stand erect and unsupported, is the thick wall of the canal, which penetrates the outer layer, where it dips into the pit of the micropyle.

*The Ruffe* (Acerina vulgaris).

The general structure of the ripe egg is the same as in *Gasterosteus*. The oil, however, forms but one large drop, the inner sac seems thicker, and the yelk-sac has an outer layer or "eikapsel." Eggs expressed from the parent may be manipulated with care if no water be added, as although very soft they do not adhere strongly; but if

water be present they imbibe it rapidly, and then become so adhesive, that every attempt to roll them and examine the surface tears off the outer layer of the yelk-sac, or "eikapsel." Before water is added they have a convoluted surface, such as a lax membrane presents, which is marked by irregular impressions of the cells which line the ovisac. When water is added it rapidly distends and effaces the convolutions, but I could not see any regular hexagonal facets such as have been described. A beautifully regular fine dotting, however, is seen arranged so as to enclose lozenge-shaped spaces.

Any attempt to move the egg while in its adhesive stage, exhibits the wonderful extensibility of the outer layer, the shreds of which are drawn out so as to appear homogeneous. This adhesiveness is lost, however, after twenty-four hours' immersion in water. The fine dottings may be best examined by placing eggs which have been thus macerated, in a solution of chloride of sodium, and after cutting them in halves, washing the yelk-sac and its capsule in a 1 per cent. solution of chromate of potash, so as to preserve it free from adhesiveness, and to remove the coagulated albumen *b*. In this way the outer layer appears to be continuous with, and of similar structure to the yelk-sac proper, from which it can in no way be separated as a distinct membrane. It seems probable, that the outer layer serves to fix or anchor the spawn upon the weeds or other bodies on which the female deposits them.

The micropyle is very similar to that seen in the pike. It is, however, not easy to find in water while the eggs are adhesive, but weak solutions of the alkaline chromates, or of chromic acid with chloride of sodium, destroy the adhesiveness of the outer layer, without otherwise changing the aspect of the egg, and thus it may be manipulated, and the micropyle easily found.

*The Perch* (Perca fluviatilis).

The eggs may be expressed from a ripe female, cohering, so as to form a long flat band. folded in zigzag. This band is a collapsed tube, a network of eggs with irregular meshes, altogether not unlike a netted bead purse.

The unimpregnated egg rapidly absorbs water so as to distend the yelk-sac and its outer layer or "eikapsel," and to form a water-chamber, while the formative yelk concentrates as in pike and salmon. The structure of the egg as a whole, is the same as in the ruffe, and the oil is in one large drop.

The yelk-sac, under which term is included the outer layer or "eikapsel" of Müller*, merits a very careful examination, but I must give only a brief description of such observations as I have been able to make upon it.

Like the outer layer of the yelk-sac in the ruffe, that of the perch is probably an organ of adhesion, but in this case the eggs adhere to each other before extrusion from the parent fish, and are not adhesive after they are expelled. The time they lie in the oviduct, free from the ovisacs, and during which they are definitely arranged to form the tube, is very short, probably about twelve hours only, as I found a female at 10 P.M.

* "Ueber zahlreiche Poren-canäle in der Eikapsel der Fische," Müller's Archiv, 1854, p. 186.

yielding on pressure but a very few free, not yet cohering eggs, mixed with some still within their ovisacs, and at 10 A.M. the next day, on pressure her spawn came out in the usual way, forming a flat zigzag band.

The outer layer, or "eikapsel," has a consistence much like that of fresh fibrine, is much thicker than the dotted yelk-sac, and is characterized by a radial striation. The striæ look like tubes, have a distinct double contour for each wall (Plate XVI. fig. 28), but are filled with a vacuolating material, and do not seem to convey anything, either fluid or solid, into or out of the egg. They are vertically set in a clear matrix, and terminate on the outer surface by expanded ends or mouths, arranged in a regular alternating order (Plate XVI. fig. 26). The surface of the outer layer is thrown into delicate folds, which radiate from the ends of the "tubes," and the aspect of these when viewed full face is seen in Plate XVI. fig. 31. I could not make out after very careful search, hexagonal outlines such as MÜLLER has figured, having the ends of the "tubes" placed in their centres. The appearance of a vertical section is shown in Plate XVI. fig. 27, which represents the point of junction of two eggs. At the free surface the profile view is crenated, the clear matrix forming lenticular elevations between the depressed expanded ends of the "tubes." The outer layer is separable only by tearing from the yelk-sac, and does not leave a clean surface. The "tubes" at their inner termination divide into branches like roots, and are in some way intimately adherent to the outside of the thick dotted yelk-sac (Plate XVI. figs. 29 & 30). They have no expanded funnel-shaped mouths at this inner termination, such as MÜLLER has described. Here and there in the substance of the outer layer, very delicate connecting branches pass from one "tube" to the other. The clear matrix is delicately shaded, as if faintly granular on its outer surface, which under high powers is seen laminated concentrically, and is sufficiently elastic to turn inside out, when a minute segment is cut off it. Its central substance is so translucent as often to escape detection in these segments. The appearance described by MÜLLER, of oil-granules passing along the "tubes," may possibly have been due to vacuolation in them. Be that as it may, I saw appearances capable of being so construed, in the tubes of segments which had been cut off the surface of the outer sac without touching the yelk-sac, so that it is certain nothing passed from the inside of the egg along them.

In various ways I tried to make out whether any absorption of fluids took place along them, but always with a negative result; for these experiments I used weak ammoniacal solution of carmine, solution of prussiate of potash, and then a salt of iron, and performed artificial impregnation in these fluids, that they might be present at the moment of the greatest inward current. The cleavage went on, the yelk-sac was dyed throughout, the clear matrix more so than the tubes, the germinal mass not at all after five hours. In short I satisfied myself that these tubes either do not at all serve for imbibition, or in a much smaller degree than the clear matrix, which has marked powers of absorption, swelling up so as very much to increase its thickness after long action of water and in various solutions. I also tried the unimpregnated egg with like results, after forty-

eight hours in carmine. Neither can they serve for admitting the spermatozooids, both because they are, if examined in ripe ovarian ova, full of a semisolid vacuolating matter; and because the micropyle exists as in other osseous fishes, and in the same position, marking the germinal pole. It is interesting to note that the eggs in the mass of spawn are all so placed that the micropyle looks directly towards the inside of the collapsed tube which forms the band. One effect of this arrangement must be to prevent its occlusion by contact.

In perch spawn, taken from the river, the micropyle is easily seen, after letting out the young embryo with a needle; a deposit of fine mud being usually deposited in the furrows around it, thus rendering it visible to the naked eye.

The dotted sac has a structure in all essential particulars like that of *Gasterosteus* (Plate XVI. figs. 32 & 33).

*The river Bullhead* (Cottus Gobio).

The eggs are held together by a viscid secretion of the oviduct, the yelk-sac is furnished with a micropyle surrounded by button-shaped processes, just as is seen in the allied genus of *Gasterosteus*.

*The Gudgeon* (Cyprinus Gobio).

The egg has a micropyle at its germinal pole, consisting of a conical pit perforated at its apex. The dotted yelk-sac is villous on its outer surface. The villi are soft, tenacious, easily deformed by pressure. The unimpregnated eggs imbibe water, form a breathing-chamber, and the formative yelk concentrates without exhibiting any active contraction.

*The Minnow* (Leuciscus phoxinus).

The eggs have a micropyle similarly placed, with a raised margin around the mouth of the funnel. In water they act just like those of the gudgeon, except that some very slow changes of the form of the yelk-ball occur.

*The Chub* (Leuciscus cephalus).

The egg has a similar micropyle, the margin of the funnel is crenated, and its sides are furrowed, it reacts in water like that of the gudgeon.

## The Impregnated Ovum.

*The Stickleback.*

I purpose first to describe the changes which follow fecundation, up to the time at which the yelk commences to contract, then to relate the experiments made to show

how impregnation is effected, and afterwards to consider the conditions of the protoplasmic movements.

1. *Earlier sequences of impregnation.*

To trace the changes which follow the action of the spermatozooids upon the egg in their earliest stages, it is necessary to fecundate artificially upon the stage of the microscope. For this purpose I used POWELL and LEALAND'S animalcule-cage, a glass ring being fastened upon its lower plate, to convert it into a cell, and a portion of the thin glass cover being cut away so as to permit the fertilizing agent to be applied at its edge. The male and female fishes about to be used may be conveniently manipulated, if the spinal cord be first divided just behind the gill-covers, after which they live very well in water for forty-eight hours or more. The semen is not easily pressed from the male, and hence in these experiments it is convenient to have a number of them ready, and to open the abdominal cavity and use a fragment of testis; this, if quite ripe, will impregnate the eggs in the cell if pushed just under the cut edge of the glass cover, so as to be not too much exposed to the action of water, which soon arrests the movements of the spermatozooids. It will not do to put the male fish into water after the abdomen has been opened, but if it be kept moist the spermatozooids live some time.

a. *Formation of the breathing-chamber.*—The earliest change which occurs after an egg is fecundated, is the formation of a space between the yelk-sac and the outer surface of the yelk. This space is the breathing-chamber of NEWPORT; it commences first close to the micropyle and gradually extends over the rest of the yelk-ball*, being complete in from three to five minutes after the spermatozooids have been applied to the edge of the glass cover, in successful experiments. It begins by a withdrawal of the funnel of the micropyle from the pit in the discus proligerus, so that as water enters the funnel is gradually shortened, and at length may be almost effaced. This withdrawal was seen to begin in about fifteen seconds after the first spermatozooid was seen to enter; but ordinarily it is visible about one minute after the testis has been applied to the eggs. After the breathing-chamber has been once formed, it for some minutes longer increases in size, by expansion of the yelk-sac, so as to efface the indents on the surface of the egg, increase its size, and render it globular, tense, and elastic, remarkably resisting, and difficult to injure, the very reverse of its state before it had imbibed water.

The formation of the breathing-chamber is, I think, not entirely due to the entrance of water, but in part to a contraction of the substance of the yelk, which commonly produces a flattening of one surface. In the eggs of *Gasterosteus* it seems certain that the water mainly enters to fill the breathing-chamber through the micropyle, and under ordinary circumstances, not by imbibition through the yelk-sac. It is somewhat difficult to conceive how the passage of such a minute body as a spermatozooid through the tube of the micropyle, closed as it seems to be only by the viscid secretion of the oviduct,

* This is only ascertained to be the fact in *Gasterosteus*. In most other osseous fishes water enters freely through the yelk-sac, and the breathing-chamber may probably commence simultaneously at all parts of the surface. In frogs I believe that I have witnessed its commencement, as in *Gasterosteus*, first near the micropyle.

can determine the prompt entry of the surrounding medium, unless it is assumed that the inward current is assisted by a contraction of that part of the protoplasm with which the spermatozooid comes into contact*. I tried in vain to observe the entry with the stream into the egg, through the micropyle, of minute particles of carmine while the breathing-chamber was forming. A solution of caramel somewhat acid decomposes the viscid layer, and stains the yelk-sac, but cannot be seen to colour the fluid in the breathing-chamber; and the same result was obtained by using a salt of iron tested afterwards with prussiate of potash. It is, however, difficult to tell the colour of the contents of the breathing-chamber seen through a dyed yelk-sac. A watery solution of the extract of safflower gave a similar result, and after twenty minutes, on rupture all the contents of the egg were colourless; the inner sac was especially noted to be colourless. It would seem that the viscid layer does not entirely prevent the absorption of watery solutions of pigments into the substance of the yelk-sac. This does not, however, invalidate the conclusion as to the mode in which water enters to form the breathing-chamber. That it is the inner sac which presents the obstacle to the imbibition of aqueous solutions of colouring-matter into the yelk, is shown by the fact that eggs which have lain for forty-eight hours in the dead body of the parent, and have become slightly decomposed, permit the tint of the caramel, and of iron when tested by prussiate of potash, to appear in the substance of the yelk, in such of them only as have the inner sac ruptured, a change which often occurs in dead eggs, and will be again referred to.

b. *Concentration of the formative yelk.*—Very soon after the funnel of the micropyle begins to shorten, the formative yelk commences to undergo the series of changes which eventually terminate in the formation of the germinal disk.

In one instance, where the spermatozooid was seen to enter, the yellow droplets were distinctly paler in $1\frac{1}{4}$ minute, and an obscure puckering was visible at the same time on the surface of the discus proligerus, which, after the completion of the act of fecundation, I would call the germinal disk. Gradually all the granular and other elements of the cortical layer or formative yelk move away from the ventral segment, and concentrate into a disk at the germinal pole, where it then covers a somewhat smaller area than that previously occupied by the discus proligerus, but is thicker. The egg becomes clearer in consequence, partly through the removal from the surface of the clear food-yelk of a granular opaque layer, partly from the distension and increased translucency of the yelk-sac. At the same time the structural elements of the cortical layer undergo certain changes, which show that some slight action of water takes place through the substance of the inner sac. The yellow droplets grow paler, and disappear without distinct vacuolation, commencing to pale first at the germinal pole. Although, in a short interval of time after these changes begin, all, or nearly all the granules of the cortex are transferred to the germinal pole, and the yellow droplets either carried with them or in some way rendered invisible, I failed to see any distinct movement of them streaming towards the germinal pole.

* Such contractions have been shown by Newport to take place in the eggs of frogs.

In one instance, seven minutes after impregnation, the germinal disk was seen not yet fully concentrated, and as it presented full face I watched in vain for several minutes some of the granules at a short distance from its outer margin, to see their progress towards it. This attempt was often repeated with a like result. But as from the first moment there are slight contractions of the protoplasm, minute displacements of such granules, if observed, would not be conclusive evidence of a streaming movement. That such streaming does, however, take place is, I think, certain from the ultimate position in which the granular matter of the formative yelk is found, and I have frequently seen the granules of the cortex arranged in lines radially placed around the periphery of the then concentrating germinal disk. The germinal disk is visibly increased in bulk three minutes after fecundation, but I have no doubt that it begins to concentrate much sooner; it continues to increase until all but a very few scattered remnants of the formative yelk are collected, and it is complete some time before cleavage, for which it is the necessary preliminary.

That the disappearance of the yellow droplets from the cortical layer is due to the action of the water which has entered into the breathing-chamber, is shown by the fact that not only does it begin near the micropyle where the water enters, but it proceeds more slowly in eggs which are too scantily supplied with water.

As the concentration of the formative yelk goes on, and the discus germinativus increases preparatory to cleavage, the accumulation of minute oil-granules distinct from the large reserve oil-drops at the under surface of the discus increases.

When at various stages after impregnation the egg was ruptured, and the germinal disk in process of formation examined, which was done in various media, it was found to contain no additional structural elements beyond those in the discus proligerus before fecundation; but the yellow droplets were very few in number, or absent altogether, unless in cases where the egg had been treated with too little water, in which cases they were numerous. The same vacuolation and pseudo cell-formation were seen as were met with in the matter of the formative yelk, but no true vesicles or cells. The mass is essentially granular with a clear matrix in very small proportion, and is somewhat more solid than before impregnation.

The inner sac during these first stages appears to get thicker and firmer, at least it is more easy to observe; it is adherent to the germinal disk, over the outer surface of which it passes, and of which it probably constitutes the clear matrix.

c. *Mode of effecting impregnation.*—To ascertain the function of the micropyle, the following observations were made.

I first sought to close it by gentle pressure, while allowing the spermatozooids to have free access to all other parts of the surface of the egg. This was done with the animalcule-cage, prepared as before described (p. 457), the depth of the cell being somewhat less than the diameter of the egg. In this way a power of $\times 100$ may be used, which enables one to follow the spermatozooids distinctly.

Experiment 1.—An egg was so compressed that its micropyle which presented was

seen to be closed; a fragment of testis from a ripe male was applied to the edge of the glass cover, so that the spermatozooids came at once into contact with the viscid layer. They were then watched incessantly, for about eighteen minutes, and seen vividly moving in contact with all other parts of the yelk-sac, except near the micropyle within the area pressed upon by the glass cover. My attention was then withdrawn; seventeen minutes later they were languidly moving in the same parts, twenty minutes later they were nearly all still; the pressure was continued till two hours twenty-five minutes after the testis had been applied, when no signs of impregnation appeared, and the next day the egg was addled.

Experiment 2.—Two ova were then impregnated in a similar manner, for control, without pressure. The breathing-chamber was distinct in each in three minutes and a half, and vivid contractions of the yelk began in eleven minutes.

Experiment 3.—An egg was strongly pressed, in such a way that the micropyle being in profile, was not closed; the spermatozooids were seen in active motion quite near to the aperture, and the evidence of impregnation was discoverable before removing the pressure in the changes which the cortical layer underwent, although no breathing-chamber could be seen until the pressure was removed. This egg went on to cleavage, although it was later than normal, and the cleavage masses were irregular. This experiment was intended to show that pressure alone, if it does not close the micropyle, does not prevent impregnation.

Experiment 4.—I put seven eggs into a larger but otherwise similar cell, and applied pressure so that the cover flattened an area of each egg the diameter of which was equal to half that of the egg. To these eggs I carefully applied the testis from a vigorous male on two occasions: and the spermatozooids were seen actively moving during the twenty-five minutes I watched, but I could find no indications of impregnation having occurred. I then removed the pressure and applied a fresh piece of the testis, and in three minutes five of the seven eggs showed a breathing-chamber. Of the two failures, one at least had its micropyle so placed that it might be closed by pressure against another egg (Plate XVI. fig. 34), and the other had it looking downwards in such a position on the inclined stage of the microscope, that the current would tend to carry the spermatozooids away from it.

Experiment 5.—I put four eggs of *Gasterosteus pungitius* (which are clearer and rather better for this inquiry than those of the three-spined species) into the cell without pressure and fertilized them. I watched closely one egg, which was placed with the micropyle in full face, so that the aperture at its apex was well seen. Spermatozooids were seen approaching and entering the funnel, and one was watched till it disappeared, apparently in the direction of the interior of the egg, just at the moment when it seemed to occupy the aperture at the apex of the micropyle. Immediately after, the depth of the funnel began to diminish, and a breathing chamber commenced to form; two or three more spermatozooids were less distinctly seen playing about in the apex of the funnel as it was shortening: one of these appeared to become still before it vanished, apparently

inwards. The breathing-chamber was complete in five minutes, and the funnel of the micropyle was effaced in fourteen minutes. During the first shortening of the funnel it seemed as if the aperture at its apex also became smaller, but this appearance may have been deceptive. This experiment was repeated, the spermatozooids seen moving in the apex of the micropyle, and in half a minute a breathing-chamber began to form.

Experiment 6.—I then impregnated similarly five eggs, using no pressure, and noted that one egg, which was so placed that the mouth of the micropyle was directed towards the stream carrying the spermatozooids, was the first to show indications of being impregnated. I saw in this case the spermatozooid enter the mouth of the funnel, but could not in this position of the egg trace it any further. This egg showed the breathing-chamber $1\frac{1}{2}$ minute after the testis was applied; in two minutes more all the eggs showed a breathing-chamber, and in every case those eggs which had their micropyles directed from the current were the latest to give evidence of being fertilized.

Experiment 7.—I placed four more eggs in the cage, and applied a fragment of ripe testis to the edge of the cover, using no pressure; one egg was so placed that the micropyle could be viewed full face, and the aperture at its apex was brought into focus; this egg was in the second row, so that the current being diverted by the upper row and flowing quickly in consequence of the inclination of this stage, carried the spermatozooids wide of it. I watched carefully and painfully for seven minutes; no spermatozooid approached the micropyle, and no trace of a breathing-chamber appeared. I applied a fresh fragment of testis and watched closely for nine minutes longer, still no spermatozooids were seen near the funnel, and no change was seen in the egg, although other parts of the egg were in contact with active spermatozooids. I then put another piece of testis to the edge of the cover, and turned the cell the other way upwards, so that gravity tended to bring the seminal particles back to the egg, which was constantly and carefully watched. In two minutes I saw an active spermatozooid enter the apex of the funnel and disappear as if inwards: a quarter of a minute more had not elapsed before the clear bright circle, which marks the aperture, became indistinct from shortening of the funnel: during the next two minutes I saw three more spermatozooids enter the apex, and vanish apparently inwards: $1\frac{1}{2}$ minute after the appearance of the first spermatozooid in the funnel, the yellow droplets became paler: the breathing-chamber was complete $3\frac{1}{2}$ minutes later, and the usual vivid contractions of the yelk appeared in fifteen minutes. The eggs, two in number, which were in the front row were impregnated by the first application of the testis. Thus for eighteen minutes, active, moving spermatozooids were seen in contact with the yelk-sac, but not in the micropyle, and no sign of impregnation appeared; yet in a quarter of a minute after one was seen to enter, the indications of perfected impregnation began and went on in the usual way. The fourth egg in this experiment was not impregnated; it lay in the back row, its micropyle closely pressed against one of the eggs in the front row, so that the access of spermatozooids was rendered difficult. These results leave no room to doubt that the function of

the micropyle is what its position and structure suggest, viz. to admit the spermatozooids to the surface of the yelk.

It may be here mentioned, although it adds little to the strength of the evidence adduced, that ripe eggs yet within the ovisacs cannot be impregnated. All attempts to see the spermatozooids in the breathing-chamber failed; nor is this to be wondered at, as the funnel of the micropyle dips so deeply into the pit in the granular opaque discus proligerus, that it is impossible to see its apex clearly until it is withdrawn to some extent; thus the first moments of the entry of the spermatozooids are lost, and their extreme minuteness and delicacy, as compared with the egg, add to the difficulty of the observation. It was observed during these experiments that the spermatozooids continued to move freely for twenty minutes or more in the viscid layer, but became still very soon if they had first to float a very short distance through water. The surer plan therefore is to apply the testis while only a little moisture covers the egg, and afterwards to fill the cell with water.

d. *Relation of these sequences to the surrounding medium and to the Spermatozooids.*—An accident occurred during these observations which shows how well the spermatozooids continue to move in the viscid secretion of the oviduct. Eggs yet within the parent fish were unintentionally fertilized by applying forceps which had just before held a piece of testis to the sexual orifice of the female.

I was thus led to make an experiment with a view of ascertaining what share water had in inducing the changes which follow impregnation.

I fertilized the ova yet in the oviduct of a three-spined female, by applying to the sexual aperture a fragment of testis from a ripe male. In ten minutes some of these eggs, pressed out and examined without water, were found to have a concentrated discus germinativus, and the yellow droplets had disappeared from the cortical layer; the breathing-chamber was not, however, distinct, partly in consequence of the strong refraction of light, partly from its small size; but on adding water it was at once apparent, so promptly, indeed, that it must have been present before. The fish was then covered with oiled silk and put aside, and eggs pressed from her twenty-eight minutes after impregnation were found contracting. Forty-eight minutes after they were still contracting, and then water was added, under observation, to see if it increased the activity of the movements, but such result was not observed. Two hours and a quarter after, more eggs, pressed from the fish, were just about to cleave. Water being then added, to make the object more distinct, the funnel of the micropyle was seen dipping into the deep pit of the discus germinativus, thus proving how imperfectly the yelk-sac had distended.

Five of these eggs which had not touched water were put into pure nut-oil twenty minutes after impregnation; five minutes later they were seen contracting, and at two hours ten minutes after impregnation four out of the five were cleaving. Thus concentration of the formative yelk, formation of a small breathing-chamber, and even cleavage may occur without the presence of water, if maternal fluids are present. But I ought

to add, that the contractions seemed to be scarcely so vivid as in eggs normally fecundated.

With the view of testing whether any changes were due to the mechanical action alone of the spermatozooids, I tried to cause the micropyle to be forced by minute animalculæ, but could not succeed in any instance, in consequence perhaps of the animalculæ being all somewhat larger than the spermatozooids. No better result followed similar attempts with the spermatozooids of *Lissotriton punctatus* and of *Unio tumidus*.

If eggs be exposed to water without being fertilized, the viscid layer prevents its action to a great extent, for they may be left in it two or three hours without losing their flaccidity; after a still longer time they imbibe a little, even before the viscid layer has lost its characteristic properties; in so doing they become rounded and more tense, the yellow droplets become paler, but do not vacuolate: an imperfect concentration of the formative yelk occurs. If the unimpregnated eggs be submitted to a stream of water of considerable strength, and for some time, by which means a part of the viscid layer is removed, although no effect is seen at once, yet in an hour a good breathing-chamber appears, and the formative yelk is concentrated.

Some ill-understood changes take place in the eggs after the death of the parent, which diminish the readiness with which they may be impregnated. I kept a dead ripe female moist for 19¾ hours, and found that only five, out of ten of her eggs, could be fertilized, although all the ten seemed alike; four hours later, only two out of seven could be fecundated. The testis of a male was used successfully after it had been dead twenty-one hours.

If a dead female be kept moist forty-eight hours in summer weather, the eggs inside her become a little decomposed, and then a breathing-chamber soon forms when they are put into water; the formative yelk concentrating, the yellow droplets vanishing at the same time.

Thus it appears that although in *Gasterosteus* the formation of a breathing-chamber and the concentration of the formative yelk, under normal conditions, only occur after fecundation, yet they are only an indirect consequence of the action of the spermatozooids, which act by favouring the entrance of the surrounding medium into the cavity of the egg.

2. *Later sequences of impregnation.*

a. *The yelk contractions* are the most striking of the phenomena which follow the entrance of the spermatozooids into the egg. They may be watched with a ½″ lens, or better, with the compound microscope, using a power of ×50 or ×100, and may be spoken of as rhythmic yelk contractions. From the first moment of entry of the spermatozooids, slow contractions of the yelk begin, and assist in the formation of the breathing-chamber, causing first a flattening of the surface of the yelk near the germinal pole, and afterwards slight changes of outline due to travelling waves at other parts of the surface, but not before the breathing-chamber has reached that part. Gradually more vivid contractions commence, at various times after fecundation, according to the

temperature. In warm weather they have been noted in six minutes, in cooler weather in fifteen or twenty minutes after impregnation. They cause a flattening of one side of the yelk-ball, to see which it is often necessary to roll the egg over (Plate XVI. figs. 35 & 35′). The flat surface gradually becomes a sulcus, giving a reniform outline to the yelk (Plate XVI. fig. 36). It then extends all round, giving rise to a dumbbell shape (Plate XVI. fig. 37). This sulcus, which may be termed equatorial, travels with considerable but variable rapidity towards the germinal pole, producing as it passes on, the flask form (Plate XVI. fig. 38). The sulcus is lost by passing forwards to the germinal pole, not by relaxation. It is seen for a brief space affecting the thickness of the germinal disk only, to which it gives a nipple-like form, while the food-yelk is round (Plate XVI. fig. 39). When effaced, the whole yelk-ball is globular and at rest, the germinal disk being no longer prominent (Plate XVI. fig. 40). This series of forms recurs with more or less of regularity, and with some variations both of time and form, about fifteen or twenty times; each series being the result of a travelling wave. About five waves pass in ten minutes. Sometimes a wave commences as usual near the equator, and then for a short space passes towards the ventral pole; but it soon returns, and passing forwards towards the germinal pole, is then lost; occasionally other irregularities occur, such as two or even three waves travelling at the same time, a new one having commenced before the previous one had ceased (Plate XVI. figs. 41 & 42). This is more often the case in warm weather.

The concentration of the discus germinativus is somewhat greater as each wave comes to that pole, although some diffusion occurs again, always as the round form is reproduced (Plates XVI. & XVII. figs. 35 to 49 inclusive). The contractions continue, although gradually declining in vigour, up to the period at which cleavage begins, after which I could not trace them beyond the area of the germinal disk. (See Plate XVII. figs. 44 to 49 inclusive, which are drawings made at short intervals, until the commencement of cleavage: and show some singular forms of the germinal surface of the food-yelk, which are difficult to understand, as contractions of its substance. The figures show also the constantly recurring elevation and depression of the germinal disk caused by the travelling waves.) The periphery of the germinal disk is perpetually varying, being now sharply defined, now shaded off and diffused, but it always has a circular outline.

Coincidently with these contractions, oscillation of the whole yelk-ball takes place. At first this is so slow that it requires the use of a cobweb micrometer. As the contractile waves increase in vigour and rapidity, the oscillations quicken.

During the early feeble contractions, the micropyle, except in cases where from deficient supply of water the funnel is not quite withdrawn from the pit in the germinal disk, has its position changed relatively to the germinal pole of the yelk, by a slight imperceptible swing of the latter.

With the *vivid* contractions begin visible oscillations of the yelk-ball, so that its germinal pole swings through about 60°, usually in a plane, which cuts the micropyle, and which may be vertical, horizontal, or inclined, but is not a true plane, as the germinal pole describes an ellipse.

Somewhat similar movements in the eggs of the pike have been spoken of as rotations, but in those of *Gasterosteï* there is no rotation on the polar axis; as I ascertained by carefully watching fixed points, on each side of a spider thread, placed so as to correspond to the polar axis. However, it is not intended to deny the occurrence of occasional slight rotations on any axis of the egg, during the various and irregular contractions which occur. The yelk rests on the lower part of the yelk-sac, being of greater specific gravity than the water in the breathing-chamber, and the oscillation takes place from the point of rest.

The oscillations are caused by the contractions of the yelk. Not only do they commence with the beginning contractions, and become pronounced as the latter become vivid, but each commencing wave is shortly followed by an oscillation. In one observation six to and fro oscillations were counted in thirteen minutes, each corresponding to a travelling wave; this was during the second quarter-hour after impregnation. During the next half-hour, the contractions being less vivid, thirteen to and fro oscillations, proportionally more limited in extent and rapidity, were counted, each corresponding to a wave. Gradually, as the contractions became feeble, and limited to the germinal surface of the food-yelk, the oscillations ceased, and the germinal pole became stationary, about 35° from the micropyle. In short, the contractions measure the oscillations, so that irregular contractions cause irregular oscillations. In one instance, where two waves began on the left-hand side of the yelk-ball, so that the first had not ceased before the second had begun, there were two oscillations to the right, and none to the left.

I could not always with certainty connect the direction of each oscillation with the position of each commencing wave, yet, as a rule, the germinal pole swung to the right if a sulcus appeared on the left side of the yelk-mass, and *vice versâ*, provided the sulcus was in the germinal hemisphere, or near the equator; but when the first depression of the surface was in the ventral hemisphere, the oscillation carried the germinal pole to the same side as the sulcus. The result was, however, often modified by the direction of the wave, as well as by the rapidity at which it travelled.

The oscillations were influenced occasionally by oil-drops of larger size than usual, adhering to the germinal disk, and making it float uppermost; then, the oscillations were in a vertical plane, or nearly so. The yelk-ball is of nearly equal specific gravity throughout, as it retains any position it may be placed in, the oil-drops usually floating so freely in the outer portion of the food-yelk, that they move up to the top during an oscillation or any other movement.

It is therefore a fair inference that the oscillations depend on the contractions, which by altering the form of a globular mass of nearly equal density throughout, and partly floated, displace its centre of gravity, and determine the movement to restore the equilibrium. The onward movement of the wave would further modify the result.

The unimpregnated eggs, in a ripe female which had been dead forty-eight hours and kept moist, were a little decomposed, and in many instances had the inner sac ruptured, so that the whole or a portion of the food-yelk had escaped into the cavity of the

yelk-sac, and the inner sac had shrunk to a variable amount. Over the partly emptied inner sac, where it still contained some food-yelk, a wave of contraction was sometimes seen to pass slowly, but distinctly and repeatedly. The observation was made in several eggs.

Thus the contractions of the yelk in *Gasterosteus* are independent of impregnation, although ordinarily they are only seen in fecundated eggs, and they may continue long after all vital processes might be supposed to have ceased, and while all around, and in contact with the contracting matter, is decomposing.

b. *The cleavage*, which is limited to the germinal disk, begins usually about two hours after fecundation, although sometimes as early as $1^h\ 25^m$, or as late as $4^h$, varying chiefly with the temperature.

The formative yelk, having been concentrated as described, varies considerably, as to its form, at the moment when it is about to cleave. It may be flat and somewhat diffused, with its periphery well defined, or not, it may be prominent and conical, or hemispherical; in short those modications of its form which result from the then fading remnants of the waves of contraction, are still going on at the moment when a fresh set of contractions begin, viz. those which result in cell-formation. Plate XVII. fig. 49 shows its usual aspect immediately before cleavage, and the moment when the first cleft is beginning is shown in Plate XVII. fig. 50.

Adhering to the under surface of the germinal disk is the group of minute oil-granules, which are more numerous at this stage than at an earlier one; and as there is a constant consumption of the stock of oil in the group of larger drops, during the development of the germinal mass, it appears probable that at the surface of contact between the two kinds of yelk, a digestion of oil, so to speak, goes on; the process having for one of its constant phenomena a subdivision of the oil into minute granules. The remarkable appearances which attend the vacuolation of the matter of the formative yelk have often, irrespective of the evident absorption of oil into its substance during development, led me to infer that it was a compound, containing some fatty substance, easily separable from its associated matter. As to its structural elements, the germinal disk differs in this stage in no other respect from its earlier condition; but when crushed and examined under higher powers, a few yellow droplets are seen in it, and it is more solid than it was.

The cleavage begins in a faint well-defined line, which, seen in profile, appears as a notch, dividing the germinal disk into two equal halves (Plate XVII. figs. 50 & 52). This deepens and gradually separates the germinal disk into two conical elevations (Plate XVII. figs. 51 & 53). Even during the cleavage, constantly recurring, very slight waves of contraction go on, change the form of the cleavage masses, and cause the periphery of the germinal disk to vary. The two first cleavage masses, after the stage of greatest separation (Plate XVII. fig. 51), approach each other and appear as if about to fuse; this would seem due to the yelk contractions; it is, however, common, if not constant, just before the next cleft begins (Plate XVII. fig. 54).

The inner sac is thrown into folds at the margin of the cleft during its formation,

reminding one of the "Faltenkranz" described by REICHERT*, and M. SCHULTZE†, in the frog's egg (Plate XVII. figs. 55 & 56).

The cleavage masses at no time can be seen to contain any nucleus, vesicular or solid, nor could I find any in the germinal disk prior to cleavage. After twenty hours the germinal mass consists of a cup-shaped group of cell-like corpuscles, the result of repeated segmentation, seated upon the germinal pole of the yelk (Plate XVII. fig. 57), without any differentiation of parts. It is closely connected with the inner sac, which may be seen at its outer boundary forming radially arranged folds. The mass is solid, and its elements cohere with some tenacity, but on rupture in water the cell-like corpuscles in part separate, and as they float away undergo vacuolation.

The surface of contact of the germinal mass with the food-yelk is difficult to study. It seems to be merely the corpuscles resulting from segmentation in contact with the fluid food-yelk.

It is not without interest to note, in passing, how frequently, from slight causes, among which pressure seems to be the most important, an asymmetrical cleavage occurs; and the possibility of artificially inducing the formation of monsters is thus suggested (Plate XVII. figs. 56 & 58). One egg, in which the irregular cleavage had been seen, showed on the eighth day a well developing embryo, about to burst the yelk-sac, but no visible deformity.

### 3. *Conditions of the Yelk Contractions and of Cleavage.*

a. *Poisons.*—Experiments were made on impregnated ova of the two species of *Gasterosteus*, to ascertain how far the contractions of the yelk, and the cleavage were influenced by poisonous substances.

*Hydrocyanic acid*, when very dilute, produced no visible effect upon the rhythmic contractions of the yelk, but caused a little delay in the commencement of cleavage. When used a little stronger it slowly produced rupture of the inner sac, without previously seeming to influence the rhythmic movements; and it delayed still more the cleavage in those eggs which had not ruptured the inner sac. When used still stronger it arrested the rhythmic contractions, but at the same moment caused bursting of the inner sac and opacity of the yelk.

*Atropia.*—A supersaturated aqueous solution had no apparent effect on the yelk contractions or on cleavage.

*Aconite.*—The spirituous extract, mixed with water, did not influence the yelk contractions, but retarded the cleavage. The next day the yelks had undergone chemical change.

*Strychnia.*—The aqueous solution had no apparent effect.

b. *Galvanism.*—The following observations were made to ascertain the influence exerted by the galvanic current upon the yelk contractions, and on cleavage. In my

* Archiv für Anatomie, Physiologie, &c., 1861 and 1863.

† Observationes Nonnullæ de Ovorum Ranarum Segmentatione, 1863.

previous paper it was said to have no effect on the yelk contractions. I used then a single cell containing $\frac{1}{2}''$ of gold wire, and the same of zinc, excited by a solution of chloride of sodium. A like negative result was obtained, with reference to the cleavage, while using a cell containing carbon and zinc, each 2 square inches, excited by common salt.

Later experiments in May 1864 with more powerful currents and better appliances have reversed the result.

I used an induction apparatus made by STOEHRER, which admits of nice graduation, and has as much power as is usually required for medical purposes.

A cell was prepared, the idea for which had been suggested by KÜHNE*, of sufficient depth to permit the eggs to be covered, without pressure, and the conductors were so suspended that the movements of the stage of the microscope were unimpeded. A contact breaker was introduced into the circuit, so that it could be closed for any required time without touching the stage or removing the eye from the microscope. A very weak secondary current was employed, using only $\frac{1}{2}''$ of metallic contact, and pushing the coil as far off the core as possible; the shocks were then barely felt by the moistened fingers. The primary current was found to be less suitable for these experiments, being less perfectly graduated.

An egg, fifteen minutes after impregnation, was placed between the poles in the cell, and watched for a minute or two with a power of $\times 75$. The yelk was seen to be languidly changing its form, by flattening one segment; no travelling waves were present (Plate XVII. fig. 59). On then making contact for about $\frac{1}{4}$ minute, there appeared, after a very brief interval, which I could not accurately measure, a deep notch surrounded by radial foldings of the inner sac in that part of the yelk-ball nearest to the platinode (Plate XVII. fig. 60). The tardiness of this reaction made it impossible to note the relative effect of making and breaking contact.

After another similar application of the current, another notch formed near the first, and directly after, the inner sac burst at a point distant from the notch, and the food-yelk escaped (Plate XVII. fig. 61).

Other applications of the current were followed by indentation of the yelk, on the side opposite to the first formed one, and the inner sac burst near the new indents; the escaping food-yelk showed signs of chemical change, being very granular from electrolysis (Plate XVII. fig. 62).

These excited contractions were followed by oscillations.

At the rupture in the inner sac the torn edge is well seen, as it retreats during the shrinking of the yelk-ball; it is often folded and ragged, sometimes drawn into threads.

The circuit being again closed as before, the inner sac shrunk into a lobular mass, which contained the greater part of the formative yelk, and a little food-yelk, which was seen to escape more rapidly under the influence of the current. Electrolysis was more marked in the yelk of both kinds (Plate XVIII. fig. 63). After this stage repeated

* Untersuchungen über das Protoplasma und die Contractilität, 1864, p. 147.

applications of the current were made, but they were only followed by slow shrinking of the remaining pouches of inner sac, by which the food-yelk and even the oil were squeezed out, and the whole mass at length became darkly granular (Plate XVIII. fig. 64).

I afterwards ascertained that one application of the current, sufficiently strong to cause a deep indent, was generally followed after a time by rupture of the inner sac, and all the further changes above described. Repeated applications of the current hastened them however.

It was somewhat difficult to adjust the strength of the current, so as to excite well-marked contractions, and yet not cause rupture; however, by using the smallest amount of metallic contact requisite to put the machine in motion, by pushing the coil quite off the core, and by closing the circuit for only two to five seconds, I obtained trustworthy results.

With these, which may be called zero-currents, I excited peristaltic waves, distinguishable from the normal ones by their greater depth, abruptness, rapidity of formation and of progress, by the varying directions in which they travelled, and the positions at which they originated. An excited contraction may begin near to either electrode, or distant from both; the sulcus may be in the direction of the current as in Plate XVIII. fig. 65, or at right angles to it, as in Plate XVIII. fig. 66. Such waves may be equatorial or meridional, as the same figures show. The zero-currents cause no observable electrolysis, and are slower in exciting the contractions than the stronger currents are.

The position of the rupture in the inner sac varied much; it had no constant relation to the electrodes, or to the poles of the yelk-ball. Sometimes it took place near to the indent, especially if the current was strong; sometimes at the part of the yelk-ball most remote from the contraction, and then it was preceded by a protrusion and distension of the inner sac, which exhibited a marvellous extensibility.

Ova which had arrived at that stage, when, being about to cleave, their natural contractions had nearly ceased, contracted in a similar manner, but required perhaps somewhat stronger shocks, and the interval which elapsed between the application of the galvanism and the commencement of the contraction was rather longer. Ova in the second stage of cleavage, when the normal contractions had ceased, were markedly contractile on the application of moderately strong currents. Unimpregnated ova, when submitted to a moderately strong current, soon imbibe water, form a breathing-chamber and contract; and then rupture of the inner sac and electrolytic changes are very apt to occur near the electrodes.

The excited contractions, although, like the normal ones, they began almost constantly upon some part of the outer surface of the food-yelk, extended afterwards to that surface which lies in contact with the germinal disk.

Neither the germinal disk, nor the separate cleavage masses could be made to exhibit any contractile movements by galvanic irritation, although certain changes of their form

appeared, which were due to the contractions of the food-yelk with its covering. Electrolytic changes, however, appeared very readily in the substance of the cleavage masses, which became regularly crenate at the margin, as if composed of small globular cells (Plate XVIII. fig. 70). The current used in this observation decomposed water.

Ova nine minutes after impregnation, before visibly moving contractile waves had commenced, contracted to the zero-currents.

The cortical layer of the food-yelk being carefully examined at $40^m$ and at $2^h\ 30^m$ after impregnation, with a power of $\times 195$, it was found that a few scattered granules and yellow droplets always remained, which had not been collected into the germinal disk; and these, on rupturing the inner sac by a strong current, were observed to retreat with it from the broken part, and to undergo vacuolation at the same time. In this way, better than by any other, the intimate connexion which exists between the inner sac and the remnants of the formative yelk where it is spread over the food-yelk, is shown.

c. *Heat.*—Some observations were made to ascertain how far abstraction of heat diminished the susceptibility of the contractile material of the yelk to galvanic stimulus.

Control experiment.—Ova were fertilized and kept, some in cells, others in capsules, at the temperature of the room, 58° F. After thirty-five minutes they were actively contracting and rotating in the usual way; they completed the first cleft after $2\frac{1}{2}$ hours, and the second after five hours. The germinal mass was in the fine mulberry stage of cleavage after twenty-four hours, and then no contractions of the yelk were seen, but zero galvanic currents excited them distinctly in the yelk, without causing any movements in the germinal mass.

Experiment *a*.—Some of the same ova fifteen minutes after impregnation were placed in a chamber cooled to 45° or 48° F.: forty-five minutes after, they were contracting normally but languidly, and responded to the galvanic current apparently as well as did the ova in the control experiment.

Experiment *a'*.—Some of the same ova, two hours after impregnation, were put into a chamber cooled to 40° F. The first cleft was not completed until $3^h\ 45^m$ after impregnation; so that cold, even when it does not act until late in the stages which precede cleavage, retards its progress.

Experiment *b*.—Some of the ova which had been cooled to 45°–48° F. were, one hour after impregnation, further cooled, so that the thermometer on the cell stood at 32° F., the water not being frozen. The yelk-ball became round and still, but zero-currents of galvanism somewhat slowly excited very distinct contractions. In this experiment, however, the cell was rapidly receiving heat from the stage of the microscope, as I had then no means of maintaining it at a constant temperature.

Experiment *c*.—Some of the stock of control ova were, one hour after impregnation, cooled so that the water was nearly all frozen, as well as some of the eggs, which were then allowed to thaw. Those which had been frozen were decidedly opake, and various

degrees of opalescence were seen in most of the eggs. Examined by a power of × 35, the opaque ova had their inner sacs ruptured and shrunken in degrees, varying with the opacity, the discus germinativus being lobular and darkly granular. Those ova which were faintly opalescent only, exhibited but slight shrinking of the inner sac, which had evidently healed soon after rupture; the site of this was marked by a deep pit surrounded by radial folds. Gradually, under the influence of the warmth of the room, the slight contractions natural to this stage returned. Zero-galvanic currents produced strongly marked contractions in these eggs. Those which were only thus partially ruptured, cleft for the first time about five hours after impregnation, but the masses were not symmetrically arranged, so that perhaps in this way also monsters may be formed (Plate XVIII. fig. 71).

Cold, then, delays the changes which follow impregnation, but does not, within those limits which fall short of mechanical rupture and complete derangement of the structure of the egg, destroy irritability. Observations are wanted, however, with eggs cooled down to the state described in experiment *b*, upon an insulated stage, kept at the required temperature, while the galvanic current is applied.

Observations were then made on the effects of elevated temperatures, upon the movements of the yelk, and on the cleavage.

Control experiment.—Ova impregnated and kept at the temperature of the room (58° F.), were $30^m$ after, contracting and rotating slightly, $45^m$ after, vigorously, $2^h\ 30^m$ after, were not cleft, but $6^h\ 30^m$ after, were cleft into eight masses.

Experiment *d*.—Ova ten minutes after impregnation, being warmed on the stage of the microscope to about 73° F., at first did not seem to be influenced; but after ten minutes' continuance of the warmth, they were seen to be moving more rapidly than those of the control experiment. The temperature was then raised to about 80° F., which, after some minutes, produced a state of almost complete rest, with the yelk-ball globular, the discus germinativus nipple-shaded, and the oil-drops displaced. At the same time the control ova were vigorously contracting. Being then removed, and left at the temperature of the room, they completed the first cleft at $1^h\ 50^m$ after impregnation, and by $6^h\ 30^m$ after, were cleft into the coarse mulberry stage, and much in advance of the control ova; a fact, the more remarkable, because the contractions had been arrested for a time by the highest temperature used. In this experiment the thermometer was laid upon the cell, but probably indicated a temperature somewhat higher than that reached by the eggs.

Experiment *e*.—Ova ten minutes after impregnation, put into a chamber, warmed to 83° F., and kept there for twenty minutes, were found actively contracting. In this case I had reason to think that they did not reach the temperature of the chamber, which was then heated to 102° F., and after some minutes, when the control ova were actively contracting, these eggs became relaxed and still, so that their globular yelk-balls filled the yelk-sac, and effaced the breathing-chamber, and the oil-drops were displaced and scattered. On cooling the eggs again slowly to 58° F., the contractions reappeared in

five minutes, but were languid. Cleavage began six hours after impregnation, the masses being arranged irregularly and without symmetry.

Another mode is thus suggested by which monsters may be formed. Some of these ova were left in the chamber at 102° F. for 50 minutes. They became opalescent, their inner sacs ruptured and shrunk up.

Experiment *f*.—Ova one hour after impregnation were warmed in a cell, upon a metal plate, the thermometer resting upon which stood at 103°–104° F. In 2½ minutes they began to be opalescent, and in four minutes they were opake; the yelk-ball was round and filled the yelk-sac, and the inner sac was not ruptured. The opaque yelk not being coagulated, diffused in water, on cutting an egg through. In this mode of applying the heat, the eggs approached more nearly the temperature indicated by the thermometer.

Experiment *g*.—Ova 1½ hour after impregnation, put for 5 minutes in a chamber at 90° F., had their yelk-balls globular, relaxed, so as to fill the yelk-sac, and their oil scattered and displaced towards the periphery; but were not at all opaque. Being then replaced in the chamber, warmed to 103° F., they soon became faintly opalescent to the naked eye, and when examined with a power of ×75 the yelk-balls were found to fill the yelk-sac; even the germinal mass was diffused and the oil scattered in small drops, but no coagulation was visible. These eggs were gradually cooled to 58° F., and soon contracted so as to form a breathing-chamber; no contractile waves appeared, but the germinal disk concentrated. They were again put, but only for a few minutes, into the chamber heated to 109° F., when again the yelk-balls became globular, and effaced the breathing-chamber; again, as they cooled, they contracted so as to cause its reappearance. Once more they were put into the chamber at 110° F., when a faint increase of opacity was visible, and being removed to the metal plate, at 103° F. they all shortly became opaque. In this experiment, I have no doubt that the thermometer in the chamber indicated a temperature considerably higher than that reached by the eggs.

Although the difficulties which stand in the way of warming the yelks to a given temperature, and maintaining them there, were not satisfactorily overcome in these experiments, it is, I think, fair to infer that a moderately elevated temperature quickens the yelk contractions, and hastens the commencement of cleavage. It is probable that 75° F. is somewhere about the upward limit of this temperature.

A higher temperature, which begins probably about 80° F., arrests the contractions, and relaxes the yelk-ball, which on cooling recovers itself, unless the heat has been carried too far. This limit was not made out with certainty, but is probably about 100° F. Imperfect coagulation of some of the contents of the yelk, or at least a granular precipitation, occurs at about 103° F.

d. *Oxygen and carbonic acid.*—The question which I have attempted to answer is this; Is oxygen in the surrounding medium a necessary condition of yelk-contraction and of cleavage?

Control experiment 1.—Ova five minutes after impregnation were placed, in a large

capsule, in tap-water and lightly covered. At thirty minutes and at forty minutes after impregnation, the contractions were observed to be normal; at $1^h\ 10^m$ they were almost limited to the neighbourhood of the germinal disk, and its surface directed towards the food-yelk; at two hours cleavage began; at $2^h\ 40^m$ progressing, the "Faltenkranz" distinct; at $3^h\ 30^m$ the second cleft commencing; at five hours complete; at eight hours there were thirty-two cleavage masses; at twenty-four hours the germinal mass was composed of minute cell-like corpuscles.

Control experiment 2.—About six ova were impregnated, and kept in a lightly covered cell in tap-water. At thirty-five minutes after impregnation they were contracting normally, at $2^h\ 15^m$ were about to cleave, at $3^h\ 30^m$ the first cleft was completed, and the second about to begin; at twenty hours after, the germinal mass was a cluster of corpuscles having the general aspect of cells.

Experiment 1.—Into a similar cell, five minutes after impregnation, an equal number of ova were put in ordinary aërated distilled water. The cover was sealed with hot wax and lard, an operation which lasted about two minutes. The eggs were then compared with their control ova, at thirty minutes, at forty minutes, at $1^h\ 10^m$, at two hours, at $2^h\ 40^m$, at $3^h\ 30^m$, at five hours, at eight hours, and at twenty-four hours after impregnation, and were on each occasion found to be progressing equally with them. Accidentally one or two of these eggs were injured, so that the inner sac broke, and partially emptied itself, but cleavage went on in an irregular manner, although with a rapidity equal to that observed in uninjured ova.

Experiment 2.—An equal number of ova, twenty-five minutes after impregnation, were put into a similar cell, in distilled water which had been well boiled, and the cover was sealed as before. At forty minutes, at one hour, at two hours, at $3^h\ 30^m$, and at twenty hours after impregnation, they were found to be progressing, equally well with their control eggs.

Experiment 3.—The same as the above, using water in the cell which had been saturated with hydrogen after having been boiled. At forty-five minutes, at one hour, at $1^h\ 55^m$, at $2^h\ 15^m$, at $3^h\ 30^m$, and at twenty hours after impregnation, they were also found to be progressing, like the eggs in the control experiment.

Experiment 4.—The same as the last, using distilled water impregnated with oxygen. At forty minutes, at $1^h\ 10^m$, at two hours, at $2^h\ 40^m$, at five hours, at eight hours, and at twenty-four hours after impregnation the ova were examined, and found to be quite like their control ova.

Experiment 5.—The same as the above, using water which had been moderately charged with carbonic acid, after having been well boiled. Forty minutes after impregnation, immediately after the cell had been sealed, the yelks were seen in the dumbbell form, and under observation the sulcus was effaced without travelling on, the yelk-ball becoming round, even the germinal mass ceasing to be prominent. For some minutes there was no visible movement, but afterwards, by imperceptible degrees the germinal mass was slowly reprotruded from the surface in a nipple-shaped and rather

irregular form, while the food-yelk underwent no change of form (Plate XVIII. fig. 72). None of these eggs passed on to cleavage, and all ultimately had ruptured inner sacs, shrunken into a dark granular mass.

Experiment 6.—Forty minutes after impregnation the last experiment was repeated, with a stronger but otherwise similar carbonic-acid water. The eggs were examined directly, and the yelk, which had been actively contracting, suddenly ceased to move; the sulci were effaced without travelling on, and under observation the yelk-balls became globular, in ten minutes the germinal disk being level with the surface. About fifteen minutes later the germinal mass again projected above the surface of the yelk-ball, and the further fate of these eggs was the same as in those of experiment 5.

Experiment 7 was the same as the last, using eggs $3^h\ 35^m$ after impregnation, at which time the first cleft was fully formed. They were not examined until four minutes after the action of the carbonic acid; the germinal disk was then withdrawn into the yelk-ball in a singular manner (Plate XVIII. fig. 73). In ten minutes more it was projecting again, but irregular in form; afterwards, by still slower steps, it was again flattened, but not drawn into the yelk-ball. While this went on the cleavage masses underwent a gradual fusion, which commenced as early as thirty minutes after closing the cell; and at length, $5^h\ 30^m$ after impregnation, the yelk-ball was globular and at rest (Plate XVIII. fig. 74). By degrees the diffused germinal mass became darker and more granular, and eight hours after impregnation many of the eggs had ruptured inner sacs, and the contents were changed to a darkly granular mass, which consisted chiefly of the decomposed formative yelk-substance, with some oil-drops and granules (Plate XVIII. fig. 75). All the changes of figure observed after the yelk has first effaced its sulcus, are imperceptible in progress.

Experiment 8 was the same as the last, using eggs $3^h\ 45^m$ after impregnation, the germinal disk being then cleft into four masses. The eggs were examined at once, in order to trace the steps of the first retraction of the germinal disk into the yelk, this took place under observation until the yelk-ball exhibited no prominence on its surface. (See Plate XVIII. fig. 79, which shows the complete retraction, and figures 77, 78, which show the intermediate stages: fig. 76 is given as a normal standard.) I could not ascertain that any changes of form took place in the mass of the food-yelk, and the explanation of the withdrawal of the germinal mass into the food-yelk is at present not made out. During the next ten minutes, by invisible movements, the germinal disk became again prominent, the cleavage masses being irregular in form and wanting in symmetry. Much more slowly after this, sinking of the germinal disk began, and gradual fusion of the cleavage masses; so that five hours after impregnation, or $1^h\ 15^m$ after the action of the carbonic acid, some of the eggs had still prominent and irregular germinal disks, but all showed more or less fusion of the cleavage masses, and by $6\frac{1}{2}$ hours after, the fusion was complete. It was not, however, till eight hours after, that the germinal mass was quite reduced to the state shown in Plate XVIII. fig. 74. Later still, in many, but not in all the eggs, rupture of the inner sac and dark granular precipitation occurred.

The slow reprotrusion of the germinal disk and commencing fusion of the masses are shown in Plate XVIII. figs. 80, 81 & 82, and the ultimate flattening and fusion of the cleavage masses in Plate XVIII. figs. 74 & 83.

In order to compare the developing embryos with the developing early germs in their relations to oxygen in the surrounding medium, particularly with reference to the relative need for oxygen shown by striated muscle and protoplasm during their contraction, I made the following additional experiments.

Experiment 9.—One young stickleback, three days hatched, was put into a similar cell in tap-water and sealed; $1^h\ 30^m$ after, it was quite strong, and at $4^h\ 30^m$ after, it could still swim about, although rather less vigorously.

Experiment 10.—Two such young fishes were sealed in a similar cell, in some of the boiled distilled water before used. One of them was accidentally injured, it ceased to swim in a few minutes; the gills became still first, and the heart, although it did not contract more than thirty minutes, last. The uninjured fish swam about for thirty minutes; by forty-five minutes it had turned on its back, by $1^h\ 30^m$ the gills and fins were still; the circulation ceased in two hours, but the heart continued for $2^h\ 30^m$.

Experiment 11.—The last experiment was repeated, using but one fish. In ten minutes the gills and fins ceased to move, and the trunk to be sustained in its position, while in $1^h\ 10^m$ the heart was motionless, but the tissues were not opaque.

Experiment 12 was the same as the last, using boiled water saturated with hydrogen, and taking care to choose a vigorous fish. In ten minutes the gills and fins ceased to move, the fish turned on its back, and after fifty-five minutes the heart alone was moving.

Experiment 13 was the same as the above. In ten minutes the fish turned on its back, in twenty minutes the gills and fins ceased to move, in fifty-five minutes all movements had ceased, and the tissues were opalescent.

Experiment 14 was an attempt to find chemical evidence of the presence of carbonic acid in the water in which eggs which had passed through the earlier stages of cleavage had been immersed; but although the observation was repeated no result was obtained, and the details are therefore omitted.

Although the preceding experiments were made upon such a small mass of material, and the methods of excluding oxygen from the surrounding medium were so imperfect, it may be inferred, without much risk of error, that the proportional demand for oxygen, of equal masses of organic matter, undergoing the changes of growth and development, is much less in the early germ than in the free embryo. Indeed the rhythmic protoplasmic contractions and the cleavage were, to all appearance, quite unchecked by water deprived of most of its oxygen. The movements of striated muscle, on the other hand, were shown to be very soon checked in water similarly deprived of its oxygen.

That the excess of oxygen does not hasten the cleavage, or promote or excite the protoplasmic contractions, appears also a legitimate inference.

That carbonic acid acts as a potent poison is clearly shown, and that it relaxes the con-

tractions promptly. Also that it causes afterwards an irregular deformed projection of the germinal disk, soon followed by a state of complete permanent relaxation, and fusion of the cleavage masses into one formless mass. Ultimately it causes rupture of the inner sac and chemical change in the yelk.

*The Trout.*

In this fish the impregnated ova do not exhibit any visible contractions of the yelk, although a breathing-chamber forms; the formative yelk concentrates at the germinal pole, and its elements undergo changes like those in *Gasterosteus*. Cleavage did not take place in my experiments made in January 1855 until the next day. Prior to the commencement of cleavage no distinction is visible between the impregnated and non-impregnated eggs. I was struck with the great length of time which these unimpregnated eggs remained clear and unchanged in water: if the inner sac be not injured, it is at least twenty-three days; while ova kept in moist air for the same period decompose, are fœtid, alkaline, and coagulate when put into water less than a fresh egg broken in water.

*The Ruffe.*

Impregnated eggs of the ruffe exhibit slow changes of form of the yelk-ball without distinct oscillations. Unimpregnated eggs in water form a breathing-chamber, and show similar slight changes of form of the yelk. In four minutes the formative yelk gradually concentrates.

*The Perch.*

Impregnated ova of perch undergo changes of form like those of the ruffe, and the same may be said of unfecundated ova put into water.

*The Pike.*

1. *The sequences of impregnation.*

These ova are better adapted for the examination of some points than are those of *Gasterosteus*, and their study has helped me to correct some errors which I fell into at first.

Impregnated eggs show a commencing breathing-chamber and slight changes in the droplets of the cortical layer after about a minute; unimpregnated eggs are similarly affected by water only. After twenty minutes the breathing-chamber was complete, and the formative yelk concentrated into a well-defined discus germinativus in the fertile eggs, and the barren ones appeared in all respects the same. After three hours and twenty minutes contractions began with a slow flattening of one side of the yelk-ball and a slight oscillation. After three hours and forty minutes cleavage began, and up to this time no distinction was visible between the impregnated and the unimpregnated eggs. Four hours after deposition in water, in these latter the discus proligerus was not lobulated.

The contractions and oscillations resemble those which occur in the egg of the stickleback, but they are more frequently irregular; thus the sulcus may be represented by a circular pit, it may be a furrow parallel to the equator in either hemisphere, and may travel towards either pole; or more rarely, it is a meridional one; frequently there are two or three present travelling over the surface at the same time.

At first the yelk-ball retains any position which it may have within the yelk-sac, so that its polar axis may be vertical, inclined, or horizontal, and these positions modify the direction and extent of the oscillation. Usually, about the time of the commencement of cleavage, the oil-drops tend in part to adhere to the germinal disk, and then the polar axis is always vertical. In this position it is easy to see that the germinal pole in oscillating describes a very wide ellipse, and there is at the same time a partial rotation on the polar axis; but the oscillations vary as much as the contractions. This general description applies equally to the impregnated and unimpregnated eggs.

After seven hours, in the impregnated eggs the germinal disk was cleft into sixteen regular segments.

At the same time the unimpregnated ones showed a remarkable and very interesting lobulation of the concentrated formative yelk, a sort of irregular asymmetrical cleavage. This lobulation or pseudo-cleavage continued to increase, and to become more and more irregular. After twenty-five hours it was noted that portions of the discus proligerus were pinched off and appeared either as projecting buds, or as detached masses in the breathing-chamber. (This tendency of the formative yelk material to pinch off portions of its substance, may explain the so-called "Richtungsbläschen.") Sometimes the whole mass of the concentrated formative yelk is pinched off, and lies free in the breathing-chamber, leaving a scar at the germinal pole of the yelk-ball, indicated by the collection of smaller oil-granules which do not separate with it, and by the ragged and radially puckered edges of the torn inner sac around. It is somewhat singular that, as a rule, from this scar no food-yelk escapes. Some hours later nearly all the unimpregnated eggs exhibited the separation of the discus proligerus, but no food-yelk had escaped. After fifty-seven hours some of them had their inner sacs ruptured, and a part of the fluid food-yelk had escaped into the breathing-chamber, but the diminished yelk-ball went on contracting as before. After seventy-four hours, decomposition having made considerable progress, the contractions were visible but feeble; after eighty-three hours they required the greatest care to see them at all. In other instances I found these movements continue for 105 hours, provided some acid clear food-yelk still remained in the inner sac.

In an unimpregnated egg twenty-five hours after it had been pressed from the parent, the inner sac escaped, on rupture, in a very distinct and firm state; its surface was marked by fine dots, having much the appearance of impressions of the inner surface of the yelk-sac. I examined this surface carefully for cilia, or ciliary motion, which have been considered a possible cause of the oscillation. I used a power of $\times 250$ without finding any.

The formative yelk in these eggs breaks up ultimately into a formless, dark, granular mass.

The fecundated eggs continued to contract and oscillate vividly during the progress of the cleavage, and often, where the sulcus was strongly marked, the surface of the inner sac was beautifully wrinkled, like a ripple upon water. Fifty hours after fecundation the contractions were more active than in the unfertilized eggs, which then were seen to have vibriones upon them, appearing to be liable to decomposition earlier than fertile eggs, and to suffer in consequence some diminution of their contractility. The fertilized eggs, after fifty hours, have a germinal mass composed of polygonal cell-like corpuscles, on the surface of which I failed, with a power of $\times 250$, to detect any trace of ciliary action.

To ascertain, if possible, in what part of the yelk-ball the contractile property resided, I ruptured, by pressure, one which had been fecundated fifty hours, the contractions being at the time vigorous.

An irregular shred of the inner sac was retained within the crushed yelk-sac, and a pouch of the former, filled with food-yelk, projected from the rupture in the latter. Very active contractions were seen $1\frac{1}{2}$ hour afterwards in both of these, causing a to-and-fro movement, during which the pouch was alternately protruded and retracted from the opening in the yelk-sac; and the shred of inner sac, which had partial attachments, moved right and left, each time, seeming to alter its form somewhat: the rapidity of these motions was much greater than that seen in unbroken eggs. I counted on the shred three contractions, each causing a right and left motion in one minute, and a point on this shred passed through $\frac{1}{440}''$ in ten seconds. For six hours longer contractions continued in this ruptured egg.

The torn shred seen moving within the broken yelk-sac could scarcely have been other than a portion of inner sac, and it had precisely the same appearance, examined with $\times 250$. At this stage of development of the germ there is no differentiation of parts, and no contractile property, and on its mass of cell-like corpuscles, as already stated, no cilia or ciliary movement could be found.

It was found impossible to repeat this observation exactly, at least so far as regarded the happy accidental position of the torn shred of inner sac, although a number of attempts were made. However, I often found escaped and projecting pouches of the inner sac containing food-yelk, over the surface of which contractile waves passed, with varying degrees of rapidity; and in one instance, while endeavouring to stain an unimpregnated egg with an extremely weak ammoniacal solution of carmine, these waves were seen, an hour after it had been crushed, moving with considerable velocity. The pouch of the inner sac was in this instance so large, as to contain the greater part of the food-yelk of the egg. The contractile waves had a certain rhythm, but not a very regular one; two or more waves sometimes coexisted, and as they travelled along they proceeded in the same or opposite directions, and then if two met, a very deep angular sulcus resulted, which showed a tendency to recur at the same spot. One of

these sulci, which passed nearly halfway across the field of the microscope, equal in this case to about $\frac{1}{30}''$, was effaced in forty seconds. Three hours later these contractions were still going on, but nine hours after rupture of the egg they had ceased. During their continuance there was a gradual emptying of the fluid food-yelk and shrinking of the inner sac.

So long as in impregnated ova any portion of the food-yelk remains uncovered by the gradually advancing germinal mass, its surface is constantly moving, and the waves of contraction are seen to pass beyond the margin of the germ, and under it, to the contained food-yelk. Seventy-four hours after fecundation there remains but a small area at the ventral pole uncovered by the advancing germ. But long after the whole of the yelk is covered by the germ, at least as late as the ninth day after impregnation, slow rhythmic contractions of the contained food-yelk are visible, producing alternate depressions of the lateral poles of the abdominal region, and consequent oscillations of the embryo, which have been described by Reichert*.

On the tenth day some of these eggs were hatched. I could not see any contractions in the food-yelk of free embryos, but the search was not very carefully made.

In the pike, as in the stickleback, eggs which have been allowed to remain in the dead parent for a certain time cannot be fertilized, even when they have undergone no discoverable change of structure. In the pike, after seven hours, the capacity of being impregnated was lost; but then a physical change had occurred, for water no longer passed through the yelk-sac to form a breathing-chamber.

On the whole, I think it may be said that the inner sac is essentially connected with the exercise of this contractile property. It is difficult to ascribe contractility, at least of this rhythmic kind, to a substance so fluid as the food-yelk, in which minute monads can swim about freely, and in which, when escaped or escaping, I never saw the slightest evidence of contractility. On the other hand, except the single observation above related, of a retained shred of inner sac in a crushed egg, I have no satisfactory evidence that the inner sac alone is capable of contracting; and that instance may possibly have been fallacious, for the closely connected pouch of the inner sac filled with food-yelk, might at each of its contractions have pulled at and moved the shred. The persistence of the movements of the food-yelk contained in the embryo, when one bears in mind that the inner sac is folded in during cleavage, and might therefore fairly be expected to be used up in the gradual extension of the germ over the yelk, would seem to give support to the notion held by Reichert†, that the substance of the food-yelk is the contractile matter. But it may be replied that the inner sac may possibly be retained on the surface of the contained food-yelk in the abdomen of the embryo, and to this view I incline. But while it appears probable that the contractile property resides in the inner sac, I am disposed to think that the presence on its inner surface of some of its acid yelk is an essential condition of its action.

* "Der Nahrungsdotter des Hechteies,—eine kontraktile Substanz," Müller's Archiv, 1857, p. 46.

† Müller's Archiv, *loc. cit.*

The substance of the formative yelk, at least where it is collected into a discus proligerus or germinativus, appears to possess the same contractility in a less degree (it may be that its solidity is a hindrance to its manifestation). Certainly, as the contractile waves pass forward to become lost at the germinal pole, the nipple-shaped form which the disk assumes (Plate XVI. fig. 39) is best explained on this assumption; and if, as I conceive, the substance of the inner sac is thicker at the germinal pole, so as to embrace the whole of the discus, the production of that shape is easy to be understood.

But the formative yelk possesses also another different contractile property, by which, when acted on by water or the maternal fluids, it tends to subdivide itself into smaller masses. This property it seems to be the function of the male element to regulate.

2. *Conditions of the yelk contractions and of cleavage.*

Pike ova being obtainable in greater numbers than those of the stickleback, and being in some other respects better adapted for experiment, the inquiry into the modifying and essential conditions of protoplasmic movements was continued with them.

a. *Poisons.—Morphia.* A solution of the acetate of morphia of 2 grs. to 60 grs. of water, which had a slight excess of acetic acid, was added in small proportion to the water of a cell containing several ova, vividly contracting, thirty-six hours after impregnation. In less than a minute they ceased to move, the yelk-ball became round, no rupture of the inner sac took place. On repeating this experiment, after adding carbonate of potash in slight excess to the solution of acetate of morphia, the movements again seemed to cease, the yelk-ball became round in less than a minute. But a source of fallacy always exists in these observations, viz. that normally the yelk tends to become round and remain at rest for a variable but brief space of time after each wave has passed. Half an hour later contractions were visible.

*Acetic acid.*—The above result being somewhat doubtful, a few drops of a solution of one drop of strong acetic acid, in sixty of water, were added to a cell containing several actively contracting eggs. At once an arrest of the movements took place, but the yelk-ball, instead of becoming round, which is the position of relaxation, remained for some minutes marked by the sulcus, which at the moment existed; but afterwards the movements began again, the sulci were remarkably deep and irregular, and travelled very slowly. A little more of the acetic acid solution being added no effect appeared at first, but in two hours, three eggs out of five had become opaque by coagulation of the food-yelk, the two ova which remained clear being motionless and globular.

*Acetate of potash.*—A weak, faintly alkaline, solution of acetate of potash was then added to a cell containing some eggs freely contracting. Soon the yelk-ball became round, the formative yelk changed in structure, became firmer, more opaque, and projected from its surface little rounded masses. After two hours the food-yelk was still slowly contracting. Thus a solution of a strength which chemically changes somewhat the formative yelk-matter, does not arrest the contractions, although it hinders them.

*Tincture of opium.*—On adding two or three drops of tincture of opium to a cell containing ova, vividly contracting, the movements seemed to cease for a time, but the sulcus remained. In about three minutes the movements reappeared and continued for half an hour, after which more tincture of opium was added, and still the contractions were found going on with moderate vigour an hour later.

*Spirit of wine.*—A few drops of spirit of wine added to a cell containing freely contracting ova, either did not affect the contractions and oscillations at all, or slightly quickened them.

*Tincture of cantharides* acts much the same as spirit of wine does; after twelve hours eggs treated with it were developing normally.

*Solution of potash.*—A very minute drop of Liquor Potassæ, L. P., was mixed with the water of a cell containing several freely contracting ova. The inner sac at once burst, its contents escaped into the cavity of the egg, and it was soon crumpled into a distinctly membranous bag, still marked by the ripples due to the previous contractions. No further contractile waves appeared, nor did it dissolve. After a very few minutes the yelk-sac burst, the solution having been too strong.

*Strychnia.*—A neutral solution of acetate of strychnia, 2 grs. to 480 grs. of water, was added, in small amount, to a cell containing vigorously contracting ova. No visible change appeared in the movements, and after twelve hours the eggs were normally developing. More strychnia was then added, but no result followed.

*Aconite.*—A solution of 5 grs. of extract of aconite in 60 minims of spirit of wine was added, by drops, to a cell containing freely moving ova, but no effect was observed, except perhaps a slight acceleration of the movements.

*Hydrocyanic acid.*—A few drops of dilute hydrocyanic acid (2 per cent.) added to the water in which ova were actively contracting, produced no effect in a quarter of an hour, during which time I watched.

In each of these experiments there were some unimpregnated eggs present, easily recognized by their irregular, often detached, proligerous disks. In no case was there any difference observable between them and the fertilized ones as to the action of the poisonous agents.

*Hydrochlorate of morphia.*—Two drops of a strong spirituous solution of hydrochlorate of morphia were added to some unimpregnated ova, which had been deposited nine hours, and were rapidly contracting and oscillating. No effect following, four drops more were added without result; again four drops were added, and during a quarter of an hour no result was observed; twelve hours later they were all opaque.

*Chromic acid.*—Eight days after impregnation, a healthy developing embryo was let out of the yelk-sac into a weak solution of chloride of sodium and chromic acid, and the contractions at the lateral poles, mentioned by Reichert, were seen to go on apparently unchecked.

*Ammonia.*—Contractions of the yelk continue in eggs placed in a very weak ammoniacal solution of carmine; and even when the egg is ruptured, and the inner sac

escapes into this solution, they go on, provided the inner sac is not too much injured. A 2 per cent. solution is too strong, it causes prompt rupture of the inner sac, but does not dissolve it.

*Ether.*—Unimpregnated ova, thirty hours after deposition, while freely contracting, put into a watch glass, with only enough water to moisten them, and exposed to the vapour of ether under a small bell-glass, exhibited no diminution of movements, although slight opacity of the eggs was produced. Similar eggs treated with an aqueous solution of ether, became opaque, and exactly in the same ratio the movements became slower, and ultimately ceased. The opacity preceded any visible diminution of the movement.

*Chloroform.*—Some chloroform in vapour was applied 144 hours after impregnation, at which time the heart is seen beating, the muscles of the trunk acting, and the yelk still contracting at the lateral abdominal poles. The first effect was to excite writhing movements in all the embryos, but in five minutes the muscles of the trunk and the heart had ceased to move, and the contractions of the yelk were arrested, the sulcus remaining uneffaced. Five minutes later the embryos began to recover, the yelk-contractions moved on slowly, and after adding fresh water the heart began to beat feebly; but the trunk could not be seen to move in any of the embryos for half an hour in spite of free addition of water. After 2½ hours the trunk moved freely, the heart beat regularly, and the yelk contracted vigorously.

As the general result of these observations, and those of a similar kind made on the ova of the *Gasterosteus*, it may be stated that the rhythmic contractility of the yelk is not materially influenced by any of the poisons used, which did not act chemically, with the exception of chloroform and of carbonic acid. It is true that acetate of morphia appeared in some experiments to arrest the movements, but the results were not confirmed by the later observations recorded with other solutions of morphia, and never were free from certain fallacies which have been mentioned. Whether alcohol or cantharides quicken these movements or not must also remain in some doubt, as the results obtained were not sufficiently marked to justify a positive assertion, and some fallacy might exist due to the currents which occur during the mixture of spirit with water.

b. *Galvanism.*—The effects of the application of galvanic currents to these eggs are like those already mentioned, as seen in the ova of *Gasterosteus*.

c. *Heat.*—Eggs nine hours after fecundation, when they are cleaving and actively contracting, warmed on the stage of the microscope to about 80° F., became still, or nearly so, and the oil-drops were a little displaced. At the temperature of the room (58° F.) they did not soon recover, but ultimately they cleft, although irregularly. Eggs at the same stage, gently warmed to about 70° F., moved much more quickly; on being cooled to about 40° F. the contractions ceased entirely; warmed again, the vivid movements returned, cooled again, they ceased: left at 60° F. until the next day, the impregnated ova were seen cleaving, the barren ones contracting, their proligerous disks being detached. In these experiments the temperature stated is only approximative, as a ther-

mometer on the stage of the microscope does not duly register the temperature of the eggs in the cell.

d. *Oxygen.*—The experiments made with the impregnated eggs of the stickleback having failed to show in a satisfactory manner to what extent the contractions of the yelk and the cleavage are dependent upon the presence of oxygen in the surrounding medium, I attempted to ascertain this by employing a larger number of ova of the pike, which, as they exhibit the contractions of the protoplasm irrespective of impregnation, make it easier to separate experimentally the conditions of the cleavage from those of the yelk contractions; and as they are free from any appreciable amount of maternal secretion, are more suitable for prosecuting the inquiry whether any and what product of oxidation passes into the surrounding medium, during the movements of the protoplasm, or the development of the germ.

In these experiments I divided the spinal cord of the parent fishes, without injuring the bodies of the vertebræ, just behind the edge of the gill-covers, and then wrapped them in a cloth which was kept wet, by which means they continued to live and breathe for some hours, and could be easily manipulated.

Having previously prepared a curved glass tube of about $2\frac{1}{2}''$ in length, tapered at one end so as to leave an aperture of about $\frac{1}{10}''$ to $\frac{1}{8}''$ in diameter, to which was fixed an elastic thread for securing it when in position, I filled the tube when warm with pure olive-oil and closed both ends with hot tallow, excluding air-bubbles. By this arrangement I was able to pass ova direct from the body of the parent into any fluid, without previous contact with air, or any air containing liquid.

Being unable to obtain water absolutely freed from dissolved oxygen, I prepared several small beakers of distilled water in which were fragments of broken glass, by prolonged boiling, until the bumping became so violent that they were in great danger of being broken (indeed two or three were thus broken), and then poured pure olive-oil upon the water whilst boiling to a depth of $\frac{3}{4}''$ to $1''$, and left them to cool. In this way I hoped to have water sufficiently freed from dissolved oxygen, for the purposes required, and in a state in which it could not support respiration.

Thus prepared, I attempted a first series of experiments on April 5th, 1866, to ascertain whether the rhythmic contractions of the yelk persisted in water deprived, as above described, of its dissolved oxygen; and at the same time whether the slower protoplasmic contractions which cleave the germinal disk persisted under similar conditions. As, however, it was ascertained that some dissolved oxygen still existed in the water, I sought to obviate this difficulty by repeating the experiments with varying proportions of water to ova: because if oxygen be used up during these movements they must cease sooner when the proportion of ova to water is greater.

I used distilled water in order that it might be afterwards tested for any product such as carbonic acid.

Control experiment 1.—For the sake of having a standard of comparison, I put into a vessel of ordinary distilled water some unimpregnated eggs in the proportion of one

part of ova to sixteen of water without covering the surface with oil. After twenty hours the rhythmic contractions of the yelk were vigorous, the formative yelk was lobular, and often detached in part or wholly. After fifty-three hours they were contracting freely, several inner sacs ruptured. After seventy-two hours most of the inner sacs were burst, and the yelks were more or less opake, and no contractions were seen in those few in which the inner sac was not ruptured. The water was slightly opalescent. 100 hours after, the water did not precipitate baryta-water, the broken eggs reddened blue litmus. Several of these eggs, from the deeper layers in the vessel, had not properly imbibed water; indeed the very lowest, fifty-three hours after, had the appearance of eggs freshly expressed from the fish, being still adhesive; they could not then be made to absorb water freely as fresh eggs do.

Control experiment 1′.—I fecundated ova successfully in ordinary distilled water, not covered with oil, by dropping in a fragment of ripe testis. Many of them cleft, but the water soon became turbid; and fifty-three hours after, many of them had opaque germinal masses; seventy-two hours after, all were opaque, decomposition being evidently favoured by the presence of the fragment of testis.

Control experiment 2.—I fertilized some spawn in the usual way in tap-water contained in a dish, and changed the water daily. Not all of these ova were impregnated, but the fecundated and non-fecundated were seen rhythmically contracting with equal vigour twenty-five hours after, when the former were in the mulberry stage of cleavage. At fifty-seven hours, at seventy-five hours, and 100 hours after, the same contractions were seen in all the unimpregnated eggs, of which the inner sac yet contained some yelk, and in the impregnated ones in that part of the food-yelk still uncovered by the advancing germinal mass. The unimpregnated ova were all still the next day. The fertile eggs were hatched from the 16th to the 18th of April, that is, after eleven to thirteen days.

Experiment *a*.—The fish being secured on a raised shelf, and the beakers on supports near, I passed the tube filled with oil into the sexual aperture, and held it in position by means of the elastic threads. Then by gentle pressure upon the abdomen the ova were made to pass into the tube until they had displaced the oil, the lower end being closed by a drop of oil. Unimpregnated ova were then passed through the tube, into one of the beakers, in the proportion of about one part of ova to ten of water, so that the tube dipping below the layer of oil, the eggs were scarcely, if at all, greased. After twenty-seven hours their yelks were contracting and oscillating freely; the discus proligerus was concentrated and had become irregularly lobular, and often detached in fragments, or as a whole; the detached masses being granular and opaque. The food-yelk was clear. The upper layers of the eggs in the beaker, however, alone exhibited these movements distinctly, the lower ones not having imbibed enough water to duly distend them; hence they were not clear on the general surface, and had not a well-concentrated discus proligerus. At this time the contractions of the eggs in the upper layer were as strong as in control experiment 1. After forty-nine hours the lower layers of the eggs,

still imperfectly distended with water, contracted slowly, resembling eggs three or four minutes after impregnation while the breathing-chamber is forming. The eggs of the uppermost layers contracted vigorously, and quite as much so as those of the control experiment 1. Nearly all had their proligerous disks entirely separated and broken into masses. The intermediate layers of ova in the beaker exhibited the formative yelk concentrated, but not at all lobulated; from which it would seem that the tendency of the discus proligerus to lobulate, and to be pinched off in portions or as a whole, like the rhythmic contractions, depends on a free supply of water as one of its conditions. The same may be said of the concentration of the granules of the formative yelk, and I think also of the tendency of the inner sac to rupture. Seventy-two hours after, the contractions were as vigorous as ever in the clear yelk-balls of the upper layers of eggs; the discus proligerus was entirely detached in all. 100 hours after, contractions had ceased, the water had a very faintly milky aspect, and the eggs were opaque. Three days later the water drawn off was not fœtid, it reacted faintly acid, and gave a slight precipitate with baryta-water.

Experiment *b*.—The last experiment was repeated, with one part of unimpregnated ova to about three parts of water. The contractions were observed twenty-seven, forty-nine, and seventy-two hours afterwards, and found to be as vigorous as those in experiment *a*, or in the control experiment 1. They were seen only in the eggs of the surface layers, which alone had imbibed water freely. 100 hours after, and three days later still, they resembled experiment *a*. The fact that water was not absorbed freely, except by the upper layers of these eggs, deprived the experiment of a part of its value, as the proportion of actively contracting eggs to the water had no definite relation to the numbers used, and could not be determined.

Experiment *c*.—The experiment was repeated, using equal parts of unimpregnated eggs and of water. At twenty-seven, fifty, and seventy-six hours after, the rhythmic contractions of the yelk were as vivid as in experiment *a* or in control experiment 1; but the discus proligerus was less lobular, and not so often detached. 104 hours after, all the eggs were opaque.

In this experiment mechanical disturbance was employed at first, to ensure a nearly equal action of water on all the eggs, but they were not quite fully distended, although nearly all the water was taken up; from which we may conclude that normally the eggs imbibe about their own bulk of water.

Experiment *d*.—The same as the above, using two parts of unimpregnated ova to one of water. Very few eggs of this experiment imbibed water, and those did so imperfectly. The rhythmic contractions were feeble, the formative yelk scarcely lobulated at all. The lowest layers did not visibly absorb any water, efface their indentations, or concentrate the formative yelk in the slightest degree. At 104 hours after, they were opaque.

Experiments were then attempted to be made in a similar manner with impregnated eggs, but being unable to insert a tube into the male fish, I passed a piece of ripe testis

quickly through the oil on to the eggs, directly after they had been introduced into the beaker in the way before described.

Experiment *a'*.—I used one part ova to ten parts of water, but the attempt to fecundate failed, perhaps from the action of the oil upon the testis.

Experiments *b'*, *c'*, and *d'*, made with varying proportions of ova to water, also failed, apparently from a similar cause. The results only served to confirm those obtained from the experiments upon unimpregnated eggs, as to the long duration of the rhythmic contractions.

I then sought to ascertain the comparative duration of these movements in aërated distilled water, with a layer of oil on its surface, employing varying proportions of eggs to water; but as in all these experiments the proportion of eggs which, being defended by the superincumbent layers, did not freely imbibe water was large and variable, the results obtained lost much of their value. I will mention, however, that the supernatant water, after 100 hours, did not precipitate baryta-water more freely than that from the experiment *a*, and had the same very faintly acid reaction.

These results not being conclusive, partly because of the difficulty of properly watching with the higher powers contractions going on in the eggs contained in beakers, partly on account of the eggs not being all equally acted on by water, partly in consequence of the failure to fecundate through a layer of oil, I obtained a further supply of ripe male and female pike, and on April 12th and 13th made a second series of observations, using glass cells having a depth of $\frac{1}{10}''$, and a cubic capacity of about ·05″. In these experiments I sought to ascertain, by varying the proportions of eggs to water, air being excluded, whether the duration of the rhythmic contractions of the yelk, or their vivacity and the activity of the cleavage, were inversely as the number of eggs.

Control experiment 3.—For purposes of comparison, unimpregnated ova were passed from the female fish into ordinary distilled water in a wide beaker, so that they formed only one layer on the bottom. Four hours after, they were contracting freely, and the discus proligerus was concentrated, but smooth on its surface. Twelve hours after, it was in some lobulated, and in many detached, wholly or in part. Twenty-five hours after, they were vigorously contracting and oscillating; in nearly all the discus proligerus was detached, and in many of these its substance was fused, so as to run like a stream of lava: in a majority the inner sac was ruptured and more or less emptied. Thirty-five hours after, in all the discus was detached and diffused, but where the inner sac was not ruptured, or ruptured and only partly emptied, the yelk was actively contracting and rotating. Forty-eight hours after, the water, which had not been changed, was faintly milky, of a neutral reaction, and did not precipitate baryta-water: filtered and concentrated, it was alkaline, was precipitated by baryta-water, and the flocculent precipitate was only in part dissolved by hydrochloric acid: dried and strongly heated, it charred, and gave off fumes having the smell of burning hair, and it left an alkaline ash which did not effervesce with dilute hydrochloric acid. In this case there must have been a transudation of some organic substance, which probably was limited to the eggs with ruptured inner sacs.

Control experiment 3′.—The last experiment was repeated with eggs which had been impregnated in the usual way. Ordinary distilled water was used, care being taken to wash away all adhering seminal fluid. Twelve hours after, they were contracting freely, the germinal disk was cleft into a coarse mulberry mass. Twenty-six hours after, the contractions were rapid, the germinal disk a fine mulberry mass: a few eggs were opaque. Thirty-five hours after, they were contracting freely, and the germinal mass covered nearly half the yelk. Forty-eight hours after, they were contracting as before, and the germinal mass covered nearly three-fourths of the yelk. The water, which had not been changed, was faintly milky, neutral in its reaction, did not precipitate baryta-water: on concentration it remained neutral, and contained faint traces of a phosphate. In this instance also the presence of several opaque eggs with ruptured inner sacs makes the examination of the water for products of respiration unsatisfactory.

Control experiment 4.—I fertilized in the usual way a number of eggs in a large dish in tap-water, which was changed daily; and as there were some eggs in which the impregnation had failed to take place, they could be compared with the control experiment 3, to ascertain whether distilled water modified the contractions and development of the germ. During the first forty-eight hours no difference was observed when they were compared with control experiments 3 and 3′.

Experiment *e*.—I placed thirty-five unimpregnated eggs, fresh from the pike, in one of the above-described cells, taking care that all were well bathed with distilled water, and I sealed the cover quickly with hot wax. Four hours after, they were contracting freely, but with a smooth discus proligerus. Twelve hours after, contracting as before, they showed less lobulation of the discus proligerus and rarer separation of it than did the eggs in control experiment 3. Twenty-six hours after, they were moving freely, but less so than those in control experiment 3. Unfortunately, however, the luting had got loose, and an air-bubble had entered. Thirty hours after, the contractions had ceased; and the yelk-ball was round, except in those eggs near the air-bubble, which were still contracting well. Fifty-three hours after, all were motionless.

Experiment *f* was the same as the above, with only eighteen eggs in the cell. A small air-bubble got in, but the yelks ceased to contract in thirty-five hours. Those nearest to the air-bubble continued to move the longest, and became still ultimately by rupture of the inner sac, which explains their early cessation.

Experiment *g* was the same as the above, with but nine eggs in the cell. At twenty-six hours after, greater vivacity of the yelk-contractions and more lobulation, and separation of the discus proligerus were observed than in the eggs of experiment *e* at the same time. After thirty-five hours they were still contracting. After fifty-three hours they had ceased to move. Into this cell also a very minute air-bubble found its way.

Experiment *h* was the same as the above, but with thirty eggs in the cell, tallow being used instead of wax to seal the cover. Twelve hours after, they rotated less freely than the control ova to which they belonged. Twenty-three hours after, they had all

ceased to move, except two or three which lay near a very minute air-bubble. Twenty-nine hours after, all were still, the yelk-ball was round, the discus proligerus was not detached but flat and diffused.

Experiment *j* was the same as the above, with only seven eggs in the cell. Twelve hours after, the eggs were contracting freely, and resembled the control eggs more than those in cell *h*. In twenty-three hours the movements were languid, but distinct. After twenty-nine hours the contractions were barely visible. In this cell there was no air-bubble.

Experiment *e'* was the same as experimant *e*, with forty-eight impregnated eggs in the cell. After four hours the germinal mass was cleft into two or four. After twelve hours it was cleft into eight or sixteen masses, the control experiment 3' being at the same time in the coarse mulberry stage, containing hundreds of cleavage masses. The germinal mass was flat and rather diffused, and the outlines of the separate cleavage masses indistinct, as if they were about to coalesce. The eggs looked granular, notably the germinal mass. The yelk-ball went on contracting, however, although not so strongly as in experiment *e*. After twenty-three hours the cleavage had not progressed, the yelk-ball continued to contract in a few eggs with still further impaired vigour; but in most eggs it was round and still. After twenty-six hours all were motionless, and the yelk-ball so relaxed as nearly to fill the yelk-sac. These appearances reminded me of those which resulted from the action of carbonic-acid water on the eggs of the stickleback. Into this cell a little air entered.

Experiment *f'* was the same as the last, with seventeen eggs only in the cell. After twelve hours they were in a coarse mulberry stage of cleavage, being, however, less advanced than the control ova, but more than those of experiment *e'*; the yelks were contracting freely. Twenty-three hours after, they were in a finer mulberry stage, and were well contracting. Thirty-five hours after, the germinal mass had become diffused and darkly granular, but the contractions of the yelk continued. Fifty-three hours after, all were motionless. There was an air-bubble in the cell.

Experiment *g'* was the same as the last, with only ten eggs in the cell: air-bubbles entered again. Twenty-three hours after, the cleavage was further advanced than in experiment *f'*, but not so far as in the control experiment 3; the yelks contracted freely, however. Thirty-five hours after, they had dark, granular, germinal masses and contracted slowly. After fifty-three hours all were still.

Experiment *h'* was the same as the last, with thirty-eight eggs in the cell, and tallow employed instead of wax as a luting. After twelve hours they were cleft in eight, the cleavage masses were dark, granular, and almost fused together; the contractions, however, were distinct. After twenty-three hours the cleavage had not progressed, the contractions were languid. After twenty-nine hours the cleavage masses were quite fused, the contractions had ceased, the yelk-ball was round, relaxed, and at rest. In this cell there were no air-bubbles.

Experiment *j'*.—The same as the above, with only seven eggs. After twelve hours

they were cleft into the mulberry stage, but somewhat coarser than their control ova: the contractions were vivid. After twenty-three hours the germinal mass was in the finer mulberry stage. After twenty-nine hours no further progress was made in the cleavage, but the contractions continued.

Concurrently with these suffocative experiments, as they may be termed, I tried to ascertain whether any, and what respiratory products could be detected in the surrounding medium, and I considered separately impregnated and unimpregnated ova, with the view of comparing the rhythmic contraction with the movements of cleavage.

Experiment *k*.—I placed unimpregnated ova, fresh from the fish, in a tube with two parts of distilled water, agitating to ensure that all were well exposed to the water. The contractions continued for forty-eight hours, although latterly with less vigour than in the control eggs. The water, which was then clear, and neutral when filtered, gave no precipitate with baryta-water, was not coagulated by boiling or by alcohol, but was by protonitrate of mercury.

Experiment *l* was the same as the above, with one part of eggs to five of distilled water; the results after forty-eight hours were the same.

Experiment *k'* was similar to the last, but made with impregnated eggs in the ratio of one to four of water, the semen being washed away as quick as possible. Development was arrested before the germinal mass had extended over one-third of the yelk. After fifty hours all were still. The water, which was not quite bright even when filtered, was neutral or faintly acid in its reaction, was not precipitated by baryta-water, was coagulated by heat and nitric acid and by alcohol, more freely precipitated by protonitrate of mercury. Probably in this experiment an egg may have been ruptured while introducing it into the tube.

Experiment *l'* was the same as the above, with one part of ova to eight of water. Fifty hours after, all were still. The water when filtered was clear, the reactions were the same as those in experiment *k'*, except that it was not coagulated by heat and nitric acid, or by alcohol; so that the albumen must have been accidental in *k'*.

Experiment *m*.—As the number of ova used in experiment *k* was small, I repeated it in a flask, in which about half an ounce of spawn was put into two ounces of distilled water. Agitation was kept up for a time to ensure that all the eggs were duly exposed to the water. After twenty-eight hours they were contracting freely. No ruptured eggs were seen among them. The water was bright; it was filtered and evaporated; the concentrated liquid was faintly alkaline, the dry residue resembled dry serum, contained an organic colloid, not albumen, with alkaline phosphates, chlorides and sulphates; but no evidence of carbonic acid was found. The organic matter was not further examined.

Experiment *m'* was the same as the above with fertilized ova. After thirty hours, while all the movements were active and cleavage was progressing, there being but few opaque ova in the flask, the water was filtered quite bright, and evaporated. The residue had the same reactions as had that of experiment *m*.

In order to permit a comparison of the consumption of oxygen in protoplasmic move-

ments with that which takes place in cell-development and multiplication, and in the muscular movements of the embryo, a third series of observations was made on more advanced developing ova of the pike, and on recently hatched embryos. In this series I employed the same cells, and using the same bulk of organic matter of the same kind, in different grades of development, I observed how the duration of the different activities varied.

Experiment *n*.—Two healthy developing ova were put into a cell in water, and sealed with hot tallow, seventy-six hours after impregnation, when the yelk-contractions were very vigorous, and no muscular movements of the trunk or contractions of the heart could be seen. Seven hours after, but little progress in the development was seen; the yelk-contractions were vigorous. Eighteen hours after, no further development, and no yelk-contractions were seen: the embryonic tissues were not opaque. Forty-eight hours after, the tissues were somewhat opaque, and the germs of the vertebræ were partially fused together.

Experiment *o* was similar to the above, using two eggs 101 hours after impregnation, when the yelk-contractions were vigorous, but no beating of the heart or movements of the trunk were seen. Seven and a half hours after, the contractions were visible, but somewhat reduced in vigour; a barely recognizable progress in the development had occurred. Eighteen hours after, no further progress had been made in the development of the organs, and all motion had ceased. The ova were much behind the control eggs, and the tissues were still clear.

Experiment *p* was similar to the above, using two healthy eggs 127 hours after impregnation, when the yelk-contractions were distinct. The germ of the heart was visible but was not seen to move, the trunk struggled rarely and fitfully. After $13\frac{1}{2}$ hours all movement had ceased.

Experiment *q* was similar to the above, using two eggs 150 hours after impregnation, when the yelk-contractions were vigorous, the heart was beating regularly, and the trunk frequently moving; a few circulating corpuscles were seen. After $6\frac{1}{2}$ hours the heart was still, the trunk was not seen to struggle, nor could I by using the micrometer thread detect any yelk-contractions.

Experiment *r* was similar to the above, using two healthy eggs 174 hours after impregnation, when the heart was acting vigorously, the stream of blood was seen entering the auricular opening, the trunk often moved, and the yelk contractions were well seen. After $7\frac{3}{4}$ hours the movement had ceased in one egg, in the heart, trunk, and yelk. In the other the heart was moving slowly, and feeble trunk-movements were seen, but no yelk-contractions. In this egg the heart was irregularly and slowly acting after twelve hours. After eighteen hours all was still. The discrepancy which this result shows as compared with experiment *q*, is explained by the fact that there were several deaths in the dishes from which the eggs were taken, and some parasitic growth on the yelk-sacs, which I neglected to wash off in the case of experiment *q*, but carefully attended to in experiment *r*, in which instance I also selected the eggs under a lens.

Experiment *s* was similar to the above, using two free embryos hatched twenty-four hours previously. The beating of the heart was regular, 112 per minute, the blood was circulating vigorously, no yelk-contractions being seen, and as yet no movement of the gill-covers. After three hours the heart's action was reduced to ninety per minute, the movements of the trunk seemed unimpaired. After five hours the heart beat only fifty a minute, the blood-corpuscles tended to block up the channels near the auricular aperture. After seven hours the heart's action was irregular, and failed at times for forty seconds, afterwards beating once a second; the blood formed a red coagulum near the heart; the trunk did not move. Nine and a half hours after, the heart was beating feebly at long intervals, although the trunk was rigid, and recurved as it often is in dead fry, and the tissues were opalescent. In eighteen hours all movement had ceased.

Experiment *t* was similar to the above, using two free embryos, two days hatched (I may say here that the young fry burst the yelk-sac at various stages of development, so that the number of days they have been hatched is no safe measure of the stage of development). At this time they were somewhat further advanced than those in experiment *s*, but no action of the gills was seen. After $4\frac{3}{4}$ hours they became still as regards the trunk, and the heart beat rarely and feebly.

Experiment *v*.—As a supplement to these experiments, and as a measure of the value of my tests for carbonic acid, I sought for respiratory products in the water in which free embryos had been suffocated.

Sixteen young pike, about one week old (when they are seen to move the gill-covers and to vibrate the pectoral fins), were put into 800 grains of distilled water, and the beaker, which was full, was covered by a glass plate. They soon showed indications of distress, moving the gill-covers much more than their fellows in the aquarium did. $5\frac{3}{4}$ hours after, most of them were motionless at the bottom; a few were attached to an air-bubble which had got in, and these were able to swim. Six and a quarter hours after they were all unable to move. The water was filtered and evaporated, the concentrated fluid was neutral, and gave an indistinct cloud with baryta-water.

Experiment *w*.—In order to have a physiological test of the extent to which, by prolonged boiling, I had exhausted the oxygen of the distilled water in preparing the beakers for the first series of experiments *a*, *b*, *c*, *d*, I put into one of the same beakers twelve young pike, about six or seven days old, passing them through the oil on the surface, wrapped in moist bibulous paper unrolled afterwards by a needle. In one hour all were quiet but one, although their hearts were acting feebly. In one hour and forty-five minutes they were dead. The beaker contained 700 grains of water.

Experiment *x*.—Ova passed into pure olive-oil formed no breathing-chamber, and did not undergo any change resembling those which occur in water.

Experiment *y*.—Ova passed into spirit of turpentine resembled the above, but became more opaque.

These two experiments were made in reference to NELSON'S observations on the ova of *Ascaris mystax*.

Without giving a detailed analysis of this group of experiments, I will briefly state the inferences which they appear to justify.

The contractions of the yelk, and to a somewhat less degree, the cleavage, are remarkable for the small amount of oxygen which they demand for their maintenance. This was shown indeed in the experiments upon the eggs of the stickleback (pages 472 to 475), which seemed almost to justify the view that oxygen in the surrounding medium is not an essential condition of protoplasmic contraction, until the more extended observations on the pike ova enabled me to arrive at a more correct conclusion.

The experiments *a*, *b*, *c*, also establish the fact, that the rhythmic contractions demand but little oxygen for their support, as they persisted for seventy-two to seventy-six hours in water deprived of oxygen, as far as it is possible to do, by boiling in air.

The persistence of yelk-contractions in eggs which are already in part decomposing, is probably another illustration of this general rule.

That cleavage also demands but little oxygen appeared from the experiments 1, 2, 3, on the ova of the stickleback, page 473, for it progressed in limited areas, and in water partly deprived of oxygen apparently as rapidly as it did in open vessels.

One is almost led to infer, on comparing the results of the experiments upon the pike ova with those of the stickleback, that the former require proportionally more oxygen than do those of the latter during cleavage. Nor does there appear to be any difficulty in believing that variations in this respect exist among different species of animals.

That some oxygen in the surrounding medium is, however, a necessary condition of these protoplasmic movements appears from a careful consideration of the whole of the observations here related, although some of the results are such as to require explanation, and no one of the experiments taken alone is entirely free from possible objections. I will briefly explain some of the apparently opposing results.

In the control experiment 1, the yelk-contractions ceased in less than seventy-two hours, therefore earlier than they did in the ova of experiments *a*, *b*, *c*; this was due to the early setting in of decomposition in the control experiment 1, and consequent rupture and shrinking of the inner sac, phenomena which are favoured by the presence of oxygen in the water if it be not from time to time renewed, and were shown to be delayed in the boiled water covered with oil.

But in the ova of the control experiment 2, in which the water was changed daily, the yelk-contractions persisted for more than 100 hours, or about thirty hours longer than they did in the experiments *a*, *b*, *c*.

The series of suffocation experiments *e* to *j*, and *e'* to *j'* inclusive, also support the inference that oxygen is an essential condition; for in all of them the yelk-contractions ceased long before they did in the eggs of the control experiment, and in all they persisted longest in those eggs which lay nearest to the accidentally admitted air-bubbles.

It is true that a constant inverse relation was not observed between the numbers of the eggs in the cell and the duration of the yelk-contractions, but this was explained

by the varying amounts of air in bubbles which entered the cells, and by the accidental bursting of the inner sacs in many eggs, a mode of terminating the contractions not due to the exhaustion of oxygen, and which is most apt to occur when the proportion of water is greatest, provided it be not changed. Nevertheless, in experiment *h*, with thirty eggs in the cell, and imperfect exclusion of air, the contraction ceased in twenty-three hours in eggs distant from the air-bubbles, while in experiment *j*, with only seven eggs in the cell, and complete exclusion of air, they continued for twenty-nine hours.

The experiments with impregnated eggs gave more definite results still. For in experiment *e'*, with forty-eight eggs in the cell, the yelk-contractions ceased in twenty-six hours, leaving the yelk-ball globular and relaxed, while in experiment *g'*, with only ten eggs in the cell, they persisted for more than thirty-five hours.

The pseudo-cleavage, or contractions of the concentrated formative yelk in unimpregnated eggs, and probably also the concentration of the formative yelk, seem to demand the presence of oxygen as well as a due supply of water: for in experiment *e*, after twelve hours it was less advanced than in the ova of control experiment 3. It also was shown to cease long before the yelk-contractions, and may be supposed to consume more oxygen, although there are other explanations which may be offered of this fact, especially the tendency of the matter of the discus proligerus to undergo chemical change and disintegration.

Cleavage may be said to demand more oxygen than do the yelk-contractions, as in experiments *d'* to *j'* it always ceased long before, and was more promptly checked by increasing the number of eggs in the cell. It is also more quickly arrested than pseudo-cleavage, and would seem therefore to need oxygen more.

Indirectly, bursting of the inner sac and consequent cessation of the yelk-contractions depends upon access of oxygen, which acts by favouring decomposition when the water is not changed.

Although some of the changes seen in the eggs in the suffocation experiments may be attributed to the poisonous action of some product, and not alone to the absence of oxygen, yet the general inference, that oxygen is consumed, is not thereby weakened; and it is a significant and interesting fact, that the cleavage masses in suffocated eggs undergo a species of fusion, which much resembles one of the effects of the action of carbonic acid on them.

It remains, however, a weak point, that I failed to obtain chemical evidence as to the product of oxidation, which might be expected to be in very small amount, as the consumption of oxygen was so minute.

Despite the difficulties of deciding what interchanges take place between the substance of the yelk and the surrounding medium during the functional activity of the former, the fact came out with sufficient clearness, that some non-albuminous organic matter, and some salts, passed into the water.

Experiments *n* to *t* inclusive, show that cell-multiplication and differentiation, in the

developing embryo, more urgently demand oxygen than do the earlier stages of cleavage, and *à fortiori*, than the yelk-contractions.

Thus the progress of development and of growth of the embryo ceased as early as seven hours after the cell was closed; and so rapidly had the oxygen been consumed, that even the yelk-contraction ceased in eighteen hours, although the proportion of ova to water was small: while the earlier cleavage, under less favourable conditions, had continued for twenty hours at least, and had not so exhausted the oxygen as to stop the yelk-contractions for thirty hours or more.

The higher the stage of development at which the embryos used for experiment had arrived, the sooner did they so exhaust the oxygen as to arrest the yelk-contractions.

In young embryos in which no striated muscle was seen, the trunk movements persisted as long as the yelk-contractions, and the heart, which is but a mass of protoplasmic balls, did the same, but the striated muscles in the trunk of more advanced embryos lost their contractility more rapidly when oxygen was withheld. The heart, even in hatched embryos, continued to contract longer than any other structure, perhaps, because it continually helped to renew the medium around it.

### *Concluding Remarks.*

The observations detailed in this communication seem to me to be confirmatory of the view that the egg of osseous fishes is a cell, and to be looked on as a structural unit, the prototype of those units which, variously aggregated and modified, form the mass of the higher organisms. The minute, simply constructed, early ovarian ovum would, I doubt not, be accepted as such by most observers; and the larger ripe egg can scarcely be held to differ essentially if its mode of growth and development be considered.

The complex structure of the yelk-sac cannot be urged with much force against this opinion, as analogous structure has been met with in other parts, which are admittedly cell-walls or their descendants; for example, the striations in the matrix of some cartilages, and in the surface-layer of intestinal epithelium, and the so-called pore-canals in the cuticular tissues of many lower animals. It is certainly somewhat difficult to conceive what is the true position of the egg if not that of a cell.

Assuming that the opinion now advocated is sound, the question as to the mode of growth of cell-walls receives some additional light from the evidence here brought forward, to prove that the yelk-sac grows interstitially, and not by accretion upon either surface, or by gradual transformation of the surface of the yelk-ball.

To the yelk-ball, however, as the essential cell, the greatest share of interest attaches, and it, like the cell contents, alone is capable of undergoing multiplication. A convenient definition of a cell-wall might therefore be,—the first separable covering of the protoplasmic mass, which does not take part in multiplication by fission.

The cell-wall must be considered as a living substance, at least so long as it continues to grow interstitially, although it is probable that a time occurs in the life-history of most cells, which possess such walls, when they cease to grow, when they render

only physical services to the organism, and then they are generally marked by singular stability.

The inner sac is to be looked on as the homologue of the primordial utricle, and its thicker portion with the granules of the discus proligerus would then correspond to the granular mass around the nucleus in the plant-cell.

The food-yelk is held to be the equivalent of the fluid cell-contents, and the germinal vesicle and spots hold the position of the nucleus and nucleoli.

Contractility, which there is some reason to think is a property common to all cell-contents or protoplasm, in the egg, as in Tradescantia, appears to have its seat in the surface-layer. It may be spoken of as of two kinds, Rhythmic and Fissile.

The former is met with, at least in the egg, as in very many other cells, in a portion only of its life-history, and varies very much as to the vividness of its manifestations in different organisms. Its essential conditions do not appear to differ from those which govern all other known vital actions, and its normal excitors are the same as those of higher motor structures, but it seems to be less liable to be influenced by most poisons than are the vital properties of higher tissues. It is not influenced in any manner by the spermatozooids. From the contractile matter of striated muscle it differs in one important particular, viz. that while the former is permeated by an alkaline fluid it is bathed with an acid. No explanation of its rhythmic character has yet been found, and its uses in the economy are also unknown. In the ova of osseous fishes, and in those of Batrachia, its existence has been ascertained, but usually its manifestations are slow and indistinct. That the rhythmic contractions have no essential relation to growth in the ovum of osseous fishes, is shown by the fact that they do not begin until the egg has reached its full size. I venture to suggest that they may be connected in some way with the conversion of a lower form of organic matter into a higher, such as occurs when food-yelk is transformed into formative yelk. It seems probable that the exceptional vividness of the contractions noted in some fishes, as the pike and the stickleback, is connected with the rapidity of the changes which take place in the egg. These eggs hatch in a shorter time than do those in which the contractions are slow and indistinct. Were it not for their orderly recurrence, one might be tempted to refer them to the same category as those motions which occur during the admixture of certain fluids, as of spirit and water.

The fissile contractility is also independent of the action of a male element, although so far influenced by fecundation as to owe persistence and regular progress to it. Its essential and modifying conditions are otherwise like those of the rhythmic contractility, but its normal excitors, if we except heat, are but little known. It requires for its maintenance that a portion of the lower form of protoplasm united with oil shall be continually converted into the higher. Its results are growth and development.

To show the extent and importance of the question as to the nature and properties of protoplasm, I will draw a brief parallel. The first germ of an animal, as the egg; the first stages of organic matter about to be formed into tissue in the body, as the white

blood-corpuscle; the lowest known existing organisms in the animal or vegetable kingdom, as the *Amœbæ*; the earliest ascertained traces of organic beings in the geological record, as the *Eozoon Canadense*,—are all essentially masses of protoplasm, and some of them have been shown to possess some important properties in common, as the researches of KÜHNE* and M. SCHULZE† have shown.

This cursory glance beyond the limited area which I have been hitherto examining, gives some support to a view, on other grounds probable, that the rhythmic contractions of the lower forms of protoplasm precede and lead up to the fissile movements which result in cell multiplication in the higher forms of protoplasm. Witness the amœboid stages of some monads before they encyst and multiply by fission, as described by CIENKOWSKI‡.

One is thus easily led to form the general conception that matter, in passing from the inorganic to the organic world, first takes on a homogeneous thick fluid form, the denser surface of which is endowed with a rhythmic contractility; that it then is gradually converted into a higher form, which is granular, and contains fat, which loses its power of rhythmic contraction, and acquires that of dividing into separate masses by fission.

## EXPLANATION OF THE PLATES.

### PLATE XV.

#### DIAGRAM A.

*a*. Dotted yelk-sac.
*b*. Buttons.
*c*. Micropyle.
*d*. Inner sac indicated by the dark line (it should be in contact with the yelk-sac).
*e*, *e*. Cortical layer, or matrix of the formative yelk, continuous with the inner surface of inner sac indicated by the fine shading.
*f*, *f*. Yellow droplets and granules of the formative yelk imbedded in the cortical layer, and forming at its thicker portion the discus proligerus.
*g*. Germinal vesicle with contents (introduced to show its position when last seen).
*h*. Group of large store oil-drops.
*j*. Food-yelk.
*k*. Smaller oil-granules of the formative yelk.

Figures 1 to 23 inclusive refer to eggs of *Gasterosteus*.

Fig. 1. The unimpregnated egg, indented by pressure, viewed with a low power:—*a*, the micropyle (proportionally too large in the figure); *b*, the buttons; *c*, oil-drops; *d*, the discus proligerus.

* *Loc. cit.* † Das Protoplasma, 1863.

‡ "Beiträge zur Kentniss der Monaden," Archiv für Mikroskopische Anatomie, 1865.

Fig. 2. The buttons, when not deformed by mechanical violence. × 200.

Fig. 3. Section of yelk-sac near the apex of the micropyle:—*a*, cut edge; *b*, folded edge; *c*, apex of the micropyle.

Fig. 4. The micropyle, front view, the apex in focus. ×200 (the fine dotting is too coarse).

Fig. 5. An unimpregnated egg under pressure, the micropyle (*b*) in profile projecting into the discus proligerus (*a*).

Fig. 6. An egg, five minutes after impregnation, showing the funnel of the micropyle (*b*), and the pit in the discus proligerus (*a*).

Fig. 7. Escaped and partially emptied inner sac:—*a*, food-yelk; *b*, formative yelk changed by the action of water.

Fig. 8. An egg crushed forty-five minutes after impregnation:—*a*, contents escaping at the rupture in the yelk-sac; *b*, collapsing inner sac thrown into folds; *c*, germinal disk.

Fig. 9. An escaped germinal vesicle in the fluids of the egg:—*a*, the germinal spots displaced; *b*, the colloid matrix changed and faintly granular; *c*, escaped yellow droplets; *d*, free oil-globules. ×200.

Fig. 10. The same vesicle acted on by water:—*a*, the germinal spots vacuolating, with dark hard outlines and irregular forms; *b*, the colloid content or matrix of the spots, more darkly granular and vacuolating. ×200.

Fig. 11. A portion of the same vesicle more highly magnified:—*a*, a germinal spot vacuolating. ×400.

Fig. 12. Youngest ova seen; no primitive yelk around the germinal vesicles.

Fig. 13. *a*, an ovum of the smallest size seen furnished with primitive yelk; *b*, an ovum of somewhat larger size changed by imbibition of water, so that the germinal spots have vanished; *c*, escaped germinal vesicle without contents; *d*, free granular matter from larger eggs; *e*, stroma of ovary.

Fig. 14. An ovum of group 2, showing granularity of the surface of the yelk, oil-drops distributed, and germinal spots vanished from prolonged action of water through the substance of the egg.

Fig. 15. An ovum of group 3, escaped from its yelk-sac, exhibiting the subangular form indicative of its solidity:—*a*, clearer zone around the germinal vesicle; *b*, granular aspect of the superficial part of the primitive yelk after the action of water.

Fig. 16. Germinal vesicle with its colloid matrix sustaining the germinal spots in their natural positions. Seen in the fluids of the ovary. (The drawing is faulty in showing the spots as if those nearest the observer were central.)

Fig. 17. A germinal vesicle which has imbibed water unequally, the colloid matrix retaining the germinal spots in their natural peripheral position.

Fig. 18. A free germinal vesicle uninjured mechanically, but acted on by water for seven hours, the position of the spots and the vesicular wall not changed.

The colloid matrix not granular, the germinal spots variously tailed, dark-bordered and vacuolating.

Fig. 19. Germinal spots, more highly magnified, seen in a $1\frac{1}{4}$ per cent. solution of chloride of sodium, variously tailed and vacuolating.

Fig. 20 (Plate XVI.). Germinal spots in various stages of fusion and solution, in a 5 per cent. solution of chloride of sodium.

Fig. 21 (Plate XVI.). A macerated yelk-sac:—*a*, the changed homogeneous-looking extensile outer layer; *b*, the dotted yelk-sac not changed. ×250 (drawn with a camera).

Fig. 22. Appearance presented by the extensile outer layer under a magnifying power of ×3000 diameters; the dots dark (diagrammatic).

Fig. 23. Aspect of the same when the focus is so arranged that the dots are light. ×3000 (diagrammatic).

## PLATE XVI.

Fig. 24. Egg of the Trout, showing the cup of the micropyle under a $\frac{1}{2}''$ lens.

Fig. 25. The micropyle of the egg of the Pike seen in $\frac{3}{4}$-face with reflected and transmitted light:—*a*, the trumpet-shaped tube. ×190.

The figures 26 to 33 inclusive refer to the ova of Perch.

Fig. 26. A diagram explanatory of the arrangement of the outer ends of the "tubes" in the outer layer of the yelk-sac of the ovum of the Perch:—*a*, the funnel-shaped mouths; *b*, the "tubes;" *c*, radiating curved furrows and folds of the surface.

Fig. 27. Shows the arrangement of the "tubes" in a vertical section where two eggs meet. In the figure the "tubes" are stretched and broken, and their spiral twist destroyed. ×190.

Fig. 28. Small fragments of the "tubes" more highly magnified to show the double contour of the walls. ×500.

Fig. 29. The inner branched ends of the "tubes:"—*a*, the outer laminated surface of the dotted yelk-sac; *b*, a detached "tube."

Fig. 30. The inner branched ends of the "tubes" seen from the inner surface of the yelk-sac through its substance, showing their relation to the finer dottings.

Fig. 31. The outer ends of the tubes seen full face, showing their regular arrangement in the clear matrix, and the aspect of the funnel-shaped ends. ×190.

Fig. 32. The dotted yelk-sac, showing its laminated structure at a fissure. ×500. (The drawing exaggerates somewhat.)

Fig. 33. The dotted yelk-sac, showing a cut edge. ×500 (also somewhat exaggerated).

The figures 34 to 83 inclusive refer to the eggs of *Gasterosteus*.

Fig. 34. Two ova of *Gasterosteus pungitius*, pressed together so as to seem to close the micropyle of one of them. ×100.

Fig. 35. An impregnated ovum of *G. leiurus*, just before the commencement of vivid contractions, showing the flat surface.

Fig. 35′. The same egg rolled a little.

Fig. 36. The first stage of active contraction producing the reniform figure.

Fig. 37. The dumb-bell figure.

Fig. 38. The flask-shaped figure.

Fig. 39. The wave having passed forwards, has left the food-yelk globular, but still influences the germinal disk, which is nipple-shaped.

Fig. 40. The whole yelk-ball globular and at rest. The germinal disk diffused somewhat.

Figs. 41 & 42. Abnormal forms of the yelk, caused by two or more waves present at the same time.

## PLATE XVII.

Figs. 43 & 44. Slower changes of form of the germinal disk and neighbouring part of the food-yelk.

Figs. 45 to 49 inclusive. Further successive slow changes in the form of the germinal disk and of the germinal surface of the food-yelk, shortly before the commencement of cleavage.

Figs. 50 & 51. Show the commencement and progress of the first cleft, and the separation of the masses.

Figs. 52 & 53. Also show the gradual deepening of the first cleft.

Fig. 54. The cleavage masses closely pressed together as seen before the second cleft begins.

Fig. 55. Shows the "Faltenkranz" of REICHERT, or folds of the inner sac during the progress of the first cleft.

Fig. 56. Shows similar folds in one cleft of an egg which is cleaving irregularly.

Fig. 57. An egg twenty hours after impregnation. The germinal mass is composed of cell-like corpuscles; *b*, a portion of the germinal mass more highly magnified.

Fig. 58. An egg cleaving asymmetrically.

Fig. 59. The yelk-ball before galvanism was used (given as a standard of comparison):—*a*, the micropyle; *b*, the germinal disk.

Fig. 60. Contraction, the effect of one application of the galvanic current.
P. The Platinode. Z. The Zincode.

Fig. 61. Another contraction, the effect of a second application of the current, with rupture of the inner sac and escape of the food-yelk.

Fig. 62. Further contraction, with rupture of the inner sac on the opposite side, from a third application of the current. The escaping yelk granular.

## PLATE XVIII.

Fig. 63. The inner sac contracted by repeated applications of the current. Electrolytic changes visible.

Fig. 64. Final result after repeated application of the galvanic current.

Figs. 65 & 66. Contractions, the effect of zero-galvanic currents, too weak to cause rupture of the inner sac.

Figs. 67 & 68. Contractions excited by weak galvanic currents in eggs which had been partially frozen.

Fig. 70. Electrolytic effect of the galvanic current upon the cleavage masses.

Fig. 71. Irregular cleavage in an egg of which the inner sac had been ruptured by partial freezing.

Fig. 72. Second stage of the action of a weak solution of carbonic acid upon an egg before cleavage. The yelk-ball globular; the germinal disk very prominent and conical (drawn seventy minutes after impregnation, and thirty-five minutes after the action of the carbonic acid).

Fig. 73. First stage, four minutes after the action of carbonic acid upon an egg, after the first cleft is complete. The yelk-ball globular, the germinal disk retracted.

Fig. 74. Last stage of the action of carbonic acid on an egg in the same grade of development, not ruptured or chemically changed. The food-yelk globular and still, the cleavage masses fused and diffused. (The breathing-chamber is too large.)

Fig. 75. Final stage of the same eggs when the inner sac had ruptured and chemical change taken place.

Fig. 76. Normal aspect of the germinal disk cleft in four masses (as standard of comparison).

Figs. 77 to 79 inclusive. Stages of the recession of the germinal disk into the yelk, which took place during the first few minutes after the action of the carbonic acid.

Figs. 80, 81 & 82. Stages of the gradual reprotrusion of the germinal disk, with commencing fusion of the cleavage masses.

Fig. 83. State of complete fusion of the cleavage masses, three hours after the action of the carbonic acid. (The yelk-ball should more nearly fill the yelk-sac.)

### Postscript.

Partly from an unwillingness to enter upon any discussion as to priority, and partly not to add to the length of this paper, I avoided historical references entirely. But as my silence might seem to indicate an acquiescence in claims which are unjust to other observers, I have since thought it better to append a short history of the observations relating to the micropyle in Fishes.

K. E. Von Baer (Untersuchungen über die Entwickelungsgeschichte der Fische, 1835) saw in the egg of *Cyprinus Blicca* (white Bream) a clear circle in the centre of the germ (discus proligerus), which when viewed full face had a dark halo around it and resembled an area pellucida, but when seen in profile was recognized as a funnel-shaped depression in the outer membrane. He says further, "I can only conceive the formation of this funnel taking place by the passage outwards of the germinal vesicle, through the centre of the germ, as I have seen occur in the egg of the frog."

It may be said therefore that Baer observed the funnel of the micropyle, and correctly described its position relative to the parts of the egg, but did not see the aperture through the yelk-sac or properly interpret the structure.

M. Doyère communicated a paper to the Soc. Philomathique de Paris, Dec. 15th, 1849 (see l'Institut, vol. xviii. p. 12, 1850), in which he described the micropyle in *Syngnathus Ophidium*, clearly indicated its relation to the discus proligerus, measured the aperture, and without doubt appreciated its significance in the act of impregnation, without, however, affording any experimental evidence of its uses.

My paper, read November 23, 1854, and published in the Proceedings of the Royal Society, was next in order of time.

C. Bruch (Zeitschrift für Wissensch. Zool. B. vii. 1855) announced his discovery of the micropyle in the eggs of trout and salmon in a letter to Professor Siebold, December 28, 1854. He did not make out its relation to the proligerous disk, and failed in his attempts to prove that it served for the entrance of spermatozooids. He, however, rightly interpreted the structure, and claimed to have first established on a firm basis the existence of a micropyle in vertebrata.

My observations on the eggs of trout and salmon were made in the same season, December 1854 and January 1855, and the results were communicated in the latter month to Dr. Allan Thompson (Cyclopædia of Anatomy and Physiology, vol. v. p. 104).

Reichert (Müller's Archiv für Anat. Physiologie, &c. 1856, p. 83) described the micropyle in the egg of the pike and of several other fishes, failing, however, to find it in perch. His first observation was made immediately before Professor Bruch's paper came to his hand. He did not describe the relation of the micropyle to the other parts of the egg, or give any proofs that it served to give entrance to the spermatozooids.

The credit of priority rests mainly with Doyère. His observation was unnoticed for some time, and my experiments were made and published without any knowledge of his discovery or of the earlier one by Von Baer.

Bruch worked on this subject also without a knowledge of what had been done before, and Reichert with only a partial knowledge.

*February* 23, 1867.

## XV. *On the Lunar-diurnal Variation of the Magnetic Declination, with special regard to the Moon's Declination.* By G. Neumayer. *Communicated by the President.*

Received March 11,—Read March 28, 1867.

The hourly records of the magnetic declination, systematically kept at the Flagstaff Observatory at Melbourne, Victoria, during the period from the 1st of May 1858 to the 28th of February 1863, have been discussed with a view to determine the lunar-diurnal variation to which this magnetic element is subjected. The results arrived at in the course of this discussion, eliciting, as I believe, facts hitherto unnoticed, induced me to think it important to have them brought before the Royal Society, with no other object in view but to direct the attention of scientific men to a subject of such vast import for the development of the science of terrestrial magnetism.

The process adopted in reducing the observations, in order to eliminate the solar-diurnal variation of the magnetic declination, is identical with the one generally adopted in such cases. The limit of disturbance was taken to be 3·61 minutes of arc, and all hourly directions which differed from their final normals by this value, or more, were consequently omitted from the record. This elimination of the larger disturbances having been effected, from every remaining reading (R) of the magnet's direction the respective final normal (N) was subtracted, thus causing the residuals (R—N) to be devoid of the influence of the solar-diurnal variation. When the remainders are negative, *i. e.* when the normal exceeds the reading, the north end of the needle is to the west of its mean direction, and when positive the needle deviates with its north end towards the east of its normal mean. The magnetic declination being east at Melbourne, we perceive that the negative values denote a decrease, and the positive ones an increase, with respect to the normal value of this magnetic element. The total number of observations at command amounts to 38,194, of which 4,178 single readings were excluded from the discussion, on account of their being beyond the above-mentioned limit, and there remained only 34,016 readings for the purpose of determining the lunar-diurnal variation. Of this number 15,735 were observed in the months from April to September, and 18,281 during the time from October to March.

The treatment of the residuals, with a view to classify them according to lunar hours, presents no particularly new features; so much may, however, be mentioned that, prior to entering upon any general discussion, every month's result was calculated separately and expressed in minutes. Subsequently the values for the various months were arranged, irrespective of the year, in two groups, viz., the sun's declination being South (October to March), and the sun's declination being North (April to September). Thus

we received a mean lunar-diurnal variation curve for each half of the year, from which again was derived the annual curve. A more rigid examination of the results obtained in this way showed forthwith that irregularities in the lunar-diurnal variation manifested themselves, for which it was hardly possible to account otherwise, than that they were depending to some degree on the moon's position with reference to the equator, whether her declination was South or North.

After having been once convinced of the truth of this, there could not exist a doubt as to the mode in which the observations had to be treated. The whole series was arranged in groups, "Declination of the Moon South" and "Declination of the Moon North," rejecting all days on which the moon was close to the equator, so as to cause her declination to be divided between the hours of the day. The 118 groups of lunar-diurnal variation thus formed were subsequently classified according to whether the sun's declination was South or North, which was easily accomplished, as of each group only the sums expressed in minutes of arc had been taken. The mean values for each period were derived by allowing due weight to the number of readings from which it had been derived, a practice which was made a rule in the course of these discussions. The only exception to this in the subsequent Tables occurs in those values of the lunar-diurnal variation for the winter and summer half-year in the single years, they being derived from the monthly tables by simply taking the means. This must be borne in mind when comparing the various values, as in some instances there are considerable discrepancies caused by this difference of treatment, which are, however, quite irrelevant for the purpose required. It ought to be mentioned that the mean values were subsequently reduced to seconds of arc.

Originally, as I have already described, I obtained yearly and half-yearly curves of the lunar-diurnal variation from the monthly tables; and it is evident that the results, obtained by the method just described, afforded a means of checking those of the inquiry primarily instituted, so far as both could reasonably be expected to tally with one another, the principles of classification being somewhat different in the two cases. Taking this into consideration, the agreement between the results was such as to cause us to receive the subsequent mean values with great confidence.

The values of the first Table are to be considered as derived irrespectively of the years during which the observations were made, and to represent mean values for the period from May 1858 to February 1863. We shall presently see that this point is of some moment, the single years differing widely in their results with regard to the lunar-diurnal variation of the magnetic declination.

The values of the following Table have been thrown into curves which may be readily studied by the aid of Plate XIX.; it was considered advisable to retain the plan adopted in the above Table, and exhibit the nine curves in three different groups, namely:—

TABLE I.—Lunar-diurnal Variation of the Magnetic Declination (1858–1863).

1st. From the superior to the inferior passage.

| Declination of the | | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
| S. and N. | S. and N. | + 6″38 | + 7″84 | + 5″13 | − 0″89 | − 5″86 | − 9″23 | −11″74 | −12″10 | − 9″04 | − 3″06 | + 6″16 | +10″31 |
| S. | ,, | +13·18 | +16·33 | +10·20 | + 1·15 | −10·04 | −15·96 | −20·64 | −15·55 | −10·32 | − 0·90 | +10·61 | +12·67 |
| N. | ,, | − 1·79 | − 1·98 | − 0·82 | − 3·30 | − 1·00 | − 1·43 | − 1·61 | − 8·12 | − 7·60 | − 3·39 | + 1·05 | + 7·54 |
| S. and N. | S. | + 9·08 | + 9·35 | + 7·48 | − 0·29 | − 8·23 | − 8·62 | −12·87 | − 7·35 | − 6·31 | + 4·36 | + 9·09 | + 9·74 |
| S. | ,, | +17·73 | +18·01 | +13·64 | + 1·91 | −14·39 | −18·08 | −24·18 | −13·14 | − 7·97 | + 8·85 | +15·94 | +12·27 |
| N. | ,, | − 1·68 | − 0·93 | + 0·02 | − 3·08 | − 0·77 | + 3·05 | + 0·67 | − 0·33 | − 4·29 | − 1·00 | + 1·02 | + 6·71 |
| S. and N. | N. | + 3·72 | + 6·33 | + 2·78 | − 1·50 | − 3·44 | − 9·83 | −10·62 | −16·76 | −11·83 | − 8·46 | + 3·24 | +10·88 |
| S. | ,, | + 8·56 | +14·62 | + 6·67 | + 0·33 | − 5·43 | −13·64 | −16·96 | −18·02 | −12·87 | −10·96 | + 5·17 | +13·07 |
| N. | ,, | − 1·90 | − 3·01 | − 1·64 | − 3·50 | − 1·22 | − 5·72 | − 3·76 | −15·36 | −10·74 | − 5·68 | + 1·09 | + 8·36 |

2nd. From the inferior to the superior passage.

| Declination of the | | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. |
| S. and N. | S. and N. | +11″38 | +10″38 | + 7″01 | + 2″43 | − 4″33 | − 7″42 | − 6″69 | − 7″83 | − 8″03 | − 5″15 | + 3″13 | + 6″25 |
| S. | ,, | +17·45 | +16·55 | +10·25 | + 2·29 | − 4·06 | −12·82 | −12·86 | −17·18 | −12·25 | − 4·23 | + 5·12 | +10·71 |
| N. | ,, | + 4·33 | + 3·24 | + 3·30 | + 2·60 | − 4·64 | − 1·26 | + 0·37 | + 3·07 | − 3·01 | − 6·27 | + 0·80 | + 1·04 |
| S. and N. | S. | + 8·67 | + 4·33 | + 4·84 | + 1·92 | − 3·28 | − 4·34 | − 7·01 | − 9·46 | − 7·53 | − 5·80 | + 1·73 | + 7·65 |
| S. | ,, | +15·19 | + 9·42 | + 5·83 | + 1·62 | + 0·97 | − 5·58 | −13·03 | −21·11 | −13·29 | − 5·25 | + 4·28 | +13·14 |
| N. | ,, | + 1·20 | − 1·68 | + 3·69 | + 2·27 | − 8·20 | − 3·33 | 0·00 | + 6·24 | − 0·56 | − 6·48 | − 1·39 | + 0·96 |
| S. and N. | N. | +14·10 | +16·64 | + 9·20 | + 2·96 | − 5·42 | −10·36 | − 6·35 | − 7·19 | − 8·55 | − 4·51 | + 4·51 | + 4·84 |
| S. | ,, | +19·73 | +24·06 | +14·81 | + 2·99 | − 9·28 | −20·35 | −12·68 | −13·27 | −11·18 | − 3·20 | + 5·98 | + 8·17 |
| N. | ,, | + 7·52 | + 8·22 | + 2·98 | + 2·92 | − 0·96 | + 0·83 | + 0·75 | − 0·09 | − 5·45 | − 6·06 | + 2·86 | + 1·12 |

Group I. :—Declination of the moon *south* and *north*, *i. e.* no distinction being made with reference to the moon's declination.

1. No distinction with reference to the sun's declination (this may be called the mean yearly curve).
2. The sun's declination south (summer half-yearly curve).
3. The sun's declination north (winter half-yearly curve).

Group II. :—Declination of the moon *south*.

1. No distinction with reference to the sun's declination (yearly south curve).
2. The sun's declination south (the summer half-yearly south curve).
3. The sun's declination north (winter half-yearly south curve).

Group III. :—Declination of the moon *north*.

1. No distinction with reference to the sun's declination (yearly north curve).
2. The sun's declination south (summer half-yearly north curve).
3. The sun's declination north (winter half-yearly north curve).

A glance at the curves of Plate I. shows that the lunar-diurnal variation must be regarded as being influenced by both the sun and the moon, for we perceive that, in case the declinations of both heavenly bodies are of the same name, both north or both south, the curves show greater regularity than they exhibit otherwise. For instance: the summer half-yearly south curve is far less irregular than the summer half-yearly

north curve; the winter half-yearly south curve scarcely deserves the name of such, while the winter half-yearly north curve exhibits the oscillations in a far higher degree, although they are still somewhat irregular.

The doubt so often raised, whether during the winter season any lunar-diurnal variation was traceable at all, can, I think, not be entertained any longer, if we pay due attention to the facts which may be gleaned from our Table. Indeed we shall presently see, when speaking of the lunar-diurnal variation in the various years of observation, that in some cases the variation manifests itself in a very distinct manner even during those months when the sun's declination is north.

The principal features of the curves just enumerated may be delineated as follows:—

Ad I. 1. The maximum value of the easterly deviation of the needle takes place at the time of the moon's lower transit, and the minimum between $6^h$ and $7^h$. A secondary maximum occurs at $1^h$, and a corresponding minimum between $19^h$ and $20^h$. The range of oscillation amounts to $24''{\cdot}0$.

Ad I. 2. In this case the maximum seems to occur a little after the moon's lower transit, and the minimum at $6^h$, while a secondary maximum takes place at $1^h$ and a secondary minimum at $19^h$, the greatest range amounting to $38''{\cdot}69$.

Ad I. 3. I consider it most likely that in this instance two influences, counteracting each other in some measure, make themselves manifest, and it is therefore rather difficult to determine the extreme deviations so well as in the former cases; but it appears that $11^h$ and the hour between $7^h$ and $8^h$ are marked as extremes, in the first instance the easterly deviation reaching its greatest value, and in the latter the same occurring with respect to the westerly deviation of the needle. The greatest range amounts to $15''{\cdot}66$.

The nodes, where the three curves intersect each other, are at the hours $15^h$–$16^h$, $20^h\ 45^m$, $3^h\ 20^m$, and $8^h\ 40^m$.

Ad II. 1. The maximum of the east magnetic declination occurs at $11^h$, the minimum at $6^h$, the greatest range amounting in this case to $22''{\cdot}61$. A secondary maximum takes place at $1^h$, and a secondary minimum at $19^h$.

Ad II. 2. Primary maximum about $30^m$ past the superior passage of the moon, and the corresponding minimum at $6^h$, with a range of $42''{\cdot}17$. Secondary maximum at $10^h$, and minimum at $19^h$.

Ad II. 3. There is in this case scarcely any regularity in the oscillations of the needle observable, the extreme values differing by $14''{\cdot}91$.

On account of the irregularity of the winter curve the nodes for this group are not well defined; they seem, however, to be between $14^h\ 30^m$ and $16^h\ 50^m$, $20^h\ 50^m$, $3^h\ 15^m$, and $8^h\ 30^m$.

Ad III. 1. Maximum at $13^h$ and minimum at $7^h$, giving a range of $33''{\cdot}40$; secondary extremes are noticeable at $1^h$ and $17^h$.

Ad III. 2. Maximum at $13^h$, minimum at $17^h$, greatest range $44''{\cdot}41$; secondary maximum and minimum respectively at $1^h$ and $7^h$. It is yet to be decided whether the deviation from the rule, that the minimum between the inferior and the superior pas-

sage exceeds the one between the superior and the inferior, is rather due to an accidental irregularity than to an actual change; but the latter is undoubtedly the case with reference to the maximum, comparing it with the summer semiannual curve obtained irrespectively of the moon's declination.

Ad III. 3. This curve shows but one distinctly expressed maximum and minimum, viz. at $11^h$ (or perhaps near the inferior transit) and at $7^h$, the range of oscillation amounting to $23''{\cdot}72$.

The intersecting points are in this group of curves more distinctly determined than in either of the two preceding ones; they are at $15^h$, $20^h$ $40^m$, $3^h$ $30^m$, and $9^h$ $50^m$.

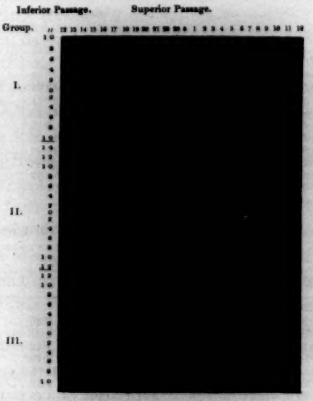
Prior to entering on the examination of the lunar-diurnal variation in various years, I may be permitted to add a few words on the semiannual inequality for the several groups of curves, taking in every one of the annexed diagrams the annual curve for the axis of projection.

Diagram I. in the annexed woodcut illustrates in the usual manner the semiannual inequality of the lunar-diurnal variation, no distinction being made respecting the name of the moon's declination.

Diagram II. shows the semiannual inequality of the lunar-diurnal variation only for declinations of the moon south of the equator.

Diagram III. represents the semiannual inequality of the lunar-diurnal variation for declinations of the moon north of the equator.

Semiannual Inequality of the Lunar-diurnal Variation.




In every one of these cases the summer branch (October to March) bears the character of the lunar-diurnal variation, even as far as the turning hours are concerned. We

notice also a marked difference between the curve when the moon is south of the equator (II.) and the one when her declination is north (III.). A glance at the diagrams will convince us of this; we need only to point out that the range of oscillation in the first instance, taking both branches into consideration, is 27″·35, in the last but 21″·18, and that the extremes occur in the latter case two hours earlier than in the former. Besides these differences there are others of minor import which also contribute towards drawing a distinction between the curves for north and south declinations of the moon; for instance, the secondary extremes differ in the south curve by 24″·85, whilst the corresponding deviation in the north curve is but 13″·20.

On examining the results of this inquiry for the several years of observation somewhat closer, we are struck by the differences which they exhibit among themselves. It was the year 1861 which first called my attention to this fact by manifesting considerable abnormities with respect to every one of the curves of the lunar-diurnal variation for the various seasons and positions of the moon. These irregularities are the more striking, as the year 1860 does not exhibit any such extraordinary deviations from the mean values for the several years, although also in this case the curve for the winter half-year shows some peculiarly interesting features (Plate XIX.).

The subjoined Table gives the lunar-diurnal variation for the years 1860 and 1861, and Plate XIX. shows the respective curves.

TABLE II.—Lunar-diurnal Variation of the Magnetic Declination for the years 1860 and 1861.

1st. From the superior to the inferior passage.

| Declination of the | | Years. | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
| | | | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| S. and N. | S. and N. | 1860 | + 7·28 | + 6·80 | + 6·03 | + 2·77 | − 3·90 | − 9·65 | − 8·09 | −14·91 | −14·45 | − 6·73 | + 2·86 | +12·92 |
| | | 1861 | − 5·87 | − 8·76 | − 6·82 | −13·38 | − 9·69 | − 8·96 | −12·88 | −10·92 | − 6·10 | − 1·09 | + 6·84 | +17·21 |
| ,, | S. | 1860 | +11·54 | + 8·60 | +10·37 | + 3·50 | − 5·39 | −11·70 | −27·16 | −21·65 | −24·92 | − 7·52 | + 2·04 | + 6·50 |
| | | 1861 | + 4·18 | − 2·47 | − 1·96 | − 5·34 | −10·59 | − 6·69 | −11·71 | + 5·10 | + 4·33 | +14·86 | +17·14 | +23·76 |
| ,, | N. | 1860 | + 3·56 | + 5·21 | + 2·14 | + 2·08 | − 2·60 | − 7·79 | − 9·73 | − 8·47 | − 4·46 | − 5·98 | + 3·67 | +18·92 |
| | | 1861 | −15·61 | −14·78 | −11·57 | −21·13 | − 8·72 | −11·37 | −14·03 | −25·98 | −15·98 | −15·91 | − 2·66 | +10·79 |
| S. | S. and N. | 1860 | +13·97 | +13·91 | + 9·88 | − 0·24 | −11·62 | −20·48 | −19·91 | −23·07 | −16·05 | − 4·06 | + 9·47 | +17·84 |
| | | 1861 | + 3·64 | + 1·74 | + 5·61 | − 1·40 | −12·78 | −18·36 | −22·62 | −13·65 | − 3·32 | − 1·59 | + 8·12 | +12·78 |
| N. | ,, | 1860 | + 2·08 | − 0·65 | + 2·76 | + 4·76 | + 1·37 | + 0·97 | − 3·97 | − 6·56 | −11·30 | −10·56 | − 2·68 | + 5·63 |
| | | 1861 | −17·98 | +19·65 | −19·96 | −25·84 | − 7·32 | + 2·30 | − 1·07 | − 5·67 | − 5·95 | + 0·12 | + 4·50 | +17·76 |

2nd. From the inferior to the superior passage.

| Declination of the | | Years. | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | | 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. |
| | | | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| S. and N. | S. and N. | 1860 | + 4·75 | + 1·66 | + 2·38 | − 0·85 | − 1·42 | − 8·02 | + 6·20 | − 5·07 | − 2·48 | + 4·82 | +13·51 | +14·30 |
| | | 1861 | +25·07 | +21·33 | +19·80 | +15·81 | + 7·60 | + 3·85 | − 5·35 | − 4·32 | −10·47 | −10·01 | − 3·01 | − 2·68 |
| ,, | S. | 1860 | − 1·04 | − 9·18 | − 6·99 | − 6·31 | − 9·43 | −14·45 | + 1·42 | −12·17 | − 6·04 | − 0·31 | +12·47 | +19·40 |
| | | 1861 | +27·28 | +21·70 | +17·03 | +11·84 | + 7·91 | + 8·37 | − 3·19 | − 2·25 | − 6·35 | − 3·24 | − 1·30 | + 2·22 |
| ,, | N. | 1860 | +10·20 | +19·13 | +11·32 | + 4·27 | + 6·34 | − 2·01 | +11·08 | + 1·79 | + 0·82 | + 9·53 | +14·43 | + 9·48 |
| | | 1861 | +22·79 | +20·96 | +22·53 | +20·30 | + 7·26 | − 0·79 | − 7·52 | − 6·21 | −14·56 | −16·38 | − 4·66 | − 7·49 |
| S. | S. and N. | 1860 | +14·93 | +12·22 | +13·34 | + 1·98 | + 5·91 | −14·15 | + 1·45 | −17·46 | −12·16 | − 2·34 | +12·43 | +16·25 |
| | | 1861 | +24·34 | +20·26 | +17·67 | +12·90 | + 1·49 | − 0·89 | −11·25 | −12·61 | −15·87 | − 5·68 | + 3·17 | + 8·53 |
| N. | ,, | 1860 | − 4·06 | − 9·39 | − 8·19 | − 4·62 | − 8·51 | − 2·50 | + 8·90 | + 7·07 | + 8·44 | +19·33 | +16·99 | +13·91 |
| | | 1861 | +20·96 | +18·10 | +20·26 | +16·84 | +12·41 | + 5·93 | + 1·02 | + 7·71 | − 6·64 | −16·16 | −10·53 | −14·84 |

One glance at this Table informs us of the great difference between the several curves for the two years. The yearly curve for 1860 shows a maximum easterly declination at $23^h$ and a minimum at $6^h$, with a secondary maximum at $11^h$ and a range of $32''{\cdot}39$; whilst the same curve for 1861 shows a maximum at the moon's lower transit, and a rather indistinctly expressed minimum at $3^h$, the greatest range being $38''{\cdot}45$. Differences, similar to those just pointed out, will be found on examining the various curves, but it may suffice for the present to single out one of the most striking anomalies, if we are allowed to speak of such, our knowledge on this point being still very indistinct. I refer to the winter half-yearly curve when the moon's declination is indiscriminately north and south. In 1860 we notice in this case a distinct maximum at $22^h$ and a minimum at $8^h$, secondary extremes occurring at $11^h$ and $13^h$. The greatest range is $28''{\cdot}23$. But on examining the same curve for 1861 we find a vast difference; a maximum deviation towards the east at the moon's inferior passage and at $3^h$, whereas it is scarcely possible to speak of secondary extremes, all other undulations appearing as accidental irregularities when compared with the main oscillation, with a range amounting to $43''{\cdot}82$. It can hardly escape our observation that both curves show a clear sweep, but in a very different sense; and we understand now the reason of the irregularity of the mean winter curve for the five years, which scarcely deserves the name of a curve. This fact frequently induced those more especially engaged in these pursuits to doubt the existence of a lunar-diurnal variation-curve during the absence of the sun from the hemisphere of observation, a notion which, after the above explanations, may safely be pronounced to be erroneous. We feel, however, also inclined to recognize in the facts above pointed out the excuse for such a conclusion.

The summer semiannual curve for 1861 exhibits a secondary maximum and minimum, respectively, at $23^h$ and $20^h$, the primary extremes occurring at the moon's lower passage and at $6^h$. As all curves for this year exhibit similar anomalies, when compared with what we adopted for the rule, the idea suggested itself that some error might have crept into the discussion; but this was soon proved to be an erroneous supposition by a perfectly independent and fresh discussion which gave in the main points results identical with those arrived at on the first occasion.

In course of the year 1861, the instruments hitherto in use at the Flagstaff Observatory, were replaced by new ones just received from Munich, and in the month of May the necessary adjustments were so far advanced as to allow of the registration of the new instruments being commenced with the beginning of the month of June. It need scarcely to be mentioned that the greatest care was taken to ensure uniformity of registration in every respect, and it is not likely that the new arrangements would have influenced the observation in any undue manner. Not satisfied, however, with such guarantees for an exact observation and discussion, I resolved to examine the anomalies of this year still further. For this purpose the lunar-diurnal variation for the period from May 1860 to April 1861 was derived with special regard to the moon's declination, as likewise for the same months in 1858 and 1859, and in comparing the respective results

we must remember that during both periods the same instruments were used; but still we perceive that towards the end of the latter period the abnormities above pointed out make themselves clearly manifest, even in the yearly curve with the moon's declination north and south (see Plate XIX.). This seems to speak strongly in favour of a progressive change rather than of an accidental irregularity. It would be premature to enter upon an explanation of these facts at present, as it is evident that for such a purpose we are in need of similar discussions from other localities; suffice it to call attention to so important a class of phenomena. The subjoined Table shows the results of this last-mentioned inquiry, and on Plate XIX. are found the lunar-diurnal variation curves for the periods above delineated.

TABLE III.—Lunar-diurnal Variation of the Magnetic Declination for the periods May 1858 to April 1859, and May 1860 to April 1861.

1st. From the superior to the inferior passage.

| Declination of the | | May to April. | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | | 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
| S. and N. | S. and N. | 1858 | +1.18 | +1.37 | +0.61 | −0.31 | −0.98 | −0.14 | −0.48 | −1.63 | −2.15 | +3.82 | +5.84 | +3.29 |
| | | 1859 | + 4·16 | + 4·57 | + 8·85 | + 4·48 | − 1·35 | − 2·89 | −10·30 | − 2·38 | − 5·98 | − 5·24 | + 3·45 | +13·55 |
| | | 1860 | + 5·47 | + 9·00 | + 9·24 | − 6·95 | −12·02 | − 2·29 | −11·75 | −16·39 | − 7·90 | + 0·55 | − 1·39 | − 4·52 |
| ,, | S. | 1858 | + 7·43 | + 5·44 | +10·96 | + 8·48 | − 1·85 | + 2·05 | − 9·56 | + 2·35 | − 7·87 | + 0·20 | + 4·49 | +11·63 |
| | | 1859 | +20·35 | +18·73 | +10·05 | + 7·52 | + 3·64 | + 2·46 | − 4·78 | − 8·42 | + 5·20 | + 7·96 | +14·83 | +12·41 |
| ,, | N. | 1860 | + 0·96 | + 3·73 | + 6·73 | + 0·28 | − 0·89 | − 8·21 | −11·08 | − 7·40 | − 3·95 | −11·05 | + 2·30 | +15·63 |

2nd. From the inferior to the superior passage.

| Declination of the | | May to April. | Lunar hours. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun. | Moon. | | 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. | 21. | 22. | 23. |
| S. and N. | S. and N. | 1858 | + 7·96 | + 1·91 | − 6·94 | − 4·36 | −12·90 | − 6·81 | − 5·44 | − 2·93 | − 5·27 | − 1·07 | + 7·85 | +10·89 |
| | | 1859 | + 9·72 | + 4·22 | + 2·26 | + 3·65 | − 2·41 | − 8·83 | − 4·73 | − 6·67 | − 6·09 | − 4·66 | + 7·06 | + 5·79 |
| | | 1860 | + 0·93 | − 9·80 | − 3·69 | + 1·29 | − 9·57 | − 4·46 | − 2·78 | + 1·66 | − 9·53 | − 7·30 | − 2·03 | + 4·00 |
| ,, | S. | 1858 | + 6·55 | − 2·03 | − 7·21 | − 2·51 | − 3·69 | −13·98 | −10·58 | −10·16 | −11·12 | − 6·87 | + 7·21 | + 6·96 |
| | | 1859 | +16·43 | +16·40 | −10·75 | −10·83 | −16·88 | − 9·55 | − 8·41 | − 8·24 | − 0·32 | + 6·08 | +19·60 | +19·51 |
| ,, | N. | 1860 | +13·13 | +11·11 | +12·43 | +10·38 | − 0·98 | − 4·04 | + 1·64 | − 2·89 | − 0·91 | − 2·42 | + 6·91 | + 4·60 |

On Plate XIX. we represent furthermore the two annual curves of the same periods without reference to the moon's declination, and we perceive that there is a depression in the curve for 1860–61 near the moon's upper transit, as likewise an increase of easterly declination near her inferior passage, while the curve for 1858–59 bears more the character of the mean curves for five years.

These few facts may for the present suffice to induce those who are engaged in similar pursuits to enter upon such a laborious task as is the classification of magnetic observations for the purpose of examining into the law and nature of the lunar-diurnal variation, according to the moon's position north or south of the equator. We may, however, rest assured that such inquiries will ultimately prove to be conducive of the greatest

benefit for the advancement of the science of terrestrial magnetism; and I hope I have been successful in showing that for such a purpose it is not enough to inquire into the nature of the lunar-diurnal curve variations only for the time of the moon's farthest deviation from the equator, as has in several instances been the practice. It is indeed indispensable for the exact investigation of the moon's influence on the magnetism of our earth, to extend the method of inquiry, which I have just explained in the example of the magnetic declination, to the other magnetic elements, for the lunar-diurnal variation of the horizontal force also shows great differences according as the moon is to the north or the south of the equator. As far as my researches have as yet gone, I feel inclined to believe that the results in this latter case are at least equally decisive with those which I have made the immediate subject of this short paper. As soon as the respective discussions are brought to a close, I shall not fail to communicate the results of my labours in this direction to the Royal Society.

## XVI. *An Eighth Memoir on Quantics.* *By* A. CAYLEY, *F.R.S.*

Received January 8,—Read January 17, 1867.

THE present Memoir relates mainly to the binary quintic, continuing the investigations in relation to this form contained in my Second, Third, and Fifth Memoirs on Quantics; the investigations which it contains in relation to a quantic of any order are given with a view to their application to the quintic. All the invariants of a binary quintic (viz. those of the degrees 4, 8, 12, and 18) are given in the Memoirs above referred to, and also the covariants up to the degree 5; it was interesting to proceed one step further, viz. to the covariants of the degree 6; in fact, while for the degree 5 we obtain 3 covariants and a single syzygy, for the degree 6 we obtain only 2 covariants, but as many as 7 syzygies; one of these is, however, the syzygy of the degree 5 multiplied into the quintic itself, so that, excluding this derived syzygy, there remain $(7-1=)$ 6 syzygies of the degree 6. The determination of the two covariants (Tables 83 and 84 *post*) and of the syzygies of the degree 6, occupies the commencement of the present Memoir.

The remainder of the Memoir is in a great measure a reproduction (with various additions and developments) of researches contained in Professor SYLVESTER'S Trilogy, and in a recent memoir by M. HERMITE*. In particular, I establish in a more general form (defining for that purpose the functions which I call "Auxiliars") the theory which is the basis of Professor SYLVESTER'S criteria for the reality of the roots of a quintic equation, or, say, the theory of the determination of the character of an equation of any order. By way of illustration, I first apply this to the quartic equation; and I then apply it to the quintic equation, following Professor SYLVESTER'S track, but so as to dispense altogether with his amphigenous surface, making the investigation to depend solely on the discussion of the bicorn curve, which is a principal section of this surface. I explain the new form which M. HERMITE has given to the Tschirnhausen transformation, leading to a transformed equation the coefficients whereof are all invariants; and, in the case of the quintic, I identify with my Tables his cubicovariants $\varphi_1(x, y)$ and $\varphi_2(x, y)$. And in the two new Tables, 85 and 86, I give the leading coefficients of the other two cubicovariants $\varphi_3(x, y)$ and $\varphi_4(x, y)$. In the transformed equation the second term (or that in $x^4$) vanishes, and the coefficient $\mathfrak{A}$ of $x^3$ is obtained as a quadric function of four indeterminates. The discussion of this form led to criteria for the character of a quintic equation, expressed like those of Professor SYLVESTER in terms of invariants, but of a

* SYLVESTER "On the Real and Imaginary Roots of Algebraical Equations; a Trilogy," Phil. Trans. vol. 154 (1864), pp. 579–666. HERMITE, "Sur l'Equation du 5e degré," Comptes Rendus, t. 61 (1866), and in a separate form, Paris, 1866.

different and less simple form; two such sets of criteria are obtained, and the identification of these, and of a third set resulting from a separate investigation, with the criteria of Professor SYLVESTER, is a point made out in the present memoir. The theory is also given of the canonical form which is the mechanism by which M. HERMITE'S investigations were carried on. The Memoir contains other investigations and formulæ in relation to the binary quintic; and as part of the foregoing theory of the determination of the character of an equation, I was led to consider the question of the imaginary linear transformations which give rise to a real equation: this is discussed in the concluding articles of the memoir, and in an Annex I have given a somewhat singular analytical theorem arising thereout.

The paragraphs and Tables are numbered consecutively with those of my former Memoirs on Quantics. I notice that in the Second Memoir, p. 126, we should have No. 26=(No. 19)$^2$−128(No. 25), viz. the coefficient of the last term is 128 instead of 1152.

Article Nos. 251 to 254.—*The Binary Quintic, Covariants and Syzygies of the degree* 6.

251. The number of asyzygetic covariants of any degree is obtained as in my Second Memoir on Quantics, Philosophical Transactions, t. 146 (1856), pp. 101–126, viz. by developing the function

$$\frac{1}{(1-z)(1-xz)(1-x^2z)(1-x^3z)(1-x^4z)(1-x^5z)},$$

as shown p. 114, and then subtracting from each coefficient that which immediately precedes it; or, what is the same thing, by developing the function

$$\frac{1-x}{(1-z)(1-xz)(1-x^2z)(1-x^3z)(1-x^4z)(1-x^5z)},$$

which would lead directly to the second of the two Tables which are there given; the Table is there calculated only up to $z^5$, but I have since continued it up to $z^{18}$, so as to show the number of the asyzygetic covariants of every order in the variables up to the degree 18 in the coefficients, being the degree of the skew invariant, the highest of the irreducible invariants of the quintic. The Table is, for greater convenience, arranged in a different form, as follows:—

## Table No. 81.

Table for the number of the Asyzygetic Covariants of any order, to the degree 18.

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252. The interpretation up to the degree 6 is as follows:—

Table No. 82.

| Degree. | Order. | No. | Constitution. Nos. in ( ) refer to Tables in the former memoirs except (83) and (84), which are given *post*. | N = new covt. S = syzygy. |
|---|---|---|---|---|
| 0 | 0 | 1 | viz. the absolute constant unity. | |
| 1 | 5 | 1 | $(13)$ | N. |
| 2 | 10 | 1 | $(13)^2$ | |
| " | 6 | 1 | $(15)$ | N. |
| " | 2 | 1 | $(14)$ | N. |
| 3 | 15 | 1 | $(13)^3$ | |
| " | 11 | 1 | $(13)(15)$ | |
| " | 9 | 1 | $(18)$ | N. |
| " | 7 | 1 | $(13)(14)$ | |
| " | 5 | 1 | $(17)$ | N. |
| " | 3 | 1 | $(16)$ | N. |
| 4 | 20 | 1 | $(13)^4$ | |
| " | 16 | 1 | $(13)^2(15)$ | |
| " | 14 | 1 | $(13)(18)$ | |
| " | 12 | 2 | $(15)^2$, $(13)^2(14)$ | |
| " | 10 | 1 | $(13)(17)$ | |
| " | 8 | 2 | $(13)(16)$, $(14)(15)$ | |
| " | 6 | 1 | $(21)$ | N. |
| " | 4 | 2 | $(20)$ $(14)^2$ | N. |
| " | 0 | 1 | $(19)$ | N. |
| 5 | 25 | 1 | $(13)^5$ | |
| " | 21 | 1 | $(13)^3(15)$ | |
| " | 19 | 1 | $(13)^2(18)$ | |
| " | 17 | 2 | $(13)^3(14)$, $(13)(15)^2$ | |
| " | 15 | 2 | $(13)^2(17)$, $(15)(18)$ | |
| " | 13 | 2 | $(13)^2(16)$, $(13)(14)(15)$ | |
| " | 11 | 2 | $(13)(21)+(14)(18)\quad-(15)(17)=0$ | S. |
| " | 9 | 3 | $(13)(14)^2$, $(13)(20)$, $(15)(16)$ | |
| " | 7 | 2 | $(14)(17)$, $(24)$ | N. |
| " | 5 | 2 | $(13)(19)$, $(14)(16)$ | |
| " | 3 | 1 | $(23)$ | N. |
| " | 1 | 1 | $(22)$ | N. |
| 6 | 30 | 1 | $(13)^6$ | |
| " | 26 | 1 | $(13)^4(15)$ | |
| " | 24 | 1 | $(13)^3(18)$ | |
| " | 22 | 2 | $(13)^3(15)^2$, $(13)^4(14)$ | |
| " | 20 | 2 | $(13)(15)(18)$, $(13)^2(17)$ | |
| " | 18 | 3 | $(18)^2+4(15)^2\quad+(13)^2(16)-(13)^2(14)(15)=0$ | S. |
| " | 16 | 2 | $(13)\{(13)(21)+(13)(14)(18)-(15)(17)\}\quad=0$ | S'. |
| " | 14 | 4 | $-6(13)(15)(16)-1(17)(18)\quad-4(14)(15)^2+(13)^2(20)\quad=0, (13)^2(14)^2$ | S. |
| " | 12 | 3 | $(13)(24)\quad+3(16)(18)\quad-2(15)(21)\quad=0, (13)(14)(17)$ | S. |
| " | 10 | 4 | $4(14)^2(15)\quad+12(13)(14)(16)-(13)^2(19)+(17)^2\quad=0, (15)(20)$ | S. |
| " | 8 | 2 | $(13)(23)\quad+2(14)(21)\quad-3(16)(17)\quad=0$ | S. |
| " | 6 | 4 | $(13)(22)\quad+2(14)(20)\quad-(14)^3\quad-(15)(19)-9(16)^3=0$ | S. |
| " | 4 | 1 | $(84)$ | N. |
| " | 2 | 1 | $(83)$ | N. |

253. For the explanation of this I remark that the Table No. 81 shows that we have for the degree 0 and order 0 one covariant; this is the absolute constant unity; for the degree 1 and order 5, 1 covariant, this is the quintic itself, being the Table No. 13 of my Second Memoir; for degree 2 and order 10, 1 covariant; this is the square of the quintic, $(13)^2$; for same degree and order 6, 1 covariant, which had accordingly to be calculated, viz. this is the Table No. 15; and similarly whenever the Table No. 81 indicates the existence of a covariant of any degree and order, and there does not exist a product of the covariants previously calculated, having the proper degree and order, then in each such case (shown in the last preceding Table by the letter N) a new covariant had to be calculated. On coming to degree 5, order 11, it appears that the number of asyzygetic invariants is only =2, whereas there exist of the right degree and order the 3 combinations (13)(21), (14)(18), and (15)(17); there is here a syzygy, or linear relation between the combinations in question; which syzygy had to be calculated, and was found to be as shown, (13)(21)+(14)(18)−(15)(17)=0, a result given in the Second Memoir, p. 126. Any such case is indicated by the letter S. At the place degree 6, order 16, we find a syzygy between the combinations $(13)^2(21)$, (13)(14)(18), (13)(15)(17); as each term contains the factor (13), this is only the last-mentioned syzygy multiplied by (13), not a new syzygy, and I have written S′ instead of S. The places degree 6, orders 18, 14, 12, 10, 8, 6 each of them indicate a syzygy, which syzygies, as being of the degree 6, were not given in the Second Memoir, and they were first calculated for the present Memoir. It is to be noticed that in some cases the combinations which might have entered into the syzygy do not all of them do so; thus degree 6, order 14, the syzygy is between the four combinations (13)(15)(16), (17)(18), $(14)(15)^2$, and $(13)^2(20)$, and does not contain the remaining combination $(13)^2(14)^2$. The places degree 6, orders 4, 2, indicate each of them a new covariant, and these, as being of the degree 6, were not given in the Second Memoir, but had to be calculated for the present Memoir.

254. I notice the following results:—

$$\begin{aligned}
&\text{Quadrinvt. (6 No. 20)} &&= 3(19)^2,\\
&\text{Cubinvt. (6 No. 20)} &&= -(19)^3+54(19)(25),\\
&\text{Disct. } (\alpha \text{ No. } 14+\beta \text{ No. } 83) &&= (-(19), (25), -3(29)\rangle\alpha, \beta)^2,\\
&\text{Jac. (No. 14, No. 20)} &&= 6(84),\\
&\text{Hess. (3 No. 16)} &&= (83),
\end{aligned}$$

the last two of which indicate the formation of the covariants given in the new Tables Nos. 83 and 84: viz. if to avoid fractions we take 3 times the Table No. 16, being a cubic $(a, \ldots)^3(x, y)^3$, then the Hessian thereof is a covariant $(a, \ldots)^6(x, y)^2$, which is given in Table No. 83; and in like manner if we form the Jacobian of the Tables Nos. 14 and 20, which are respectively of the forms $(a, \ldots)^3(x, y)^2$, and $(a, \ldots)^6(x, y)^4$, this is a covariant $(a, \ldots)^6(x, y)^4$, and dividing it by 6 to obtain the coefficients in their lowest terms, we have the new Table, No. 84. I have in these, for greater distinctness, written the numerical coefficients *after* instead of *before*, the literal terms to which they belong.

The two new Tables are—

Table No. 83.

| | | | | | |
|---|---|---|---|---|---|
| $a^2c^2f^2$ | $-1$ | $a^3cdf^3$ | $-1$ | $a^3d^2f^2$ | $-1$ |
| $a^3cdef$ | $+5$ | $a^3ce^2f$ | $+1$ | $a^3de^2f$ | $+2$ |
| $a^3ce^3$ | $-3$ | $a^3d^2ef$ | $+1$ | $a^3e^4$ | $-1$ |
| $a^3d^2f$ | $-3$ | $a^3de^3$ | $-1$ | $abcdf^3$ | $+5$ |
| $a^2d^3e^2$ | $+2$ | $ab^2df^2$ | $+1$ | $abce^2f$ | $-5$ |
| $ab^2ef^2$ | $+2$ | $ab^2e^2f$ | $-1$ | $abd^2ef$ | $-5$ |
| $ab^2def$ | $-5$ | $abe^2f^2$ | $+1$ | $abde^3$ | $+5$ |
| $ab^2e^3$ | $+3$ | $abcdef$ | $+6$ | $ac^2f^2$ | $-3$ |
| $abc^2ef$ | $-5$ | $abce^3$ | $-8$ | $ac^2def$ | $+7$ |
| $abcd^2f$ | $+7$ | $abd^2f$ | $-10$ | $ac^2e^3$ | $+2$ |
| $abcde^2$ | $-1$ | $abd^2e^2$ | $+11$ | $acd^2f$ | $-1$ |
| $abd^3e$ | $-1$ | $ac^3ef$ | $-10$ | $acd^2e^2$ | $-8$ |
| $ac^4df$ | $-1$ | $ac^3d^2f$ | $+11$ | $ad^2e$ | $+3$ |
| $ac^3e^2$ | $+6$ | $ac^2de^2$ | $+18$ | $b^2df^2$ | $-3$ |
| $ac^2d^2e$ | $-8$ | $acd^3e$ | $-28$ | $b^2e^2f$ | $+3$ |
| $acd^4$ | $+3$ | $ad^4$ | $+9$ | $b^2c^2f^2$ | $+2$ |
| $b^4f^2$ | $-1$ | $b^2cf^2$ | $-1$ | $b^2cdef$ | $-1$ |
| $b^2cef$ | $+5$ | $b^2def$ | $-8$ | $b^2ce^3$ | $-3$ |
| $b^2d^2f$ | $+2$ | $b^2e^3$ | $+9$ | $b^2d^3f$ | $+6$ |
| $b^2de^2$ | $-3$ | $b^2c^2ef$ | $+11$ | $b^2d^2e^2$ | $-4$ |
| $b^2c^3df$ | $-8$ | $b^2cd^2f$ | $+18$ | $bc^2ef$ | $-1$ |
| $b^2c^2e^2$ | $-4$ | $b^2cde^2$ | $-37$ | $bc^2d^2f$ | $-8$ |
| $b^2cd^2e$ | $+7$ | $b^2d^3e$ | $+8$ | $bc^2de^2$ | $+7$ |
| $b^2d^4$ | $-1$ | $bc^3df$ | $-28$ | $bcd^2e$ | $+5$ |
| $bc^4f$ | $+3$ | $bc^2e^2$ | $+8$ | $bd^4$ | $-3$ |
| $bc^3de$ | $+5$ | $bc^2d^2e$ | $+37$ | $c^4df$ | $+3$ |
| $bc^2d^3$ | $-4$ | $bcd^3$ | $-17$ | $c^4e^2$ | $-1$ |
| $c^5e$ | $-3$ | $c^5f$ | $+9$ | $c^3d^2e$ | $-4$ |
| $c^4d^2$ | $+2$ | $c^4de$ | $-17$ | $c^3d^4$ | $+2$ |
| | | $c^3d^3$ | $+8$ | | |
| | $+52$ | | $+167$ | | $+52$ |

$\}(x, y)^3.$

Table No. 84.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $a^2df^3$ | $-1$ | $a^2bdf^3$ | $-4$ | $a^2ce^2f$ | $+6$ | $a^2cef^2$ | $+4$ | $a^2cf^3$ | $+1$ |
| $a^2e^2f$ | $+1$ | $a^2b^2ef$ | $+4$ | $a^2d^2ef$ | $-12$ | $a^2d^2f^2$ | $-4$ | $a^2def^2$ | $-3$ |
| $a^2bcf^2$ | $+3$ | $a^2c^2f^2$ | $+4$ | $a^2de^2$ | $+6$ | $a^2de^2f$ | $-4$ | $a^2e^2f$ | $+2$ |
| $a^2bdef$ | $+2$ | $a^2cdef$ | $-8$ | $a^2bdf^2$ | $-6$ | $a^2e^4$ | $+4$ | $ab^2f^2$ | $-1$ |
| $a^2be^3$ | $-5$ | $a^2ce^3$ | $+4$ | $abc^2f^2$ | $+12$ | $ab^2ef^2$ | $-4$ | $abcef^2$ | $-2$ |
| $a^2c^2ef$ | $-8$ | $ab^2cf^2$ | $+4$ | $abce^3$ | $-36$ | $abcdf^2$ | $+8$ | $abd^2f^2$ | $+8$ |
| $a^2cd^2f$ | $+2$ | $ab^2def$ | $+16$ | $abd^2f$ | $+48$ | $abce^2f$ | $-16$ | $abde^2f$ | $+2$ |
| $a^2cde^2$ | $+12$ | $ab^2e^3$ | $-24$ | $abd^2e^2$ | $-12$ | $abd^2ef$ | $+48$ | $abe^4$ | $-6$ |
| $a^2d^3e$ | $-6$ | $abc^2ef$ | $-48$ | $ac^3ef$ | $-48$ | $abde^3$ | $-32$ | $ac^2df^2$ | $-2$ |
| $ab^2f^3$ | $-2$ | $abcd^2f$ | $+40$ | $ac^2de^2$ | $+156$ | $ac^2def$ | $-40$ | $ac^2e^2f$ | $+6$ |
| $ab^2cef$ | $-2$ | $abcde^2$ | $+40$ | $acd^3e$ | $-168$ | $ac^2e^3$ | $+56$ | $acd^2ef$ | $-20$ |
| $ab^2d^2f$ | $-6$ | $abd^3e$ | $-24$ | $ad^4$ | $+54$ | $acd^2f^2$ | $+8$ | $acde^2$ | $+12$ |
| $ab^2de^2$ | $+13$ | $ac^3df$ | $-8$ | $b^3cf^2$ | $-6$ | $acd^2e^2$ | $-40$ | $ad^2f$ | $+9$ |
| $abc^2def$ | $+20$ | $ac^3e^2$ | $+56$ | $b^3def$ | $+36$ | $ad^2e$ | $+12$ | $ade^3$ | $-6$ |
| $abc^2e^3$ | $+4$ | $ac^2d^2e$ | $-88$ | $b^3c^2ef$ | $+12$ | $b^2df^2$ | $-4$ | $b^2ef^2$ | $+5$ |
| $abcd^2e$ | $-52$ | $acd^4$ | $+36$ | $b^3cd^2f$ | $-156$ | $b^2e^2f$ | $+24$ | $b^2cdf^2$ | $-12$ |
| $abd^4$ | $+24$ | $b^4f^2$ | $-4$ | $b^3d^2e$ | $+60$ | $b^2cdef$ | $-40$ | $b^2ce^2f$ | $-13$ |
| $ac^4f$ | $-9$ | $b^3cef$ | $+32$ | $bc^3df$ | $+168$ | $b^2ce^3$ | $-60$ | $b^2d^2ef$ | $-4$ |
| $ac^3de$ | $+20$ | $b^3d^2f$ | $-56$ | $bc^2e^2$ | $-60$ | $b^2d^2f$ | $-56$ | $b^2de^3$ | $+15$ |
| $ac^2d^3$ | $-10$ | $b^3de^2$ | $+60$ | $bcd^4$ | $-30$ | $b^2d^3e^2$ | $+100$ | $be^2f^2$ | $+6$ |
| $b^4ef$ | $+6$ | $b^3c^2df$ | $+40$ | $c^5f$ | $-54$ | $bc^2ef$ | $+24$ | $bc^2def$ | $+52$ |
| $b^3cdf$ | $-12$ | $b^3c^2e^2$ | $-100$ | $c^4de$ | $+30$ | $be^2d^2f$ | $+88$ | $be^2e^3$ | $-10$ |
| $b^3ce^3$ | $-15$ | $b^3cd^2e$ | $-80$ | | | $b^2c^2de^2$ | $+80$ | $bcd^3f$ | $-20$ |
| $b^3d^2e$ | $+10$ | $b^3d^4$ | $+60$ | | | $bcd^2e$ | $-200$ | $bcd^2e^2$ | $-30$ |
| $b^3c^2f$ | $+6$ | $bc^4f$ | $-12$ | | | $bd^5$ | $+60$ | $bd^4e$ | $+15$ |
| $b^3c^2de$ | $+30$ | $bc^3de$ | $+200$ | | | $c^4df$ | $-36$ | $c^4ef$ | $-24$ |
| $b^3cd^3$ | $-20$ | $bc^2d^2$ | $-120$ | | | $c^4e^3$ | $-60$ | $c^3d^2f$ | $+10$ |
| $bc^4e$ | $-15$ | $c^5e$ | $-60$ | | | $c^3d^2e$ | $+120$ | $c^3de^2$ | $+20$ |
| $bc^3d^2$ | $+10$ | $c^4d^2$ | $+40$ | | | $c^3d^4$ | $-40$ | $c^3d^3e$ | $-10$ |
| | $+163$ | | $+636$ | | $+588$ | | $+636$ | | $+163$ |

$\}(x, y)^4.$

Article No. 255. *Formulæ for the canonical form* $ax^5+by^5+cz^5=0$, *where* $x+y+z=0$.

255. The quintic $(a, b, c, d, e, f)(x, y)^5$ may be expressed in the form

$$ru^5+sv^5+tw^5,$$

where $u, v, w$ are linear functions of $(x, y)$ such that $u+v+w=0$. Or, what is the same thing, the quintic may be represented in the canonical form

$$ax^5+by^5+cz^5,$$

where $x+y+z=0$; this is $=(a-c, -c, -c, -c, -c, b-c)(x, y)^5$, and the different covariants and invariants of the quintic may hence be expressed in terms of these coefficients $(a, b, c)$.

For the invariants we have

$$\text{No. 19} \quad =\mathrm{J} =b^3c^3+c^3a^3+a^3b^3-2abc(a+b+c).$$

$$\text{No. 25} \quad =\mathrm{K}=a^3b^3c^3(bc+ca+ab).$$

$$-\text{No. 29} \quad =\mathrm{L}=a^4b^4c^4.$$

$$\text{No. 29A}=\mathrm{I} =4a^3b^3c^3(b-c)(c-a)(a-b).$$

Hence, writing for a moment

$$a+b+c=p, \text{ and } \therefore \mathrm{J}=q^3-4pr,$$

$$bc+ca+ab=q \qquad \mathrm{K}=r^3q,$$

$$abc \quad =r \qquad \mathrm{L}=r^4,$$

we have

$$(a-b)^2(b-c)^2(c-a)^2=p^3q^3-4q^3-4p^3r+18pqr-27r^2,$$

and thence

$$\mathrm{I}^2=16r^{10}(p^3q^3-4q^3-4p^3r+18pqr-27r^2),$$

and

$$\begin{aligned}\mathrm{J}(\mathrm{K}^2-\mathrm{JL})^2+8\mathrm{K}^3\mathrm{L}-72\mathrm{JKL}^2-432\mathrm{L}^3 &=r^{10}\{(q^3-4pr)16p^3+8q^3-(q^3-4pr)72q-432r^3\}\\ &=8r^{10}\{(q^3-4pr)(2p^3-9q)+q^3-54r^3\}\\ &=16r^{10}\{p^3q^3-4q^3-4p^3r+18pqr-27r^2\},\end{aligned}$$

that is,

$$\mathrm{I}^2=\mathrm{J}(\mathrm{K}^2-\mathrm{JL})^2+8\mathrm{K}^3\mathrm{L}-72\mathrm{JKL}^2-432\mathrm{L}^3,$$

which is the simplest mode of obtaining the expression for the square of the 18-thic or skew invariant I in terms of the invariants J, K, L of the degrees 4, 8, 12 respectively.

$$\begin{aligned}\text{No. 26}=\mathrm{D}&=\{b^3c^3+c^3a^3+a^3b^3-2abc(a+b+c)\}^2-128a^3b^3c^3(bc+ca+ab),\\ &=q^4-8q^3pr-128qr^3+16p^3r^2,\end{aligned}$$

$$\mathrm{D}=\mathrm{Norm}\left((bc)^{\frac{1}{2}}+(ca)^{\frac{1}{2}}+(ab)^{\frac{1}{2}}\right).$$

And we have also the following covariants:

$$\begin{aligned}\text{No. 14}&=(-ac, ab-ac-bc, -bc)(x, y)^3\\ &=bcyz+cazx+abxy.\end{aligned}$$

$$\text{No. } 15 = (-ac,\ -3ac,\ -3ac,\ ab-ac-bc,\ -3bc,\ -3bc,\ -bc)(x, y)^6$$
$$= bcy^3z^3 + caz^3x^3 + abx^3y^3.$$

$$\text{No. } 16 = (0,\ -abc,\ -abc,\ 0)(x, y)^3 = abc\,xyz.$$

$$\begin{aligned}\text{No. } 17 = {} & (\ a^2b - \ ac^2 + \ bc^2 - a^2c - 2abc)x^5 \\ & +(\ \ - 5ac^2 + 5bc^2 \ \ - 5abc)x^4y \\ & +(\ \ -10ac^2 + 10bc^2 \ \ - 2abc)x^3y^2 \\ & +(\ \ -10ac^2 + 10bc^2 \ \ + 2abc)x^2y^3 \\ & +(\ \ - 5ac^2 + 5bc^2 \ \ + 5abc)xy^4 \\ & +(-ab^2 - \ ac^2 + \ bc^2 + b^2c + 2abc)y^5 \\ = {} & (b-c)a^2x^5 + (c-a)b^2y^5 + (a-b)c^2z^5 \\ & - abc(y-z)(z-x)(x-y)(yz+zx+xy).\end{aligned}$$

Article No. 256.—*Expression of the* 18*-thic Invariant in terms of the roots.*

256. It was remarked by Dr. SALMON, that for a quintic $(a, b, c, d, e, f)(x, y)^5$ which is linearly transformable into the form $(a, 0, c, 0, e, 0)(x, y)^5$, the invariant I is $=0$. Now putting for convenience $y=1$, and considering for a moment the equation

$$x(x-\beta)(x-\gamma)(x-\delta)(x-\varepsilon)=0,$$

then writing herein $\frac{x}{mx+n}$ for $x$, the transformed equation is

$$x(x-\beta')(x-\gamma')(x-\delta')(x-\varepsilon')=0,$$

where

$$\beta' = \frac{n\beta}{1-m\beta},\ \gamma' = \frac{n\gamma}{1-m\gamma},\ \&c.;$$

hence $m$ may be so determined that $\beta'+\gamma'$ may be $=0$; viz. this will be the case if $\beta+\gamma=2m\beta\gamma$, or $m=\frac{\beta+\gamma}{2\beta\gamma}$. In order that $\delta'+\varepsilon'$ may be $=0$, we must of course have $m=\frac{\delta+\varepsilon}{2\delta\varepsilon}$, and hence the condition that simultaneously $\beta'+\gamma'=0$ and $\delta'+\varepsilon'=0$ is $\frac{\beta+\gamma}{2\beta\gamma}=\frac{\delta+\varepsilon}{2\delta\varepsilon}$; that is, $(\beta+\gamma)\delta\varepsilon-\beta\gamma(\delta+\varepsilon)=0$. Or putting $x-\alpha$ for $x$ and $\beta-\alpha$, $\gamma-\alpha$, &c. for $\beta$, $\gamma$, &c., we have the equation

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)(x-\varepsilon)=0,$$

which is by the transformation $x-\alpha$ into $\frac{x-\alpha}{m(x-\alpha)+n}$ changed into

$$(x-\alpha')(x-\beta')(x-\gamma')(x-\delta')(x-\varepsilon')=0$$

(where $\alpha'=\alpha$), and the condition in order that in the new equation it may be possible to have simultaneously $\beta'+\gamma'-2\alpha'=0$, $\delta'+\varepsilon'-2\alpha'=0$, is

$$(\beta+\gamma-2\alpha)(\delta-\alpha)(\varepsilon-\alpha)-(\delta+\varepsilon-2\alpha)(\beta-\alpha)(\gamma-\alpha)=0,$$

or, as this may be written,

$$\begin{vmatrix} 1, & 2\alpha & , & \alpha^2 \\ 1, & \beta+\gamma, & & \beta\gamma \\ 1, & \delta+\varepsilon, & & \delta\varepsilon \end{vmatrix} = 0.$$

Hence writing $x+\alpha'$ for $x$, the last-mentioned equation is the condition in order that the equation

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)(x-\varepsilon)=0$$

may be transformable into

$$x(x-\beta')(x-\gamma')(x-\delta')(x-\varepsilon')=0,$$

where $\beta'+\gamma'=0$, $\delta'+\varepsilon'=0$, that is, into the form $x(x^2-\beta'^2)(x^2-\delta'^2)=0$. Or replacing $y$, if we have

$$(a, b, c, d, e, f\,(x, y)^5 = a(x-\alpha y)(x-\beta y)(x-\gamma y)(x-\delta y)(x-\varepsilon y),$$

then the equation in question is, the condition in order that this may be transformable into the form $(a', 0, c', 0, e', 0\,(x, y)^5$, that is, in order that the 18-thic invariant I may vanish. Hence observing that there are 15 determinants of the form in question, and that any root, for instance $\alpha$, enters as $\alpha^2$ in 3 of them and in the simple power $\alpha$ in the remaining 12, we see that the product

$$\alpha^{18}\Pi \begin{vmatrix} 1, & 2\alpha & , & \alpha^2 \\ 1, & \beta+\gamma, & & \beta\gamma \\ 1, & \delta+\varepsilon, & & \delta\varepsilon \end{vmatrix}$$

contains each root in the power 18, and is consequently a rational and integral function of the coefficients of the degree 18, viz. save as to a numerical factor it is equal to the invariant I. And considering the equation $(a, \ldots(x, y)^5 = 0$ as representing a range of points, the signification of the equation $I=0$ is that, the pairs $(\beta, \gamma)$ and $(\delta, \varepsilon)$ being properly selected, the fifth point $\alpha$ is a focus or sibiconjugate point of the involution formed by the pairs $(\beta, \gamma)$ and $(\delta, \varepsilon)$.

Article Nos. 257 to 267.—*Theory of the determination of the Character of an Equation; Auxiliars; Facultative and Non-facultative space.*

257. The equation $(a, b, c \ldots(x, y)^n = 0$ is a *real* equation if the ratios $a:b:c, \ldots$ of the coefficients are all real. In considering a given real equation, there is no loss of generality in considering the coefficients $(a, b, c \ldots)$ as being themselves real, or in taking the coefficient $a$ to be $=1$; and it is also for the most part convenient to write $y=1$, and thus to consider the equation under the form $(1, b, c \ldots(x, 1)^n = 0$. It will therefore (unless the contrary is expressed) be throughout assumed that the coefficients (including the coefficient $a$ when it is not put $=1$) are all of them real; and, in speaking of any functions of the coefficients, it is assumed that these are rational and integral real functions, and that any values attributed to these functions are also real.

258. The equation $(1, b, c \ldots)(x, 1)^n = 0$, with $\alpha$ real roots and $2\beta$ imaginary roots, is said to have the character $\alpha r + 2\beta i$; thus a quintic equation will have the character $5r$, $3r+2i$, or $r+4i$, according as its roots are all real, or as it has a single pair, or two pairs, of imaginary roots.

259. Consider any $m$ functions (A, B, ... K) of the coefficients, ($m =$ or $< n$). For given values of (A, B, ... K), *non constat* that there is any corresponding equation (that is, the corresponding values of the coefficients $(b, c, \ldots)$ may be of necessity imaginary), but attending only to those values of (A, B, ... K) which have a corresponding equation or corresponding equations, let it be assumed that the equations which correspond to a given set of values of (A, B, ... K) have a determinate character (one and the same for all such equations): this assumption is of course a condition imposed on the form of the functions (A, B, ... K); and any functions satisfying the condition are said to be "auxiliars." It may be remarked that the $n$ coefficients $(b, c, \ldots)$ are themselves auxiliars; in fact for given values of the coefficients there is only a single equation, which equation has of course a determinate character. To fix the ideas we may consider the auxiliars (A, B, ... K) as the coordinates of a point in $m$-dimensional space, or say in $m$-space.

260. Any given point in the $m$-space is either "facultative," that is, we have corresponding thereto an equation or equations (and if more than one equation then by what precedes these equations have all of them the same character), or else it is "non-facultative," that is, the point has no corresponding equation.

261. The entire system of facultative points forms a region or regions, and the entire system of non-facultative points a region or regions; and the $m$-space is thus divided into facultative and non-facultative regions. The surface which divides the facultative and non-facultative regions may be spoken of simply as the bounding surface, whether the same be analytically a single surface, or consist of portions of more than one surface.

262. Consider the discriminant D, and to fix the ideas let the sign be determined in such wise that D is $+$ or $-$ according as the number of imaginary roots is $\equiv 0 \pmod 4$, or is $\equiv 2 \pmod 4$; then expressing the equation $D = 0$ in terms of the auxiliars (A, B, ... K), we have a surface, say the discriminatrix, dividing the $m$-space into regions for which D is $+$, and for which D is $-$, or, say, into positive and negative regions.

263. A given facultative or non-facultative region may be wholly positive or wholly negative, or it may be intersected by the discriminatrix and thus divided into positive and negative regions. Hence taking account of the division by the discriminatrix, but attending only to the facultative regions, we have positive facultative regions and negative facultative regions. Now using the simple term region to denote indifferently a positive facultative region or a negative facultative region, it appears from the very notion of a region as above explained that we may pass from any point in a given region to any other point in the same region without traversing either the bounding surface or the discriminatrix; and it follows that the equations which correspond to the several

points of the same region have each of them one and the same character; that is, to a given region there correspond equations of a given character.

264. It is proper to remark that there may very well be two or more regions which have corresponding to them equations with the same character; any such regions may be associated together and considered as forming a kingdom; the number of kingdoms is of course equal to the number of characters, viz. it is $=\frac{1}{2}(n+2)$ or $\frac{1}{2}(n+1)$ according as $n$ is even or odd; and this being so, the general conclusion from the preceding considerations is that the whole of facultative space will be divided into kingdoms, such that to a given kingdom there correspond equations having a given character; and conversely, that the equations with a given character correspond to a given kingdom. Hence (the characters for the several kingdoms being ascertained) knowing in what kingdom is situate a point (A, B, ... K), we know also the character of the corresponding equations.

265. Any conditions which determine in what kingdom is situate the point (A, B, ... K) which belongs to a given equation $(1, b, c \ldots \chi(x, 1)^n = 0$, determine therefore the character of the equation. It is very important to notice that the form of these conditions is to a certain extent indeterminate; for if to a given kingdom we attach any portion or portions of non-facultative space, then any condition or conditions which confine the point (A, B, ... K) to the resulting aggregate portion of space, in effect confine it to the kingdom in question; for of the points within the aggregate portion of space it is only those within the kingdom which have corresponding to them an equation, and therefore, if the coefficients $(b, c, \ldots)$ of the given equation are such as to give to the auxiliars (A, B, ... K) values which correspond to a point situate within the above-mentioned aggregate portion of space, such point will of necessity be within the kingdom.

266. In the case where the auxiliars are the coefficients $(b, c, \ldots)$, to any given values of the auxiliars there corresponds an equation, that is, all space is facultative space. And the division into regions or kingdoms is effected by means of the discriminatrix, or surface D=0, alone. Thus in the case of the quadric equation $(1, x, y\chi(\theta, 1)^2 = 0$ the $m$-space is the plane. We have $D = x^2 - y$, and the discriminatrix is thus the parabola $x^2 - y = 0$. There are two kingdoms, each consisting of a single region, viz. the positive kingdom or region $(x^2 - y = +)$ outside the parabola, and the negative kingdom or region $(x^2 - y = -)$ inside the parabola, which have the characters $2r$ and $2i$, or correspond to the cases of two real roots and two imaginary roots, respectively. And the like as regards the cubic $(1, x, y, z\chi(\theta, 1)^3 = 0$; the $m$-space is here ordinary space, $D = -4x^3z + 3x^2y^2 + 6xyz - 4y^3 - z^2$, and the division into kingdoms is effected by means of the surface D=0; but as in this case there are only the two characters $3r$ and $r+2i$, there can be only the two kingdoms D=+ and D=− having these characters $3r$ and $r+2i$ respectively, and the determination of the character of the cubic equation is thus effected without its being necessary to proceed further, or inquire as to the form or number of the regions determined by the surface D=0: I believe that there are only two regions, so that in this case also each kingdom consists of a single region. But pro-

ceeding in the same manner, that is, with the coefficients themselves as auxiliars, to the case of a quartic equation, the $m$-space is here a 4-dimensional space, so that we cannot by an actual geometrical discussion show how the 4-space is by the discriminatrix or hypersurface $D=0$ divided into kingdoms having the characters $4r$, $2r+2i$, $4i$ respectively. The employment therefore of the coefficients themselves as auxiliars, although theoretically applicable to an equation of any order whatever, can in practice be applied only to the cases for which a geometrical illustration is in fact unnecessary.

267. I will consider in a different manner the case of the quartic, chiefly as an instance of the actual employment of a surface in the discussion of the character of an equation; for in the case of a quintic the auxiliars are in the sequel selected in such manner that the surface breaks up into a plane and cylinder, and the discussion is in fact almost independent of the surface, being conducted by means of the curve (Professor SYLVESTER'S Bicorn) which is the intersection of the plane and cylinder.

Article Nos. 268–273.—*Application to the Quartic equation.*

268. Considering then the quartic equation $(a, b, c, d, e\chi\theta, 1)^4=0$ (I retain for symmetry the coefficient $a$, but suppose it to be $=1$, or at all events positive), then if I, J signify as usual, and if for a moment

$$\Theta=a^2d-3abc+2b^3,$$
$$X=3aJ+2(b^2-ac)I,$$

we have identically

$$\tfrac{1}{4}(3a^2J^2+X^2)\Theta^2=9(b^2-ac)^2X^2-a^2(b^2-ac)^2(I^3-27J^2)-a^2X^3$$

(see my paper, "A discussion of the Sturmian Constants for Cubic and Quartic Equations," Quart. Math. Journ. t. iv. (1861) pp. 7–12). And I write

$$x=b^2-ac,$$
$$y=3aJ+2(b^2-ac)I,$$
$$z=I^3-27J^2(=D).$$

269. I borrow from STURM'S theorem the conclusion (but nothing else than this conclusion) that $(x, y, z)$ possess the fundamental property of auxiliars (that is, that the quartic equations (if any) corresponding to a given system of values of $(x, y, z)$ have one and the same character). The foregoing equation gives $9x^3y^2-x^2z-y^3=$ a square function, and therefore positive; that is, the facultative portion of space is that for which $9x^3y^2-x^2z-y^3$ is $=+$. And the equation

$$x^2(9y^2-z)-y^3=0$$

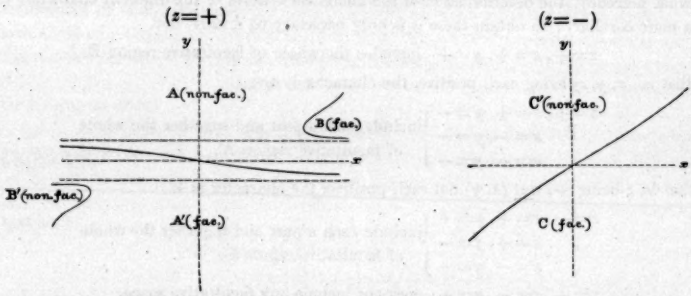
is that of the bounding surface, dividing the facultative and non-facultative portions of space.

270. To explain the form of the surface we may imagine the plane of $xy$ to be that of the paper, and the positive direction of the axis of $z$ to be in front of the paper. Taking $z$ constant, or considering the sections by planes parallel to that of $xy$,

$z=0$, gives $y^2(9x^3-y)=0$, viz. the section is the line $y=0$, or axis of $x$ twice, and the cubical parabola $y=x^3$.

$z=+$, the curve $x^3=\frac{y^3}{9y^3-z}$ has two asymptotes $y=\pm\frac{1}{3}\sqrt{z}$, parallel to and equidistant from the axis of $x$, and consists of a branch included between the two parallel asymptotes, and two other portions branches outside the asymptotes, as shown in the figure ($z=+$).

$z=-$, the curve $x^3=\frac{y^3}{9y^3-z}$ has no real asymptote, and consists of a single branch, resembling in its appearance the cubical parabola as shown in the figure ($z=-$).




Taking $x$ as constant, or considering the sections by planes parallel to that of $zy$, the equation of the section is $z=9y^3-\frac{y^3}{x^3}$, which is a cubical parabola, meeting the plane of $xy$ in a point on the cubical parabola $y=9x^3$, and also in a twofold point on the axis of $x$, that is, touching the plane of $xy$ at the last-mentioned point.

271. The surface consists of a single sheet extending to infinity, the form of which is most easily understood by considering the sections by a system of spheres having the origin of coordinates for their common centre. These sections have all of them the same general form; and one of them is shown (Plate XX. fig. 1), the projection being orthogonal on the plane of $xy$ or plane of the paper, and the spherical curve being shown, the portion of it above the plane of the paper by a continuous line, that below it by a dotted line (the double point in the figure is thus of course only an apparent one): the same figure shows also the sections by planes parallel to that of $xy$ previously shown in the figures ($z=+$) and ($z=-$).

272. Now considering the discriminatrix $D=0$, in this case the plane $z=0$, it appears that the bounding surface and this plane divide space into six regions, viz. above the plane of the paper we have the four regions, A non-facultative, B facultative, A′ facultative, B′ non-facultative, and below it the two regions, C facultative, C′ non-facultative. There are thus in all three facultative regions A′, B, C, and since A′ and B correspond

to D=+, these must have the characters $4r$ and $4i$, and it is easy by considering a particular case to show that B has the character $4r$, and A′ the character $4i$; C corresponds to D=−, and can therefore only have the character $2r+2i$. Hence, for any given equation, $(x, y, z)$ will lie in one of the regions (B, A′, C), and if $(x, y, z)$

is in the region B, the character is $4r$,
" A′, " $4i$,
" C, " $2r+2i$.

273. It is right to notice that the determination of the character is really made in what precedes; the determination of the analytical criteria of the different characters is a mere corollary; to obtain these it is only necessary to remark that

$z=+, x=+, y=+$ includes the whole of facultative region B,

that is, $(x, y, z)$ being each positive, the character is $4r$;

$z=+, x=+, y=-$
$x=-, y=-$
$x=-, y=-$
} include each a part and together the whole of facultative region A′,

that is, $z$ being +, but $(x, y)$ not each positive, the character is $4i$;

$z=-, x=+, y=+$
" $x=+, y=-$
" $x=-, y=-$
} include each a part and together the whole of facultative region C,

$z=-, x=-, y=+$ does not include any facultative space,

that is, $z$ being −, the character is $2r+2i$; and the combination of signs $z=-, x=-, y=+$ is one which does not exist.

The results thus agree with those furnished by STURM'S theorem; and in particular the impossibility of $z=-, x=-, y=+$ appears from STURM'S theorem, inasmuch as his combination would give a gain instead of a loss of changes of sign.

Article Nos. 274 to 285.—*Determination of the characters of the quintic equation.*

274. Passing now to the case of the quintic, I write

J = No. 19,
K = No. 25,
D = No. 26,
L = −No. 29,
I = No. 29A;

viz. J is the quartinvariant, K and D are octinvariants (D the discriminant), L is 12-thic invariant, and I is the 18-thic or skew invariant. Hence also J, D, $2^{11}L-J^3$ are invariants of the degrees 4, 8, 12 respectively; and forming the combinations

$$x=\frac{2^{11}L-J^3}{J^3},\ y=\frac{D}{J^2},\ z=J,$$

I assume that $(x, y, z)$ are auxiliars, reserving for the concluding articles of the present memoir the considerations which sustain this assumption.

275. The separation into regions is effected as follows:—We have identically (see *ante*, No. 255)

$$16I^3 = JK^4 + 8LK^3 - 2J^3LK^2 - 72JL^2K - 432L^3 + J^3L^2,$$

or putting for K its value $= \frac{1}{1\,2\,8}(J^3 - D)$, this is

$$\begin{aligned} 2^{23}I^3 = {} & J(J^3 - D)^4 + \&c. \\ = {} & (J^3 - 2^{11}L)^2(J^3 - 3^3.2^{16}L) \\ & + DJ(-4J^6 + 61.2^{10}J^3L + 144.2^{20}L^2) \\ & + D^2J^3(\ 6J^3 - 2^{16}.29L) \\ & + D^3\ (-4J^3 - \ 2^{16}L) \\ & + D^4J. \end{aligned}$$

Or writing as above

$$x = \frac{2^{11}L - J^3}{J^3},\quad y = \frac{D}{J^3},$$

whence also

$$1 + x = \frac{2^{11}L}{J^3},$$

this is

$$\begin{aligned} 2^{23}\frac{I^3}{J^9} = {} & -x^3\{\tfrac{3}{2}(1+x) - 1\} \\ & + y\ \{36(1+x)^2 - \tfrac{61}{2}(1+x) - 4\} \\ & - y^2\{\tfrac{329}{2}(1+x) - 6\} \\ & + y^3\{-\tfrac{1}{2}(1+x) - 4\} \\ & + y^4, \end{aligned}$$

or, what is the same thing,

$$\begin{aligned} 2.2^{23}\frac{I^3}{J^9} = {} & -3x^3 - x^3 \\ & + y\,(72x^3 + 205x + 125) \\ & + y^2(-29x - 17) \\ & + y^3(-x - 9) \\ & + y^4.2 \\ & = \phi(x, y) \text{ suppose.} \end{aligned}$$

276. Hence also writing $z = J$, we have

$$z\phi(x, y) = 2.2^{23}\frac{I^3}{J^9} = +$$

or the equation of the bounding surface may be taken to be

$$z\phi(x, y) = 0,$$

that is, the bounding surface is composed of the plane $z = 0$, and the cylinder $\phi(x, y) = 0$. Taking the plane of the paper for the plane $z = 0$, the cylinder meets this plane in a

curve $\varphi(x, y)=0$, which is Professor SYLVESTER's Bicorn: this curve divides the plane into certain regions, and if we attend to the solid figure and instead of the curve consider the cylinder, then to each region of the plane there correspond *in solido* two regions, one in front of, the other behind the plane region, and of these regions *in solido*, one is facultative, the other is non-facultative (viz. for given values of $(x, y)$, whatever be the sign of $\varphi(x, y)$, then for a certain sign of $z$, $z\varphi(x, y)$ will be positive or the solid region will be facultative, and for the opposite sign of $z$, $z\varphi(x, y)$ will be negative or the region will be facultative). It hence appears that we may attend only to the plane regions, and that (the proper sign being attributed to $z$, that is to J) each of these may be regarded as facultative. It is to be added that the discriminatrix is in the present case the plane $y=0$, or, if we attend only to the plane figure, it is the line $y=0$; so that in the plane figure the separation into regions is effected by means of the Bicorn and the line $y=0$.

277. Reverting to the equation of the Bicorn, and considering first the form at infinity, the intersections of the curve by the line infinity are given by the equation $y^3(2y-x)=0$, viz. there is a threefold intersection $y^3=0$, and a simple intersection $2y-x=0$; the equation $y^3=0$ indicates that the intersection in question is a point of inflexion, the tangent at the inflexion (or stationary tangent) being of course the line infinity; the visible effect is, however, only that the direction of the branch is ultimately parallel to the axis of $x$. The equation $2y-x=0$ indicates an asymptote parallel to this line, and the equation of the asymptote is easily found to be $2y-x+5=0$.

278. The discussion of the equation would show that the curve has an ordinary cusp; and a cusp of the second kind, or node-cusp, equivalent to a cusp and node; the curve is therefore a unicursal curve, or the coordinates are expressible rationally in terms of a parameter $\varphi$; we in fact have

$$x=\frac{-(\varphi+2)(\varphi^3-\varphi^2+2\varphi-4)}{\varphi^3(\varphi+1)}, \quad y=\frac{(\varphi+2)^2(\varphi-3)}{\varphi^2(\varphi+1)},$$

whence also

$$\frac{dy}{dx}=\tfrac{1}{2}\varphi(\varphi+2).$$

279. The curve may be traced from these equations (see Plate, fig 2, where the bicorn is delineated along with a cubic curve afterwards referred to): as $\varphi$ extends from an indefinitely small positive value $\epsilon$ through infinity to $-1-\epsilon$, we have the upper branch of the curve, viz.

$\varphi=\epsilon$, gives $x=\infty$, $y=-\infty$, point at infinity, the tangent being horizontal.

$\varphi=\infty$, gives $x=-1$, $y=+$, the node-cusp, tangent parallel to axis of $y$.

$\varphi=-2$, gives $x=0$, $y=0$, the tangent at this point being the axis of $x$.

$\varphi=-1-\epsilon$, gives $x=\infty$, $y=+$, point at infinity along the asymptote.

And as $\varphi$ extends from $x=-1+\epsilon$ to $x=-\epsilon$, we have the lower branch, viz.

$\varphi=-1+\epsilon$, gives $x=-\infty$, $y=-\infty$, point at infinity along the asymptote

$\phi = -\frac{3}{4}$, $x = -76\frac{23}{27}$, $y = -41\frac{2}{3}$; the cusp, shown in the figure out of its proper position (observe that for $x = -76\frac{23}{27}$, we have for the asymptote $y = -40\frac{23}{27}$, so that the distance below the asymptote is $= \frac{10}{27}$; Professor SYLVESTER's value $y = -25$ for the ordinate of the cusp is an obvious error of calculation).

$\phi = -\epsilon$, gives $x = -\infty$, $y = -\infty$, point at infinity, the tangent being horizontal.

The class of the curve is $= 4$.

280. The node-cusp counts as a node, a cusp, an inflexion, and a double tangent; the node-cusp absorbs therefore $(6+8+1=)$ 15 inflexions, and the other cusp 8 inflexions; there remains therefore $(24-15-8=)$ 1 inflexion, viz. this is the inflexion at infinity, having the line infinity for tangent; there is not, besides the tangent at the node-cusp, any other double tangent of the curve.

281. The form of the Bicorn, so far as it is material for the discussion, is also shown in the Plate, fig 3, and it thereby appears that it divides the plane into three regions; viz. these are the regions PQR and S, for each of which $\phi(x, y)$ is $= -$, and the region TU, for which $\phi(x, y)$ is $= +$; that is, for PQR and S we must have $J = -$, and for TU we must have $J = +$. Hence in connexion with the bicorn, considering the line $y = 0$, we have the six regions P, Q, R, S, T, U. It has just been seen that for P, Q, R, S we have $J = -$, and for T, U we have $J = +$; and the sign of J being given, the equations $x = \frac{2^{11}L - J^3}{J^3}$, $y = \frac{D}{J^9}$, then fix for the several regions the signs of $2^{11}L - J^3$ and D, as shown in the subjoined Table; by what precedes each of the six regions has a determinate character, which for R, S, and U (since here D is $= -$) is at once seen to be $3r+2i$, and which, as will presently appear, is ascertained to be $5r$ for P and $r+4i$ for Q and T.

282. We have thus the Table

$$\left.\begin{array}{llll} \text{P}, & D = +, & J = -, & 2^{11}L - J^3 = + \end{array}\right\} 5r,$$

$$\left.\begin{array}{llll} \text{Q}, & D = +, & J = -, & 2^{11}L - J^3 = - \\ \text{T}, & D = +, & J = +, & 2^{11}L - J^3 = \pm \end{array}\right\} r+4i,$$

$$\left.\begin{array}{llll} \text{R}, & D = -, & J = -, & 2^{11}L - J^3 = \pm \\ \text{S}, & D = -, & J = -, & 2^{11}L - J^3 = + \\ \text{U}, & D = -, & J = +, & 2^{11}L - J^3 = \pm \end{array}\right\} 3r+2i;$$

so that we have the kingdom $5r$ consisting of the single region P, the kingdom $r+4i$ consisting of the regions Q and T, and the kingdom $3r+2i$ consisting of the regions R, S, and U.

283. For a given equation if D is $= -$, the character is $3r+2i$; if $D = +$, $J = +$, the character is $r+4i$; if $D = +$, $J = -$, then, according as $2^{11}L - J^3$ is $= +$ or is $= -$, the character is $5r$ or $r+4i$. But in the last case the distinction between the characters $5r$ and $r+4i$ may be presented in a more general form, involving a parameter $\mu$, arbitrary between certain limits. In fact drawing upwards from the origin, as in Plate, fig. 3, the lines $x-2y=0$ and $x+y=0$, and between them any line whatever $x+\mu y=0$, the point

$(x, y)$, assumed to lie in the region P or Q, will lie in the one or the other region according as it lies on the one side or the other side of the line in question, viz. in the region P if $x+\mu y$ is $=-$, in the region Q if $x+\mu y$ is $=+$. But we have

$$x+\mu y=\frac{2^{11}\mathrm{L}-\mathrm{J}^3+\mu\mathrm{JD}}{\mathrm{J}^3},$$

and J being by supposition negative, the sign of $2^{11}\mathrm{L}-\mathrm{J}^3+\mu\mathrm{JD}$ is opposite to that of $x+\mu y$. The region is thus P or Q according to the sign of $2^{11}\mathrm{L}-\mathrm{J}^3+\mu\mathrm{JD}$; and completing the enunciation, we have, finally, the following criteria for the number of real roots of a given quintic equation, viz.

If $\mathrm{D}=-$, the character is $3r+2i$,
If $\mathrm{D}=+$, $\mathrm{J}=+$, then it is $r+4i$.

But if $\mathrm{D}=+$, $\mathrm{J}=-$, then $\mu$ being any number at pleasure between the limits $+1$ and $-2$, both inclusive, if

$2^{11}\mathrm{L}-\mathrm{J}^3+\mu\mathrm{JD}=+$, the character is $5r$,
$2^{11}\mathrm{L}-\mathrm{J}^3+\mu\mathrm{JD}=-$, „ „ „ $r+4i$.

284. The characters $5r$ of the region P and $r+4i$ of the regions Q and T may be ascertained by means of the equation $(a, 0, c, 0, e, 0)(\theta, 1)^5=0$, that is

$$\theta(a\theta^4+10c\theta^2+5e)=0;$$

there is always the real root $\theta=0$, and the equation will thus have the character $5r$ or $r+4i$ according as the reduced equation $a\theta^4+10c\theta^2+5e=0$ has the character $4r$ or $4i$. It is clear that $(a, e)$ must have the same sign, for otherwise $\theta^2$ would have two real values, one positive, the other negative, and the character would be $2r+2i$. And $(a, e)$ having the same sign, then the character will be $4r$, if $\theta^2$ has two real positive values, that is, if $ae-5c^2$ is $=-$, and the sign of $c$ be opposite to that of $a$ and $e$, or, what is the same thing, if $ce$ be $=-$; but if these two conditions are not satisfied, then the values of $\theta^2$ will be imaginary, or else real and negative, and in either case the character will be $4r$.

285. Now, for the equation in question, putting in the Tables $b=d=f=0$, we find

$$\mathrm{D}=256\ ae^3(ae-5c^2)^3,$$
$$\mathrm{J}=\ 16\ ce\ (ae+3c^2),$$
$$2^{11}\mathrm{L}-\mathrm{J}^3=\ 2^{12}\ ce^3\{2(ae-\ c^2)^4-c^2(ae+3c^2)^3\}$$
$$=\ 2^{13}\ ce^3(ae-5c^2)(2a^3e^3+a^2c^2e^2+8ac^4e+5c^6).$$

We have by supposition $\mathrm{D}=+$, that is, $ae=+$; hence J has the same sign as $ce$; whence if $\mathrm{J}=+$, then also $ce=+$, and the character is $4i$; that is the character of the region T is $r+4i$. But if $\mathrm{J}=-$, then also $ce=-$. But $ae$ being $=+$, the sign of $2^{11}\mathrm{L}-\mathrm{J}^3$ is the same as that of $ce(ae-5c^2)$, and therefore the opposite of that of $ae-5c^2$: hence $\mathrm{D}=+$, $\mathrm{J}=-$, the quartic equation has the character $4r$ or $4i$ according as $2^{11}\mathrm{L}-\mathrm{J}^3$ is $=+$ or $=-$. Whence the region P has the character $5r$ and the region Q the character $r+4i$; and the demonstration is thus completed.

Article Nos. 286 to 293.—HERMITE'S *new form of* TSCHIRNHAUSEN'S *transformation, and application thereof to the quintic.*

286. M. HERMITE demonstrates the general theorem, that if $f(x, y)$ be a given quantic of the $n$-th order, and $\varphi(x, y)$ any covariant thereof of the order $n-2$, then considering the equation $f(x, 1)=0$, and writing

$$z=\frac{\varphi(x, 1)}{f'_x(x, 1)}$$

(where $f'_x(x, 1)$ is the derived function of $f(x, 1)$ in regard to $x$), then eliminating $x$, we have an equation in $z$, the coefficients whereof are all of them invariants of $f(x, y)$.

287. In particular for the quintic $f(x, y)=(a, b, c, d, e, f)(x, y)^5$, if

$$\varphi_1(x, y),\ \varphi_2(x, y),\ \varphi_3(x, y),\ \varphi_4(x, y)$$

are any four covariant cubics, writing

$$z=\frac{t\varphi_1(x, 1)+u\varphi_2(x, 1)+v\varphi_3(x, 1)+w\varphi_4(x, 1)}{f'_x(x, 1)}$$

(viz. the numerator is a covariant cubic involving the indeterminate coefficients $t, u, v, w$) then, in the transformed equation in $z$, the coefficients are all of them invariants of the given quintic. Conducting the investigation by means of a certain canonical form, which will be referred in the sequel, he fixes the signification of his four covariant cubics, these being respectively covariant cubics of the degrees 3, 5, 7, and 9, defined as follows; viz. starting with the form

$$-3\left|\begin{array}{cccc} y^3, & -y^2x & yx^2, & -x^3 \\ a, & b, & c, & d \\ b, & c, & d, & e \\ c, & d, & e, & f \end{array}\right|,$$

$=-3$ No. 16,

$=-3(\mathrm{A, B, C, D})(x, y)^3$, or $(-3\mathrm{A}, -\mathrm{B}, -\mathrm{C}, -3\mathrm{D})(x, y)^3$, suppose,

and considering also the quadric covariant

$$(\alpha, \beta, \gamma)(x, y)^2, = \text{No. } 14,$$

then $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ are derived from the form

$$(\mathrm{A, B, C, D})(\zeta x-\eta(\beta x+2\gamma y), \zeta y+\eta(2\alpha x+\beta y))^3,$$

viz. we have

$$\begin{aligned} \varphi_1(x, y) &= -3(\mathrm{A, B, C, D})(x, y)^3, \\ \varphi_2(x, y) &= +3(\mathrm{A, B, C, D})(x, y)^2(-\beta x-2\gamma y,\ 2\alpha x+\beta y), \\ \{\varphi_3(x, y)\} &= -3(\mathrm{A, B, C, D})(x, y)\,(-\beta x-2\gamma y,\ 2\alpha x+\beta y)^2, \\ \{\varphi_4(x, y)\} &= +3(\mathrm{A, B, C, D})\quad\ (-\beta x-2\gamma y,\ 2\alpha x+\beta y)^3, \end{aligned}$$

where $\{\varphi_3(x, y)\}$ and $\{\varphi_4(x, y)\}$ are the functions originally called by him $\varphi_3(x, y)$ and

$\varphi_4(x, y)$: those ultimately so called by him are

$$*\varphi_3(x, y) = 4\{\varphi_3(x, y)\} + \mathrm{J}\varphi_1(x, y) \quad (\mathrm{J} = \text{No. } 19),$$

$$\varphi_4(x, y) = 4\{\varphi_4(x, y)\} + 3\mathrm{J}\varphi_3(x, y) + 96\psi_1(x, y),$$

where $\psi_1(x, y)$ is the cubicovariant $(-27\mathrm{A}^2\mathrm{D} + 9\mathrm{ABC} - 2\mathrm{B}^3, \ldots)(x, y)^3$ of $\varphi_1(x, y)$, $= (-3\mathrm{A}, -\mathrm{B}, -\mathrm{C}, -3\mathrm{D})(x, y)^3$, *ut suprà*.

The covariant $\varphi_3(x, y)$ has the property that if the given quintic $(a, \ldots)(x, y)^5$ contains a square factor $(lx+my)^2$, then $\varphi_3(x, y)$ contains the factor $lx+my$: $\{\varphi_3(x, y)\}$ and $\{\varphi_4(x, y)\}$ are covariants not possessing the property in question, and they were for this reason replaced by $\varphi_3(x, y)$ and $\varphi_4(x, y)$ which possess it, viz. $\varphi_3(x, y)$ contains the factor $lx+my$, and $\varphi_4(x, y)$ contains $(lx+my)^3$, being thus a perfect cube when the given quintic contains a square factor.

288. The covariants $\varphi_1(x, y)$ and $\varphi_2(x, y)$ are included in my Tables, viz. we have

$$\varphi_1(x, y) = -3\text{No. } 16,$$

$$\varphi_2(x, y) = - \text{ No. } 23$$

(observe that in No. 23 the first coefficient vanishes if $a=0$, $b=0$, which is the property just referred to of $\varphi_3(x, y)$); the other two covariants, as being of the degree 7 and 9, are not included in my Tables, but I have calculated the leading coefficients of these covariants respectively, viz.

Table No. 85 gives leading coefficient (or that of $x^3$) in $\varphi_3(x, y)$, and

Table No. 86 gives leading coefficient (or that of $x^3$) in $\varphi_4(x, y)$.

The coefficients in question vanish for $a=0$, $b=0$, that is, $\varphi_3(x, y)$ and $\varphi_4(x, y)$ then each of them contain the factor $y$; if the remaining coefficients of $\varphi_4(x, y)$ were calculated, it should then appear that for $a=0$, $b=0$, those of $x^2y$, $xy^2$ would also vanish, and thus that $\varphi_4(x, y)$ would be a mere constant multiple of $y^3$.

Table No. 85.

| | | | |
|---|---|---|---|
| $a^3cef^2$ + 1 | $a^2b^2ef^2$ − 1 | $ab^3df^2$ + 64 | $b^4cf^2$ ∞ |
| $a^3d^2f^2$ +15 | $a^2bcdf^2$ − 94 | $ab^3e^2f$ − 54 | $b^4def$ − 144 |
| $a^3de^2f$ −32 | $a^2bce^2f$ + 86 | $ab^2c^2f^2$ − 48 | $b^4e^3$ + 135 |
| $a^3e^4$ +16 | $a^2bd^2ef$ +106 | $ab^2cdef$ + 184 | $b^3c^2ef$ + 108 |
| | $a^2bde^3$ − 96 | $ab^2ce^3$ − 135 | $b^3cd^2f$ + 288 |
| | $a^2c^2f^2$ + 63 | $ab^2d^2f$ − 272 | $b^3cde^2$ − 450 |
| | $a^2c^2def$ −188 | $ab^2d^2e^2$ + 243 | $b^3d^3e$ + 80 |
| | $a^2ce^3$ + 32 | $abc^3ef$ − 66 | $b^2c^3df$ − 360 |
| | $a^2cd^2f$ + 60 | $abc^2d^2f$ + 212 | $b^2c^2e^2$ + 135 |
| | $a^2cd^2e^2$ + 68 | $abc^2de^2$ + 148 | $b^2c^2d^2e$ + 360 |
| | $a^2d^4e$ − 36 | $abcd^3e$ − 412 | $b^2cd^4$ − 160 |
| | | $abd^5$ + 144 | $bc^3f$ + 108 |
| | | $ac^4df$ − 36 | $bc^2de$ − 180 |
| | | $ac^3e^2$ − 48 | $bc^2d^3$ + 80 |
| | | $ac^2d^3e$ + 124 | |
| | | $ac^2d^4$ − 48 | |
| +32 | +415 | +1119 | +1294 |

* M. Hermite, p. 17, has erroneously written $\varphi_3(x, y) + 4\mathrm{A}\varphi_1(x, y)$, instead of $4\varphi_3(x, y) + \mathrm{A}\varphi_1(x, y)$; the latter expression is that which he really makes use of, and the formula in the text is correct.

Table No. 86.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $a^4cef^3$ | + 9 | $a^3bef^3$ | − 9 | $a^2b^2df^3$ | + 120 | $ab^4cf^3$ | − 576 | $b^4f^3$ | + 192 |
| $a^4d^2f^3$ | +21 | $a^3bcdf^3$ | − 162 | $a^2b^2e^2f^3$ | − 21 | $ab^4def^3$ | + 672 | $b^4cef^3$ | − 1440 |
| $a^4def^3$ | −78 | $a^3bce^2f^3$ | + 99 | $a^2b^2c^2f^3$ | + 486 | $ab^4e^2f^3$ | − 359 | $b^4d^2f^3$ | − 192 |
| $a^4e^4f$ | +48 | $a^3bd^2ef^3$ | + 309 | $a^2b^2cdef^3$ | − 2160 | $ab^3c^2ef^3$ | + 3456 | $b^4de^2f$ | − 1080 |
| | | $a^3bde^2f$ | + 12 | $a^2b^2ce^2f$ | + 1023 | $ab^3cd^2f^3$ | − 864 | $b^4e^4$ | + 2025 |
| | | $a^3be^4$ | − 240 | $a^2b^2d^2f^3$ | + 120 | $ab^3cde^2f$ | + 2094 | $b^4c^2df^3$ | + 2592 |
| | | $a^3c^2f^3$ | − 81 | $a^2b^2d^2e^2f$ | − 1053 | $ab^3ce^4$ | − 3915 | $b^4c^2e^2f$ | + 3546 |
| | | $a^3c^2def^3$ | +1026 | $a^2b^2de^4$ | + 1314 | $ab^3d^2ef$ | + 528 | $b^4cd^2ef$ | + 5280 |
| | | $a^3c^2e^2f^3$ | − 768 | $a^2bc^2ef^3$ | − 1863 | $ab^3d^2e^3$ | − 45 | $b^4cde^3$ | −13500 |
| | | $a^3cd^2f^3$ | − 738 | $a^2bc^2d^2f^3$ | + 2538 | $ab^3c^2df^3$ | − 2592 | $b^4d^4f$ | − 4800 |
| | | $a^3cd^2e^2f$ | − 564 | $a^2bc^2de^2f$ | + 2340 | $ab^3c^2e^2f$ | − 9747 | $b^4d^3e^2$ | + 7800 |
| | | $a^3cde^4$ | +1056 | $a^2bc^2e^4$ | + 672 | $ab^3c^2d^2ef$ | − 8496 | $b^3c^4f^3$ | − 648 |
| | | $a^3d^4ef$ | + 756 | $a^2bcd^3ef$ | + 2820 | $ab^3c^2de^3$ | + 26610 | $b^3c^3def$ | −14040 |
| | | $a^3d^3e^3$ | − 696 | $a^2bcd^2e^3$ | − 7812 | $ab^3cd^4f$ | + 8544 | $b^3c^2e^3$ | + 3075 |
| | | | | $a^2bd^3f$ | − 3024 | $ab^3cd^3e$ | − 16650 | $b^3c^2d^2f$ | + 9120 |
| | | | | $a^2bd^4e^2$ | + 4572 | $ab^3d^5e$ | + 720 | $b^3c^2d^2e^2$ | +16350 |
| | | | | $a^2c^4df^3$ | − 324 | $abc^5f^3$ | + 972 | $b^3cd^4e$ | −19200 |
| | | | | $a^2c^4e^2f$ | + 3888 | $abc^4def$ | + 24048 | $b^3d^6$ | + 4800 |
| | | | | $a^2c^3d^2ef$ | − 8748 | $abc^4e^3$ | − 4464 | $b^3c^4ef$ | + 4860 |
| | | | | $a^2c^3de^3$ | − 4800 | $abc^3d^3f$ | − 15984 | $b^3c^3d^2f$ | − 3240 |
| | | | | $a^2c^3d^4f$ | + 4248 | $abc^3d^2e^3$ | − 30108 | $b^3c^3de^3$ | − 8100 |
| | | | | $a^2c^2d^3e^2$ | +14520 | $abc^3d^4e$ | + 35088 | $b^3c^2d^3e$ | + 9000 |
| | | | | $a^2cd^5e$ | −11448 | $abcd^6$ | − 8640 | $bc^3d^3$ | − 2400 |
| | | | | $a^2d^7$ | + 2592 | $ac^6ef$ | − 7776 | | |
| | | | | | | $ac^5d^2f$ | + 5184 | | |
| | | | | | | $ac^5de^2$ | + 12960 | | |
| | | | | | | $ac^4d^2e$ | − 14400 | | |
| | | | | | | $ac^3d^5$ | + 3840 | | |
| ±78 | | ±3258 | | ±41253 | | ± 124716 | | ±68640 | |

289. The equation in $z$ is of the form

$$z^5+\frac{\mathfrak{A}}{D}z^3+\frac{\mathfrak{B}}{D}z^2+\frac{\mathfrak{C}}{D}z+\frac{\mathfrak{D}}{D}=0,$$

where D is the discriminant of the quintic and $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ denote rational and integral functions of the coefficients $(a, b, c, d, e, f)$. And the covariants $\varphi_1(x, y)$, $\varphi_2(x, y)$, $\varphi_3(x, y)$, $\varphi_4(x, y)$ having the values given to them above, the actual value of $\mathfrak{A}$ is obtained as a quadric function of the indeterminates $(t, u, v, w)$, viz. this is

$$=[D_1t^2-6BDtv-D(D_1-10AB)v^2]+D[-Bu^2+2D_1uw+9(BD-10AD_1)w^2],$$

where $D_1=25AB+16C$, these quantities, and the quantity

$$N(=D_1^2-10ABD_1+9B^2D)$$

afterwards spoken of, being in the notation of the present Memoir as follows:

| | | |
|---|---|---|
| A = | J | (= No. 19), |
| B = | −K | (= − No. 25), |
| C = | 9L+JK | (= − 9 No. 29 + No. 19. No. 25), |
| D = | D | (= No. 26), |
| $D_1$ = | 9(16L − JK), | |
| N = | 1152(18L² − JKL − K³). | |

290. If by establishing two linear relations between the coefficients $(t, u, v, w)$ the equation $\mathfrak{A}=0$ can be satisfied (which in fact can be done by the solution of a quadric equation), then these quantities can be by means of the relations in question expressed as linear functions of any two of them, say of $v$ and $w$; and then the next coefficient $\mathfrak{B}$ will be a cubic function $(v, w)^3$, and the equation $\mathfrak{B}=0$ will be satisfied by means of a cubic equation $(v, w)^3=0$, that is, the transformed equation in $z$ can be by means of the solution of a quadric and a cubic equation reduced to the trinomial form

$$z^5+\frac{\mathfrak{C}}{\mathfrak{D}}z+\frac{\mathfrak{E}}{\mathfrak{D}}=0,$$

and M. HERMITE shows that the equation $\mathfrak{A}=0$ can be satisfied as above very simply, and that in two different ways, viz.

291. 1°. $\mathfrak{A}=0$ if

$$\mathrm{D}_1t^3-6\mathrm{BD}tv-(\quad \mathrm{D}_1-10\mathrm{AB}\ )v^3=0,$$
$$\mathrm{B}u^3-2\mathrm{D}_1uw-(9\mathrm{BD}-10\mathrm{AD}_1)w^3=0,$$

that is, N denoting as above, if

$$t=\frac{3\mathrm{BD}+\sqrt{\mathrm{ND}}}{\mathrm{D}_1}v,\ u=\frac{\mathrm{D}_1+\sqrt{\mathrm{N}}}{\mathrm{B}}w.$$

292. 2°. Writing the expression for $\mathfrak{A}$ in the form

$$\mathrm{D}_1(t^3-\mathrm{D}v^3+2\mathrm{D}uw-10\mathrm{AD}w^3)+\mathrm{BD}(10\mathrm{A}v^3-6tv-u^3+9\mathrm{D}w^3),$$

then $\mathfrak{A}=0$, if

$$t^3-\mathrm{D}v^3+2\mathrm{D}uw-10\mathrm{AD}w^3=0,$$
$$10\mathrm{A}v^3-6tv-u^3+9\ \mathrm{D}w^3=0.$$

These equations, writing therein

$$t=\frac{1}{\sqrt{2}}\sqrt{\mathrm{D}}\mathrm{T},\ u=\mathrm{U}+5\mathrm{AW},\ v=\frac{1}{\sqrt{2}}\mathrm{V},\ w=\mathrm{W},$$

become

$$\mathrm{T}^3-\mathrm{V}^3+4\mathrm{UW}=0,$$
$$-5\mathrm{AV}^3+3\sqrt{\mathrm{D}}\mathrm{TV}+\mathrm{U}^3+10\mathrm{AUW}+(25\mathrm{A}^2-9\mathrm{D})\mathrm{W}^3=0,$$

the first of which is satisfied by the values

$$\mathrm{T}=\varrho\mathrm{W}-\frac{1}{\varrho}\mathrm{U},\ \mathrm{V}=\varrho\mathrm{W}+\frac{1}{\varrho}\mathrm{U};$$

and then substituting for T and V, the second equation will be also satisfied if only

$$\varrho^3=5\mathrm{A}+3\sqrt{\mathrm{D}}.$$

Article Nos. 293 to 295.—HERMITE'S *application of the foregoing results to the determination of the Character of the quintic equation.*

293. By considerations relating to the form

$$\frac{1}{\mathrm{D}}\Big\{[\mathrm{D}_1t^3-6\mathrm{BD}tv-\mathrm{D}(\mathrm{D}_1-10\mathrm{AB})v^3]+\mathrm{D}[-\mathrm{B}u^3+2\mathrm{D}_1uw+9\mathrm{BD}-10\mathrm{AD}_1w^3]\Big\},$$

M. HERMITE obtains criteria for the character of the quintic equation $f(x, 1)=0$.

294. If $D=-$, the character is $3r+2i$, but if $D=+$, then expressing the foregoing form as a sum of four squares affected with positive or negative coefficients, the character will be $5r$ or $2+4i$, according as the coefficients are all positive, or are two positive and two negative. Whence, if N denote as above, then for

$$D=+,\ N=-,\ D_1=+,\ B=-,\ \text{character is } 5r,$$

and

$$\left.\begin{array}{l} D=+,\ N=-,\ BD_1=+ \\ D=+,\ N=+ \end{array}\right\} \quad \text{character is } r+4i;$$

and further, the combination $D=+$, $N=-$, $D_1=-$, $B=+$ cannot arise (HERMITE'S first set of criteria).

295. Again, from the equivalent form

$$\frac{1}{D}\left\{D_1(t^2-Dv^2+2Duw-10Aw^2)+BD(10Av^2-6tv-u^2+9Dw^2)\right\},$$

which, if $\omega$, $\omega'$ are the roots of the equation $9\theta^2-10A\theta+D=0$, is

$$=\frac{1}{D}\left\{\frac{D_1\omega-BD}{\omega-\omega'}\left[(t-3\omega v)^2-\omega'\left(u-\frac{D}{\omega}w\right)^2\right]+\frac{D_1\omega'-BD}{\omega'-\omega}\left[(t-3\omega v)^2-\omega\left(u-\frac{D}{\omega}w\right)\right]^2\right\};$$

then by similar reasoning it is concluded that

$$\left.\begin{array}{l} D=+,\ 25A^2-9D=+,\ A=-,\ N=-,\ \text{character is } 5r, \\ D=+,\ 25A^2-9D=+,\ A=-,\ N=+, \\ D=+,\ 25A^2-9D=+,\ A=+, \\ D=+,\ 25A^2-9D=-, \end{array}\right\} \quad ,, \quad r+4i.$$

(HERMITE'S second set of criteria).

Article Nos. 296 to 303.—*Comparison with the Criteria* No. 283: *the Nodal Cubic.*

296. For the discussion of HERMITE'S results, it is to be observed that in the notation of the present Memoir we have

$$\begin{aligned} A &= J, \\ B &= -K = -\tfrac{1}{12}(J^3-D), \\ D &= D, \\ D_1 &= 16L-JK = \tfrac{1}{12}(2^{11}L-J^3+JD), \\ N &= 18L^2-JKL-K^3 \\ &= \tfrac{1}{2^{21}}\{3^3.2^{22}L^2-14JL(J^3-D)-(J^3-D)^3\}, \end{aligned}$$

or, putting as above,

$$x=\frac{2^{11}L-J^3}{J^3},\ y=\frac{D}{J^3},\ \text{and } \therefore\ 1+x=\frac{2^{11}L}{J^3},\ 1-y=\frac{J^6-D}{J^3},$$

we have

$$\begin{aligned}
A &= J,\\
B &= \tfrac{1}{12}J^3(y-1),\\
D &= J^3y,\\
D_1 &= \tfrac{1}{12}J^3(x+y),\\
N &= \tfrac{1}{2^{11}}J^6\{9(1+x)^3-8(1+x)(1-y)-(1-y)^3\},\\
&= \tfrac{1}{2^{11}}J^6.\{y^3-3y^2+8xy+9x^2+11y+10x\}.
\end{aligned}$$

It thus becomes necessary to consider the curve

$$\psi(x, y)=y^3-3y^2+8xy+9x^2+11y+10x=0,$$

the equation whereof may also be written

$$9x+4y+5=(y-1)\sqrt{25-9y}.$$

297. This is a cubic curve, viz. it is a divergent parabola having for axis the line $9x+4y+5=0$, and its ordinates parallel to the axis of $x$; and having moreover a node at the point $x=-1$, $y=+1$, that is, at the node-cusp of the bicorn; the curve is thus a nodal cubic; we may trace it directly from the equation, but it is to be noticed that *quà* nodal cubic it is a unicursal curve; the coordinates $x$, $y$ are therefore rationally expressible in terms of a parameter $\psi$; and it is easy to see that we in fact have

$$\begin{aligned}
81(x+1) &= \psi^2(\psi-8),\\
9(y-1) &= -\psi(\psi-8),
\end{aligned}$$

whence also

$$\frac{dy}{dx}=\frac{-18(\psi-4)}{\psi(3\psi-16)}.$$

298. We see that

$\psi=\infty$, gives $x=\infty$, $y=-\infty$, point at infinity, the direction of the curve parallel to axis of $x$.

$\psi=9$, „ $x=0$, $y=0$, the origin.

$\psi=8$, „ $x=-1$, $y=+1$, the node, tangent parallel to axis of $y$.

$\psi=\frac{16}{3}$, „ $x=\frac{4352}{2187}$, $y=\frac{304}{81}$, tangent parallel to the axis of $y$.

$\psi=4$, „ $x=-\frac{145}{81}$, $y=\frac{25}{9}$, tangent parallel to axis of $x$.

$\psi=0$, „ $x=-1$, $y=+1$, the node.

$\psi=-1$, „ $x=-\frac{10}{9}$, $y=0$.

$\psi=-16$, „ $x=-76\frac{23}{27}$, $y=-41\frac{2}{3}$, the cusp of the bicorn.

$\psi=-\infty$, „ $x=-\infty$, $y=-\infty$, point at infinity, direction of curve parallel to axis of $x$.

299. The Nodal Cubic is shown along with the Bicorn, Plate, fig. 2; it consists of one

continuous line, passing from a point at infinity, through the cusp of the bicorn, on to the node-cusp, then forming a loop so as to return to the node-cusp, again meeting the bicorn at the origin, and finally passing off to a point at infinity, the initial and ultimate directions of the curve being parallel to the axis of $x$.

300. It may be remarked that, inasmuch as one of the branches of the cubic touches the bicorn at the node-cusp, the node-cusp counts as $(4+2=)$ 6 intersections; the intersections of the cubic with the bicorn are therefore the cusp, the node-cusp, and the origin, counting together as $(2+6+1=)$ 9 intersections, and besides these the point at infinity on the axis of $x$, counting as 3 intersections. This may be verified by substituting in the equation of the cubic the bicorn $\phi$-values of $x$ and $y$. We must, however, to include all the proper factors, first write the equation of the cubic in the homogeneous form

$$(9x+8y+5z)^3z-(y-z)^3(25z-9y)=0,$$

and herein substitute the values

$$x : y : z=-(\phi+2)(\phi^3-\phi^2+2\phi-4) : (\phi+2)^3(\phi-3)\phi : (\phi+1)\phi^3;$$

the result is found to be

$$\phi^3\{(\phi+1)(4\phi^3+6\phi-9)^3-(2\phi+3)^3(4\phi^3+4\phi^2+18\phi+27)\}=0,$$

that is

$$-9\phi^3(\phi+2)(4\phi+3)^3=0;$$

and considering this as an equation of the order 12, the roots are $\phi=0$, 3 times, $\phi=-2$, 1 time; $\phi=-\frac{3}{4}$, 2 times, and $\phi=\infty$, 6 times.

301. The cubic curve divides the plane into 3 regions, which may be called respectively the loop, the antiloop, and the extra cubic; for a point within the loop or antiloop, $\psi(x, y)$ is $=--$, for a point in the extra cubic $\psi(x, y)$ is $=+$. If in conjunction with the cubic we consider the discriminatrix, or line $y=0$, then we have in all six regions, viz. $y$ being $=+$, three which may be called the loop, the triangle, and the upper region; and $y$ being $=-$, three which may be called the right, left, and under regions respectively; the triangle and the under region form together the antiloop.

302. It is now easy to discuss HERMITE'S two sets of criteria; the first set becomes

$$\begin{array}{llll} y=+, & y-1=-, & \mathrm{J}(x+y)=+, & \psi(x, y)=-, \text{ character } 5r \\ y=+, & & \mathrm{J}(y-1)(x+y)=+, & \psi(x, y)=- \\ y=+, & & & \psi(x, y)=+ \end{array} \Big\} \text{character } r+4i,$$

$$y=+, \quad y-1=+, \quad \mathrm{J}(x+y)=-, \quad \psi(x, y)=-, \text{ cannot exist.}$$

Referring to the Plate, fig. 4, which shows a portion of the cubic and the bicorn, then 1° the conditions $y=+$, $\psi(x, y)=-$ imply that the point $(x, y)$ is within the loop or within the triangle of the cubic; the condition $y-1=-$ brings it to be within the triangle, and for any point within the triangle we have $x+y=-$, whence also the condition $\mathrm{J}(x+y)=+$ becomes $\mathrm{J}=-$; hence the conditions amount to $\mathrm{J}=-$, $(x, y)$ within the triangle; but by the general theory $(x, y)$, being within the triangle, that is, in the region P or T, if $\mathrm{J}=-$, will of necessity be within the region P; so that the condi-

tions give $J=-$, $(x, y)$ within the region P; the corresponding character being $5r$, which is right.

2°. $y=+$, $\psi(x, y)=-$, the point $(x, y)$ must be within the loop, or within the triangle; if $(x, y)$ is within the loop, then $y-1=+$, $x+y=1$, and the condition $J(y-1)(x+y)=+$ becomes $J=-$, that is, we have $J=-$ and $(x, y)$ within the loop, that is, in the region T. And again, if $(x, y)$ be within the triangle, then $y-1=-$, $x+y=+$, and the condition $J(y-1)(x+y)=+$ still gives $J=-$; but $J=-$, and $(x, y)$ within the triangle, that is, in the region T or P, will of necessity be in the region T; so that in either case we have $J=-$, $(x, y)$ in the region T, which agrees with the character $r+4i$.

3°. $y=+$, $\psi(x, y)=+$, $(x, y)$ is in the upper region, that is, in the region Q or T; if $(x, y)$ is in the region Q, then of necessity $J=-$, and if in the region T, then of necessity $J=+$, that is, we have

$J=-$, $(x, y)$ in the region Q, or
$J=+$, $(x, y)$ in the region P,

which agrees with the character $r+4i$.

And it is to be observed that the portions of T under 2° and 3° respectively make up the whole of the region T, and that 3° relates to the whole of the region Q, so that the conditions allow the point $(x, y)$ to be anywhere in Q or T, which is right.

4°. $y=+$, $\psi(x, y)=-$, $(x, y)$ is in the loop or the triangle, and then $y-1=+$ implies that it is in the loop, whence $x+y=+$, and the condition $J(x+y)=-$ becomes $J=-$; we should therefore if the combination existed have $J=-$, $(x, y)$ within the loop, that is, in the region T; but this is impossible.

303. HERMITE'S second set of criteria are

$y=+$, $\frac{2}{9}5-y=+$, $J=-$, $\psi(x, y)=-$, character $5r$.

$y=+$, $\frac{2}{9}5-y=+$, $J=-$, $\psi(x, y)=+$
$y=+$, $\frac{2}{9}5-y=+$, $J=+$
$y=+$, $\frac{2}{9}5-y=-$,
} character $r+4i$.

1°. If $y=+$, $\psi(x, y)=-$, then the point $(x, y)$ must be situate within the loop or within the triangle; and recollecting that at the highest point of the loop we have $y=\frac{2}{9}5$, the condition $\frac{2}{9}5-y=+$ is satisfied for every such point, and may therefore be omitted. The conditions therefore are $J=-$, $(x, y)$ within the loop, that is, in the region T, or within the triangle, that is, in the region P or the region T; but for any point of T the general theory gives $J=+$, and the conditions are therefore $J=-$, $(x, y)$ within the region P; which agrees with the character $5r$.

2°. $y=+$, $\psi(x, y)=+$, that is, $(x, y)$ is within the upper region, that is, in the region Q or T; and $\frac{2}{9}5-y=+$, $(x, y)$ will be within the portions of Q and T which lie beneath the line $y=\frac{2}{9}5$; but $J=-$, and therefore $(x, y)$ cannot lie in the region T; hence the conditions amount to $J=-$, $(x, y)$ within that portion which lies beneath the line $y=\frac{2}{9}5$ of the region Q.

3°. $y=+$, $\frac{3}{5}-y=+$, $(x, y)$ lies beneath the line $y=\frac{3}{5}$, viz. in one of the regions P, Q or T; but $J=+$, $(x, y)$ cannot lie in the region P or Q; hence the conditions give $J=+$, $(x, y)$ within the portion which lies beneath the line $y=\frac{3}{5}$ of the region T.

4°. $y=+$, $\frac{3}{5}-y=-$, that is, $(x, y)$ lies above the line $y=\frac{3}{5}$, and therefore in one of the regions T or Q; and by the general theory, according as $(x, y)$ lies in T or in Q, we shall have $J=+$ or $J=-$, hence the conditions give

$J=-$, $(x, y)$ within the portion which lies above the line $y=\frac{3}{5}$, of the region Q.

$J=+$, $(x, y)$ within the portion which lies above the line $y=\frac{3}{5}$, of the region T.

2°, 3°, and 4°, each of them agree with the character $r+4i$, and together they imply $J=-$, $(x, y)$ any where in the region Q, or else $J=+$, $(x, y)$ anywhere in the region T; which is right.

Article Nos. 304 to 307.—HERMITE'S *third set of Criteria; comparison with* No. 283, *and remarks.*

304. In the concluding portion of his memoir, M. HERMITE obtains a third set of criteria for the character of a quintic equation; this is found by means of the equation for the function

$$a^4(\theta_0-\theta_1)(\theta_1-\theta_2)(\theta_2-\theta_3)(\theta_3-\theta_4)(\theta_4-\theta_0)$$

of the roots $(\theta_0, \theta_1, \theta_2, \theta_3, \theta_4)$ of the given quintic equation $(a, b, c, d, e, f)(\theta, 1)^5=0$. The function in question has 12 pairs of equal and opposite values, or it is determined by an equation of the form $(u^2, 1)^{12}=0$, which equation is decomposable, not rationally but by the adjunction thereto of the square root of the discriminant, into two equations of the form $(u^2, 1)^6=0$; viz. one of these is

$$\begin{aligned}
&u^{12}\\
&+u^{10}(\mathrm{a}+3\sqrt{\Delta})\\
&+u^8\left[\tfrac{1}{4}(\mathrm{a}-\sqrt{\Delta})^2+\Delta\right]\\
&-u^6\ \mathrm{d}\\
&+u^4\left[\tfrac{1}{4}(\mathrm{a}+\sqrt{\Delta})^2+\Delta\right]\Delta\\
&+u^2(\mathrm{a}-3\sqrt{\Delta})\Delta^2\\
&+\ \Delta^3=0,
\end{aligned}$$

and the other is of course derived from it by reversing the sign of $\sqrt{\Delta}$. I have in the equation written (a, d) instead of HERMITE'S writing capitals A, D; the sign $-$ of the term in $u^6$ instead of $+$, as printed in his memoir, is a correction communicated to me by himself. The signification of the symbols is in the author's notation

$$\mathrm{a}=5^4\mathrm{A},$$
$$\mathrm{d}=4.5^9(\mathrm{AD}-\tfrac{8}{9}\mathrm{D}_1),$$
$$\Delta=5^8\mathrm{D},$$

whence, in the notation of the present memoir, the expressions of these symbols are

$$\begin{aligned} a&= 5^4 J, \\ d&= -\tfrac{1}{2}5^{10}(2^{11}L - J^3 - \tfrac{3}{5}JD), \\ \Delta&= 5^5 D. \end{aligned}$$

305. From the equation in $u$, taking therein the radical $\sqrt{\Delta}$ as positive, M. HERMITE obtains ($d<0$ a mistake for $d>0$) the following as the necessary and sufficient conditions for the reality of all the roots,

$$\Delta=+,\ a+3\sqrt{\Delta}=-,\ d=+,\ \text{character } 5r$$

(HERMITE's third set of criteria).

306. It is clear that $a+3\sqrt{\Delta}=-$ is equivalent to ($a=-$ and $a^2-9\Delta=+$), and we have $a^2-9\Delta=5^8(125J^3-9D)$, so that these conditions for the character $5r$ are

$$D=+,\ J=-,\ 125J^3-9D=+,\ 2^{11}L-J^3-\tfrac{3}{5}JD=+.$$

Now, writing as above,

$$x=\frac{2^{11}L-J^3}{J^3},\quad y=\frac{D}{J^2},$$

these are $y=+$, $J=-$, $\frac{125}{9}-y=+$, $x-\frac{3}{5}y=-$; the conditions $y=+$, $J=-$ imply that $(x, y)$ is in the region P or the region Q; and the condition $x-\frac{3}{5}y=-$ (observe the line $x-\frac{3}{5}y=0$ lies between the lines $x+y=0$, $x-2y=0$, and so does not cut either the region P or the region Q) restricts $(x, y)$ to the region P; and for every point of P $y$ is at most $=1$, and the condition $\frac{125}{9}-y=+$ is of course satisfied. The condition, $125J^3-9D=+$, is thus wholly unnecessary, and omitting it, the conditions are

$$D=+,\ J=-,\ 2^{11}L-J^3-\tfrac{3}{5}JD=0,\ \text{character } 5r,$$

which, $-\frac{3}{5}$ being an admissible value of $\mu$, agrees with the result *ante*, No. 283.

307. It may be remarked in passing that if 12345 is a function of the roots $(x_1, x_2, x_3, x_4, x_5)$ of a quintic equation, which function is such that it remains unaltered by the cyclical permutation 12345 into 23451, and also by the reversal (12345 into 15432) of the order of the roots, so that the function has in fact the 12 values

$$\begin{aligned} \alpha_1&=12345, & \beta_1&=24135, \\ \alpha_2&=13425, & \beta_2&=32145, \\ \alpha_3&=14235, & \beta_3&=43125, \\ \alpha_4&=21435, & \beta_4&=13245, \\ \alpha_5&=31245, & \beta_5&=14325, \\ \alpha_6&=41325, & \beta_6&=12435, \end{aligned}$$

then $\phi(\alpha, \beta)$ being any unsymmetrical function of $(\alpha, \beta)$, the equation having for its roots the six values of $\phi(\alpha, \beta)$ (viz. $\phi(\alpha_1, \beta_1)$, $\phi(\alpha_2, \beta_2) \ldots \phi(\alpha_6, \beta_6)$) can be expressed ratio-

nally in terms of the coefficients of the given quintic equation and of the square root of the discriminant of this equation. In fact, $v$ being arbitrary, write

$$L=\Pi_\theta\{v-\phi(\alpha, \beta)\}, \quad M=\Pi_\theta\{v-\phi(\beta, \alpha)\},$$

then the interchange of any two roots of the quintic produces merely an interchange of the quantities L, M; that is,

$$L+M \text{ and } (L-M)\div\zeta^5(x_1, x_2, x_3, x_4, x_5)$$

are each of them unaltered by the interchange of any two roots, and are consequently expressible as rational functions of the coefficients; or observing that $\zeta^5(x_1, x_2, x_3, x_4, x_5)$ is a multiple of $\sqrt{D}$, we have L a function of the form $P+Q\sqrt{D}$; the equation $L=0$, the roots whereof are $v=\phi(\alpha_1, \beta_1) \ldots v=\phi(\alpha_6, \beta_6)$, is consequently an equation of the form $P+Q\sqrt{D}=0$, viz. it is a sextic equation $(*)(v, 1)^6=0$, the coefficients of which are functions of the form in question. Hence in particular

$$u^3=12345=(x_1-x_2)^2(x_3-x_5)^2(x_3-x_4)^2(x_4-x_5)^2(x_5-x_1)^2$$

is determined as above by an equation $(*)(u^3, 1)^6=0$. Another instance of such an equation is given by my memoir "On a New Auxiliary Equation in the Theory of Equations of the Fifth Order," Phil. Trans. t. 151 (1861), pp. 263–276.

Article Nos. 308 to 317.—HERMITE'S *Canonical form of the quintic.*

308. It was remarked that M. HERMITE'S investigations are conducted by means of a canonical form, viz. if A (=J, = No. 19 as above) be the quartinvariant of the given quintic $(a, b, c, d, e, f)(x, y)^5$, then he in fact finds (X, Y) linear functions of $(x, y)$ such that we have

$$(a, b, c, d, e, f)(x, y)^5=(\lambda, \mu, \sqrt{k}, \sqrt{\bar{k}}, \mu', \lambda')(X, Y)^5$$

(viz. in the transformed form the two mean coefficients are equal; this is a convenient assumption made in order to render the transformation completely definite, rather than an absolutely necessary one); and where moreover the quadricovariant (Table No. 14) of the transformed form is

$$=\sqrt{A}XY,$$

or, what is the same thing, the coefficients $(\lambda, \mu, \sqrt{k}, \sqrt{\bar{k}}, \mu', \lambda)$ of the transformed form are connected by the relations

$$\left.\begin{aligned} \lambda\mu'-4\mu\sqrt{k}+3k&=0,\\ \lambda'\mu-4\mu'\sqrt{k}+3k&=0,\\ \lambda\lambda'-3\mu\mu' \quad +2k&=\sqrt{A}, \end{aligned}\right\}$$

the advantage is a great simplicity in the forms of the several covariants, which simplicity arises in a great measure from the existence of the very simple covariant operator $\frac{d}{dX}\cdot\frac{d}{dY}$ (viz. operating therewith on any covariant we obtain again a covariant).

309. Reversing the order of the several steps, the theory of M. HERMITE's transformation may be established as follows:—

Starting from the quintic

$$(a, b, c, d, e, f)(x, y)^5,$$

and considering the quadricovariant thereof

$$(\alpha, \beta, \gamma)(x, y)^2 \quad \ldots\ldots\ldots \quad \text{Tab. No. 14}$$

($(\alpha, \beta, \gamma)$ are of the degree 2), and also the linear covariant

$$Px+Qy \quad \ldots\ldots\ldots \quad \text{Tab. No. 22}$$

((P, Q) are of the degree 5), we have

$$\beta^2-4\alpha\gamma=A, \quad \ldots\ldots\ldots \quad \text{Tab. No. 19,}$$

and moreover

$$(\alpha, \beta, \gamma)(Q, -P)^2=-C,$$

viz. the expression on the left hand, which is of the degree 12, and which is obviously an invariant, is $=-C$, where C is (*ut suprà*)

$$C=9L+JK=-9 \text{ No. } 29+(\text{No. } 19)(\text{No. } 25).$$

The Jacobian of the two forms, viz.

$$\begin{vmatrix} 2\alpha x+\beta y, & \beta x+2\gamma y \\ P & , \quad Q \end{vmatrix},$$

$$=x(2\alpha Q-\beta P)+y(\beta Q-2P\gamma),$$

is a linear covariant of the degree 7, say it is

$$=P'x+Q'y,$$

and it is to be observed that the determinant $PQ'-P'Q$ of the two linear forms is $=-2(\alpha, \beta, \gamma)(Q, -P)^2$, that is, it is $=2C$.

310. Hence writing

$$T=\frac{1}{2\sqrt{C}}(Px+Qy)=\frac{1}{2\sqrt[4]{A}}(\ X+Y),$$

$$U=\frac{1}{2\sqrt{C}}(P'x+Q'y)=\frac{\sqrt[4]{A}}{2}\ (-X+Y),$$

whence also

$$X=T\sqrt[4]{A}-\frac{U}{\sqrt[4]{A}},$$

$$Y=T\sqrt[4]{A}+\frac{U}{\sqrt[4]{A}},$$

the determinant of substitution from (X, Y) to (T, U) is $=2$, that from (T, U) to $(x, y)$ is $\frac{1}{4C}\,2C$, $=\frac{1}{2}$, and consequently that from (X, Y) to $(x, y)$ is $=1$.

We have

$$AT^2-U^2=\frac{1}{4C}\left\{(\beta^2-4\alpha\gamma)(Px+Qy)^2-(P'x+Q'y)^2\right\};$$

or putting for P', Q' their values, this is $=\frac{1}{4C}$ into $4(\alpha, \beta, \gamma\text{)(Q, }-\text{P)}^2(\alpha x^2+2\beta xy+\gamma y^2)$, that is, we have

$$AT^2-U^2=\alpha x^2+\beta xy+\gamma y^2;$$

and we have also

$$AT^2-U^2=\tfrac{1}{4}\sqrt{A}[(X+Y)^2-(X-Y)^2]=\sqrt{A}XY,$$

consequently

$$\alpha x^2+\beta xy+\gamma y^2=AT^2-U^2=\sqrt{A}XY.$$

311. We have

$$x=\frac{1}{\sqrt{C}}(\ \ Q'T-QU),$$

$$y=\frac{1}{\sqrt{C}}(-P'T+PU),$$

so that, pausing a moment to consider the transformation from $(x, y)$ to (T, U), we have

$$(a, b, c, d, e, f\text{)(}x, y)^4=\frac{1}{\sqrt{C}^4}(a, b, c, d, e, f\text{)(}Q'T-QU, -P'T+PU)^4$$

$$=\frac{1}{\sqrt{C}^4}(\text{a, b, c, d, e, f)(T, U)}^4 \text{ suppose,}$$

where (a, b, c, d, e, f) are *invariants*, of the degrees 36, 34, 32, 30, 28, 26 respectively; it follows that b, d, f each of them contain as a factor the 18thic invariant I, the remaining factors being of the orders 16, 12, 8 respectively.

312. That (a, b, c, d, e, f) are invariants is almost self-evident; it may however be demonstrated as follows. Writing

$$\{y\partial_x\}=a\partial_b+2b\partial_c+3c\partial_d+4d\partial_e+5e\partial_f=\delta \text{ suppose,}$$

$$\{x\partial_y\}=5b\partial_a+4c\partial_b+3d\partial_c+2e\partial_d+f\partial_e=\delta_1 \quad ,,$$

then $Px+Qy$, $P'x+Q'y$ being covariants, we have $\delta P=0$, $\delta Q=P$, $\delta P'=0$, $\delta Q'=P'$, whence, treating T, U as constants, $\delta(Q'T-QU)=P'T-PU$, $\delta(-P'T+PU)=0$. Hence

$$\begin{aligned}\delta(a, b, c, d, e, f\text{)(}Q'T-QU, -P'T+PU)^4 \\ =5(a, b, c, d, e\text{)(}Q'T-QU, -P'T+PU)^4\cdot(-P'T+PU) \\ +5(a, b, c, d, e\text{)(} \quad ,, \quad ,, \quad )^4\cdot(\ P'T-PU) \\ +5(b, c, d, e, f\text{)(} \quad ,, \quad ,, \quad )^4 \quad .0,\end{aligned}$$

the three lines arising from the operation with $\delta$ on the coefficients $(a, b, c, d, e, f)$ and on the facients $Q'T-QU$ and $-P'T+PU$ respectively; the third line vanishes of itself, and the other two destroy each other, that is,

$$\delta\,(a, b, c, d, e, f\text{)(}Q'T-QU, -P'T+PU)^4=0, \text{ and similarly}$$

$$\delta_1(a, b, c, d, e, f\text{)(}Q'T-QU, -P'T+PD)^4=0,$$

or the function $(a, b, c, d, e, f\text{)(}Q'T-QU, -PT+PU)^4$, treating therein T and U as constants, is an invariant, that is, the coefficients of the several terms thereof are all invariants.

313. The expressions for the coefficients (a, b, c, d, e, f) are in the first instance obtained in the forms

$$\begin{aligned} a&= 2(L+5MC+10C^3),\\ b&=-2(L'+3M'C)A,\\ c&= 2(L+MC-2C^3)A^{-1},\\ d&=-2(L'-M'C),\\ e&= 2(L-3MC+2C^3)A^{-2},\\ f&=-2(L'-5M'C)A^{-1}, \end{aligned}$$

where, developing M. HERMITE's expressions,

| $72L=$ | $24M=$ | $24L'=$ | $24M'=$ |
|---|---|---|---|
| $A^7B + 1$ | $A^4B - 1$ | $ABI + 1$ | $I + 1$ |
| $A^6C^2 + 1$ | $A^3C - 1$ | $CI + 5$ | |
| $A^5B^2 + 6$ | $A^2B^2 - 3$ | | |
| $A^4BC - 24$ | $ABC + 12$ | | |
| $A^3B^3 + 9$ | &c. | | |
| $A^3C^3 - 39$ | $C^3 + 24$ | | |
| $A^2B^2C + 9$ | | | |
| $ABC^3 + 108$ | | | |
| $C^3 + 72$ | | | |

and substituting these values, we find

| $36a=$ | $36b=$ | $36c=$ | $36d=$ | $36e=$ | $36f=$ |
|---|---|---|---|---|---|
| $A^7B + 1$ | $A^3BI - 3$ | $A^4B + 1$ | $ABI - 3$ | $A^4B + 1$ | $BI - 3$ |
| $A^6C^2 + 1$ | $ACI - 24$ | $A^3C + 1$ | $CI - 12$ | $A^3C + 1$ | |
| $A^5B^2 + 6$ | | $A^2B^2 + 6$ | | $A^2B^2 + 6$ | |
| $A^4BC - 39$ | | $A^2BC - 27$ | | $A^2BC - 15$ | |
| $A^3B^3 + 9$ | | $A^2C^2 + 9$ | | $AB^3 + 9$ | |
| $A^3C^3 - 54$ | | $AB^3 - 42$ | | $AC^2 - 30$ | |
| $A^2BC - 36$ | | &c. | | $B^2C + 36$ | |
| $ABC^2 + 288$ | | $BC^2 + 144$ | | | |
| $C^3 + 1152$ | | | | | |

I have not thought it worth while to make in these formulæ the substitutions $A=J$, $B=-K$, $C=9L+JK$, which would give the expressions for (a, b, c, d, e, f) in terms of J, K, L.

314. Substituting for $(x, y)$ their values in terms of (X, Y), we have

$$\begin{aligned} &(a, b, c, d, e, f)(x, y)^4\\ &=(a, b, c, d, e, f)\left\{\frac{1}{2\sqrt{C}}\left(\frac{Q'}{\sqrt[4]{A}}+Q\sqrt[4]{A}\right)X+\frac{1}{2\sqrt{C}}\left(\frac{Q'}{\sqrt[4]{A}}-Q\sqrt[4]{A}\right)Y,\right.\\ &\qquad\left.\frac{1}{2\sqrt{C}}\left(\frac{-P'}{\sqrt[4]{A}}-P\sqrt{A}\right)X+\frac{1}{2\sqrt{C}}\left(-\frac{P'}{\sqrt[4]{A}}+P\sqrt[4]{A}\right)Y\right\}^4\\ &=(\lambda, \mu, \nu, \nu', \mu', \lambda')(X, Y)^4 \text{ suppose,} \end{aligned}$$

and by what precedes

$$\alpha x^2+\beta xy+\gamma y^2=\sqrt{A}XY;$$

this gives

$$\alpha\partial_y^2-\beta\partial_y\partial_x+\gamma\partial_x^2=-\sqrt{A}\partial_X\partial_Y,$$

and thence

$$(a\partial_y^2-\beta\partial_y\partial_x+\gamma\partial_x^2)^5(a, b, c, d, e, f)(x, y)^5$$
$$=A\partial_X^5\partial_Y^5(\lambda, \mu, \nu, \mu', \nu', \lambda')(X, Y)^5$$
$$=120A(\nu X+\nu' Y);$$

the left hand side is a linear covariant of the degree 5, it is consequently a mere numerical multiple of $Px+Qy$; and it is easy to verify that it is $=120(Px+Qy)$. (In fact writing $b=d=e=0$, the expression is $(3c^2\partial_y^2-af\partial_y\partial_x)^5(ax^3+10cx^2y^2+fy^3)$, and the only term which contains $x$ is $a^2f^3\,.\,\partial_y^2\partial_x^8\,.\,10cx^2y^3=120a^2cf^3\,.\,x$; but for $b=d=e=0$, Table No. 22 gives $Px=a^2cf^3x$, and the coefficient 120 is thus verified.) But $Px+Qy$ is $=\frac{\sqrt{C}}{\sqrt[4]{A}}(X+Y)$, and we have thus $A\nu=A\nu'=\frac{\sqrt{C}}{\sqrt[4]{A}}$, whence not only $\nu=\nu'$, $=\sqrt{k}$ suppose, but we have further $k=\frac{C}{\sqrt{A^5}}$, a result given by M. HERMITE.

315. Substituting for $\nu=\nu'$ the value $\sqrt{k}$, we have

$$(a, b, c, d, e, f)(x, y)^5$$
$$=(a, b, c, d, e, f)\Big(\frac{1}{2\sqrt{C}}\Big(\frac{Q'}{\sqrt[4]{A}}+Q\sqrt[4]{A}\Big)X+\frac{1}{2\sqrt{C}}\Big(\frac{Q'}{\sqrt[4]{A}}-Q\sqrt[4]{A}\Big)Y,$$
$$\frac{1}{2\sqrt{C}}\Big(\frac{-P'}{\sqrt[4]{A}}-P\sqrt[4]{A}\Big)X+\frac{1}{2\sqrt{C}}\Big(-\frac{P'}{\sqrt[4]{A}}+P\sqrt[4]{A}\Big)Y)^5,$$
$$=(\lambda, \mu, \sqrt{k}, \sqrt{k}, \mu', \lambda')(X, Y)^5,$$

and we have then $\alpha x^5+\beta xy+\gamma y^5=\sqrt{A}XY$, viz. the left-hand side being the quadricovariant of $(a, b, c, d, e, f)(x, y)^5$, the equation shows that the quadricovariant of the form $(\lambda, \mu, \sqrt{k}, \sqrt{k}\,\mu', \lambda')(X, Y)^5$ is $=\sqrt{A}XY$, and we thus arrive at the starting-point of HERMITE'S theory.

316. The coefficients $(\lambda, \mu, \sqrt{k}, \sqrt{k}, \mu', \lambda')$ of HERMITE'S form are by what precedes *invariants*; they are consequently expressible in terms of the invariants A, B, C (and I). M. HERMITE writes

$$\lambda\lambda'=g, \quad \mu\mu'=h,$$

and he finds

$$\sqrt{A}=g-3h+2k, \quad \frac{B}{\sqrt{A^3}}=h-k, \quad \frac{C}{\sqrt{A^5}}=k,$$

or, what is the same thing,

$$g=\frac{A^2+3AB+C}{\sqrt{A^5}}, \quad h=\frac{AB+C}{\sqrt{A^5}}, \quad k=\frac{C}{\sqrt{A^5}},$$

which give $g$, $h$, $k$ in terms of A, B, C, and then putting

$$\Delta=(9k^2+16hk-gh)^2-24hk^3, \quad =\frac{I^2}{A^7}$$

(the equation $I^2=A^7\Delta$ is in fact equivalent to the before-mentioned expression of $I^2$

in terms of the other invariants), the coefficients $(\lambda, \mu, \mu', \lambda')$ are expressed in terms of $g, h, k$, that is of A, B, C, viz. we have

$$\begin{cases} 72\sqrt{k^2}\lambda = h(g-16k)^2 - 9k(g+16k) + (g-16k)\sqrt{\Delta}, \\ 24\sqrt{k^2}\mu = 9k^3 + 16hk - gh - \sqrt{\Delta}, \\ 24\sqrt{k^2}\mu' = 9k^3 + 16hk - gh + \sqrt{\Delta}, \\ 72\sqrt{k^2}\mu' = h(g-16k)^2 - 9k(g+16k) - (g-16k)\sqrt{\Delta}; \end{cases}$$

these values of $(\lambda, \mu, \mu', \lambda')$ could of course be at once expressed in terms of (J, K, L), but I have not thought it necessary to make the transformation.

317. It has been already noticed that the linear covariant (No. 15, $= Px + Qy$), was

$$= \sqrt{A}\,(\sqrt{k}, \sqrt{k}\,\Big|\,X, Y),$$

it is to be added that the septic covariant $(P'x + Q'y)$ is

$$= \sqrt{A^3}(\sqrt{k}, -\sqrt{k}\,\Big|\,X, Y),$$

and that the canonical forms of the cubicovariants $\phi_1(x, y)$, &c. are as follows:

$$\phi_1(X, Y) = \sqrt{A}\,(\mu, \quad 3\sqrt{k}, \quad 3\sqrt{k}, \quad \mu'\,\Big|\,X, Y)^3,$$

$$\phi_3(X, Y) = A\,(\mu, \quad \sqrt{k}, \quad -\sqrt{k}, \quad -\mu'\,\Big|\,X, Y)^3,$$

$$\{\phi_3(X, Y)\} = \sqrt{A^3}(\mu, \quad -\sqrt{k}, \quad -\sqrt{k}, \quad \mu'\,\Big|\,X, Y)^3,$$

$$\{\phi_4(X, Y)\} = A^3(\mu, \quad -3\sqrt{k}, \quad 3\sqrt{k}, \quad -\mu'\,\Big|\,X, Y)^3,$$

$$\psi_1(X, Y) = \sqrt{A^3}\left\{\begin{matrix} (2\sqrt{k^3} - 3\mu k \quad + \mu'\mu^2), \\ 3(\sqrt{k^3} + \mu\mu'\sqrt{k} - 2\mu k), \\ -3(\sqrt{k^3} + \mu\mu'\sqrt{k} - 2\mu' k), \\ -(2\sqrt{k^3} - 3\mu' k \quad + \mu\mu'^2), \end{matrix}\right\}(X, Y)^3,$$

$$\phi_5(X, Y) = \sqrt{A^3}(5\mu, \quad -\sqrt{k}, \quad \sqrt{k}, \quad 5\mu'\,\Big|\,X, Y)^3,$$

$$\phi_4(X, Y) = \sqrt{A^3}\left\{\begin{matrix} (7\sqrt{A}\mu \quad + 96(2\sqrt{k^3} - 3\mu k \quad + \mu'\mu^2)), \\ -3(3\sqrt{A}\sqrt{k} - 96(\sqrt{k^3} + \mu\mu'\sqrt{k} - 2\mu k)), \\ +3(3\sqrt{A}\sqrt{k} - 96(\sqrt{k^3} + \mu\mu'\sqrt{k} - 2\mu' k)), \\ -(7\sqrt{A}\mu' \quad + 96(2\sqrt{k^3} - 3\mu' k \quad + \mu\mu'^2)) \end{matrix}\right\}(X, Y)^3$$

or, as the last formula may also be written,

$$\phi_4(X, Y) = \sqrt{A^3}\left\{\begin{matrix} ((7g - 53h + 110k)\mu \quad - 64\lambda\mu'\sqrt{k}), \\ -3((3g + 151h - 90k)\sqrt{k} - 64\lambda'\mu^2 \quad ), \\ +3((3g + 151h - 90k)\sqrt{k} - 64\lambda\mu'^2 \quad ), \\ -((7g - 53h + 110k)\mu' \quad - 64\lambda'\mu\sqrt{k}) \end{matrix}\right\} X, Y)^3.$$

It is in fact by means of these comparatively simple canonical expressions that M. HERMITE was enabled to effect the calculation of the coefficient $\mathfrak{A}$.

Article Nos. 318 to 326.—*Theory of the imaginary linear transformations which lead to a real equation.*

318. An equation $(a, b, c, \ldots)(x, y)^n = 0$ is real if the ratios $a : b : c$, &c. of the coefficients are all real. In speaking of a given real equation there is no loss of generality in assuming that the coefficients $(a, b, c, \ldots)$ are all real; but if an equation presents itself in the form $(a, b, c, \ldots)(x, y)^n = 0$ with imaginary coefficients, it is to be borne in mind that the equation may still be real; viz. the coefficients may contain an imaginary common factor in such wise that throwing this out we obtain an equation with real coefficients.

In what follows I use the term *transformation* to signify a linear transformation, and speak of equations connected by a linear transformation as *derivable* from each other. An imaginary transformation will in general convert a real into an imaginary equation; and if the proposition were true universally,—viz. if it were true that the transformed equation was always imaginary—it would follow that a real equation derivable from a given real equation could then be derivable from it only by a real transformation, and that the two equations would have the same character. But any two equations having the same absolute invariants are derivable from each other, the two real equations would therefore be derivable from each other by a real transformation, and would thus have the same character; that is, all the equations (if any) belonging to a given system of values of the absolute invariants would have a determinate character, and the absolute invariants would form a system of auxiliars.

But it is not true that the imaginary transformation leads always to an imaginary equation; to take the simplest case of exception, if the given real equation contains only even powers or only odd powers of $x$, then the imaginary transformation $x : y$ into $ix : y$ gives a real equation. And we are thus led to inquire in what cases an imaginary transformation gives a real equation.

319. I consider the imaginary transformation $x : y$ into

$$(a+bi)x+(c+di)y : (e+fi)x+(g+hi)y,$$

or, what is the same thing, I write

$$x=(a+bi)\mathrm{X}+(c+di)\mathrm{Y},$$
$$y=(e+fi)\mathrm{X}+(g+hi)\mathrm{Y},$$

and I seek to find P, Q real quantities such that $\mathrm{P}x+\mathrm{Q}y$ may be transformed into a linear function $\mathrm{RX}+\mathrm{SY}$, wherein the ratio $\mathrm{R} : \mathrm{S}$ is real, or, what is the same thing, such that $\mathrm{RX}+\mathrm{SY}$ may be the product of an imaginary constant into a real linear function of (X, Y). This will be the case if

$$\mathrm{P}x+\mathrm{Q}y = (1+\theta i)\{\mathrm{P}(a\mathrm{X}+c\mathrm{Y})+\mathrm{Q}(e\mathrm{X}+g\mathrm{Y})\},$$

that is if,

$$\mathrm{P}(b\mathrm{X}+d\mathrm{Y})+\mathrm{Q}(f\mathrm{X}+h\mathrm{Y}) = \theta\{\mathrm{P}(a\mathrm{X}+c\mathrm{Y})+\mathrm{Q}(e\mathrm{X}+f\mathrm{Y})\},$$

which implies the relations

$$bP+fQ=\theta(aP+eQ),$$
$$dP+hQ=\theta(cP+gQ),$$

or, what is the same thing,

$$(b-a\theta)P+(f-e\theta)Q=0,$$
$$(d-c\theta)P+(h-g\theta)Q=0,$$

and if the resulting value of $P:Q$ be real, the last-mentioned equations give

$$(ag-ce)\theta^2-(ah+bg-cf-de)\theta+bh-df=0,$$

and $\theta$ being known, the ratio $P:Q$ is determined rationally in terms of $\theta$.

320. The equation in $\theta$ will have its roots real, equal, or imaginary, according as

$$(ah+bg-cf-de)^2-4(ag-ce)(bh-df),$$

that is

$$\begin{aligned}&a^2h^2+b^2g^2+c^2f^2+d^2e^2\\&-2ahbg-2ahcf-2ahde-2bgcf-2bgde-2cfde\\&+4adfgh+4bceh\end{aligned}$$

is $=+$, $=0$, or $=-$; and I say that the transformation is subimaginary, neutral, and superimaginary in these three cases respectively. In the subimaginary case there are two functions $Px+Qy$ which satisfy the prescribed conditions; in the neutral case a single function; in the superimaginary case no such function. But in the last-mentioned case there are two conjugate imaginary functions, $Px+Qy$, which contain as factors thereof respectively two conjugate imaginary functions $UX+VY$.

321. Hence replacing the original $x$, $y$, $X$, $Y$ by real linear functions thereof, the subimaginary transformation is reduced to the transformation $x:y$ into $kX:Y$, where $k$ is imaginary; and the superimaginary transformation is reduced to $x+iy:x-iy$ into $k(X+iY):(X-iY)$, where $k$ is imaginary. As regards the neutral transformation, it appears that this is equivalent to

$$\begin{aligned}x&=(a+bi)X+(c+di)Y,\\y&=\qquad\quad(g+hi)Y,\end{aligned}$$

with the condition $0=(ah+bg)^2-4agbh$, $=(ah-bg)^2$, that is, we have $ah-bg=0$, or without any real loss of generality $g=a$, $h=b$, or the transformation is

$$\begin{aligned}x&=(a+bi)X+(c+di)Y,\\y&=\qquad\quad(a+bi)Y,\end{aligned}$$

that is, $x:y=X+kY:Y$, $k$ being imaginary.

322. The original equation after any real transformation thereof, is still an equation of the form

$$(a,\ldots)(x,y)^n=0;$$

and if we consider first the neutral transformation, the transformed equation is

$$(a,\ldots)(X+kY,Y)^n=0;$$

this is not a real equation except in the case where $k$ is real.

323. For the superimaginary transformation, starting in like manner from $(a, \ldots \chi(x, y)^n = 0$, this may be expressed in the form

$$(\alpha+\beta i,\ \gamma+\delta i,\ \ldots,\ \gamma-\delta i,\ \alpha-\beta i\chi(x+iy,\ x-iy)^n = 0,$$

viz. when in a real equation $(x, y)^n = 0$ we make the transformation $x : y$ into $x+iy : x-iy$, the coefficients of the transformed equation will form as above pairs of conjugate imaginaries. Proceeding in the last-mentioned equation to make the transformation $x+iy : x-iy$ into $k(X+iY) : X-iY$, I throw $k$ into the form

$$\cos 2\phi + i \sin 2\phi,\ = (\cos\phi + i\sin\phi) \div (\cos\phi - i\sin\phi)$$

(of course it is not here assumed that $\phi$ is real), or represent the transformation as that of $x+iy : x-iy$ into $(\cos\phi+i\sin\phi)(X+iY) : (\cos\phi-i\sin\phi)(X-iY)$; the transformed equation thus is

$$(\alpha+\beta i, \ldots \alpha-\beta i\chi(\cos\phi+i\sin\phi)(X+iY),\ (\cos\phi-i\sin\phi)(X-iY))^n = 0.$$

The left-hand side consists of terms such as $(X^2+Y^2)^{n-s}$ into

$$(\gamma+\delta i)(\cos s\phi + i\sin s\phi)(X+iY)^s + (\gamma-\delta i)(\cos s\phi - i\sin s\phi)(X-iY)^s,$$

viz. the expression last written down is

$$= (\gamma\cos s\phi - \delta\sin s\phi)\{(X+iY)^s + (X-iY)^s\}$$
$$-(\gamma\sin s\phi + \delta\cos s\phi)\left\{\frac{(X+iY)^s-(X-iY)^s}{s}\right\},$$

and observing that the expressions in { } are real, the transformed equation is only real if $(\gamma\cos s\phi - \delta\sin s\phi) \div (\gamma\sin s\phi + \delta\cos s\phi)$ be real, that is, in order that the transformed equation may be real, we must have $\tan s\phi$ = real; and observing that if $\tan s\phi$ be equal to any given real quantity whatever, then the values of $\tan\phi$ are all of them real, and that $\tan\phi$ real gives $\cos\phi$ and $\sin\phi$ each of them real, and therefore also $\phi$ real, it appears that the transformed equation is only real for the transformation

$$x+iy : x-iy = (\cos\phi+i\sin\phi)(X+iY) : (\cos\phi-i\sin\phi)(X-iY),$$

wherein $\phi$ is real; and this is nothing else than the *real* transformation $x : y$ into $X\cos\phi - Y\sin\phi : X\sin\phi + Y\cos\phi$. Hence neither in the case of the neutral transformation or in that of the superimaginary transformation can we have an imaginary transformation leading to a real equation.

324. There remains only the subimaginary transformation, viz. this has been reduced to $x : y$ into $kX : Y$, the transformed equation is

$$(a, \ldots \chi(kX, Y)^n = 0,$$

and this will be a real equation if some power $k^p$ of $k$ ($p$ not greater than $n$) be real, and if the equation $(a, \ldots \chi(x, y)^n = 0$ contain only terms wherein the index of $x$ (or that of $y$) is a multiple of $p$. Assuming that it is the index of $y$ which is a multiple, the form of the equation is in fact $x^\alpha(x^p, y^p)^m = 0$, $(n = mp+\alpha)$, and the transformed equation is $X^\alpha(k^pX^p, Y^p)^m = 0$, which is a real equation.

325. It is to be observed that if $p$ be odd, then writing $k^p = \mathrm{K}$ (K real) and taking $k'$ the real $p$-th root of K, then the very same transformed equation would be obtained by the real transformation $x : y$ into $k'\mathrm{X} : \mathrm{Y}$; so that the equation obtained by the imaginary transformation, being also obtainable by a real transformation, has the same character as the original equation.

326. Similarly if $p$ be even, if K be real and positive, the equation $k^p = \mathrm{K}$ has a real root $k'$ which may be substituted for the imaginary $k$, and the transformed equation will have the same character as the original equation; but if K be negative, say $\mathrm{K} = -1$ (as may be assumed without loss of generality), then there is no real transformation equivalent to the imaginary transformation, and the equation given by the imaginary transformation has not of necessity the same character as the original equation; and there are in fact cases in which the character is altered. Thus if $p = 2$, and the original equation be $x(x^2, y^2)^m = 0$, or $(x^3, y^3)^m = 0$, then making the transformation $x : y$ into $i\mathrm{X} : \mathrm{Y}$, the transformed equation will be $\mathrm{X}(\mathrm{X}^2, -\mathrm{Y}^2)^m = 0$ or $(\mathrm{X}^3, -\mathrm{Y}^3)^m = 0$, giving imaginary roots $\mathrm{X}^3 + a\mathrm{Y}^3 = 0$ corresponding to real roots $x^3 - ay^3 = 0$.

Article No. 327.—*Application to the auxiliars of a quintic.*

327. Applying what precedes to a quintic equation $(a, \ldots)(x, y)^5 = 0$, this after any real transformation whatever will assume the form $(a', \ldots)(x', y')^5 = 0$; and the only cases in which we can have an imaginary transformation producing a real equation of an altered character is when this equation is $(a', 0, c', 0, e', 0)(x', y')^5 = 0 (c' \text{ not} = 0)$, or when it is $(a', 0, 0, 0, e', 0)(x', y')^5 = 0$, viz. when it is $x'(a'x'^4 + 10c'x'^2y'^2 + 5e'y'^4) = 0$, or $x'(a'x'^4 + 5e'y'^4) = 0$. In the latter case the transformation $x', y'$ into $\mathrm{X}\sqrt[4]{-1} : \mathrm{Y}$ gives the real equation $\mathrm{X}(a'\mathrm{X}^4 - 5e'\mathrm{Y}^4) = 0$. I observe however that for the form $(a', 0, 0, 0, e', 0)(x, y)^5$, and consequently for the form $(a, \ldots)(x, y)^5$ from which it is derived we have $\mathrm{J} = 0$; this case is therefore excluded from consideration. The remaining case is $(a', 0, c', 0, e', 0)(x', y')^5 = 0$, which is by the imaginary transformation $x' : y'$ into $i\mathrm{X} : \mathrm{Y}$ converted into $(a', 0, -c', 0, e', 0)(\mathrm{X}, \mathrm{Y})^5 = 0$; for the first of the two forms we have $\mathrm{J} = 16a'c'e'^3$, and for the second of the two forms $\mathrm{J} = -16a'c'e'^3$, that is, the two values of J have opposite signs. Hence considering an equation $(a, b, c, d, e, f)(x, y)^5 = 0$ for which J is not $= 0$, whenever this is by an imaginary transformation converted into a real equation, the sign of J is reversed; and it follows that, given the values of the absolute invariants and the value of J (or what is sufficient, the sign of J), the different real equations which correspond to these data must be derivable one from another by real transformations, and must consequently have a determinate character; that is, the Absolute Invariants, and J, constitute a system of auxiliars.

ANNEX.—*Analytical Theorem in relation to a Binary Quantic of any Order.*

The foregoing theory of the superimaginary transformation led me to a somewhat remarkable theorem. Take for example the function

$$(a, b, c)(x+k, 1-kx)^2,$$

or, as this may be written,

$$\begin{array}{c|ccc} & k^2 & k & 1 \\ x^2 & c, & 2b, & a \\ x & 2b, & 2a-2c, & -2b \\ 1 & a, & -2b, & c, \end{array} \quad \text{or} \quad \left(\begin{array}{ccc} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{array}\right)(k, 1)^2(x, 1)^2,$$

then the determinant

$$\begin{vmatrix} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{vmatrix}$$

is a product of linear functions of the coefficients $(a, b, c)$; its value in fact is

$$= -2(a+c)(a+2bi+ci^2)(a-2bi+ci^2), = -2(a+c)[(a-c)^2+4b^2].$$

To prove this directly, I write

$$\begin{aligned} a' &= a-2bi+ci^2, \\ b' &= a \qquad -ci^2, \\ c' &= a+2bi+ci^2, \end{aligned}$$

and we then have

$$\begin{vmatrix} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{vmatrix} \begin{vmatrix} 1, & 2, & 1 \\ i, & 0, & -i \\ i^2, & -2i^2, & i^2 \end{vmatrix}$$

$$= \left(\begin{array}{ccc} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{array}\right) \overbrace{\begin{array}{ccc} (1, i, i^2), & (2, 0, -2i^2), & (1, -i, i^2) \end{array}}$$

$$= \begin{vmatrix} i^2a', & -2i^2b', & i^2c' \\ 2ia', & 0b', & -2ic' \\ a', & 2b', & c' \end{vmatrix} = a'b'c' \begin{vmatrix} i^2, & -2i^2, & i^2 \\ 2i, & 0, & -2i \\ 1, & 2, & 1 \end{vmatrix}$$

whence observing that the determinants

$$\begin{vmatrix} 1, & 2, & 1 \\ i, & 0, & -i \\ i^2, & -2i^2, & i^2 \end{vmatrix}, \quad \begin{vmatrix} i^2, & -2i^2, & i^2 \\ 2i, & 0, & -2i \\ 1, & 2, & 1 \end{vmatrix}$$

are as $1:-2$, we have the required relation,

$$\begin{vmatrix} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{vmatrix} = -2a'b'c' = -2(a+c)\{(a-c)^2+4b^2\}.$$

It is to be remarked that the determinant

$$\begin{vmatrix} 1, & 2, & 1 \\ i, & 0, & -i \\ i^2, & -2i^2, & i^2 \end{vmatrix},$$ taken as the multiplier of $$\begin{vmatrix} c, & 2b, & a \\ 2b, & 2a-2c, & -2b \\ a, & -2b, & c \end{vmatrix}$$

is obtained by writing therein $a=b=c, =1$; and multiplying the successive lines thereof by $1, \frac{1}{2}i, i^2$ $(1, \frac{1}{2}, 1$ are the reciprocals of the binomial coefficients $1, 2, 1)$, the proof is the same, and the multiplier is obtained in the like manner for a function of any order; thus for the cubic

$$(a, b, c, d)(k+x, 1-kx)^3,$$

| | $k^3$ | $k^2$ | $k$ | $1$ |
|---|---|---|---|---|
| $=x^3$ | $-d$, | $3c$, | $-3b$, | $a$ |
| $x^2$ | $3c$, | $-6b+3d$, | $3a-6c$, | $3b$, |
| $x$ | $-3b$, | $3a-6c$, | $6b-3d$, | $3c$ |
| $1$ | $a$, | $3b$, | $3c$, | $d$ |

the multiplier is obtained from the determinant by writing therein $a=b=c=d=1$, and multiplying the successive lines by $1, \frac{1}{3}i, \frac{1}{3}i^2, i^3$, viz. the multiplier is

$$\begin{vmatrix} -1 & 3 & -3 & 1 \\ i & -i & -i & i \\ -i^2, & -i^2, & i^2 & i^2 \\ i^3 & 3i^3, & 3i^3, & i^3 \end{vmatrix}$$

and the value of the determinant is found to be

$$9(a-3bi+3ci^2-di^3)(a-bi-ci^2+di^3)(a+bi-ci^2-di^3)(a+3bi+3ci^2+di^3),$$
$$=9\big((a-3c)^2+(3b-d)^2\big)\big((a+c)^2+(b+d)^2\big).$$

But the theory may be presented under a better form; take for instance the cubic, viz. writing $\frac{x}{y}$ and $\frac{k}{l}$ in place of $x$ and $k$ respectively, we then have

$$(a, b, c, d)(ky+lx, ly-kx)^3$$

| | $k^3$ | $k^2l$ | $kl^2$ | $l^3$ |
|---|---|---|---|---|
| $= x^3$ | $-d$, | $3c$, | $-3b$, | $a$ |
| $x^2y$ | $3c$, | $-6b+3d$, | $3a-6c$, | $3b$ |
| $xy^2$ | $-3b$, | $3a-6c$, | $6b-3d$, | $3c$ |
| $y^3$ | $a$, | $3b$, | $3c$, | $d$ |

a bipartite cubic function $(*)(k, l)^3(x, y)^3$; and the determinant formed out of the matrix is at once seen to be an invariant of this bipartite cubic function.

Assume now that we have identically

$$(a, b, c, d)(x, y)^3 = (a', b', c', d')(\tfrac{1}{2}x+iy, \tfrac{1}{2}(x-iy))^3,$$

viz. this equation written under the equivalent form

$$(a', b', c', d')(X, Y)^3 = (a, b, c, d)(X+Y, \quad i(X-Y))^3,$$

determines $(a', b', c', d')$ as linear functions of $(a, b, c, d)$, it in fact gives

$$\begin{aligned}
a' &= (a, b, c, d)(1, -i)^3 &&= a - 3bi + 3ci^2 - di^3,\\
b' &= (a, b, c, d)(1, -i)^2(1, i) &&= a - bi - ci^2 + di^3,\\
c' &= (a, b, c, d)(1, -i)(1, i)^2 &&= a + bi - ci^2 - di^3,\\
d' &= (a, b, c, d)(1, i)^3 &&= a + 3bi + 3ci^2 + di^3,
\end{aligned}$$

then observing that $ky+lx \pm i(ly-kx) = (x \pm iy)(\mp ik+l)$, we have

$$(a, b, c, d)(ky+lx, \ ly-kx)^3 = (a', b', c', d')(\tfrac{1}{2}(x+iy)(-ik+l), \ \tfrac{1}{2}(x-iy)(ik+l))^3,$$

and if in the expression on the right-hand side we make the linear transformations

$$\begin{aligned}
x+iy &= \ \ x'\sqrt{2}, \quad -ik+l = \ \ k'\sqrt{2},\\
x-iy &= -iy'\sqrt{2}, \quad ik+l = -il'\sqrt{2},
\end{aligned}$$

which are respectively of the determinant $+1$, the transformed function is

$$= (a', b, c', d')(k'x', \ -l'y')^3,$$

that is, we have

$$(a, b, c, d)(ky+lx, \ ly-kx)^3 = (a', b', c', d')(k'x', \ -l'y')^3.$$

The last-mentioned function is

| | $k'^3$ | $k'^2l'$ | $k'l'^2$ | $l'^3$ |
|---|---|---|---|---|
| $x'^3$ | $a'$ | . | . | . |
| $x'^2y'$ | . | $-3b'$ | . | . |
| $x'y'^2$ | . | . | $+3c'$ | . |
| $y'^3$ | . | . | . | $-d'$ |

and (from the invariantive property of the determinant) the original determinant is equal to the determinant of this new form, viz. we have

$$\begin{vmatrix} -d, & 3c, & -3b, & a \\ 3c, & -6b+3d, & 3a-6c, & 3b \\ -3b, & 3a-6c, & 6b-3d, & 3c \\ a, & 3b, & 3c, & d \end{vmatrix} = 9a'b'c'd'$$

$$= 9[(a-3c)^2+(3b-d)^2][(a+c)^2+(b+d)^2],$$

which is the required theorem. And the theorem is thus exhibited in its true connexion, as depending on the transformation

$$(a, \ldots)(x, y)^n = (a', \ldots)(\tfrac{1}{2}(x+iy), \tfrac{1}{2}(x-iy))^n.$$

ADDITION, 7th October, 1867.

Since the present Memoir was written, there has appeared the valuable paper by MM. CLEBSCH and GORDAN "Sulla rappresentazione tipica delle forme binarie," Annali de Matematica, t. i. (1867) pp. 23–27, relating to the binary quintic and sextic. On reducing to the notation of the present memoir the formula 95 for the representation of the quintic in terms of the covariants $\alpha$, $\beta$, which should give for (a, b, c, d, e, f) the values obtained *ante*, No. 312, I find a somewhat different system of values; viz. these are

| 36a= | 36b= | 36c= | 36d= | 36e= | 35f= |
|---|---|---|---|---|---|
| $A^7B$ + 1 | *$A^4I$ − 1 | $A^4B$ + 1 | *$A^3I$ − 1 | $A^4B$ + 1 | *$A^3I$ −1 |
| $A^6C$ + 1 | $A^3BI$ − 3 | $A^4C$ + 1 | $ABI$ − 3 | $A^4C$ + 1 | $ABI$ −3 |
| $A^5B^2$ + 6 | *$ACI$ +24 | $A^4B^2$ + 6 | *$CI$ +12 | $A^3B^2$ + 6 | |
| $A^4BC$ − 39 | | $A^3BC$ − 27 | | $A^3BC$ −15 | |
| $A^3B^3$ + 9 | | $A^3B^3$ + 9 | | $AB^3$ + 9 | |
| $A^3C^2$ − 54 | | $A^3C^2$ − 42 | | $AC^2$ −30 | |
| $A^2B^2C$ − 126 | | *$AB^2C$ − 90 | | *$B^2C$ −54 | |
| $ABC^2$ + 288 | | $BC^2$ +144 | | | |
| $C^3$ +1152 | | | | | |

where I have distinguished with an asterisk the terms which have different coefficients in the two formulæ. I cannot at present explain this discrepancy.

# XVII. *On the Chemical Intensity of Total Daylight at Kew and Pará*, 1865, 1866, and 1867. *By* Henry E. Roscoe, *F.R.S.*

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## Part I.—THE KEW OBSERVATIONS.

In the year 1864 I communicated to the Royal Society* the description of a method for the Meteorological Registration of the Chemical Intensity of Total Daylight, founded upon an exact measurement of the tint which standard sensitive paper assumes when exposed for a given time to the action of daylight.

During the last two years measurements of the chemical intensity, according to this plan, have, through the kindness of Dr. Balfour Stewart, been made regularly every day at the Kew Observatory by Mr. T. W. Baker, and thus the practicability of carrying out a continued series of observations according to this method has been effectually and satisfactorily tested.

Owing to the press of regular work at the observatory only three separate registrations of chemical intensity could be made at different hours each day. Hence the results obtained do not in any way indicate the hourly variation of chemical intensity, nor can even the individual integrals of daily intensity, giving the mean chemical action each day, be said to do more than exhibit approximately the changes which go on from day to day. The monthly integrals, on the other hand, each calculated from a large number of observations, show with a great degree of accuracy the rise and fall of the chemical intensity with the changing seasons of the year, and enable us to deduce from this the first series of observations of the kind, the mean monthly and yearly chemical intensities at Kew for 1865, 1866, and 1867.

The hours at which the chemical intensity was registered each day were those chosen for the reading of the meteorological instruments, viz. $9^h\ 30^m$ A.M., $2^h\ 30^m$ P.M., and $4^h\ 30^m$ P.M. The condition of the sun's surface as regards freedom from cloud, the amount of the cloud, the temperature (wet and dry bulb) and the atmospheric pressure were also noted.

As an example of the results thus obtained, the observations made in the month of July 1866 have been chosen as exhibiting well the great changes in chemical intensity produced by varying cloud and sunshine.

* Bakerian Lecture, Philosophical Transactions, 1865, Part II. p. 605.

## Table I.

### Observations of Chemical Intensity at Kew, July 1866.

| Date. | Time. | Chemical intensity. | Sun's surface. | Clouds. | Thermometers Fahr. | | Barometer at 32° F. |
|---|---|---|---|---|---|---|---|
| | | | | | Dry. | Wet. | |
| 1866. | h m | | | | | | inches. |
| July 2. | 9 30 | Rain. | | | | | |
| | 2 30 | 0·270 | Light clouds. | 8 | 63·7 | 54·9 | 29·267 |
| 3. | 9 45 | 0·335 | Clouded over. | 9 | 57·1 | 51·1 | ·384 |
| | 2 40 | 0·335 | Id. | 9 | 59·8 | 54·6 | ·374 |
| | 4 50 | 0·227 | Light clouds. | 4 | 58·6 | 54·5 | ·338 |
| 4. | 9 50 | 0·430 | Clouded over. | 10 | 56·8 | 53·8 | ·451 |
| | 2 45 | 0·190 | Id. | 9 | 63·5 | 55·7 | ·477 |
| | 4 30 | 0·126 | Id. | 9 | 60·7 | 55·7 | ·499 |
| 5. | 9 40 | 0·320 | Id. | 6 | 60·8 | 53·2 | ·481 |
| | 3 0 | 0·355 | Unclouded. | 5 | 60·1 | 56·4 | ·513 |
| | 5 0 | 0·180 | Clouded over. | 5 | 59·7 | 52·9 | ·524 |
| 6. | 9 30 | Rain. | | | | | |
| | 4 30 | 0·187 | Light clouds. | 4 | 60·3 | 53·7 | ·665 |
| 7. | 9 45 | 0·122 | Clouded over. | 8 | 57·0 | 50·1 | ·961 |
| | 2 0 | 0·143 | Unclouded. | 5 | 63·8 | 53·7 | 30·013 |
| 9. | 9 40 | 0·142 | Clouded over. | 10 | 63·6 | 59·9 | ·179 |
| | 2 30 | 0·315 | Very light clouds. | 5 | 72·8 | 65·2 | ·179 |
| | 4 30 | 0·160 | Id. | 6 | 73·8 | 66·0 | ·188 |
| 10. | 9 30 | 0·390 | Unclouded. | 0 | 71·3 | 64·7 | ·295 |
| 11. | 2 40 | 0·165 | Hazy. | 0 | 74·8 | 66·0 | ·265 |
| | 4 30 | 0·062 | Clouded over. | 9 | 75·5 | 66·0 | ·249 |
| 12. | 9 40 | 0·560 | Light haze. | 0 | 75·7 | 67·5 | ·204 |
| | 2 40 | 0·630 | Unclouded. | 4 | 81·1 | 69·2 | ·151 |
| | 5 0 | 0·237 | Id. | 1 | 82·2 | 69·5 | ·129 |
| 13. | 9 40 | 0·640 | Id. | 0 | 78·3 | 70·5 | ·057 |
| | 4 50 | 0·280 | Id. | 5 | 79·8 | 66·5 | ·055 |
| 14. | 9 30 | 0·380 | Haze. | 4 | 69·2 | 63·3 | ·141 |
| | 2 0 | 0·550 | Unclouded. | 4 | 80·6 | 69·6 | ·125 |
| 16. | 2 50 | 0·177 | Clouded over. | 5 | 71·8 | 62·0 | ·044 |
| 17. | 9 40 | 0·107 | Id. | 10 | 65·9 | 58·5 | ·050 |
| | 2 40 | 0·185 | Id. | 9 | 68·1 | 59·6 | ·012 |
| | 4 30 | 0·156 | Id. | 10 | 65·9 | 58·5 | 29·997 |
| 18. | 9 50 | 0·305 | Light clouds. | 4 | 62·4 | 55·3 | ·996 |
| | 2 40 | 0·197 | Unclouded. | 3 | 68·1 | 56·8 | ·952 |
| | 4 30 | 0·177 | Id. | 3 | 68·6 | 57·4 | ·936 |
| 19. | 3 0 | 0·185 | Clouded over. | 8 | 65·6 | 59·0 | ·886 |
| 20. | 9 40 | 0·257 | Id. | 9 | 57·8 | 50·6 | 30·058 |
| | 2 30 | 0·355 | Unclouded. | 4 | 65·7 | 56·8 | ·054 |
| | 4 30 | 0·203 | Id. | 1 | 68·7 | 58·3 | ·036 |
| 21. | 9 40 | 0·177 | Unclouded: very thin haze. | 0 | 64·2 | 58·2 | ·095 |
| | 2 0 | 0·345 | Unclouded. | 1 | 75·5 | 61·2 | ·051 |
| 23. | 9 30 | 0·217 | Clouded over. | 10 | 56·0 | 52·3 | ·042 |
| | 2 40 | 0·280 | Unclouded. | 1 | 65·4 | 56·7 | 29·985 |
| | 4 30 | 0·237 | Id. | 1 | 65·7 | 56·9 | ·986 |
| 24. | 9 30 | 0·247 | Clouded over. | 10 | 57·8 | 54·1 | 30·037 |
| | 4 30 | 0·089 | Id. | 9 | 61·7 | 53·9 | ·056 |
| 25. | 9 40 | 0·140 | Id. | 9 | 58·4 | 53·0 | ·204 |
| | 2 40 | 0·187 | Id. | 10 | 62·6 | 55·9 | ·212 |
| | 4 30 | 0·064 | Id. | 10 | 61·5 | 55·7 | ·217 |
| 26. | 2 30 | 0·177 | Id. | 10 | 66·6 | 58·5 | ·083 |
| | 4 30 | 0·063 | Id. | 10 | 64·6 | 57·5 | ·063 |
| 27. | 9 50 | 0·063 | Id. | 10 | 61·4 | 59·7 | 29·823 |
| | 2 45 | 0·092 | Id. | 10 | 63·2 | 59·8 | ·754 |
| | 4 40 | 0·088 | Id. | 10 | 64·8 | 60·8 | ·719 |
| 28. | 9 30 | 0·193 | Id. | 8 | 65·2 | 59·7 | ·654 |
| | 1 40 | 0·112 | Id.; light rain. | 10 | 64·8 | 61·1 | ·645 |
| 30. | ..... | Rain. | | | | | |
| 31. | ..... | Rain. | | | | | |

The integrals of daily mean chemical intensity obtained from the numbers in column 3 of the preceding Table by the method described in the above-mentioned memoir, are as follows:—

Daily Mean Chemical Intensity.

(Intensity 1·0 acting for 24 hours = 1000.)

| | | | |
|---|---|---|---|
| July 3rd, 1866 | 138·1 | July 18th, 1866 | 119·4 |
| „ 4th „ | 125·1 | „ 20th „ | 119·5 |
| „ 5th „ | 140·7 | „ 23rd „ | 106·6 |
| „ 9th „ | 91·6 | „ 25th „ | 67·7 |
| „ 12th „ | 229·0 | „ 27th „ | 35·8 |
| „ 17th „ | 60·6 | | |

Monthly mean . . . . 112·2

On the days omitted the number of observations made was too small to enable a determination of daily mean intensity to be made.

In a similar manner the daily mean chemical intensity for every day on which a sufficient number of observations were made, has been determined from April 1, 1865, to April 1, 1867. Table II. contains the numbers thus obtained.

TABLE II.—Daily Mean Chemical Intensities at Kew, 1865–66–67.

(Intensity 1·0 acting for 24 hours = 1000.)

| Date. | Intensity. | Date. | Intensity. | Date. | Intensity. | Date. | Intensity. | Date. | Intensity. |
|---|---|---|---|---|---|---|---|---|---|
| 1865. | | 1865. | | 1865. | | 1865. | | 1865. | |
| April 1. | 69·2 | May 12. | 105·7 | June 23. | 128·1 | Aug. 24. | 101·3 | Oct. 10. | 38·9 |
| 3. | 41·0 | 15. | 40·1 | 26. | 16·7 | 25. | 62·3 | 11. | 29·5 |
| 4. | 35·9 | 16. | 99·4 | 27. | 79·5 | 28. | 44·3 | 12. | 37·9 |
| 5. | 28·6 | 17. | 60·6 | 28. | 87·2 | 29. | 85·7 | 13. | 19·4 |
| 6. | 66·0 | 18. | 129·8 | 29. | 15·2 | 30. | 80·3 | 16. | 18·5 |
| 7. | 42·9 | 19. | 109·1 | July 3. | 179·5 | 31. | 70·1 | 17. | 25·9 |
| 8. | 96·7 | 22. | 220·8 | 4. | 120·5 | Sept. 1. | 88·1 | 20. | 24·0 |
| 10. | 158·2 | 24. | 122·1 | 5. | 73·1 | 6. | 195·5 | 25. | 26·8 |
| 11. | 78·1 | 26. | 160·0 | 6. | 103·9 | 7. | 244·7 | Nov. 3. | 14·8 |
| 12. | 50·5 | 29. | 115·5 | 7. | 132·0 | 8. | 189·5 | 4. | 16·6 |
| 13. | 86·8 | 30. | 100·0 | 10. | 110·3 | 11. | 64·2 | 6. | 12·0 |
| 15. | 36·3 | 31. | 64·6 | 11. | 133·3 | 12. | 113·4 | 7. | 9·2 |
| 18. | 110·9 | June 1. | 53·1 | 14. | 124·7 | 14. | 129·8 | 8. | 12·0 |
| 20. | 73·6 | 2. | 38·0 | 19. | 103·9 | 15. | 165·7 | 9. | 15·7 |
| 21. | 125·3 | 6. | 76·0 | 20. | 110·4 | 18. | 113·4 | 10. | 16·7 |
| 24. | 82·4 | 7. | 177·0 | 21. | 64·9 | 19. | 75·4 | 11. | 17·6 |
| 25. | 87·9 | 8. | 64·6 | 24. | 26·0 | 20. | 102·2 | 13. | 12·9 |
| 26. | 89·0 | 9. | 144·5 | 28. | 90·9 | 22. | 97·9 | 15. | 13·9 |
| 27. | 106·6 | 12. | 108·7 | Aug. 1. | 46·7 | 25. | 50·7 | 23. | 12·9 |
| May 1. | 54·9 | 13. | 135·5 | 4. | 74·0 | 28. | 64·2 | 24. | 12·9 |
| 2. | 91·6 | 14. | 96·2 | 8. | 100·0 | 29. | 24·9 | 27. | 13·9 |
| 3. | 84·0 | 15. | 68·0 | 14. | 88·1 | Oct. 2. | 44·0 | 30. | 6·5 |
| 4. | 59·1 | 19. | 89·7 | 17. | 100·0 | 3. | 34·3 | Dec. 1. | 2·8 |
| 5. | 61·1 | 20. | 61·5 | 18. | 74·0 | 4. | 12·9 | 2. | 8·3 |
| 8. | 68·8 | 21. | 98·7 | 21. | 137·6 | 5. | 44·5 | 4. | 9·2 |
| 9. | 115·9 | 22. | 53·8 | 22. | 114·3 | 6. | 39·8 | 8. | 5·6 |

TABLE II. (continued.)

| Date. | Intensity. | Date. | Intensity. | Date. | Intensity. | Date. | Intensity. | Date. | Intensity. |
|---|---|---|---|---|---|---|---|---|---|
| 1865. | | 1866. | | 1866. | | 1866. | | 1866. | |
| Dec. 11. | 4·6 | Feb. 21. | 21·3 | April 26. | 39·1 | July 7. | 91·6 | Dec. 14. | 19·5 |
| 12. | 8·3 | 23. | 25·4 | 27. | 105·1 | 12. | 229·0 | 18. | 8·8 |
| 13. | 6·5 | 24. | 36·1 | 30. | 27·3 | 17. | 60·6 | 19. | 18·5 |
| 14. | 12·0 | 28. | 28·9 | May 2. | 48·0 | 18. | 119·4 | 20. | 9·6 |
| 16. | 6·4 | March 2. | 43·8 | 3. | 47·3 | 20. | 119·5 | 27. | 18·2 |
| 18. | 2·8 | 3. | 34·4 | 4. | 80·9 | 23. | 106·6 | 31. | 14·6 |
| 20. | 9·2 | 5. | 31·0 | 8. | 94·5 | 25. | 67·7 | 1867. | |
| 21. | 12·0 | 7. | 31·0 | 9. | 56·9 | 27. | 35·8 | Jan. 16. | 13·9 |
| 22. | 5·6 | 8. | 23·1 | 14. | 60·7 | Aug. 5. | 71·6 | 17. | 12·4 |
| 28. | 9·3 | 9. | 24·1 | 15. | 61·9 | 13. | 81·0 | 22. | 7·6 |
| 1866. | | 10. | 19·8 | 17. | 76·1 | 21. | 92·0 | 23. | 4·8 |
| Jan. 1. | 19·7 | 13. | 30·5 | 18. | 75·6 | 23. | 69·0 | 24. | 5·7 |
| 2. | 15·2 | 14. | 41·3 | 22. | 63·8 | 27. | 118·1 | 25. | 1·6 |
| 3. | 14·2 | 15. | 37·2 | 23. | 98·0 | 31. | 143·7 | 29. | 12·4 |
| 5. | 9·3 | 16. | 37·4 | 28. | 89·8 | Sept. 3. | 215·0 | Feb 1. | 8·6 |
| 6. | 11·3 | 19. | 9·3 | 29. | 86·9 | 10. | 88·5 | 2. | 12·8 |
| 9. | 22·1 | 20. | 13·2 | 30. | 60·7 | 13. | 69·5 | 5. | 19·0 |
| 10. | 7·9 | 23. | 55·9 | 31. | 49·8 | 20. | 45·1 | 6. | 12·4 |
| 12. | 17·2 | 26. | 42·4 | June 1. | 66·1 | 27. | 90·3 | 7. | 21·9 |
| 15. | 22·1 | 27. | 18·2 | 2. | 134·5 | 28. | 93·8 | 8. | 8·6 |
| 20. | 20·1 | 28. | 26·5 | 4. | 93·8 | Oct. 1. | 65·9 | 11. | 13·3 |
| 23. | 22·6 | 29. | 32·0 | 5. | 52·1 | 2. | 35·4 | 12. | 10·5 |
| 24. | 19·2 | April 3. | 24·9 | 7. | 114·1 | 3. | 80·1 | 13. | 16·7 |
| 25. | 13·2 | 5. | 24·9 | 9. | 86·5 | 5. | 35·8 | 14. | 18·6 |
| 26. | 9·8 | 6. | 28·6 | 13. | 33·1 | 16. | 23·9 | 15. | 26·7 |
| 27. | 8·8 | 7. | 7·7 | 14. | 94·6 | 17. | 27·0 | 19. | 18·6 |
| 29. | 23·6 | 9. | 5·9 | 15. | 48·0 | 23. | 19·5 | 20. | 28·0 |
| 30. | 16·2 | 10. | 38·5 | 16. | 79·3 | 24. | 34·5 | 25. | 28·6 |
| Feb. 1. | 16·7 | 11. | 25·4 | 19. | 46·7 | Nov. 14. | 20·8 | March 4. | 13·3 |
| 5. | 20·4 | 12. | 60·7 | 20. | 106·4 | 20. | 13·7 | 5. | 20·0 |
| 6. | 25·0 | 13. | 52·2 | 21. | 90·6 | 21. | 19·5 | 6. | 20·0 |
| 8. | 37·5 | 14. | 38·5 | 22. | 111·6 | 23. | 16·6 | 8. | 6·2 |
| 9. | 20·0 | 17. | 67·4 | 25. | 47·5 | 24. | 16·6 | 15. | 29·5 |
| 10. | 24·0 | 18. | 39·8 | 26. | 100·2 | 28. | 19·5 | 20. | 36·2 |
| 12. | 19·7 | 19. | 75·2 | 27. | 99·5 | 29. | 19·5 | 21. | 23·8 |
| 13. | 26·4 | 20. | 38·9 | 28. | 127·6 | 30. | 15·6 | 26. | 42·8 |
| 15. | 20·0 | 21. | 109·7 | 29. | 104·0 | Dec. 1. | 9·0 | 28. | 50·9 |
| 17. | 13·7 | 22. | 80·4 | July 3. | 138·1 | 8. | 20·1 | | |
| 19. | 29·5 | 24. | 83·6 | 4. | 125·1 | 10. | 14·1 | | |
| 20. | 24·0 | 25. | 73·7 | 5. | 140·7 | 13. | 7·8 | | |

The first result which presents itself from the daily observations is that the mean chemical intensity for hours equidistant from noon is found to be constant; that is, for equal altitudes of the sun the chemical intensities are equal. Thus the mean of all the morning observations in 1865 (207 in number) was at $9^h\ 34^m$ A.M. $=0{\cdot}153$; that of the afternoon observations in the same year (197 in number) was at $2^h\ 27^m$ P.M. $=0{\cdot}159$; whilst in 1866 the mean of the morning observations (283 in number) was at $9^h\ 49^m$ A.M. $=0{\cdot}119$, and the afternoon observations (274 in number) at $2^h\ 29^m$ P.M. $=0{\cdot}116$. The morning observations in 1867 (62 in number) at $9^h\ 50^m$ gave 0·044, the afternoon (58 in number) at $2^h\ 26^m$ gave 0·047. These give

Chemical intensity.

Mean of 552 morning observations in 1865–67 at $9^h\ 41^m$ A.M. $=0{\cdot}105$

Mean of 529 afternoon observations in 1865–67 at $2^h\ 27^m$ P.M. $=0{\cdot}107$

Hence we may with certainty conclude that when the disturbing causes of cloud &c. are eliminated, the daily maximum of chemical intensity corresponds to the maximum of the sun's altitude, and that the chemical intensity exhibits no sign of a post-meridian maximum, as is observed in the measurements of hourly temperature.

In order to obtain an expression for the relation existing between the sun's altitude and the chemical intensity of total daylight, a much larger number of observations than the foregoing must be made at widely differing altitudes, either on the same day or on consecutive days. Such a series of observations was made at Heidelberg (see Proceedings Roy. Soc. No. 81, 1866) on a cloudless day. The relation between the sun's altitude and the chemical intensity as found in these determinations is graphically represented in fig. 1A, Plate XXI., and is seen to be a straight line, the abscissæ representing the altitude and the ordinates the corresponding chemical intensity. The formula

$$CI_a = CI_0 + \text{const} \times a$$

represents this relation, where $CI_a$ signifies the chemical intensity at any altitude ($a$) in circular measure, $CI_0$ the chemical intensity at the altitude 0, and const. is a number to be calculated from the observations. That this formula closely represents the relation in the case of the Heidelberg observations is seen from the agreement of the observed with the calculated intensities.

| Altitude. | Chemical Intensity. | |
|---|---|---|
| | Observed. | Calculated from formula. |
| 7° 15′ | 0·050 | 0·050 |
| 24 43 | 0·200 | 0·196 |
| 34 34 | 0·306 | 0·276 |
| 53 37 | 0·437 | 0·435 |
| 62 30 | 0·518 | 0·506 |

A similar series of observations made at Pará (see page 565 of this paper) under a tropical sun in April last, in the middle of the rainy season, shows that a similar relation holds good between the chemical intensity and the sun's altitude even when the sky is not cloudless.

| No. of expts. | Sun's mean altitude. | Chemical Intensity. | |
|---|---|---|---|
| | | Observed. | Calculated from formula. |
| 22 | 73° 40′ | 0·964 | 0·959 |
| 11 | 60 40 | 0·769 | 0·800 |
| 11 | 49 28 | 0·685 | 0·666 |
| 10 | 22 58 | 0·344 | 0·338 |

This relation is graphically represented in fig. 1 B, Plate XXI.

Assuming, as we may fairly do, that the same relation between the sun's altitude and chemical intensity holds good at Kew as at Heidelberg and Pará, the value of the inten-

sity at noon can be calculated from the observations at $2^h\ 30^m$ and $4^h\ 30^m$ P.M. The observed values of the monthly mean chemical intensities at $9^h\ 30^m$ A.M., $2^h\ 30^m$ P.M., and $4^h\ 30^m$ P.M., from April 1865 to April 1867, are given in Table III.; the values of the intensities at noon have been calculated by help of the foregoing formula.

TABLE III.

| Month. | Hour. | Mean intensity. | Month. | Hour. | Mean intensity. | Month. | Hour. | Mean intensity. |
|---|---|---|---|---|---|---|---|---|
| 1865. | h m | | 1865. | h m | | 1866. | h m | |
| April. | 9 30 | 0·195 | Dec. | 9 33 | 0·029 | August. | 9 34 | 0·194 |
| | 12 0 | 0·297 | | 12 0 | | | 12 0 | 0·280 |
| | 2 25 | 0·215 | | 2 26 | 0·020 | | 2 30 | 0·210 |
| | 4 38 | 0·112 | 1866. | | | | 4 42 | 0·115 |
| May. | 9 30 | 0·211 | January. | 9 34 | 0·038 | Sept. | 9 45 | 0·172 |
| | 12 0 | 0·356 | | 12 0 | | | 12 0 | 0·286 |
| | 2 21 | 0·240 | | 2 26 | 0·047 | | 2 32 | 0·187 |
| | 4 30 | 0·115 | Feb. | 9 39 | 0·051 | | 4 38 | 0·058 |
| June. | 9 33 | 0·192 | | 12 0 | 0·094 | October. | 9 41 | 0·085 |
| | 12 0 | 0·313 | | 2 26 | 0·065 | | 12 30 | 0·088 |
| | 2 26 | 0·223 | | 4 31 | 0·021 | | 2 30 | 0·059 |
| | 4 39 | 0·116 | March. | 9 35 | 0·081 | | 4 34 | 0·019 |
| July. | 9 35 | 0·218 | | 12 0 | 0·101 | Nov. | 9 37 | 0·042 |
| | 12 0 | 0·283 | | 2 30 | 0·075 | | 12 0 | 0·057 |
| | 2 30 | 0·214 | | 4 31 | 0·041 | | 2 27 | 0·035 |
| | 4 30 | 0·129 | April. | 9 37 | 0·129 | | 4 21 | 0·002 |
| August. | 9 39 | 0·177 | | 12 0 | 0·163 | Dec. | 9 43 | 0·028 |
| | 12 0 | 0·254 | | 2 31 | 0·116 | | 12 0 | |
| | 2 28 | 0·187 | | 4 43 | 0·057 | | 2 32 | 0·016 |
| | 4 44 | 0·104 | May. | 9 37 | 0·167 | 1867. | | |
| Sept. | 9 39 | 0·236 | | 12 0 | 0·259 | January. | 9 50 | 0·033 |
| | 12 0 | 0·397 | | 2 28 | 0·164 | | 12 0 | |
| | 2 38 | 0·271 | | 4 48 | 0·067 | | 2 31 | 0·019 |
| | 4 35 | 0·106 | June. | 9 43 | 0·205 | Feb. | 9 46 | 0·042 |
| October | 9 31 | 0·066 | | 12 0 | 0·248 | | 12 0 | 0·080 |
| | 12 0 | 0·063 | | 2 33 | 0·183 | | 2 27 | 0·053 |
| | 2 32 | 0·042 | | 4 43 | 0·106 | | 4 30 | 0·012 |
| | 4 29 | 0·013 | July. | 9 38 | 0·229 | March. | 9 53 | 0·057 |
| Nov. | 9 37 | 0·046 | | 12 0 | 0·330 | | 12 0 | 0·099 |
| | 12 0 | | | 2 32 | 0·238 | | 2 21 | 0·071 |
| | 2 29 | 0·025 | | 4 39 | 0·141 | | 4 36 | 0·033 |

The relations existing between the sun's altitude and the mean monthly chemical intensities are graphically represented (for 1865) in fig. 2, Plate XXI.; and (for 1866) in fig. 3, Plate XXI. The ordinates denote the intensity, and the abscissæ the corresponding altitude of the sun.

From the variation in direction of the straight lines representing the relation of intensity to altitude for the different months, it is clear that in each month a different value exists for the constant of the formula, which in fact represents the degree of atmospheric opalescence, the amount of cloud, and the various other factors which, in addition to the sun's altitude, influence the chemical intensity.

That the simple relation which has been shown to hold good when the sun has reached a certain altitude does not apply in the case of low altitudes, is distinctly seen from the above-mentioned figures. When the sun is only a few degrees above the

horizon, the disturbing phenomena of opalescence come into play, and the values of the further and yet undetermined terms of the expression become so large as materially to affect the result. It is only in the case of the Heidelberg observations that the first terms of the series express the relation as low as 8° of altitude, and this is to be explained by the fact that the observations at Heidelberg were made at an elevation of 1900 feet above the sea-level, and therefore at a situation above a great portion of the denser layers of the atmosphere in which the phenomena of opalescence are most marked.

The curves on fig. 4, Plate XXII., show the rise and fall of monthly chemical intensity with the hour of the day for the months of January, February, and March 1867, and April, May, and June 1865; those on fig. 5, Plate XXII., give the same for the last six months of 1865. Figs. 6 and 7 show the same for the twelve months of 1866.

The mean monthly integrals of chemical intensity for each month from April 1865 to April 1867, as obtained from these curves, are contained in the first column of the following Table. In the second column are given approximations to this integral obtained by taking the average of the daily means as given on Table II. (pp. 557 and 558). The third column contains the average amount of moisture for the month, in grains per cubic foot; the fourth the relative humidity for the month; the fifth the average amount of cloud at the times of observation; and the sixth the relation between the number of observations on which the sun was overcast (=1) and those made in sunshine.

TABLE IV.

| Date. | Chemical Intensity. | | Humidity. | | Cloud. | |
|---|---|---|---|---|---|---|
| | I. | II. | III. | IV. | V. | VI. |
| 1865. | | | | | | overcast=1. |
| April | 97·8 | 77·1 | 3·32 | 0·71 | 4·1 | 1·9 |
| May | 117·8 | 98·6 | 3·63 | 0·72 | 6·3 | 0·5 |
| June | 82·3 | 83·9 | 4·23 | 0·73 | 4·5 | 1·6 |
| July | 114·4 | 105·6 | 4·82 | 0·74 | 6·0 | 1·0 |
| August | 88·9 | 84·2 | 4·50 | 0·78 | 6·9 | 0·6 |
| September | 107·8 | 114·6 | 4·81 | 0·72 | 2·4 | 3·6 |
| October | 23·4 | 30·4 | 3·68 | 0·83 | 4·0 | 1·9 |
| November | 17·8 | 13·2 | 3·12 | 0·85 | 6·7 | 0·5 |
| December | ..... | 8·0 | 2·98 | 0·88 | 7·5 | 0·3 |
| 1866. | | | | | | |
| January | 15·0 | 15·9 | 2·82 | 0·85 | 6·0 | 0·5 |
| February | 24·3 | 24·2 | 2·63 | 0·81 | 6·4 | 0·5 |
| March | 34·5 | 30·6 | 2·49 | 0·81 | 5·6 | 0·4 |
| April | 52·4 | 49·9 | 3·02 | 0·80 | 6·3 | 0·7 |
| May | 78·9 | 70·0 | 2·83 | 0·67 | 5·0 | 0·8 |
| June | 92·3 | 86·1 | 4·52 | 0·76 | 6·6 | 1·0 |
| July | 106·9 | 111·9 | 4·33 | 0·73 | 6·0 | 0·9 |
| August | 94·5 | 95·2 | 4·29 | 0·74 | 7·2 | 0·5 |
| September | 70·1 | 100·3 | 4·13 | 0·83 | 6·4 | 0·7 |
| October | 29·5 | 40·2 | 3·82 | 0·88 | 6·3 | 0·7 |
| November | 15·6 | 17·7 | 2·96 | 0·83 | 5·3 | 0·9 |
| December | ..... | 14·0 | 3·09 | 0·88 | 6·9 | 0·4 |
| 1867. | | | | | | |
| January | 13·0 | 8·3 | ..... | 0·86 | 7·8 | 0·3 |
| February | 21·7 | 17·5 | 2·86 | 0·82 | 7·2 | 0·4 |
| March | 30·6 | 27·0 | 2·33 | 0·83 | 7·7 | 0·2 |

Although the curves of mean daily chemical intensity showing the variation from hour to hour are symmetrical, the chemical action for hours equidistant from noon being the same, this relation appears by no means to hold good for the curves of yearly chemical intensity. This is distinctly seen if we compare the monthly means for the two months about the vernal with the two about the autumnal equinox, for 1865, 1866, and 1867.

| 1865–67. | Mean Chem. Int. | 1866. | Mean Chem. Int. |
|---|---|---|---|
| March 1867 . . . . | 30·5 | March 1866 . . . . | 34·5 |
| April 1865 . . . . . | 97·8 | April 1866 . . . . . | 52·4 |
| September 1865 . . . | 107·8 | September 1866 . . . | 70·1 |
| August 1865 . . . . | 88·9 | August 1866 . . . . | 94·5 |

Or for 100 chemically active rays falling in the months of March and April 1865, 1866, and 1867 at Kew, there fell in the months of September and August 1865–66 167 rays, the sun's mean altitude being the same in both cases.

The curve, fig. 8, Plate XXII., exhibits the biennial variation of chemical intensity at Kew for the two years ending April 1, 1867. The yearly integral for the twelve months January–March 1867 and April–December 1865 is 55·7; whilst that for the twelve months of 1866 is 54·7.

The marked differences between the chemical intensities in spring and autumn must be caused by corresponding differences either in the amount of cloud or in the transparency of the atmosphere. From Table IV. (p. 561) it is seen that the mean amount of cloud in March 1867 and April 1865 is 5·9, and that in August and September 1865 =4·7; whilst the mean cloud for March and April 1866 is 5·9, and that for the corresponding autumn months=6·8. If the number of observations made when the sun is shining be compared with those made when the sun's surface is obscured by clouds, it is seen that of sixty-nine observations made in April 1865 the proportion between cloud and sunshine was as 1 to 1·9, whereas in the months of August and September, out of 130 observations the proportion between cloud and sunshine was 1 to 2·1. In 1866 out of 123 spring observations the relation is 1 cloud to 0·55 sunshine, and out of 122 autumn observations the relation was found to be 1 cloud to 0·60 sunshine. Hence it appears that the effect of varying amount of cloud has been eliminated by the number of the observations, and that the difference in chemical intensity cannot be ascribed to the presence of more cloud in the spring than in the autumn.

The only other possible explanation is to be sought in the difference in atmospheric transparency in spring and autumn, and the only indication which we at present possess of such variation in transparency is afforded by measurements of the hygrometric condition of the air, the increased transparency of moist air for the visible rays being well known. In March 1867 and April 1865 the mean amount of moisture was found to be 2·82 grains per cubic foot; in August and September 1865 it was 4·65 grains. In March and April 1866 the moisture was 2·8 grains, and in August and September 4·21 grains. This gives a relation of 1 to 1·65 for spring and autumn moisture 1865, and 1 to 1·50

for the same in 1866. Another important factor as influencing the transparency must not be overlooked, viz. the presence of finely divided solid particles which floating about produce the phenomenon of atmospheric opalescence. This, taken in connexion with the well known fact of the greater velocity of the winds in spring than in autumn, thus increasing the quantity of these floating particles in the spring, points to an explanation of the high autumnal and low vernal chemical intensity.

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### Part II.—THE PARÁ OBSERVATIONS.

Our knowledge concerning the distribution of the chemically active rays on the earth's surface is as yet very limited, and any conclusions with respect to the intensity of the chemical rays in the tropics have been, up to the present time, based upon the vague and unsatisfactory statements of photographers. According to photographic observations it would appear that in advancing from England towards the equator the difficulty of obtaining good pictures is increased, and more time is said to be required to produce the same effect upon a sensitive film under the full blaze of a tropical sun than in the gloomier atmosphere of London*. It is likewise stated† that in Mexico, where the light is very intense, from twenty minutes to half an hour was required to produce photographic effects which in England occupy but a minute; and it is said that travellers engaged in copying the antiquities of Yucatan have on several occasions abandoned the use of the photographic camera and taken to their sketch-books. Dr. Draper has also noticed certain differences of a similar kind between the light of New York and that of Virginia, and hence a supposition has been thrown out of the existence of a peculiar retarding action exerted by the luminous and the calorific rays upon the more refrangible and chemically active portion of the sunlight. In order to test the validity of these statements, it becomes a matter of great interest to determine directly the intensity of the chemically active rays in the tropics. Through the kindness of Messrs. Alfred Booth and Co., of Liverpool, and thanks to the zeal and ability of my assistant, Mr. T. E. Thorpe, I have been able to obtain such a set of measurements made at Pará, situated nearly under the equator in the northern province of the Brazils, and lying upon a branch of the Amazons, in longitude 48° 30′ West, and latitude 1° 28′ South.

The observations, the results of which are given below, were made at Pará by Mr. Thorpe from the 4th to the 26th of April 1866, in a situation possessing a clear horizon. Owing to the rainy season having set in when the experiments were commenced, the changes in the chemical intensity as observed from hour to hour, and even from minute to minute, are very sudden and remarkable, and render a large number of daily observations necessary. These sudden changes are well represented in the curves, figs. 9, 10, 12, and 13, Plate XXI., showing the variation of chemical intensity at Pará during the

* Golding Bird, 'Natural Philosophy,' p. 622, 5th Edit.

† Robert Hunt, 'Researches on Light,' p. 366.

days of April 18th, 23rd, 25th, and 26th. The curves for these days, compared with the dotted lines below, indicating the corresponding action at Kew, show the enormous variation in chemical intensity which occurs under a tropical sun in the rainy season. Regularly every afternoon, and sometimes at other hours of the day, enormous thunder-clouds obscure the sun, and discharging their contents in the form of deluging rain, reduce the chemical action nearly to zero. The storm quickly passes over and the chemical intensity rapidly rises to its normal value.

If we compare the daily mean intensities at Pará and Kew on the same days, we gain some idea of the true chemical action of the tropics, and it becomes at once evident that the alleged failure of photographers cannot, at any rate, be ascribed to a diminution in the sun's chemical intensity, but must rather be referred either to overexposure of the plate, or more probably to the difficulty of obtaining a distinct image owing to constant variation in the density of the layers of air intervening between the plate and the object. The curves, figs. 9 to 14, Plate XXI., exhibit graphically the relation of chemical intensity at Kew and Pará on the 18th, 23rd, 24th, 25th, and 26th of April 1866, these being chosen from the other sets of observations as being the most complete. The data for these five days' observations are found in the Tables at the end of this paper.

The following numbers give the Daily Mean Chemical Intensities at Kew and Pará for fifteen days in April 1866.

| Date. | Daily mean Intensity. | | Ratio. |
|---|---|---|---|
| | Kew. | Pará. | |
| 1866. | | | |
| April 4 ... | ..... | 269·4 | |
| 6 ... | 28·6 | 242·0 | 8·46 |
| 7 ... | 7·7 | 301·0 | 39·09 |
| 9 ... | 5·9 | 326·4 | 55·25 |
| 11 ... | 25·4 | 233·2 | 9·18 |
| 12 ... | 55·8 | 203·1 | 3·66 |
| 13 ... | 52·2 | 337·8 | 6·46 |
| 14 ... | 38·5 | 265·5 | 6·89 |
| 18 ... | 39·8 | 350·1 | 8·80 |
| 19 ... | 75·2 | 352·3 | 4·68 |
| 20 ... | 38·9 | 385·0 | 9·90 |
| 23 ... | 80·4 | 350·1 | 4·35 |
| 24 ... | 83·6 | 362·7 | 4·34 |
| 25 ... | 73·7 | 307·8 | 4·17 |
| 26 ... | 39·1 | 261·1 | 6·67 |
| Mean intensity. | 46·06 | 303·2 | |

Hence it appears that the chemical action of total daylight in the month of April 1866 was 6·58 times as great at Pará as at Kew.

In order to form an idea of the march of the daily chemical intensity under the equator in the sunshine, all the observations made when the sun's disk was unobscured by clouds have been collected, and a curve plotted out from the means thus obtained.

The following Table gives the results, and the curve, fig. 14, Plate XXI., exhibits the regular nature of the increase before and after noon. The curve is a symmetrical one, and exhibits a maximum at noon; the dotted curve is the curve of mean chemical intensity for April at Kew, and the relation between these two intensities is as 52·4 to 313·3, or a ratio of 1 to 5·98.

| Mean time. | No. of observations. | Intensity. | Mean time. | No. of observations. | Intensity. |
|---|---|---|---|---|---|
| h m | | | h m | | |
| 7 3 | 11 | 0·196 | 12 54 | 13 | 0·981 |
| 7 54 | 11 | 0·389 | 2 5 | 17 | 0·820 |
| 9 24 | 8 | 0·789 | 2 54 | 14 | 0·664 |
| 10 1 | 19 | 0·871 | 3 57 | 7 | 0·406 |
| 11 5 | 27 | 0·983 | 4 49 | 4 | 0·223 |
| 12 1 | 21 | 1·028 | | | |

In a future communication I propose to discuss the relation between the chemical intensity of direct and diffuse sunlight at Kew, Pará, and Lisbon.

Chemical Intensity of Total Daylight at Pará, April 12th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 9 30 | 0·348 | Clouded; dull. | 9–10 | 28·1 | 25·7 | 764·4 |
| 9 45 | Rain. | | | | | |
| 9 55 | 0·731 | ..... | 9–10 | 28·4 | 26·4 | |
| 10 0 | Rain. | | | | | |
| 10 25 | 0·947 | Unclouded. | 8 | 29·4 | 28·3 | |
| 10 35 | Rain. | | | | | |
| 10 55 | Id. | | | | | |
| 11 2 | 0·971 | Unclouded. | 7–8 | 29·4 | 27·3 | |
| 11 30 | 1·019 | | 7 | 31·1 | 27·2 | |
| 11 55 | 1·019 | Unclouded. | 5 | 30·1 | 25·9 | 764·2 |
| 1 14 | 0·968 | Id. | 6 | 27·8 | 26·1 | |
| 2 20 | ..... | Very heavy rain. | ..... | ..... | ..... | 763·0 |
| 2 45 | 0·744 | | 10 | 27·8 | 26·6 | |
| 3 0 | 0·190 | Gloomy; thunder. | 10 | 26·6 | 25·0 | |
| 3 13 | Rain. | | | | | |
| 4 30 | ..... | Heavy rain. | | | | |

Chemical Intensity of Total Daylight at Pará, April 13th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 7 13 | 0·336 | Unclouded. | 2 | 24·6 | 34·0 | 763·5 |
| 8 0 | ..... | ..... | 3 | 25·8 | 24·4 | |
| 9 30 | 0·851 | ..... | 3 | 29·1 | 25·9 | |
| 10 0 | | | | | | |
| 10 20 | | | | | | |
| 10 45 | 0·565 | Clouded. | 7–8 | 30·0 | 25·9 | |
| 11 10 | 0·570 | ..... | ..... | 30·0 | 26·1 | 765·0 |
| 11 20 | ..... | Rain. | 8 | 30·5 | 26·7 | |
| 11 35 | 1·079 | Sunshine. | ..... | 31·4 | 27·6 | |
| 11 50 | 0·980 | ..... | 7–8 | 31·7 | 27·2 | |
| 12 10 | 0·665 | Clouded. | ..... | 32·0 | 27·2 | |
| 12 37 | 0·474 | | ..... | 30·9 | 27·2 | |
| 1 0 | 1·080 | Sunshine. | 7 | 30·4 | 26·9 | |
| 1 10 | ..... | Rain. | 8 | | | |
| 1 42 | 0·210 | Clouded. | 8–9 | 26·8 | 25·6 | |
| 1 45 | ..... | Thunderstorm. | | | | |
| 2 23 | 0·425 | Sunshine cloud. | 8 | 26·9 | 25·1 | 762·0 |
| 2 43 | 0·743 | Unclouded. | 7–8 | 28·2 | 25·9 | |
| 3 2 | 0·420 | Sunshine cloud. | ..... | 28·3 | 25·6 | |
| 3 15 | 0·378 | ..... | 8 | 28·1 | 25·3 | |
| 3 37 | 0·248 | Clouded. | 8 | 27·9 | 25·3 | |
| 4 0 | ..... | Heavy rain. | | | | |

Chemical Intensity of Total Daylight at Pará, April 19th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 6 50 | 0·227 | Clouded. | 5 | 24·5 | 24·3 | 766·5 |
| 7 15 | 0·333 | Id. | 5 | 25·0 | 24·4 | |
| 7 40 | 0·360 | Id. | 5 | 25·6 | 25·0 | |
| 8 0 | 0·416 | Id. | 9–10 | 26·6 | 25·6 | |
| 9 25 | 0·850 | Unclouded. | 6–7 | 28·4 | 26·0 | |
| 9 49 | 0·839 | Id. | 6–7 | 30·6 | 27·4 | |
| 9 52 | 0·803 | Id. | 6–7 | 30·6 | 27·4 | |
| 10 30 | 0·791 | ..... | 6–7 | 30·9 | 27·3 | |
| 10 47 | 1·266 | Unclouded. | 4–5 | 31·8 | 27·6 | 765·0 |
| 10 49 | 1·115 | Id. | 4–5 | 31·8 | 27·6 | |
| 11 25 | 0·900 | Id. | 5 | 32·6 | 27·6 | |
| 11 27 | 1·050 | Id. | 5 | 32·6 | 27·6 | |
| 12 41 | 0·940 | Id. | 3–4 | 33·3 | 27·8 | |
| 1 50 | 0·564 | Clouded. | 6–7 | 29·1 | 26·3 | |
| 2 15 | 1·000 | ..... | 6–7 | 28·1 | 25·6 | 762·5 |
| 2 46 | 0·739 | ..... | 9 | 29·4 | 26·3 | |
| 3 10 | ..... | Heavy rain. | | | | |
| 3 30 | 0·260 | Clouded. | 9–10 | 26·2 | 25·3 | |
| 3 50 | ..... | Rain. | | | | |

## Chemical Intensity of Total Daylight at Pará, April 20th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 7 40 | 0·456 | Unclouded. | 1 | 26·1 | 25·0 | 764·5 |
| 9 53 | 0·940 | Id. | 1 | 29·4 | 26·1 | |
| 10 21 | 1·000 | Id. | 3 | 30·8 | 27·1 | |
| 10 48 | 0·768 | Id. | 3–4 | 31·7 | 27·5 | |
| 11 31 | 0·893 | Id.; hazy. | 4–5 | 32·2 | 27·2 | 764·5 |
| 12 0 | 0·900 | Id.; id. | 4–5 | 34·1 | 28·1 | |
| 12 32 | 0·960 | Id. | 4–5 | 34·3 | 27·8 | |
| 1 2 | 0·908 | Id. | ..... | 33·7 | 28·2 | |
| 3 5 | 0·336 | Clouded; gloomy. | 8 | 31·9 | 27·2 | |
| 3 16 | 0·237 | Id.; id. | 8–9 | 31·3 | 27·5 | |
| 3 40 | 0·539 | ..... | 7–8 | 29·4 | 26·4 | |
| 4 0 | 0·452 | Unclouded. | 7–8 | 27·8 | 25·7 | |
| 4 20 | 0·333 | Clouded. | 7–8 | 28·4 | 26·1 | |

## Chemical Intensity of Total Daylight at Pará, April 23rd, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 9 5 | 0·761 | ..... | 7 | 28·1 | 25·9 | 766·5 |
| 9 15 | 0·532 | Clouded. | 8 | 28·1 | 25·8 | |
| 9 30 | 1·079 | ..... | 6 | 28·4 | 26·1 | |
| 9 45 | 0·725 | Clouded. | 5 | 29·4 | 26·5 | |
| 10 0 | 1·402 | Unclouded. | 4 | 29·6 | 26·5 | |
| 10 15 | 1·019 | ..... | 5 | 29·4 | 26·2 | |
| 10 30 | 1·105 | Unclouded. | 4 | 30·6 | 26·4 | |
| 10 45 | 1·114 | Id. | 4 | 29·6 | 27·0 | |
| 11 0 | 1·148 | Id. | 4 | 31·6 | 27·2 | |
| 11 17 | 1·318 | Id. | 4 | 32·9 | 27·4 | |
| 11 30 | 0·674 | Clouded. | 5 | 32·5 | 27·2 | 765·8 |
| 11 45 | 1·019 | Unclouded. | 3 | | | |
| 12 0 | 1·019 | Id. | 3 | 32·8 | 26·1 | |
| 12 15 | 1·054 | Id. | 3–4 | 32·2 | 25·7 | |
| 12 30 | 1·344 | Id. | 4 | 32·1 | 25·3 | |
| 12 45 | 0·689 | Clouded. | 8 | 31·7 | 25·4 | |
| 1 0 | Rain. | | | | | |
| 1 12 | 0·444 | Clouded. | 8 | ..... | ..... | 764·5 |
| 1 30 | 1·002 | Unclouded. | 4 | 29·0 | 26·1 | |
| 1 50 | 0·874 | ..... | 7 | 30·0 | 26·1 | |
| 2 5 | 0·925 | ..... | 7 | 30·0 | 26·1 | |
| 2 15 | 0·968 | ..... | 7–8 | 30·6 | 26·6 | |
| 2 30 | 0·925 | ..... | 7–8 | 30·9 | 26·6 | |
| 2 45 | 0·977 | ..... | 7–8 | 31·1 | 26·6 | |
| 3 0 | 0·856 | Unclouded. | 6 | 30·9 | 26·6 | |
| 3 15 | 0·280 | Overcast; gloomy. | 8–9 | 28·9 | 26·1 | 763·5 |
| 3 30 | 0·384 | Id.; id. | 9–10 | 27·8 | 25·7 | |
| 3 45 | 0·352 | Overcast. | 9–10 | 27·2 | 25·1 | |
| 4 0 | Rain. | | | | | |
| 5 0 | 0·233 | Overcast. | 8–9 | 25·7 | 24·8 | |
| 5 10 | 0·200 | Id. | 8 | 25·7 | 25·0 | |
| 5 20 | Rain. | ..... | ..... | ..... | ..... | 764·0 |

Chemical Intensity of Total Daylight at Pará, April 24th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 6 55 | 0·151 | Sun shining through mist. | ..... | 24·4 | 24·2 | 763·5 |
| 7 30 | 0·213 | Clouded. | 9–10 | 25·6 | 24·9 | |
| 8 0 | 0·359 | Id. | 8 | 26·6 | 25·6 | |
| 9 31 | 0·633 | Thin haze. | 2 | 29·4 | 26·4 | |
| 10 2 | 0·684 | Unclouded. | 2 | 30·0 | 26·4 | 767·0 |
| 10 30 | 0·719 | Id. | 2 | 30·6 | 26·1 | |
| 11 3 | 0·951 | Id. | 3 | 32·3 | 27·8 | |
| 12 0 | 1·019 | Id. | 4 | 31·7 | 25·9 | |
| 1 0 | 0·942 | Id. | ..... | 31·7 | 25·3 | |
| 2 0 | 0·754 | Id. | 4 | 32·2 | 26·1 | 764·5 |
| 3 0 | 0·492 | Id. | 2 | 32·2 | 25·1 | |
| 3 51 | 0·389 | Id. | 3 | 30·1 | 26·6 | 762·5 |
| 4 29 | 0·306 | Id. | 3–4 | 29·0 | 26·2 | |

Chemical Intensity of Total Daylight at Pará, April 25th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 6 48 | 0·116 | Unclouded. | 0·5 | 24·2 | 23·6 | 765·5 |
| 7 31 | 0·312 | Id. | 0·5 | 25·9 | 25·0 | |
| 9 41 | 0·490 | Clouded over. | ..... | 29·4 | 25·4 | 766·8 |
| 10 3 | 0·762 | Unclouded. | 3–4 | 31·3 | 27·0 | |
| 10 29 | 0·944 | Id. | 5 | 31·9 | 26·5 | |
| 10 53 | 0·529 | Clouded over. | 4–5 | 31·1 | 26·2 | |
| 10 55 | 0·959 | Unclouded. | 4–5 | 31·1 | 26·2 | |
| 11 29 | 0·475 | Clouded over. | 6–7 | 32·7 | 27·4 | |
| 11 30 | 0·976 | Unclouded. | 6–7 | 32·7 | 27·4 | |
| 11 45 | 0·479 | Clouded over. | 5 | 32·2 | 26·6 | 766·0 |
| 12 0 | 1·011 | Unclouded. | 5–6 | | | |
| 12 16 | 0·977 | Id. | 5 | 32·0 | 26·1 | |
| 12 47 | 0·882 | ..... | ..... | 31·3 | 26·9 | |
| 1 0 | 0·335 | Overcast; gloomy. | ..... | 29·7 | 26·6 | |
| 1 23 | 0·365 | Id.; id. | ..... | 29·7 | 26·7 | |
| 1 47 | 0·774 | Unclouded. | 5 | 30·8 | 27·5 | |
| 2 39 | 0·236 | Clouded over. | 8 | 28·3 | 26·1 | |
| 2 45 | Rain. | | | | | |
| 3 6 | 0·677 | Unclouded. | 7–8 | 29·7 | 27·2 | 763·0 |
| 3 45 | Rain. | | | | | |
| 3 49 | 0·210 | Clouded. | 9 | 28·3 | 26·1 | |
| 4 5 | Rain. | | | | | |

## Chemical Intensity of Total Daylight at Pará, April 26th, 1866.

| Solar time. | Chemical intensity. | Condition of solar disk. | Clouds. | Temperature °C. | | Barometer. |
|---|---|---|---|---|---|---|
| | | | | Dry. | Wet. | |
| h m | | | | | | millims. |
| 7 43 | 0·360 | Unclouded. | 2 | 26·3 | 25·3 | 766·0 |
| 8 19 | 0·408 | Clouded. | 7–8 | 26·7 | 25·0 | |
| 10 0 | 0·958 | ..... | 7–8 | 30·8 | 26·6 | 767·0 |
| 10 15 | 0·354 | Clouded; gloomy. | 10 | 29·4 | 26·1 | |
| 10 30 | 0·608 | Id.; id. | 9 | 28·9 | 25·6 | |
| 10 45 | 0·650 | Id. | 9 | 30·0 | 26·0 | |
| 11 0 | 0·822 | ..... | 8–9 | 29·9 | 26·4 | 766·5 |
| 11 15 | 1·037 | ..... | 7–8 | 31·7 | 27·2 | |
| 11 30 | 1·088 | Unclouded. | 7 | 32·2 | 27·5 | |
| 11 45 | 1·011 | Id. | 5 | 31·1 | 25·6 | |
| 12 0 | 0·539 | Clouded. | 5–6 | 30·8 | 25·0 | 765·5 |
| 12 10 | 1·036 | Unclouded. | 3–4 | 31·0 | 25·5 | |
| 12 30 | 0·976 | Id. | 3 | 32·2 | 26·6 | |
| 1 20 | 0·831 | Id. | 5 | 32·3 | 25·8 | |
| 2 13 | 0·608 | Id. | 5 | 31·9 | 25·0 | |
| 2 33 | 0·540 | Id. | 6–7 | 29·7 | 26·4 | 763·8 |
| 2 53 | 0·336 | Clouded. | 8 | 27·4 | 24·7 | |
| 3 0 | Rain. | | | | | |
| 3 30 | Rain. | | | | | |
| 3 34 | 0·200 | ..... | 8–9 | 26·3 | 25·3 | 763·5 |
| 3 46 | 0·166 | Clouded. | 8–9 | 26·3 | 25·3 | |

# XVIII. The Croonian Lecture.—*On the Influence exercised by the Movements of Respiration on the Circulation of the Blood.* By J. Burdon Sanderson, *M.D., F.R.C.P. Communicated by Dr.* Sharpey, *Sec. R.S.*

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Having in the course of an inquiry relating to the order of cessation of the vital phenomena in apnœa, the results of which I propose shortly to submit to this Society, been led to doubt the truth of the received opinion as to the influence exercised by the movements of the chest in respiration on the circulation, and having found that similar doubts were entertained by others who had given attention to the subject, I thought it necessary before proceeding further to endeavour to obtain a solution of this most important question by experiment.

## Part I.—PREVIOUS RESEARCHES.

It is to Professor Ludwig that we owe the first application of exact methods in the investigation of the influence of the thoracic movements on the action of the heart. In 1846 he performed a series of experiments, the results of which were published the following year in Müller's 'Archiv'*. In these experiments he employed an instrument (called by him a Kymographion) by which the readings of a hæmadynamometer attached to an arterial trunk were inscribed on a cylinder revolving by clockwork at a uniform rate. He found that in ordinary respiration the tracing of the kymographion always exhibited characters which were distinctive, consisting of large undulations or waves produced by the thoracic movements, the contours of which were broken by smaller waves expressing the contractions of the heart. As regards the relation between the

* "Beiträge zur Kenntniss des Einflusses der Respirationsbewegungen auf den Blutlauf im Aortensysteme," Müller's Archiv, 1847, p. 242.

greater or respiratory wave and the thoracic movements, it appeared to LUDWIG that the descending limb (or to speak more shortly, the *descent*) of the wave was coincident with the expansion of the chest in inspiration, and that the ascending limb (or *ascent*) corresponded to the time occupied by the collapse of the chest in expiration and the succeeding pause, so that the summit of the wave indicated the commencement of inspiration. And with respect to the smaller or cardiac waves, he observed that in the ascent of the great wave they were more frequent and more abrupt than in the descent, whence he inferred that during expiration the contractions of the heart succeed each other with great rapidity and are separated by short periods of relaxation, whereas during inspiration the heart remains relaxed for some time between each contraction and its successor.

In most of these experiments the arterial pressure only was traced on the cylinder of the kymographion; as, however, it was found that under certain circumstances the relations between the respiratory movements and the undulations of the tracing were obscure, the experiment was modified. An elastic bag containing air was introduced into the cavity of the pleura by an aperture to which the mouth of the bag fitted air-tight. The bag was connected by a flexible tube with a second bag, the expansive movements of which were transmitted by a lever to the cylinder of the kymographion, on which a tracing was inscribed simultaneously with that which indicated the arterial pressure. It was found that the expansion of the bag connected with the pleura did not always coincide with increased arterial pressure. These inconsistencies did not, however, appear to be of such a nature as to render it necessary to abandon the theory which had been adopted. Although LUDWIG had evidently some misgivings as to its truth, he did not continue the investigation; for in a prefatory note to a paper by EINBRODT, published in 1859, LUDWIG states that during the preceding twelve years he had abandoned the inquiry although convinced that he had misunderstood the connexion of the facts*.

[*Note.*—In the last edition of his 'Lehrbuch der Physiologie,' LUDWIG gives the following account of the changes of arterial pressure consequent on the respiratory movements:—"At the beginning of expiration the contractions of the heart become more frequent, the mean tension of the blood increases, so that even during the relaxation of the heart it sinks very inconsiderably or not at all. Every new contraction induces a higher tension than its predecessor. At the close of the expiratory movement, when the narrowed thorax resumes its normal size, a long pause in the heart's action suddenly occurs, during which the tension sinks considerably, and the movements of the heart are in consequence retarded." This passage shows that until 1861 the author entertained the same views as in 1859.]

In 1860 LUDWIG's experiments were repeated by EINBRODT, who published his results

* "Ich hatte mich überzeugt dass ich in meiner früheren Arbeit die an und für sich richtigen Thatsachen nicht richtig verknüpft hatte."

in MOLESCHOTT'S 'Untersuchungen'*. Of these experiments, which were performed in LUDWIG'S laboratory, and apparently at his suggestion, there were two series. The first related to the effect produced by the artificial production and maintenance for various periods in the air-tubes of pressures which either exceeded or fell short of the pressure of the atmosphere by several inches of mercury. The second series, which alone directly concerns the present inquiry, consisted of experiments with the kymographion, in which either the movements of the thoracic walls or those of the respired air were recorded on the cylinder simultaneously with the variations of arterial pressure. The animals employed (dogs) and all the other conditions were the same as in the previous experiments of LUDWIG, and the difference in the mode of recording was insignificant. The arterial tracings were of the same form as those obtained by LUDWIG, but the relation of the arterial to the respiratory wave was such as to lead EINBRODT to opposite conclusions. According to EINBRODT, "the mean arterial pressure is slightly diminished at the very commencement of inspiration, but immediately afterwards gradually and constantly rises, while the action of the heart is accelerated. The pressure continues to rise until the beginning of expiration, when it reaches its maximum. Thereupon a pause is usually, but not constantly, observed in the contractions of the heart, in which of course the arterial pressure sinks considerably. It always happens that during the remainder of expiration the pressure is diminished while the action of the heart is retarded."

The only other observations by which our knowledge of this subject has been materially advanced are those of MAREY, made with the aid of the instrument invented by him, and described in his work on the Physiology of the Circulation, under the name of "Sphygmographe." By this instrument, the purpose of which is to measure the arterial tension in the living human subject and to record its rhythmical changes by mechanical means, a tracing may be obtained (provided that the individual under observation breathes largely or with effort) which exhibits characters essentially the same as those described by LUDWIG, that is to say, by taking corresponding points in a series of arterial oscillations and connecting them with each other, a curve is produced consisting of larger waves, which are in relation with the thoracic movements. MAREY found that the nature of this relation differs according as the individual breathes with widely open or partially closed respiratory apertures. When respiration is performed largely and without obstruction, the ascent of the respiratory wave coincides with inspiration, the descent with expiration; whereas when the movement of air is obstructed by the narrowing of the air-channels, and each respiration is performed with great effort though with little effect as regards the quantity of air introduced, the curve descends in inspiration, ascends in expiration. These differences MAREY thus explains: in free breathing the diaphragm presses on the aorta and thereby increases the arterial pressure. In restricted breathing the movement of the diaphragm and consequently its influence is diminished, while on the other hand those changes of tension in the thoracic cavity, which under

* "Über den Einfluss der Athembewegungen auf Herzschlag und Blutdruck," MOLESCHOTT'S Untersuch. B. vii. 1860.

ordinary circumstances are very inconsiderable, are manifestly increased whenever the influx and efflux of air are obstructed*.

## Part II.—EXPERIMENTS.

The inquiry was commenced in 1864 and continued at various periods during 1865 and 1866. Dogs were exclusively employed. On account of the mobility of their chests, and the great variety they exhibit in the mode of breathing, these animals are especially adapted for the purpose required. In every instance the respiratory movements and the changes of arterial pressure were recorded mechanically on paper moving horizontally by clockwork.

### 1. *Description of the Apparatus.*

a. *Respiratory movements.*—For the purpose of recording the respiratory movements, a disk-shaped bag of caoutchouc about 6 inches in diameter is used. In the earlier experiments the material employed was vulcanized, but subsequently non-vulcanized india-rubber was preferred. Each of the opposite sides of the bag is glued to a circular disk of wood, which being of smaller circumference leaves a free margin of half an inch round the edge. Of the two boards the lower is fixed, the upper moveable. The latter is screwed to a double horizontal arm of whalebone, the effect of which is to support it horizontally about half an inch above the level of its fellow. By means of this arrangement the expansive and vertical movements of the bag are limited in such a manner that an equal resistance is afforded by the elastic whalebone to the ingress and egress of air. The centre of the moveable disk is connected, by means of a vertical rod, with a lever of the third kind, which is made of light wood terminating in whalebone, and has a total length of 25 inches. The lever works on a steel axis fitted to it by a socket of brass; the axis is supported by a framework of brass which slides up and down on two rectangular brass rods, and is so arranged as to be readily adjusted and fixed by a screw at any desired height. The lever bears at its extremity a fine sable brush which is fixed horizontally at right angles to its length. Communication is made between the respiratory passages of the animal and the caoutchouc bag by a ⊤-shaped tube of

* The present state of opinion on the question may be gathered from the following quotations from the most recent physiological works:—

"During expiration the external surface of the heart is subjected to a stronger pressure than during inspiration, which is expressed in the greater frequency of its pulsation and in the rise of the mercury in the dynamometer."—Budge, *Lehrbuch der spec. Physiol.* 1862, p. 350.

After referring to the most recent researches, particularly those of Marey, the editor of the last edition of Dr. Carpenter's 'Physiology' says, "During the act of expiration the frequency of the pulse is considerably augmented, whilst the line of mean pressure rapidly rises, indicating increased tension in the arterial walls. .... During the act of inspiration, on the contrary, the pulsation becomes slower, the curves much bolder, and the line of mean pressure gradually falls; for then the blood readily enters the thorax, and, as a consequence, the great veins, capillaries, and arterial walls become comparatively flaccid."—Carpenter's *Physiology*, last edition, 1864, p. 245.

gutta percha, one arm of which is connected with the trachea, the other with the elastic bag by a flexible tube of equal width with itself. The third arm remains open. The slight resistance afforded by this arrangement to the flow of air produces sufficient movements of the lever to indicate the exact duration and relative intensity of the respiratory movements.

b. *Variations of arterial pressure.*—As the purpose of the investigation was not to measure the absolute arterial pressure, but to determine the variations of pressure with reference to their duration and order of succession, it appeared unnecessary to employ a more complicated apparatus than the following, from the construction of which it will be seen that the results obtained by it are subject to the error due to the difference of level between the arterial aperture of the animal and the surface of the mercury at the commencement of the observation. The error in question would rarely exceed a tenth of an inch of mercury. The hæmadynamometer employed is a U-shaped tube of glass, of which the longer arm measures 15 inches, the shorter 10 inches. It differs from the hæmadynamometer of POISEUILLE in this respect, that the attached arm, which is the longer of the two, is of smaller calibre than the open arm, the area of the mercurial column contained in the latter being about twelve times as great as that in the other. Hence for every variation of an inch of pressure the surface of the mercury in the open tube moves only one-thirteenth of an inch. The movements of the mercury are transmitted to the recording cylinder, by a lever of the same length and supported on a moveable bearing in the same manner as the one described above. This lever is connected by a vertical rod of iron wire with a conical cork float which rests on the surface of the mercury contained in the wide arm, its base being concave, so as to correspond with the convexity of the meniscus. The joint by which the vertical rod is connected with the lever is so arranged that for every inch of variation in pressure a nearly vertical movement of the extremity of the lever carrying the brush, amounting to three-tenths of an inch, takes place. In order that the lever may accurately follow all the oscillations of the mercury, its dead weight is nearly neutralized by a weight of lead suspended beyond the fulcrum. A similar counterpoise is attached to the lever for recording the respiratory movements.

c. *The recording apparatus* consists of two cylinders, each of 10 inches in diameter, both of which revolve vertically. They are connected together by the band of paper on which the tracing is to be made; this band as it is delivered from the one is wound round the other. The motion of the receiving-cylinder is produced by the descent of a weight and regulated by clockwork to which the feeding-cylinder is adapted. The movements of the levers are inscribed on the paper in its transit from one cylinder to the other. The rate of movement of the paper admits of being varied according to the requirements of the experiment, but in most instances it was such that 10 inches were delivered in a minute. It is obvious that although the greatest care may be taken to adjust the levers in such a manner at the commencement of each observation that the points of the brushes are in the same vertical line, this relation can only be maintained

so long as the two levers are parallel. When therefore great variations of arterial pressure take place in the course of an experiment, it is necessary in order to determine the synchronism of the events recorded, to adopt some method of marking synchronical points in the two tracings inscribed simultaneously on the paper. This object is best attained by making simultaneous momentary interruptions by withdrawing the levers from the paper at the same instant. With this view the stand on which the lever-apparatus and manometer are fixed is so constructed as to have a horizontal rotatory movement round a fixed pin, the position of which nearly coincides with the base of the rectangular brass rods on which the bearings of the levers are supported. By this movement, which is regulated by a screw, the operator can at will approximate or withdraw the points of the brushes.

### 2. *Experiments as to the relation between the arterial pressure and the movements of the thorax in the normal animal.*

The first experiment was made in the Museum of Middlesex Hospital on the 14th of September, 1864. The dog having been secured in the usual manner, the dynamometer was adjusted to the femoral artery, and as soon as the breathing had become tranquil, a mask of gutta percha of a suitable form was placed loosely over his snout and connected by a tube with the vulcanite bag. In this preliminary experiment it was observed that "each inspiratory descent of the lever was accompanied and followed by an arterial ascent, that is to say, by a succession of short and quick oscillations, which imply that during the period they express, the ventricles of the heart became fuller and fuller, their systole more vigorous but less complete, and their diastole accelerated; while in the interval between each inspiration and its successor the arterial pressure sank, the contractions of the ventricles were more rare, and the diastole of longer duration."

The investigation was resumed in the physiological laboratory of University College, the apparatus having been in the meantime entirely reconstructed. Preliminary experiments were made on the 2nd, 3rd, 9th, and 11th of March, which yielded results in accordance with those previously and subsequently obtained. The apparatus was not, however, brought to a satisfactory state of completeness till the end of May, after which the observations given in the Plates were made.

#### Observation I.—June 10th, 1865 (Plate XXIII. fig. 1).

A male terrier of moderate size was secured on its back in the usual way, viz. by ligatures attached to each extremity. The femoral artery having been exposed, a silver canula, previously filled with saturated solution of carbonate of soda, was introduced and secured. The trachea was then laid bare about 1 inch from its upper end, and partially cut across. A glass tube as large as it would admit was inserted and secured by a ligature, which was then connected with the caoutchouc bag by means of the T-shaped tube above described, while the communication between the artery and the dynamometer was

completed by a vulcanite tube filled with solution of carbonate of soda*. It having been ascertained that the apparatus was in order, and the dog breathing quietly, the clockwork was set in motion, the rate of horizontal movement of the paper being 1 inch in six seconds. The tracings show that there were about nine respirations and 108 pulsations per minute. The upper or arterial tracing is crossed by a faint horizontal line, which indicates a pressure of 6·2 inches of mercury†. During the progress of the observations the brushes were simultaneously withdrawn from the paper at short intervals, so that synchronical points could be accurately taken. The tracings show that the period occupied by each respiratory act is divisible into two parts, one of which (about two-fifths of the whole) is occupied by the thoracic movements, the remainder by the pause. Of the first part, two-thirds correspond to inspiration, one-third to expiration. It is further seen that the effects of the thoracic movements are as readily discernible in the respiratory as in the arterial tracing. The part of each arterial tracing corresponding to a single respiratory interval consists, as described by LUDWIG, of a great wave, the contour of which is broken by smaller waves, each representing a contraction of the heart. During the whole period of the pause the arterial pressure gradually sinks. The commencement of inspiration is immediately followed by an increase of pressure, which becomes still more marked during expiration; but no sooner is the expiratory act completed than it again subsides. The apex of the greater or respiratory wave in the arterial tracing is therefore coincident with the end of expiration. As regards the effect of the thoracic movements on the duration of each cardiac revolution, it is no less distinctly seen that the interval between each two succeeding contractions is about three times as great in those pulsations which immediately follow the end of expiration as in those which precede it, and that this interval gradually diminishes until the next corresponding period. At one part of the tracing, where the inspiration lasted longer and was deeper than usual, it is seen that the consequent elevation of pressure and acceleration of pulse was greater‡.

Observation II.—June 16th, 1865 (Plate XXIII. fig. 2).

The animal employed was a male mongrel dog of moderate size. The experimental method was the same as before, with the exception that the rate of movement of the paper was about twice as quick. The synchronical points are marked by simultaneous interruptions, and by dots indicating the relative position of the brushes§. The faint

* In my earlier experiments, although a saturated solution was always used, the blood sometimes coagulated before the observation was completed, rendering it necessary to remove, cleanse, reinsert, and reconnect the silver canula. To prevent this result a pressure of about 5 inches of mercury, *i. e.* nearly equal to that usually existing in the arteries of a dog, was first produced in the dynamometer (by a mechanical arrangement which it is not necessary to describe) before completing the connexion. When this precaution was taken, very little blood passed beyond the silver tube in the artery, and even there was mixed with the saline solution in such proportion that no coagulation took place.

† The absolute value of this result is subject to the exception made at p. 575.

‡ This animal was afterwards employed in an experiment on asphyxia.

§ These faint horizontal lines and the dots have been omitted in the engravings.

horizontal line denotes an arterial pressure of 5 inches. The rate of breathing was 11–12 per minute, and of the heart's contractions 94. It is to be noted that in this animal the time occupied by the thoracic movements was greater as compared with the duration of the pause. Hence probably the variations of the arterial tracing are less abrupt. It is further to be noted that the diastolic interval is shortest a little after the commencement of expiration, and longest immediately after its termination*.

Observation III.—June 13th, 1865 (Plate XXIII. fig. 3).

The animal employed was a small male English terrier. Experimental method as before; rate of movement 1 inch in six seconds; arterial pressure 5 inches; eleven respirations and seventy pulsations per minute. From the relative length of the respiratory intervals and the great regularity of the arterial undulations, the precise relation between the two tracings can be determined with great exactitude.

Observation IV.—May 31st, 1865 (Plate XXIII. fig. 4).

Animal used, a smooth black male English terrier. In this case a ⊤ tube with a very wide aperture was employed; hence the oscillations of the lever were so slight that the commencement of inspiration cannot be distinguished. The moment at which the expiratory act commences and terminates is, however, clearly indicated. Rate of movement 1 inch in 3·3 seconds; arterial pressure 5 inches; the respirations were more frequent than in any of the previous cases, being 32 per minute; as there were 100 pulsations in the same time, three occurred during each respiratory act. Notwithstanding this peculiarity the relation between the two remained the same; the highest points in the arterial tracing corresponding to the end of expiration, and the diastolic interval immediately after that event being twice as long as the one preceding it†.

Observation V.—June 21st, 1865 (Plate XXIII. fig. 5).

A male mongrel terrier of good size was employed. Arterial pressure 6 inches (indicated by horizontal line); rate of movement 1 inch in three seconds; respirations 16, pulsations 102 per minute. The tracings exhibit the following peculiarities:—the respiratory movements were more irregular, and the time occupied by them was greater as compared with the pause; so much so that it several times happened that there was no interval at all between expiration and inspiration. The effects of this mode of breathing are well seen in one part of the tracing, where two inspiratory descents succeed each other almost immediately, in consequence of which a high arterial pressure is maintained for several seconds. The irregularities of the respiratory tracing are, so to speak, reflected in the arterial, each respiratory undulation having the same duration as the arterial undulation which corresponds to it‡.

* The animal was used for other experimental purposes.

† The animal was further used for an experiment relating to the poisonous effects of carbonic acid.

‡ This animal was subsequently used in an experiment on poisoning by hydrocyanic acid.

Observation VI.—May 30th, 1865.

The animal used was a large-sized male mongrel terrier. Rate of movement 1 inch in 3·3 seconds; arterial pressure 5 inches; respirations seventeen, pulsations seventy-six per minute. From the irregularity of the respiratory movements it is more difficult to make out the relation between the two tracings, which, however, is in conformity with previous observations. Those inspirations which were deepest and longest produced the most marked effect*.

Observation VII.—June 20th, 1865.

The animal used was of the same description as in the last experiment. Rate of movement 1 inch in 3·3 seconds; arterial pressure 5 inches; respirations twenty to twenty-two, pulsations sixty per minute. In this animal the variations of arterial pressure were very slight; but as regards the diastolic intervals the facts of previous observations were confirmed†.

Observation VIII.—June 17th, 1865.

A female mongrel cur of moderate size was employed. Rate of movement 1 inch in three seconds; arterial pressure 5 inches; twenty-one respirations and fifty-eight pulsations per minute. In this animal the respiratory movements were unusually frequent as compared with the contractions of the heart. Occasionally the cardiac and respiratory intervals are shown in the tracings to be nearly equal, in which case it is obvious that the thoracic movements could not produce any effect on the variations of arterial pressure. If, however, the respiration was retarded, the usual variations manifested themselves, as is well seen in the second part of the tracing, where the frequency of the pulse happens to be twice as great as that of the breathing. The diastolic interval which follows inspiration is here found to be not more than half the length of that which coincides with it. Again, towards the end of the observation, where the animal is seen to have sighed deeply, a succession of short diastolic intervals with increasing arterial pressure is seen to follow the expansion of the chest, and to be of corresponding duration‡.

Observation IX.—June 24th, 1865.

A male cur was employed in this experiment. Rate of movement 1 inch in three seconds; arterial pressure 5 inches; respirations nineteen, pulsations ninety-six per minute. The influence of the thoracic movements on the arterial pressure was not marked, excepting when the inspirations were deeper than usual. It is worthy of notice that there was a distinct pause between inspiration and expiration, as illustrated in the first part of the tracing, which serves to show that the expiratory act, unless it is unnaturally forcible, is without effect on the arterial pressure§.

* The animal was further used for other experimental purposes.

† The animal was subsequently used in an experiment as to the toxic effects of hydrocyanic acid.

‡ This animal was further used for an experiment as to the influence of strychnia on the thoracic movements.

§ The animal was subsequently employed for an experiment as to the toxic effects of hydrocyanic acid.

Observation X.—June 28th, 1865.

A full-sized male mongrel. Rate of movement 1 inch in six seconds. The animal struggled excessively on being secured, and breathed during the period of observation 120 times in a minute; the frequency of the heart's action being from 190 to 200. The respiration was occasionally interrupted by deep sighs, the effect of which on the arterial pressure is well shown in the tracing. Each sigh was accompanied by an immediate and rapid increase of pressure, which was followed by a corresponding acceleration of the action of the heart*.

Observation XI.—August 9th, 1865 (Plate XXIII. fig. 6).

A male English terrier was employed in this experiment. The trachea was not opened, the communication between the air-passages and the caoutchouc bag being effected by means of a moist bladder tied over the snout. The mouth of the bladder was adapted to the T tube already described. In order to ensure the animal's breathing freely, the teeth were kept apart by a wooden wedge. Rate of movement 1 inch in three seconds; respirations eighteen, pulsations sixty-four per minute. The observation illustrates the effect of a mode of breathing which, although not strictly normal, is frequently seen in the dog. The animal inspired suddenly, the time between each inspiration and its successor being for the most part occupied by a prolonged whine terminating in a short expulsive movement which was immediately succeeded by inspiration. During the whine the respiratory lever remained at the same level or ascended very gradually. As regards the arterial pressure, the tracing shows that in this instance the rapid rise of the mercurial column coincided with the commencement of the whine, and that it began to fall one or two seconds before inspiration, so that the period of decline corresponded to the last half of the period of expiration and to the whole of the period of inspiration.

The preceding observations afford conclusive evidence that in dogs the expiratory act is not the cause of the elevation of the arterial pressure which is associated with each respiration, for they show that the elevation invariably commences and is sometimes at an end before inspiration is completed. It can therefore scarcely be doubted that the effect in question is due to the expansion of the chest; for if not caused by expiration, there is no other possible agency to which it could be attributed. It is now necessary to show that the conditions of experiment, in so far as they were unnatural, were not such as to interfere with the natural performance of the respiratory function, and that whatever explanation is applicable to the phenomena recorded, must necessarily be equally applicable to ordinary breathing in the dog.

The mode in which the animals were secured, and the contrivances employed for transferring to paper the movements of the air in and out of the respiratory cavity, have been already described. It might be objected that the natural breathing would be interfered with, either by the pain and terror of the operation, by the absence of the controlling influence of the larynx, or by the resistance offered to the influx and efflux of air in its

* The animal was afterwards used for an experiment on carbonic oxide.

passage through the breathing-tube. The first of these objections is, I think, answered by the tracings. It is seen that in almost every instance they were performed with perfect regularity, and that in several they were remarkably slow as compared with the ordinary rate of breathing in the dog. Although no anæsthetic was used, yet from the simplicity of the operations performed and the rapidity with which they were completed, the animals were in a perfectly tranquil condition during the periods of observation. Importance is also to be attached to the circumstance that dogs of impure breed were always employed. The second objection is of little value; for it is well known that in the tranquil breathing the rhythmical laryngeal movements have no appreciable influence on those of the thorax. But the third is of sufficient weight to demand not only consideration but special inquiry.

In preliminary experiments I found that when the caoutchouc bag was connected with a mask placed loosely over the animal's snout, the movements communicated to the lever were of the same nature as those represented in the tracings, although of greater extent. I had also repeatedly found that the apparatus was so sensible that if the aperture of the ⊤ tube was placed opposite the nostril at an inch distance, the opposite aperture being closed, movements of the lever were produced by the air passing in and out in ordinary breathing. By these facts, as well as by the comparison of the width of the ⊤ tube with that of the natural air-passages, I was convinced that, so far from the resistance afforded being greater, it was considerably less than in ordinary breathing through the nostrils. Further, from what I believed I had ascertained as to the mode in which the results are produced, I was led to expect that they would not have been in the slightest degree modified even if the resistance had been many times as great as it actually was. For the purpose of testing the truth of this assumption I made the following experiment.

Observation XII.—June 15th, 1866 (Plate XXIV. figs. 1–4).

A male brindled mongrel terrier, weighing 30 lbs., was employed. 3·1 cubic centimetres of a solution of hydrochlorate of morphia, containing one grain per cubic centimetre, was injected into the cellular tissue of the axilla. The animal became torpid almost immediately; there were no convulsions. The usual operation was then performed. The connexions having been completed, the clockwork was set in motion twenty minutes after the injection of the morphia; the rate of movement of the paper being 1 inch in 2·9 seconds, and the arterial pressure 5 inches. Observations were continued for an hour, during the whole of which period the breathing was regular. The ⊤ tube which was employed had an internal diameter of four-tenths of an inch; and in order to produce various degrees of resistance cylindrical corks were used, the transverse sections of which were reduced by slicing them in planes parallel to their axes, but at various distances therefrom. Thus, between the cork and the tube containing it, an air-passage was left, of which the section was an arc of a circle of four-tenths of an inch in diameter. The corks so prepared were severally marked A, B, C, and D. The form of the section of the cork used in the production of each tracing is shown in the

1 (fig. 1). Cork B was inserted in the tube, but by inadvertence a second aperture in the caoutchouc bag intended for a different purpose was left open. As it was of equal diameter with the tube by which the bag is connected with the trachea, the resistance offered by the apparatus in this instance was extremely small. This is indicated by the inconsiderable extent of the oscillations of the respiratory lever. Measurements show that the acceleration of the contractions of the heart, and the increase of arterial pressure, occurred invariably two-thirds of a second after the commencement of each inspiration.

2 (not engraved). Cork B was again used, but the second opening in the caoutchouc bag was closed. The increased resistance is denoted by the depth of the inspiratory descent. The characters of the arterial tracing remain unaltered. It is to be observed that whenever inspiration was performed with greater suddenness than usual, its influence on the arterial tracing was more transitory, and the interval between the inspiratory act and its effect was shorter.

3 (fig. 2). Cork C was employed. The extent of movement of the lever was much increased, but the characters of the arterial tracing are the same.

4 (fig. 3). By inserting the cork D, which almost filled the aperture of the ⊤ tube, the resistance was increased to the utmost. The main features of the arterial tracing are unchanged, but the variations of arterial pressure occur at a shorter interval, viz. four-tenths of a second after the thoracic movements. The breathing was slightly accelerated, the number of respirations per minute being thirteen as compared with eleven at the beginning of the period of observation.

5 (not engraved). The cork A was substituted for D. The breathing appeared to be perfectly free, the extent of movement of the lever being scarcely greater than when the aperture was left entirely open.

6 (not engraved). Soon after the last observation, viz. an hour and five minutes after the commencement of the experiment, the breathing became irregular, the respiratory movements becoming unequal both in duration and depth, although no change was made in the apparatus. The effects of these variations are seen in the tracing.

7 (fig. 4). In order to carry the investigation one step further the open arm of the ⊤ tube was completely closed, so that the limited quantity of air contained in the connecting tube and in the caoutchouc bag was repeatedly respired. The tracing was made during a period commencing twenty-three seconds after this had been done, and shows that the movements of the respiratory lever were still more ample, and particularly that the inspiratory efforts were so energetic that the caoutchouc bag was emptied each time the chest expanded. This is indicated by the horizontal line between the descending and ascending limb of the curve. The breathing was scarcely at all accelerated (12–14 per minute), and the arterial tracing retains the same character as before. The increased arterial tension and acceleration of the pulse lasted in each case during the whole of the period of inspiration.

The series of results just stated afford evidence that the relation previously observed

between the thoracic movements and those of the heart do not depend on mechanical conditions peculiar to the mode of experiment; and they furnish additional proof that it is not affected by the abnormal psychical conditions of the animal, which in this instance was throughout under the influence of morphia. It may therefore be assumed that in the dog, so long as the respiratory passage is sufficiently open to allow of the entrance of air into the chest, the act of inspiration is invariably followed in normal breathing by increase of tension and shortening of the diastolic interval, *i. e.* acceleration of the heart's action*. It remains to be considered by what instrumentality this influence is exercised. The facts indicate that the result consists in an alteration of the mode of contraction of the heart. That part therefore of the nervous system which presides over the movements of that organ must be concerned in its production. But the effect may be brought about either by agencies which are entirely mechanical, *i. e.* altered relations between the pressures existing in different parts of the circulation, or may be also more or less due to changes in the chemical state of the circulating fluid. For this reason the proper course seems to be, first to determine to what extent the increased activity of the heart which follows each inspiration may be accounted for as a mechanical effect of the expansion of the chest. Then, even if it be found that the whole of the observed phenomena may be thus explained, it will still be open to question how far the chemical consequences of each respiration may be also concerned in their production.

The effect of the respiratory movements on the arterial pressure stands in relation to the fact, demonstrated by DONDERS, that all the organs contained in the chest are kept when its walls are at rest (as *e. g.* after death) in a state of distension, so that the mass of the thoracic viscera has constantly a tendency to shrink to a smaller volume than that of the cavity in which they are contained. As all of these organs possess elasticity, they must necessarily all participate in any expansion of the whole mass, but inasmuch as they resist expansion in very different degrees, their participation is unequal. Of the four principal kinds of organs contained in the chest, viz. the lungs, veins, arteries, and heart, the arteries and heart (when contracted) are by far least capable of distension, for they are already distended by an internal pressure equal to that of 5 to 7 inches of mercury. Consequently in inspiration the arteries and contracting heart take little or no part in the amplification of the chest; so that the increase of bulk produced by dilatation of the thorax is for the most part divided between the lungs, the great veins, and the heart when in a state of relaxation. The actual ratio between the resistance to expansion of the arteries and that of the veins may be inferred, from what we know of the relative tension of the blood in the two systems of vessels, to be about 20 : 1.

In ordinary inspiration with free access of air two effects are produced. The tension of the air contained in the respiratory cavity is reduced, and the resistance to expansion of the lung is increased. These two conditions exercise a similar influence on the

* If, however, the communication between the chest and the atmosphere is completely closed, the relation is reversed. The variations of blood-pressure in the arteries then become coincident in time and of similar extent with those of air-pressure in the thoracic cavity. This will be shown in my paper on apnœa.

thoracic veins and heart. The former, however, is of little importance, for its operation is inconsiderable in itself, and is moreover confined to the period during which the thorax is actually expanding. It is to the latter condition (the resistance offered by the contractility of the lung to expansion) that the effect of inspiration is mainly referable. If the veins, like the lungs, contained air, and communicated freely with the atmosphere, they would evidently expand as rapidly. Actually their expansion is much slower, so that during the act of inspiration the relation between their expansibility and that of the lung is altered, the *proportion* of the thoracic space occupied by the lungs being increased, that occupied by the veins being diminished. Correspondingly the resistance to dilatation of the lungs, as compared with the resistance offered by the mass of intra-thoracic organs, is increased, that of the veins diminished. If the chest continues expanded the balance between the two resistances is gradually restored, that is to say, the veins fill with blood until their distension attains the same proportion to that of the lungs which it possessed before inspiration. Hence it follows that the repletion of the veins produced by inspiration varies in degree according to the length of the period during which the expansion of the chest continues, so that by a short inspiration, however deep it may be, scarcely any effect will be produced on the circulation. As during diastole the cavities of the heart are affected by the movements of the thorax, in precisely the same manner and probably in about the same degree, the preceding considerations are as applicable to them as to the veins, *mutatis mutandis*.

In expiration a slight increase of tension of the air contained in the air-passages takes place. But if the efflux of air is free and unrestrained, this influence is so inconsiderable as to be without influence on the thoracic organs. The only way in which expiration can materially affect the circulation is by diminishing the capacity of the thorax. In *regular* breathing its effect must be always equal to that of inspiration; for whatever increase of the calibre of the veins results from the expansion of the chest, must be reduced when it collapses.

But if in any expiratory act more air is expelled by the forcible contraction of the expiratory muscles than has previously been inhaled, the capacity of the veins will be thereby reduced in a degree proportional to the diminution of the capacity of the chest itself. Thus, if it were possible for the chest to be so contracted by the action of the expiratory muscles as to allow the lungs to collapse to a bulk equal to that which they assume when left to themselves, their tendency to contract would be in abeyance and their distending influence on other organs contained in the chest would no longer be exercised. Similarly, in all less degrees of contraction, the distension of these organs must be proportionably diminished. In other words, *the difference between the pressure to which the thoracic veins are exposed and that of the atmosphere* (the so-called negative pressure), *varies with the volume of the thoracic cavity provided that the air-passages are open.*

The preceding considerations lead to the conclusion that the dilatation of the chest in inspiration aids the expansion of the heart during the diastole, and of the thoracic veins.

The explanation cannot, however, be regarded as complete unless reference is made to other conditions not yet taken into account, *i. e.* to variations of the velocity of the circulation, and of the pressure existing in the systemic veins. For although there can be no doubt as to the *direction* in which increase or diminution of the external pressure to which the thoracic veins are exposed must affect the quantity of blood which they convey to the heart, the *degree* in which their influence is exercised must depend on the fulness of the veins outside of the chest. In healthy animals these two conditions are inseparably associated together. The more rapid the circulation, the fuller are the veins as compared with the arteries. When the veins are distended and the movement of the blood is rapid, the filling of the cavities of the heart, in diastole, takes place in a shorter period, while at the same time the contraction of both auricle and ventricle is more sudden and effective in consequence of the diminished arterial resistance. In the opposite case, when the veins are empty and the movement of the blood within them is sluggish, the cavities of the heart fill slowly, and empty themselves imperfectly in consequence of the excessive arterial resistance.

This being admitted, it may be readily understood that the effect of inspiration is likely to be materially influenced by the relative velocity and tension of the arterial and venous circulations. In the one case the right auricle, at the moment of commencing diastole, is still full of blood (*i. e.* when the arterial tension is high, the veins empty and the circulation retarded), in the other the right auricle is empty at the end of systole. In other words, when the thoracic veins are almost emptied by the heart, at each contraction the effect of thoracic expansion is far greater than when the intrathoracic veins, even in their emptiest condition, are much fuller than those that lie outside of the chest.

It being admitted that the expansion of the chest not only aids the filling of the heart during diastole, but affords it an abundant supply of blood, the shortening of the diastolic period, and the increase of arterial tension may be readily understood. Inasmuch as the heart possesses the property of contracting the instant that its walls are dilated with blood to the proper degree, it is manifest that the more rapidly the heart fills the shorter must be its interval of relaxation, and the more frequent its contractions. It is no less obvious that increased pressure must be produced by the same agency; for, provided that the ventricles are well filled with blood at the moment that each systole commences, the more frequently they contract the greater will be the quantity of blood forced into the systemic arteries, and hence the higher will be the arterial tension.

I venture to think that the explanation I have offered of the phenomena observed is complete and satisfactory, and that it will be found to be consistent with all that has been previously ascertained. But I do not deem it the less necessary to pursue the investigation further, for by so doing I shall certainly strengthen the basis on which my theory is founded, and anticipate objections which might otherwise be made to it. Admitting, then, that the influence of the respiratory movements on the heart is partly mechanical, I proceed to inquire whether it is not also partly chemical. This may be

tested by observing the results obtained when a mode of breathing is induced in which the mechanism is reversed or altered while the chemical changes are the same. For this purpose I have availed myself of the well-known properties of woorara, under the influence of which the respiratory movements cease, while those of the heart remain unaltered.

### 3. *Experiments as to the relation between the arterial pressure and the thoracic movements in artificial respiration.*

Observations XIII.–XVI.—August 11th and 16th, 1865, and June 2nd and 8th, 1866 (Plates XXIV. & XXV.).

The animals employed on these occasions were (1) a short-legged spotted cur, (2) a small English terrier, (3) a bull terrier weighing 13 lbs., (4) a black and tan terrier weighing 18½ lbs. All were males. With the exception that in the last experiment the pneumogastric nerves were divided, the procedure was the same in each case. The rates of movement of the paper were as follows:—August 11th, 1 inch in 3·2 seconds; August 16th, 1 inch in 3·5 seconds; June 2nd and 8th, 1 inch in 2·8 seconds. The solution of woorara employed in 1865 was kindly given by Professor HARLEY; that used in 1866 was obtained some years ago from Professor PÉLOUZE, and is believed to have been derived from the same source as that used by BERNARD in his investigations of the toxic properties of woorara. The solution contains 0·01 gramme of the substance in each cubic centimetre. The solution was always injected into the subcutaneous cellular tissue. In the two more recent experiments the quantity used corresponded to one-tenth of a gramme of solid woorara. The respiratory movements ceased at periods varying from twelve to fifteen minutes after the injection.

The apparatus was modified so as to admit of artificial respiration. For this purpose a common pair of bellows was employed, which could be adapted at will to the open end of the T tube. The caoutchouc bag was provided with a second tube of the same size as that by which it was connected with the T tube. This additional tube was kept closed so long as the animal continued to respire naturally. As soon as it was desired to practise artifical respiration, air was injected by the bellows; of this air a sufficient proportion inflated the chest, while the remainder passed out through the caoutchouc bag. Immediately after each stroke of the bellows the air introduced was expelled by the elastic reaction of the thoracic walls. By this arrangement the too forcible inflation of the lungs was effectually prevented, and the complete removal of breathed air from the apparatus was ensured. As the experiments were all of the same nature, I prefer to enumerate the results obtained rather in their relation to each other than in the order of time.

1. Under certain circumstances, and particularly when artificial respiration is practised at long intervals, the inflation of the lungs appears to produce analogous effects to those of ordinary breathing. This was well seen during the observations made on the 16th

of August, when immediately after the thoracic movements had ceased, air was injected for some time at regular intervals, at first of ten seconds, and subsequently of fifteen seconds. In both cases each inflation was followed by an increase of arterial tension, and an acceleration of the pulse. The result of one of these experiments is shown in Plate XXIV. fig. 6, the interval between each inflation and its successor being fifteen seconds.

2. It soon, however, appeared that the relation observed on this occasion was not constant; for even in the same animal it was subsequently found that it could no longer be traced when the interval was shortened to five seconds; the variations of arterial pressure and of pulse-frequency in this case resembled those previously seen, but it was only every third stroke of the bellows that appeared to be effectual (see Plate XXV. fig. 4). A similar effect had been previously observed in the experiment of the 11th of August. Again, it was repeatedly noticed that even when artificial respiration was entirely discontinued, rhythmical variations in the force and frequency of the heart's action manifested themselves. This is well seen in Plate XXV. fig. 3, taken about an hour after the solution had been injected. Similar undulations were observed during apnœa in all the animals experimented upon, the intervals varying from five to fifteen seconds.

3. The results stated in the preceding paragraphs seem to show (1) that there is a marked tendency to periodical variations in the activity of the heart of animals under the influence of woorara, and (2) that these variations are for the most part independent of external agencies; and it seems not improbable that the apparent relation observed between the artificial thoracic movements and the fluctuations of the mercurial column in certain cases, may be due to the mechanical stimulation of the heart by the sudden inflation of the chest. However this may be, subsequent observations show that a much more marked and constant influence is exercised by the injection of air into the chest under other circumstances. In my experiment of June 2nd artificial respiration was commenced about a minute and a half after the natural breathing had ceased, and the inflations were continued at intervals of ten seconds, just as in the experiment of the previous year. About five seconds after each stroke, as is well seen in the tracing (Plate XXIV. fig. 5), the arterial pressure rose. Here the length of the interval between the two events at once suggested that the relation between them could not possibly be mechanical.

4. In other observations which were made after several minutes' discontinuance of artificial respiration this was still more evident. In all the animals I found that the heart was much less affected by the privation of air than in the normal state. Thus the inflations could be suspended for three or four minutes without making any material alteration in the character of the oscillations of the mercurial column; but when apnœa was still further prolonged the arterial pressure gradually subsided from 5 or 6 inches to 2 or 3. If under these circumstances air was injected no immediate effect was produced, excepting a slight elevation of the arterial lever simultaneous with the inflation. After the expiration of six or seven seconds the pressure began to rise, while the heart's

contractions became more frequent. This effect usually lasted for several seconds, during which the mercurial column attained an elevation which sometimes equalled and sometimes exceeded that which existed before apnœa was produced; in the latter case it soon relapsed to the normal level. Observations of this nature were made on each occasion. The tracing on Plate XXV. fig. 5 was obtained on the 8th of June. After about three minutes of apnœa, a slight inflation was made, twenty-two minutes having elapsed since the injection of the solution. The arterial pressure had sunk from 5 inches to 3. The interval between inflation and its effect was six seconds. The increased action of the heart lasted fourteen seconds, after which the mercury subsided to its former level. Again, on the 2nd of June (Plate XXV. fig. 6), twelve minutes after the injection of woorara, artificial respiration was discontinued for four minutes, at the end of which time the arterial pressure had sunk to 3 inches. The effect followed the cause at an interval of seven seconds, and lasted for about ten seconds. About half a minute after, the clockwork was stopped, and the animal was again deprived of air for four minutes. The pressure having again sunk to 3 inches, the inflation was repeated with a perfectly similar result. An observation of the same nature was made in 1865. Apnœa had existed for four minutes, the injection of woorara having been made one hour and ten minutes previously. The usual mechanical effect accompanied the inflation of the chest, but no more permanent elevation of arterial pressure occurred until about eight seconds later. It is to be noted that the effects described above could only be obtained when the arterial pressure had been considerably reduced by apnœa; for if the inflation of the chest had been discontinued for shorter periods no material increase of tension was produced by resuming it.

5. As in all the observations recorded in the preceding paragraph the interval between each inflation and its effect was too long to admit of any mechanical explanation, the phenomenon can only be referred to the chemical action of the injected air on the circulating blood. To test this I repeated the experiment on two occasions, with this difference, that hydrogen was substituted for air. In each case the result was the same. An observation of this nature was recorded on the 2nd of June; artificial respiration had been suspended until the arterial pressure had sunk to 3·4 inches. The chest was then fully inflated with hydrogen, when it was observed that the mercury, instead of rising at the sixth or seventh second, remained at the same level. Fourteen seconds after, a very slight injection of air was made, which was followed by the usual result. Still later the lungs were fully inflated with air, in consequence of which the pressure rose to 6 inches.

6. It is obvious that if artificial respiration exercises any mechanical effect at all on the circulation, that effect must be not only different from that produced by the natural movements, but of an opposite nature. For when air is injected it is evident that the expansion of the thoracic veins must be diminished, just as it is increased by natural inspiration. I have had several opportunities of observing that this is actually the case; *i. e.* that the invariable effect of inflation is to induce an immediate but very slight

increase of pressure. Thus towards the close of my first experiment, August 11th, when the animal had been for a long time under the influence of the poison, and the pulse had increased in frequency, it was found that if artificial respiration was vigorously performed at regular intervals, a tracing could be obtained of the character shown in Plate XXV. fig. 1, in which it is seen that each injection of air is accompanied by a simultaneous but very slight elevation of the arterial lever, which, however, was not associated with acceleration of the heart's contractions. This experiment was repeated under similar conditions on the 8th of June, when a corresponding but not so marked mechanical effect was observed. Ten minutes afterwards both pneumogastrics were divided in the neck. The arterial pressure at once rose to 9 inches, and the pulse attained a frequency of 240 per minute. The mercurial column remained at the same height for several minutes, during which the tracing (Plate XXV. fig. 2) was taken. Here, as in fig. 1, it is seen that the diminution of arterial pressure produced by each inflation is not accompanied with any acceleration of the heart's contractions*.

The facts related in the preceding paragraphs afford no answer to the question whether the phenomena observed are due to the direct influence of aërated blood on the heart itself, or to its indirect influence through the nervous centres over the rhythmical movements of the heart. Their principal significance in relation to the present inquiry consists in their affording ground for the inference that, whereas the effects of the thoracic movements in ordinary inspiration are almost immediate, a considerable time is required for the production of those which are due to chemical changes in the circulating blood.

4. *Experiments as to the relation between the arterial pressure and the thoracic movements after section of the pneumogastric nerves.*

It is the opinion of Ludwig (Lehrbuch der Physiologie des Menschen, Bd. ii. p. 163) that the increased frequency of the contractions of the heart which follows ordinary inspiration is in part owing to excitation of the pneumogastric nerves. This view he supports on the ground that, although an increase of arterial tension corresponding exactly to the increased expansion of the thorax is observed to accompany the expiratory act in animals in which the vagus has been divided on each side, this effect is not associated with any acceleration of the pulse. I have already endeavoured to show that the ordinary respiratory variations of frequency of the heart's action admit of a more simple explanation; it is, however, not the less necessary to investigate the facts in question, which certainly at first sight seem to suggest such an inference as that drawn from them by Ludwig.

It is well known that the immediate effect of section of the vagi in the neck is to diminish the frequency of respiration, and to accelerate the contractions of the heart. There are, however, several points relating to these changes which must be referred to here in their bearing on our present inquiry. The mode of respiration, after section, is

* From the extreme frequency of the contractions of the heart the oscillations of the arterial lever are not distinguishable in the engraving, although they are well defined in the original.

peculiar. Thus if you watch the movements of the thorax, you observe, first, that the chest is unnaturally dilated, and secondly, that in inspiration its further dilatation is performed slowly and with effort, and is immediately followed by an expiratory movement of so sudden and violent a character, that it resembles the collapse of an elastic bag or bladder distended with gas which takes place when its stopcock is opened. The inspiratory act, on the other hand, is not only slow but comparatively fruitless; for although the animal breathes so much less frequently than before, the quantity of air taken in at each expiration is scarcely, if at all, increased*. As regards the action of the heart, it does not appear to have been noticed by physiologists that the acceleration is not the only result produced; along with the increased frequency there is a great increase of the arterial pressure, amounting in some instances to 2 or 3 inches. This effect, although it occurs immediately after the operation, is a continuous one, and may be observed at any time after its completion.

The experiments on which these statements are founded were made at various periods during the last two years. I submit the tracings of two observations, made severally on the 29th of July and the 1st of August, 1865.

Observation XVII.—July 29th (Tracings not engraved).

The animal employed was a large, rough, mongrel terrier (male). The rate of movement of the paper was 1 inch in 7·4 seconds. The experimental procedure was the same as usual, with the exception that the two pneumogastrics were exposed low in the neck, and ligatures passed round each of them. Immediately before dividing the nerves the animal was breathing tolerably regularly fifteen times per minute, the arterial pressure being 5 inches and the rate of the pulse seventy-eight. Six seconds elapsed between the sections of the two nerves. After the division the respiratory movements became irregular and then excessive, the arterial pressure increasing at once to 6·2 inches. In less than half a minute after the first nerve was divided the breathing had again become regular, but had diminished in frequency to nine per minute, and had assumed the character usually observed after section. So long as the respiratory movements were regularly performed the mean arterial pressure remained unaltered, the variations during each respiratory act being as follows:—the highest point coincided with the end of expiration; the pressure then sank during a period of about four-thirds of a second, then gradually rose until the commencement of the next expiratory act, which was accompanied by a much more decisive increase of arterial tension, lasting for about two-thirds of a second. Immediately after section of the first nerve the frequency of the pulse increased to 150, and subsequently to 165, which latter rate was maintained. After a time the respiratory movements became irregular, occasionally, however, resuming their original character. The irregularity principally consisted in the increased extent and frequency of the inspiratory movements, in consequence of which the arterial pres-

* This statement is founded on the observations of Rosenthal, "Die Athembewegungen und ihrer Beziehungen zum Nervus Vagus," p. 109 *et seq.*

sure was repeatedly reduced below the original level, rising again as soon as the former method of breathing was resumed.

Observation XVIII.—August 1st (Tracings not engraved).

On this occasion a rough mongrel terrier was used. Rate of movement 1 inch in 7·1 seconds. Before section the animal was breathing somewhat irregularly, twenty-seven times in a minute, the rate of pulsation being forty-four, and the height of the mercurial column being 5 inches; twenty-three seconds elapsed between the section of the two nerves. During this interval the respiratory movements gradually increased in extent and became irregular; but immediately after the second nerve was divided the thoracic movements became so violent as to shake the table and apparatus, while the arterial pressure rose to 8 inches, remaining at that height for several minutes. The animal continued to breathe violently for about sixty seconds, during which period about twenty inhalations took place. After this the respiration assumed a more regular character, its rate varying from five to six per minute. After the first section the pulse gradually increased in frequency, finally attaining a rate of 120 per minute. This rate was maintained for two minutes and a half, when it suddenly diminished to fifty.

In all the animals I have observed after section of the pneumogastrics I have found (1) that the arterial pressure tends to increase during the slow inspiration, and to decline during the pause; (2) that a more rapid increase of tension occurs simultaneously with expiration; and (3) that this last effect, as I have ascertained by repeated measurements, never lasts for more than a second. In order to arrive at a satisfactory explanation of these facts, it is necessary to consider what is the condition of the heart and circulation after section of the vagi. The arterial tracings obtained indicate extreme abbreviation of the diastolic period combined with high arterial tension. The contractions of the heart follow each other so rapidly that the organ is in a state of continuous thrill, while at the same time they are sufficiently vigorous to maintain an arterial pressure several inches higher than the normal. In other words, the heart, although it relaxes between each contraction and its successor, never has time to empty itself, so that the whole systemic circulation is unduly distended. All this is sufficiently explained if we assume that after section of the pneumogastrics the action of the heart is intensified, the effect being altogether analogous to that which results from the injection of air into the lungs of a partially asphyxiated animal (see Observations XIII.–XVI.). It is to be further noticed that the thoracic cavity is also in a state of permanent distension, in consequence of the excessive action of the inspiratory muscles—that is to say the thoracic walls during more than half of the respiratory act remain expanded to such a degree that their elasticity is more than counterbalanced by that of the lungs.

This being understood, it appears that there are two distinct reasons why the effect of inspiration is less marked after section of the vagi than in the normal animal. It is so, first, because the diastolic period is already so abbreviated that there is no room for further abbreviation; and secondly, because the veins of the chest being already expanded

beyond their normal capacity admit of no further amplification. On the other hand, the marked rise of pressure which accompanies expiration, is no doubt, as LUDWIG says, due to the compression of the air contained in the air-passages produced by the sudden contraction of the chest. It resembles the elevation of arterial pressure, which has been already described as the immediate mechanical result of sudden inflation of the lungs. Similar effects always accompany violent expiratory movements.

*Conclusions.*

1. The force and frequency of the contractions of the heart may be influenced either by variations in the intrathoracic pressure—viz. the pressure to which its own surface or that of the intrathoracic blood-vessels is exposed—or by variations in the chemical state of the circulating blood.

2. In natural breathing the influence exercised on the heart by each expansion of the chest is entirely mechanical. This may be inferred from its being no longer observed when the mechanism is altered, as in artificial respiration, as well as from the shortness of the interval by which the effect is separated from the cause.

[This conclusion is strongly confirmed by observations which I have made in the course of my experiments on apnœa, in which it was found that in animals asphyxiated by the continued inhalation of a limited quantity of air from a bladder, the normal relation between the thoracic movements and the arterial pressure and frequency of the pulse remains unaltered.]

3. The degree in which this influence is exercised varies according to the state of the circulation. It is greatest when the systemic veins are full, the circulation rapid, and the arterial pressure low. Under the opposite conditions it can scarcely be recognized.

4. In tranquil breathing the influence exercised by variations of air-pressure in the bronchial tubes and vesicles of the lung (amounting to about 0″·02 of mercury) (DONDERS) is so slight as to be inappreciable; and even when the extent of variation is much increased by narrowing the aperture through which air passes in and out of the chest (as in the experiments related in Observation XII.), no effect is observed which can be attributed to the impeded influx and efflux of air.

5. In forcible breathing the effects of variations of air-pressure may be recognized. This is particularly the case as regards violent expulsive movements; for whereas the effect of normal expiration is not appreciable, violent expiration is always accompanied by a simultaneous increase of arterial pressure, as *e. g.* in animals in which the vagi have been divided. In this case the elastic contractility of the lung being expended in ext pelling the air contained in the chest, the negative pressure on the external surface of the great arteries is correspondingly diminished, and hence the positive pressure against their internal surfaces is increased.

6. The increased action of the heart which results from chemical changes produced in the circulating fluid by exposure to air resembles the mechanical effect of inspiration; both being indicated by increase of arterial tension and acceleration of the pulse. The

former may, however, be distinguished from the latter, first, by the length of the interval which elapses after the introduction of air before the effect manifests itself; and secondly, by the circumstance that it is not produced at all unless the animal has been for some time deprived of air, so as to weaken the action of the heart and diminish the arterial pressure by several inches.

Finally, the relation between the respiratory movements, the arterial pressure, and the frequency of the contractions of the heart, in the dog, has been shown to be the opposite of that hitherto supposed to exist. Inasmuch as many of the conditions on which this relation depends are not the same in animals of different species, the relation itself is no doubt subject to corresponding modifications; but it may be assumed that in all animals having hearts of the same structure, variations in the quantity of blood contained in the venæ cavæ influence the mode of contraction of the ventricles, and consequently the arterial tension, in the same way as in the dog.

*Note.*—Professor VALENTIN, of Berne, in his recently published work (Versuch einer physiologischen Pathologie des Herzens, Leipzig, 1866) gives the results of observations made on Marmots during their winter sleep as to the relation between the respiratory movements and the arterial pressure. In the Marmot, when in profound sleep, the contractions of the heart occur at long intervals, which, however, are much exceeded in duration by those which separate the respirations. By connecting the carotid artery with the kymographion (which can be done without waking the animal), a tracing was obtained from which it appeared that the mercurial column sank during the first third of inspiration, rose during the second two thirds, continued to rise during the beginning of expiration, and again fell during the remainder. Of these facts he gives the following theoretical explanation:—"The negative inspiration-pressure not only sucks air into the lungs, but blood towards the heart. The greater impletion of the heart enables it to propel more blood into the arteries, and increases its frequency, while its contraction as compared with its relaxation is prolonged. Expiration produces an opposite result; for it facilitates the emptying of the lungs of blood, and adds to the systemic pressure. But inasmuch as the heart becomes less and less full during expiration, the increase of pressure is limited to the commencement of the expiratory act, a smaller quantity of blood being injected into the arteries. At the same time the duration of the contraction diminishes, while that of relaxation increases" (p. 353). Here the author appears to attach much more importance to the direct influence of expiration than the facts warrant.

## EXPLANATION OF THE PLATES.

In each figure the upper tracing is that produced by the lever connected with the Dynamometer, and expresses the variations of arterial pressure; the lower by the lever connected with the caoutchouc bag, and expresses the movements of air in and out of the chest.

In the arterial tracing three-tenths of an inch of vertical measurement correspond to

one-tenth of arterial pressure. The breaks indicate synchronical points in the two tracings.

Plate XXIII. figs. 1, 2, 3, 4 & 5. Varieties of natural breathing: fig. 6 represents that form of breathing in which expiration commences with a prolonged whine.

Plate XXIV. figs. 1–4. Obstructed respiration: figs. 1, 2 & 3, partial closure of breathing-tube, the degree of obstruction being indicated by the shaded portions of the circles B, C, D; fig. 4, complete closure.

Plate XXIV. figs. 5 & 6, and Plate XXV. figs. 1, 2, & 4. Artificial respiration after subcutaneous injection of woorara: fig. 3, period during which artificial respiration was arrested; figs. 5 & 6, effects of single injections of air into the chest after several minutes' discontinuance of artificial respiration.

# XIX. *Contribution to the Anatomy of* Hatteria (Rhynchocephalus, Owen).

*By* Albert Günther, *M.A., Ph.D., M.D. Communicated by Professor* Owen, *F.R.S.*

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The remarkable Saurian which forms the subject of this memoir, appears to have been first mentioned in a diary by Mr. Anderson, the companion of Captain Cook, to whom "a monstrous animal of the lizard kind" was described by the two New Zealand boys who joined the expedition whilst staying in Queen Charlotte's Sound (Cook's Third Voyage, 2d edit., 1785, vol. i. p. 153). Polack (New Zealand, 1838, vol. i. p. 317) speaks of it as a creature well known to the settlers. "The gigantic lizard or guana exists principally in the island of Victoria. Some are found in the isles of the Bay of Plenty. The natives relate ogre-killing stories of this reptile, but doubtless it is harmless."

Dr. Dieffenbach has the merit of having first made us acquainted with it. In his 'Travels in New Zealand,' vol. ii. (1843) p. 205, he has the following notes:—"I had been apprized of the existence of a large lizard, which the natives called Tuatera, or Narara, with a general name, and of which they were much afraid. But although looking for it at the places where it was said to be found, and offering great rewards for a specimen, it was only a few days before my departure from New Zealand that I obtained one, which had been caught at a small rocky islet called Karewa, which is about two miles from the coast, in the Bay of Plenty. From all that I could gather about this Tuatera, it appears that it was formerly common in the islands; lived in holes, often in sand-hills near the sea-shore; and the natives killed it for food. Owing to this latter cause, and no doubt also to the introduction of pigs, it is now very scarce; and many even of the older residents of the islands have never seen it. The specimen from which the description is taken I had alive, and kept for some time in captivity; it was extremely sluggish, and could be handled without any attempt at resistance or biting."

This specimen was presented by Dieffenbach to the British Museum, where it still is—in the most perfect state of preservation. Dr. Gray recognized it at once as the type of a distinct genus, which he characterized in the 'Zoological Miscellany,' March 1842, p. 72, referring it to the family of *Agamidæ*, and naming it *Hatteria punctata*. The same diagnosis is republished in the 'Catalogue of Lizards,' 1845, p. 249; and an excellent figure of the entire animal was given in the 'Zoology of the Erebus and Terror,' together with a drawing of the skull* exhibiting its general configuration. Unfortunately no letterpress accompanies this figure.

* This skull is still in the British Museum. Mr. Ford has introduced into this drawing an erect process of the lower jaw, just below the tympanic condyle; this, however, is merely the remainder of a dried ligament.

About that period several other examples or parts of such reached England: the British Museum received four (adult and young) from Dr. F. Knox, Captain Drury, and the Collection of Haslar Hospital. Sir A. Smith obtained two living specimens, which he kept for some time; and finally the Royal College of Surgeons came into possession of a skull and some vertebræ, by which Professor Owen was enabled to point out some of the peculiarities which render this lizard so highly interesting (*Rhynchocephalus*, in Trans. Geol. Soc. vol. vii., 1845, p. 64, pl. 6, figs. 5–7 (skull), and in Catal. Osteol. Ser. Collect. Roy. Coll. of Surg. vol. i., 1853, pp. 142, 143).

No other specimen appears to have reached Europe; indeed, as far as I am aware, no Museum out of England appears to possess *Hatteria*. French Herpetologists do not even mention it; Stannius appears to have overlooked Professor Owen's observations. Evidently restricted in its distribution, exposed to easy capture by its sluggish habits, esteemed as food by the natives, pursued by pigs, it is one of the rarest objects in zoological and anatomical collections, and may one day be enumerated among the forms of animal life which have become extinct within the memory of man.

I may be allowed to pass over a detailed description of the *external* characters of *Hatteria*, which are sufficiently known from the diagnosis and figure given by Dr. Gray (*loc. cit.*).

*The Skull* (figs. 1–7)*.

The *occipital* arch is distinguished by its unusual shortness, the basioccipital being, on its lower surface, only 5 millims. long†. The foramen occipitale has the not very usual appearance of being higher than broad; more than one-third of its circumference is formed by the superoccipital, the exoccipitals‡ contributing but little to the formation of the condyle, which in our specimen is of the usual width, not broader than in *Monitor*, *Iguana*, *Grammatophora*, *Crocodilus*. The superoccipital is raised into a short mesial crest, entirely separate from the superstructure of the parietal. The exoccipital (*a*) (most closely united with the alisphenoid, *b*) is dilated and swollen at its base to receive the acoustic cavity, and emits its lateral process in an oblique but only slightly backward direction. This process is styliform, though strong, deeply grooved below along its entire length, to receive the long stapes (*c*); it is strengthened by a paroccipital (*d*), which covers nearly the entire side of the process, and is united with the occipital part by only partly distinct sutures.

The *basisphenoid* is comparatively long, the posterior pair of hypapophyses (*e*) (tubercles for insertion of the musculus rectus capitis anticus) being in close proximity to the occipital condyle, but rather remote from the anterior pair (*f*) for the articulation of the pterygoids). The brain-capsule being much compressed, the *parietal* bone (*g*) is very narrow, and elevated into a strong mesial crest, which, although appearing simple in an individual

* The figures in this Paper refer to Plates XXVI., XXVII., and XXVIII.

† As I shall have occasion to give the dimensions of some parts of the skull in millimetres, I must remark that the description is taken from a skull 57 millims. long (between end of premaxillary and occipital condyle) and 47 millims. broad (between the outer surfaces of the tympanic bones).

‡ The sutures between these bones are so indistinct that they could not be represented in the drawing.

of advanced age, evidently consisted of two lateral halves in youth, and which diverges into two slight arched ridges in front, running across the posterior part of the postfrontals, and into two blade-like processes behind. Each of the latter joins a portion of the mastoid (*h*), forming the parieto-tympanic bars; but, whilst in other Lizards (*Iguana*, *Varanus*) the parietal process forms the predominant portion of this bar, in *Hatteria* it is comparatively short, and far surpassed in extent by the mastoid portion. This bar is also much less backwardly directed than in other Lizards, being nearly at right angles with the parietal crest. The fontanelle in front of the sagittal suture, present in many but not in all Lizards, is also present here; it is entirely within the parietals, although these bones are, at this place, overlapped by the frontals and postfrontals.

The *frontal* bones are united by a distinct suture; they are narrow, elongate, tapering in front and behind, forming but a small part of the orbit; a very distinct groove runs along their lower edge for the reception of the olfactory nerve. *Nasal* bones large, trapezoid, forming the greater part of the upper surface of the snout, the nasal openings being entirely lateral and of rather inconsiderable width.

The prefrontal (*i*) is narrow, with the outer margin nearly straight, not forming a projection in front of the orbit; on the inner upperside of the orbit it extends backwards to its middle, but does not reach so far on the upper surface of the skull; below it terminates as soon as it reaches the maxillary and palatine, covering the lacrymal (*k*) entirely, so that, of the latter bone, a small part only, above the ascending branch of the maxillary, is visible.

We have now arrived at a portion of the skull which differs remarkably from that of other lizards, the homologies of which have been differently interpreted by authors, and which, therefore, deserves our particular attention—viz. the bones intercalated between the frontals and maxillary on one side, and the quadrate bone on the other. These bones form in *Hatteria* a vertical orbital bar which is connected by an upper (temporal) and lower (zygomatic) horizontal bar with the os quadratum. In all other Lizards (as far as they have been examined at present) the lower bar is absent, in some of them (Geckos) also the upper; and, again, in others (*Varanus*) even the orbital ring is incomplete, whilst in Crocodiles we find the same arrangement as in *Hatteria*. The bones composing these bars are the following:—

1. The postfrontal (*l*) has a considerable extent, forming the hinder part of the roof of the orbit, and extending backwards to the commencement of the parietal crest; therefore it participates, at least externally, in the formation of the brain-capsule; its posterior portion is crossed by an arched ridge. It is united by a suture with

2. A three-branched bone (*m*), the upper branch of which contributes to the formation of the upper part of the orbit; the second is directed backwards, forming one half of the temporal bar; and, finally, the lower descends to the zygomatic, to complete the orbital ring.

3. The temporal bar is completed by a portion of the mastoid (*h*), a branch of which descends along the anterior outer edge of the quadrate bone.

4. The zygomatic (*n*) is much developed; it emits one process to meet the lower branch of bone *m*, and a second to the os quadratum (*o*), forming the lower horizontal bar.

As regards the second of the bones described (*m*), it is evidently the same which we find more or less separate from the postfrontal, and forming part of the temporal bar, in Lizards generally. It is described by Stannius (Vergl. Anat. Wirbelth. vol. ii. p. 159) as *os quadrato-jugale*; but from the way in which he speaks of this bone as being found, "in most Saurians and the Crocodiles, connected with the os zygomaticum and frontale posterius," and as "constantly reaching the tympanic," it is evident that he confounded* the temporal bar of the Lizards with the zygomatic bar of Crocodilians, and the bone in question with the squamosal of Crocodilians (see Cuvier, Oss. Foss. vol. v. pl. 3. fig. 1, *p*, or Owen, Anat. Vertebr. vol. i. p. 145, fig. 95, bone marked 27). The squamosal, as it exists in Crocodilians, belongs to the lower, zygomatic bar, and completes the connexion between the zygomatic and quadrate bones; this squamosal is absent in Lizards generally, and also in *Hatteria*, where the zygomatic is in immediate connexion with the quadrate. On the other hand, the bone, more or less closely attached to the postfrontal in Lizards, does not exist in the Crocodile as an independent bone, the postfrontal entering into direct sutural connexion with the mastoid (temporal bar) and with the zygomatic: but from the position and form of the Crocodile's postfrontal it is perfectly clear that this bone of Lacertians is nothing but a detached portion of the postfrontal; and for such it has been taken by Cuvier, at least in *Iguana* (Oss. Foss. vol. v. pl. 16. fig. 23, *i*); in *Monitor* and *Varanus* it is also present, although its sutural connexion with the postfrontal has been left unnoticed by Cuvier; in *Grammatophora* it is absent. Professor Owen does not describe it as a separate bone (Osteol. Catal. vol. i. p. 663, or Anat. Vertebr. vol. i. p. 154), but mentions it as a continuous portion of the postfrontal†.

Hollard, who has made researches into the developments and homologies of these bones, has come to the conclusion that the bone considered by Cuvier to be the mastoid in Reptiles and Fishes, is in fact the squamosal. A comparison of the skull of the Crocodile with that of *Hatteria* seems to support this view, inasmuch as Cuvier's "partie écailleuse du temporal" of Crocodiles appears merely as a segment of the zygomatic‡, with which it is reunited in *Hatteria*.

Very remarkable is the form of the *os quadratum* (*o*) and its junction with the hind part of the pterygoid (*r*); both bones are much dilated, forming a vertical plate composed of two laminæ, the laminæ being immoveably united by suture, the quadrate being the anterior plate, the pterygoid the posterior. This sutural kind of union appears to be unique among Lizards, which have those two bones united by a joint allowing of moveability to a more or less considerable extent. The condyle of the quadrate has a deep

* This view is maintained also in the 2nd edition, pp. 52 and 57.

† It must be mentioned that Rathke (Untersuchungen über die Entwickelung der Crocodile, 1866, p. 33) does not appear to have observed a division of the postfrontal into two parts in embryos of an Alligator.

‡ Rathke (*loc. cit.* p. 34) has found it and the zygomatic in an equally advanced state of ossification whilst the tympanic was still nearly entirely cartilaginous.

depression across its middle: and there is a very singular wide foramen above the condyle; it is filled with loose cellular tissue.

The *columella* (*p*) arises from above the suture between the pterygoid and quadrate bones; it differs in its form from that of other Lizards, being very broad (8 millims.), spatulate at its upper and lower ends, and constricted in the middle. Being fixed by an irregular suture to the pterygoid and quadrate, and attached to the parietal by a narrow strip of cartilage, it contributes materially to the solidity and immoveability of the pterygo-tympanic arch. A crescentic space between alisphenoid and columella remains cartilaginous; and the fore part of the cranial cavity is closed by fibro-cartilaginous membrane without a trace of ossification.

The *maxillo-palatal* portion of the skull offers no less peculiarities than the parts described before. The *intermaxillaries* are paired; their posterior portion is tapering and wedged in between the nasals; there is no discontinuity of the osseous substance whatever on the upperside of the snout, the nostril being entirely lateral; the fore part of the nasal opening is formed by the intermaxillary. This bone resembles in some measure that of a Rodent, each half being armed with an extremely strong incisor as broad as the bone to which it is anchylosed. The horizontal part of the *maxillary* is extremely narrow, reduced to a simple although strong alveolar ridge; its ascending nasal process is (with the nostril) much advanced forwards, in the anterior third of the bone. The foramina maxillaria superiora are present as in other Lizards.

The *vomer* (*q*) is paired, each half being of an elongate triangular shape, broader behind than in front; it forms nearly entirely the inner edges of the choanæ, and its posterior extremity is even behind their level. Most singularly it is in immediate contact with the front part of the pterygoids (*r*)*, which thus prevent the palatines (*s*) from reaching the median line of the palate.

The *palatine* bones (*s*) are entirely separate from each other, each being closely united by suture to the posterior two-thirds of the maxillary; and being armed along its maxillary margin with a series of teeth similar and parallel to that of the maxillary, the two series are in close proximity, so as to give to the maxillary the appearance of having a double dentigerous ridge. The inner part of the palatine is unusually broad, participating but little in the formation of the choana, from which it is shut out by the vomer, but forming about one half of the bottom of the orbit, which is rendered almost completely osseous by the accession of the pterygoid, zygomatic, and os transversum.

The *pterygoid* bones (*r*) are suturally connected with the vomer, separated in the middle by a comparatively narrow cleft, *meeting again* where they are immoveably joined to the basisphenoid processes (*f*) which are close together, and finally *suturally* attached to the quadrate bone as described above. They are toothless, but traversed in their middle

* The perfect preservation of the sutures in a specimen prepared by myself has enabled me to point out this very unusual arrangement of the bones of the palate. Professor Owen designated, in *Rhynchocephalus*, these bones according to the position usually occupied by them in Lizards, viz. the front part of the pterygoids as palatines, and the palatines as palatal plates of the maxillary.

by a low, sharp ridge, the two ridges converging behind. The process for the attachment of the ectopterygoid (*t*) is unusually long, rectangular at the base; two of the branches of the ectopterygoid receive this angular process between them.

Before I pass to the description of the remaining part of the skull, the mandible, I may be allowed to notice the appearance of the *palate* whilst covered with the soft parts and the mucous membrane. Corresponding to the interpterygoid vacuity in the skull, there is a longitudinal arrow-shaped deep depression in the median line of the palate, commencing on a level with the hinder end of the palatine teeth. This depression is generally widened behind in Lizards, or at least continuous with the plane of the base of the skull; but in *Hatteria* it is entirely closed behind, in consequence of the proximity of the hypapophyses of the basisphenoid. Another, small but very deep, recess exists immediately behind the palatine series of teeth, in front of the long styliform process of the pterygoid and ectopterygoid. The boundary between palatine and pterygoid is marked by a slight ridge of the mucous membrane, nearly corresponding to the suture between those two bones; this ridge runs forward to the inner hinder angle of the choana; and posteriorly it is arched outwards, to form the gingival fold of the palatine series of teeth; it is split into two folds in front of this series, one to accompany the alveolar edge of the maxillary, whilst the other runs along the outer margin of the choana, which can be nearly entirely covered or closed by it. The choanæ are rather narrow slits not extending backwards to the palatine teeth. The auditory recess is rather shallow, eustachian tubes being absent with the tympanic cavity.

The symphysis of the *mandibles* is formed by a fibrous ligament allowing of some mobility; the persistent cartilage of Meckel is entirely replaced by fibrous tissue near the symphysis*. A part of the sutures between the bones of which the lower jaw of Lizards is generally composed have entirely disappeared (if they ever existed), so that the following bones only can be distinguished. The *dentary* (*u*) forms nearly entirely the outer surface of the mandible, a comparatively small articular portion and the top of the coronoid process excepted. The foramina mentalia vary in number from two to four, and are small. There is a very distinct foramen between the dentary and articular, penetrating to the inner surface of the mandible; it is identical with the large vacuity of the lower jaw of the Crocodile, and very indistinct or entirely closed on the outer surface in the Lizards. The *splenial* (*v*) is narrow elongate, behind twisted downwards to the lower side of the mandible and terminating about 3 millims. from its extremity. The *coronoid* (*x*) is triangular, covering with one angle the cartilage of Meckel, and forming with another the coronoid process. The *articular* bone (*w*) is very peculiar: if an angular bone was present at an early age, it has now entirely coalesced with the splenial, there being scarcely any osseous projection behind the articulary surface†. The articular

* Indeed *Heloderma* appears to be the only Lizard in which this cartilage forms the symphysis (Troschel, Wiegm. Archiv, 1853, p. 301).

† In *Grammatophora*, *Monitor*, and in a great number of Lizards the angular bone projects far beyond the articulary surface, frequently turned upwards in an oblique direction. Two muscles arise from this projection,

surface itself does not correspond in form with the condyle of the quadrate bone, being much elongate in the direction of the longitudinal axis of the body, and, in fact, nearly four times as long as the opposite articular surface.

When the jaws are closed, the upper condyles rest on the hindmost part of the lower articulary surface; but when in action, the lower jaw can be moved *backwards* to a considerable extent, the condyles advancing to the front part of the joint. As the upper condyles are concave, moving on a longitudinal convexity of the lower surface, they cannot escape sidewards, but press the hind part of the mandibles outwards; this again would not be possible, if the mandibles were anchylosed at the symphysis; so that their ligamentous junction finds now its explanation in the peculiar structure of the maxillary joints. When the jaws are at rest, the distance between the posterior extremities of the mandibles is 28 millims.; when the lower jaw is drawn backwards, the distance is 34 millims. It is evident that this structure is connected with the mode of feeding of this animal, which leads us next to a description of its dentition.

### *Dentition* (figs. 8–15).

*Dentition of full-grown specimens.—Hatteria* is an acrodont in the strictest meaning of the term, the teeth being so intimately anchylosed with the cutting alveolar edge of the maxillary and palatine, as to appear mere prominences of these bones. *The alveolar edges themselves are highly polished like the teeth, and perform the function of teeth* when these are ground down to the edge in advanced age. This, however, is not the case with the premaxillaries, each of which is armed with a single broad smooth tooth, notched at the crown in individuals of middle age (fig. 9); this notch disappears when the tooth is somewhat ground down (fig. 8), and the teeth have then the appearance of the upper incisors of a Rodent. The teeth of the maxillary and palatine (figs. 2 & 4) are rather short, triangular, pointed, longitudinally compressed; there are originally about eighteen in each maxillary, and eleven in each palatine. However, those of the anterior half of the maxillary appear to be soon ground down to the alveolar edge, with the exception of one midway between incisor and first palatine tooth, a trace of which is visible in the oldest example examined*. From the level of the first palatine tooth, the maxillary teeth are persistent, although more worn than those of the palatine. The

the outer of which is the *M. digastricus*, generally very distinctly separated into two portions; the inner is a simple short supplementary muscle. In *Hatteria*, where the *M. digastricus* has only a narrow space for its origin, it is thin and simple, vertically ascending to the mastoid; there is no supplementary muscle beneath it, but a strong fascia connecting the mandibular joint with the point of junction of exoccipital and quadrate bones, and covering the terminal cartilage of the stapes.

* The examples figured by Gray and Owen had their teeth complete; and although the one figured in the 'Zoology of the Erebus and Terror' was not only full-grown, but unusually large, it does not appear to have been old. The example also of which a full figure has been given by Dr. Gray has its teeth complete. I have well considered the question whether I had not to deal with two species (perhaps one from the northern, and the other from the southern island); but, after a very careful examination, no characters to justify a specific distinction were revealed.

first palatine tooth is much stronger than, and separated by a short interspace from, the succeeding. Each mandible is armed in front by two incisors and a canine tooth, more or less confluent at the base, so that the three teeth together, when ground down by the action of the upper incisor, present an uninterrupted oblique edge, most deeply chiselled out towards the symphysis (figs. 6 & 7). The alveolar edge of the mandible is polished, bearing about sixteen teeth as long as the number is complete; but (as in the maxillary) the teeth are gradually lost from the front backwards, and there is, corresponding to the remaining anterior maxillary tooth, a very shallow and long concavity scooped out by the vertical and *horizontal* action of this tooth. The toothed, or rather serrated hinder portion of the dentary fits closely between the maxillary and palatine series.

*Dentition of young examples.*—I have mentioned above that the upper incisors of a full-grown example are notched; this is explained by the remarkable circumstance that each is, in fact, composed of two teeth, which are conical and perfectly distinct in young examples about 7 inches long (figs. 10 & 11). Also the incisors of the lower jaw are separate to their base, and the lateral canine tooth is somewhat removed from them. The other mandibulary, maxillary, and palatine teeth do not differ peculiarly from those of older examples. No polished surface can be distinguished on the alveolar edges*.

Although the peculiarities mentioned render the dentition of *Hatteria* unique, I find on examining a number of acrodont Lizards, that a recent and a fossil genus approach it closely in this respect. *Uromastyx*, an Agamoid genus inhabiting sandy plains of Northern Africa and Hindostan, has a very narrow, strong, single intermaxillary, arched downwards and more or less overlapping the symphysis of the lower jaw, more so in adult examples than in immature (figs. 12, 13, 15). In a half-grown *Uromastyx hardwickii* (fig. 14) the intermaxillary does not project below the level of the maxillary, and is armed with four small, closely set, incisor-like teeth. The maxillary and mandibulary teeth are similar, and occupy in an uninterrupted series the alveolar edges of the jaws. This dentition is in time worn away: in the adult of the same species (figs. 12 & 13) the intermaxillary and the fore part of the maxillaries and mandibles are edentulous, the sharpened alveolar edges performing the function of teeth; the intermaxillary and the upper anterior angle of each mandible project, and these projections have their surfaces polished and bevelled as true teeth. In an old specimen of *Uromastyx spinipes* (fig. 15) I find the teeth complete in both jaws, but the intermaxillary projects considerably below the level of the maxillary, overlapping the mandibulary symphysis. The osteological characters of *Uromastyx* do not indicate a further affinity with *Hatteria*. It is worthy of notice that *Uromastyx hardwickii* is strictly herbivorous; I am not acquainted with the food of *U. spinipes*.

The fossil genus which I would compare with these recent forms is *Rhynchosaurus* (Owen), from the New Red Sandstone of Shropshire. In this Lizard the premaxillaries are paired, and bent downwards over the symphysis of the lower jaw; are these produced

* I cannot speak quite positively on this point, on account of the specimen having lain for some time in some fluid which affected the bones.

ends of the premaxillaries dilated teeth anchylosed to the bone as in *Hatteria*? And even if they really be part of the bone itself, the step from the premaxillary dentition of *Hatteria* to this beak of *Rhynchosaurus* can scarcely be regarded as greater than that from the dentition of the young *Hatteria* to that of the fully developed. However, a real affinity between the recent Lizards mentioned and *Rhynchosaurus* can only be conjectured, as its toothless lateral alveolar edges have been seen in one specimen only; nothing is known of the palate, or of the arrangement of the bones in the temporal region; its orbit is complete.

It will be a point of great interest to know whether those extinct Saurians which are distinguished either by entirely edentulous jaws, or by a combination of teeth with an edentulous and cutting alveolar edge (*Cryptodontia* and *Dicynodontia* of Owen), are completely toothed when young. This does not appear to be at all improbable; and if it should prove to be the case, the transition from the normal Saurian dentition to that of the Turtle will be complete*. Among Fishes the family of *Labridæ* offers a strikingly similar series of forms of dentition; and the observations made on *Hatteria* and *Uromastyx* go far to prove the correctness of the views advocated in the 'Catalogue of Fishes,' vol. i.v, viz. that small fishes with complete dentition (referred by other authors to different species and genera) are merely the young of others with partly edentulous jaws, and that the Scaroid and Odacoid fishes cannot be separated from the Labroids on account of their dentition. Thus in this most natural family we find the majority of generic forms provided with a normal complete dentition; in others (*Chærops*, *Xiphochilus*, *Pseudodax*, &c.) the lateral teeth are gradually and normally replaced by a more or less cutting edge of the mandible; and finally, in the *Scarina* and *Odacina* the entire mass of the teeth and jaws are coalesced, forming a beak with sharp cutting edges, the single teeth being still visible in the true *Scarus*, whilst they have entirely disappeared in adult *Pseudoscarus* and *Odax*. Unfortunately, up to this time, we are unacquainted with the dentition of very young Scaroid fishes.

All Lizards masticate their food in some degree; in so doing their lower jaw is moved, in a vertical direction only, towards the upper. In full-grown specimens of *Hatteria* a great portion of the side of the jaws has a *sharp cutting* edge, whilst the toothed portion also has more the appearance of a *cutting serrated* edge than of series of teeth. The force in cutting and sawing is, of course, considerably increased by the property of moving the lower jaw backwards and forwards, a property dependent on the peculiarity of the maxillary joint as described above.

All the specimens examined had the stomach and upper part of the intestine empty; but the rectum of one contained a great mass of the remains of the young of some bird which probably builds on the ground; the plumes of the feathers without quills (which being still soft had been digested), together with the condition of the bills and bones, could not leave any doubt as to the age of the bird. The same mass contained also one fragment of the elytron of a beetle. It is known, from actual observation of living speci-

* The embryo of *Trionyx* shows numerous rudiments of teeth.—Owen, Odontography, p. 179.

mens, that *Hatteria* is very slow and sluggish in its movements*; therefore its food must chiefly consist of other Lizards of similarly sluggish habits, and young birds or insects accidentally approaching its mouth. Young specimens feed probably on insects only.

Before I proceed to the description of the trunk of the skeleton, I may be allowed to recapitulate the peculiarities in the structure of the skull; they are—

1. Persistence of the sutures, especially of those between the lateral halves of the skull, combined with a great development of its ossified parts as it appears in the expanse of the bones forming the upperside of the facial portion, in the completeness of an orbital ring with a temporal and zygomatic bar (Crocodilia), in the much expanded columella, in the completely osseous bottom of the orbit, and in the almost continuous roof of the palate—the palato-narial and interpterygoid vacuities being very narrow.

2. Firm and solid union of the os quadratum with the skull, and of the bones of the palate with the quadrate, as shown by the sutural connexion of quadrate and pterygoid, broad sutural connexion of the columella with quadrate and pterygoid, immoveable pterygo-sphenoid joint, firm and extensive attachment of pterygoid to ectopterygoid.

3. This restriction of the mobility of the bones named is compensated by an increased and modified mobility of the lower jaw, the mandibles being united by a ligament and provided with an elongate articular surface.

4. Displacement of the palatine bones which are separated by the pterygoids, and replace a palatal portion of the maxillaries.

5. Dentition unique, viz.:—two large cutting teeth above, formed by the confluence of two pairs of conical "milk"-teeth; alveolar edges of the jaws and palatines cutting and polished (in the adult), only partially armed with teeth (forming a serrature); palatine teeth in close proximity and parallel to maxillary series, both series receiving between them in a groove the similarly serrated edge of the mandible.

6. Finally, perforation of quadrate bone and extremely short postarticular process of mandible.

### *The Vertebral Column.*

Professor Owen has discovered the biconcavity of the vertebræ† (figs. 19, 22, 23). Those of the trunk (from the third cervical vertebra to the fourth or fifth caudal) are distinguished by their uniformly developed, strong and compressed neural spines, which become more slender and remote on the tail, disappearing only on about the last ten vertebræ. The total number of vertebræ is 63, viz.:—

3 cervical (1–3) without pleurapophyses.
5 cervical (4–8) with pleurapophyses.
3 dorsal (9–11) with ribs attached to sternum.
11 dorsal (12–22) with ribs and abdominal ribs.
3 lumbar (23–25).

* I am indebted to Sir Andrew Smith for this observation.

† Catal. Osteol. Ser. Roy. Coll. of Surgeons, vol. i. p. 142 (1853).

2 sacral (26, 27).
36 caudal (28–63).

As regards the first five vertebræ (fig. 17), Professor Owen (*l. c.*) has given so detailed a description of them, that I may be allowed to quote it in full.

"The atlas consists chiefly of the hypapophysis and neurapophyses; the former is in the form of a transverse arched bar, concave upwards, with the anterior border cut obliquely to receive the under part of the occipital condyle; the posterior border is convex vertically, and adapted to the transverse trochlear groove in the fore part of the odontoid process; the neurapophyses have a small articular surface at the fore part of their base for the occipital condyle, and a second at their inner and back part for their proper centrum, the odontoid process. They are expanded superiorly, develope a small posterior zygapophysis, exterior to which is a tubercle; and they come in contact, but do not coalesce, above the neural canal. The odontoid process [fig. 18] has coalesced with the body of the axis, which it equals in height and exceeds in breadth; it is convex from side to side, concave vertically at its lower half, having, as it were, a channel scooped out from side to side; this kind of joint will allow of great extent and freedom of motion of the atlas with the head from side to side; whilst the vertical movements would be restricted. The neurapophyses of the axis have coalesced with the centrum below, and with each other above, where they develope a strong ridge or spine, which is most produced in the antero-posterior direction. An autogenous hypapophysis [fig 17, *e*] is wedged into the inferior insterspace between the centrum of the axis and the third vertebra. The centrum and neurapophyses of the third vertebra have coalesced; a short diapophysis projects from the line of union. The anterior and posterior zygapophyses form the angles of the broad base of the neural spine; this spine is of moderate length, thick and trihedral. There is a small wedge-shaped hypapophysis beneath the interspace of the third and fourth vertebræ. The fourth vertebra has a short pleurapophysis on each side, with a bifurcate proximal end articulated by a broad tubercle to the diapophysis and by a slender neck and head to a rudimental parapophysis; but this is very feebly marked off from the diapophysis. In the fifth vertebra the parapophysis and diapophysis form together an oblique ridge, chiefly extended vertically, and to which the expanded head of the pleurapophysis articulates by a single surface. There is a wedge-shaped hypapophysis at the interspace of the fourth and fifth vertebræ."

This description agrees in every point with the three skeletons examined by myself, except that, in one example, the pleurapophysis of the fourth vertebra is not bifurcate, the lower branch being replaced by a ligament, and no trace of a parapophysis can be distinguished.

The hindmost autogenous hypapophysis (fig. 17, *e'*) corresponds to the seventh and eighth vertebræ. The dorsal vertebræ differ scarcely from the middle and posterior cervical; the zygapophyses are more distant from one another; and the oblique ridge formed by coalesced diapophysis and parapophysis is less prominent, receding nearly to the level of the centrum, on the hinder dorsal vertebræ. The centrum, the lower half

of which is rather compressed in the vertebræ of the anterior two-thirds of the trunk, becomes more flattened on its abdominal surface in the lumbar region, and still more in the sacral and two or three anterior caudal vertebræ.

The three *lumbar* vertebræ are distinguished merely by the shortness of the hæmapophysis, which does not reach the abdominal sternum, and is quite rudimentary at the second and third lumbar vertebræ; their pleurapophyses form horizontal transverse processes, not entirely anchylosed to the diapophysis, a suture being visible; however they do not appear to be moveable.

The pleurapophyses of the two sacral vertebræ are about thrice as strong as the preceding, constricted in the middle, and swollen at both ends, the sutures by which they are united with correspondingly increased diapophyses and with the ilium being very distinct; their iliac extremities are in contact with each other.

*Hatteria* is one of those Lizards in which the tail, when mutilated, is reproduced*; however, it is much less easily broken than in the *Lacertidæ*, *Geckonidæ*, &c., its external integuments being less distinctly divided into segments or verticelli, and strengthened by a thick layer of strong subcutaneous fibrous tissue. The centrum of each caudal vertebra (fig. 23) is divided into an anterior and a posterior portion, as in other Lizards with verticellated tail†, the epiphysial line passing *through the middle*, and *behind* the transverse process; this line corresponds to the external vertical furrow between two verticelli. The neural spines, which on the three anterior caudal vertebræ are as strong as those of the sacral region, become gradually shorter and thinner, and disappear entirely on the posterior third of the tail. The hæmapophyses, coalesced as in other Lizards, appear first between the third and fourth caudal vertebræ (fig. 21), are there rather longer but much narrower than the neural spines, and, becoming more feeble towards the extremity of the tail, disappear entirely on the last six or seven vertebræ. The transverse processes of the first two caudal vertebræ are nearly vertical to the longitudinal axis of the vertebral column, and not much shorter than those of the sacrum; all the remainder are obliquely directed forwards, and become rudimentary with the eighth vertebra.

### *Ribs and Sternal Apparatus of Thorax and Abdomen.*

The pleurapophyses of the fourth and fifth vertebræ have been described above (p. 11); those of the sixth and seventh are not much longer than that of the fifth; but their distal ends are considerably more dilated (fig. 17). All the pleurapophyses mentioned have cartilaginous hæmapophyses slightly ossified at the base. The anterior (of fourth vertebra) is quite rudimentary; but the third and also the fourth (of sixth and seventh vertebræ) are much dilated at the base, the dilatation extending some way upwards along the *posterior* edge of the pleurapophysis, and overlapping the *succeeding* pleurapophysis. There is no anterior projection as in Crocodiles. The pleurapophysis of the eighth vertebra is slender,

* One specimen, in which the posterior third is reproduced, has a rudimentary appendage at the base of the reproduced portion, an anomaly frequently observed in other Lizards.

† Cuvier, Recherches, vol. x. p. 13.

twice as long as the preceding one, terminating in a short terminal cartilaginous or semiossified hæmapophysis; *in the middle of its length, on its posterior edge, it is (like all the following ribs) provided with an apophysis directed obliquely backwards and upwards, overlapping the following rib, of the same form as, and homologous with, the processus uncinatus of birds.* The first of these apophyses is fibro-cartilaginous; the two or three following are semiossified, the following entirely osseous, and the posterior, again, less ossified than the middle. None of them are completely anchylosed to the rib, but attached to it by a short suture strengthened by ligaments*. When we consider the transition from the dilated hæmapophysis of the sixth and seventh vertebræ to the completely ossified uncinate (epipleural) process of one of the middle dorsal vertebræ, we shall be inclined to regard these uncinate processes as parts of the hæmapophyses, separate and removed from the distal end of the pleurapophyses in proportion as the latter increase in length†.

All the complete ribs, fourteen in number, have a broad, compressed head, joined to a low oblique ridge of the centrum of the vertebræ; they are entirely smooth, without any tubercle in their arched basal region, which has a shallow longitudinal groove in front; they are slender, becoming gradually a little broader towards their distal extremity. The anterior are but little, the posterior considerably, shorter than the middle, the length of which is nearly equal to the extent of five dorsal vertebræ. The hæmapophyses of all (the last two excepted) are divided into two semiossified pieces united by a joint (fig. 24); the upper (*a*), thinner and shorter piece (of about one-fourth or one-fifth the length of the rib) follows the direction of the rib, whilst the lower is directed inwards and forwards. The hæmapophyses of the first three ribs reach the sternum, the two foremost not showing any peculiarity of form, and the third approaching the succeeding in form by having a slight expansion in front and behind. The lower pieces of the hæmapophyses of all the following ribs are much dilated and imbricate (fig. 24, *b*, and fig. 20), each with a rounded wing-like expansion in front and behind; they have the form of a trapezoid situated obliquely on each side of the medial line of the abdomen; its upper anterior angle is one of the expansions overlapping the preceding hæmapophysis; the upper posterior angle is produced to meet the costal piece of the hæmapophysis; the lower posterior angle is the other expansion overlapped by the produced lower anterior angle of the succeeding hæmapophysis; and this latter angle reaches one of the bones of the abdominal sternum. The hæmapophysis of the two hindmost ribs consists of one long bent semiossified cartilage only, and the expansions are less developed than in the other ribs.

* In young pheasants, just on the point of being hatched, I have found the uncinate processes of the anterior and posterior ribs cartilaginous, ossification having commenced only in those of the middle ribs, at some distance from the rib; the body of the ribs was completely ossified; of course, no anchylosis of the processes to the bone had taken place. The same I found to be the case in a *Talegalla* of the same age, a bird which is capable of flying almost as soon as it leaves the egg-shell.

† I am confirmed in this view by a comparison of the same parts of the Crocodile, where a similar process is developed near the distal end of the ribs; in young individuals this process is still confluent with the hæmapophysis, as in the fourth rib of *Hatteria*.

The *sternum* (fig. 26, *a*) does not essentially differ from that of other Lizards; it is a five-sided plate with a very thin ossified layer imbedded in its cartilaginous substance; no division into lateral halves is perceptible. Its hinder margin passes into a broad ligament connecting it with the first abdominal rib, to the posterior lateral margins the hæmapophyses of the first three ribs are joined; the anterior lateral margins are entirely cartilaginous, and grooved for the reception of the coracoid (fig. 26, *e d*); and, finally, at its front angle enters the episternum (fig. 17, *b*; fig. 26, *b*), which is very long, styliform, in continuity with a median keel of the sternal plate, and terminates in a transverse bar which is suturally united with the clavicles (fig. 17, *c*; fig. 26, *c*).

The eleven posterior ribs are connected by means of their double hæmapophyses with a series of bones crossing the abdominal region and situated in the subcutaneous ligamentous tissue extending from the sternum to the pelvis (fig. 26). This system of bones is similar to, but essentially different from, that observed in Crocodiles and some Lizards (*Chamæleon*, *Polychrus*, &c.), known as abdominal ribs or abdominal sternum, and considered to be the ossified inscriptiones tendineæ of the abdominal muscles.

The first point of interest is, that in *Hatteria* the number of these abdominal ribs does not equal that of the corresponding true ribs and vertebræ, being nearly double that number (I have counted from twenty-five to twenty-six); it equals rather the number of transverse series of plates into which the external integument of the abdomen is divided, so that each abdominal rib runs along, and is firmly attached by tissue to, the anterior margin of one of those transverse series of plates. Each abdominal rib has the form of an angular, thin, very slender bone, tapering at the extremities, and somewhat dilated at the angle, which is directed forwards and obtuse. The angles of all these ribs lie in the same line, in the median line of the abdomen. Each consists of three bones, united by the closest juxtaposition—a central and a pair of lateral, the lateral being about as long as one of the halves of the central; however, these three bones are so firmly united that it is very difficult to separate them*. Normally the ribs are entirely separate from one another, and only exceptionally two or three coalesced by a narrow osseous strip in the median line. Every alternate abdominal rib (fig. 20) is suspended from the hæmapophyses of a pair of ribs, the suspension being effected by a short ligament in which the produced extremity of the dilated piece of the hæmapophysis terminates. The point of attachment is on the inner side of the abdominal rib, near the junction of its central and lateral pieces. The intermediate abdominal ribs are "floating," but otherwise not distinguished from the others.

* The first example examined by me showed a very curious anomaly as regards the union of the three bones of which the abdominal ribs consist; *they were united by joints*. That these joints are not the result of some accident—of fracture of the ribs at a former period—is proved by the circumstance that they exist only in every alternate rib, viz. in those which are connected with hæmapophyses, and, secondly, that they occupy exactly the same place, namely, at a short distance from the attachment of the hæmapophysis, towards the median line of the belly. The intermediate abdominal ribs have no joints, their bones being juxtaposed as I have described above, I cannot offer any explanation of this singularity, which I must consider anomalous, not having met with it in five other examples. It is represented in fig. 20.

Before I proceed to the next portion of the skeleton, I shall describe what I have noticed with regard to the muscles connected with this complex apparatus of ribs, in order to determine its probable function.

After the skin and the *latissimus dorsi* (which, thinned into a cutis-muscle, descends to the middle of the side of the trunk) are removed, the series of the uncinate apophyses of the ribs are seen imbedded (fig. 33), and forming the boundary line between two large masses of muscles, the dorsal of which corresponds to the *longissimus dorsi* (including the *sacro-lumbaris*), the ventral to the *obliquus externus*. Their effects are antagonistic. The *longissimus dorsi* is divided by *ligamenta intermuscularia* (in number equal to the dorsal and lumbar vertebræ), each of them terminating at and attaching itself to the hinder edge of a rib and to the extremity of its apophysis. This division of the mass of this muscle renders it possible that certain portions of it can be called into action independently of the others. It raises the ribs, or part of them. Towards the middle of the back it is covered by a strong aponeurosis, from the median line of which arises a series of short, erect, conical muscles (fig. 33, *b*), each enclosed in an aponeurotic sheath, destined to move parts or the whole of the dorsal crest.

The ventral muscular mass forms a much thinner layer; and although it can be divided into three strata, the whole must be regarded as *M. obliquus externus*, as all the fascicles run backwards and downwards (fig. 33, *c*). The outer stratum (*c'*) is fixed to the abdominal aponeurotic band; and immediately below this line of attachment lies the hinder portion of the *pectoralis major* (*d*), which thus is received in a sheath, formed by a separation of the two outer strata of the *obliquus externus*. The innermost stratum is very thin, and frequently interrupted. The three strata coalesce as they approach the costal apophyses, the mass being divided into fifteen parts, each of which is inserted into the lower edge of a costal apophysis. The function of this muscle is to depress and draw backwards the ribs.

Beside the *intercostales*, the outer stratum of which runs backwards and downwards, the inner having a backward and upward direction, irregular muscles, passing two or three ribs to insert themselves at the third and fourth, assist the function of the true intercostales; they are found both on the inside and the outside of the ribs.

Each of the dilated costal hæmapophyses has two muscles: one (fig. 33, *g*) is flat, thin and broad, and arises from the concave anterior edge of its ventral portion, and is attached behind the insertion of the corresponding muscle of the preceding cartilage; its fibres run obliquely forward and inward. The second muscle (*h*) arises from the outer surface of the dorsal portion of the hæmapophysis, and is attached to the inner side of the joint between the next following cartilage and its rib; its fibres run obliquely backward and outward. Sometimes a small fascicle is detached from this muscle, coalescing with the intercostalis of the same rib. I shall speak subsequently of the use of these muscles.

On examining the *ventral region of the trunk* (fig. 32), we find that the cutis, which is so easily detached in other Saurians, is most intimately attached to a layer of strong fibrous tissue (*c c*) extending over the entire lower surface of the abdomen to, and passing into,

the aponeurotic borders of the *MM. pectoralis major* and *obliqui externi*, the latter being situated entirely on the sides of the trunk. The fibrous tissue is so strong, that the cutis can only be removed with the knife; and it adheres not less firmly to the outer surface of the abdominal ribs; the portions stretched from one rib to another are, although firm, very pliable, allowing of an easy approximation of the ribs. Immediately below this fibrous layer is the *M. rectus* (*d*), in the superficial substance of which the abdominal ribs are imbedded, and which transversely extends from one extremity of an abdominal rib to the other. Below the *M. rectus* is the *fascia abdominalis* (*f*), covered by the black peritoneum, and passing laterally into a very thin *M. transversus abdominis*. The dilated hæmapophyses with their special muscles lie between the *M. rectus* and the *fascia abdominalis*. Finally, it must be mentioned that the extremities of the fifteen anterior abdominal ribs are attached to the hinder portion of the *pectoralis major*.

Abdominal ribs are developed in many Saurians and in a great number of Teleosteous Fishes; they may serve merely to strengthen the abdominal muscle and to afford a firmer and larger base for the attachment of muscular fibres, forming, as for instance in Saurians living on the ground, a kind of abdominal sole. When in continuity with the ribs proper (as in the Chameleon), they will essentially contribute to the support of the contents of the abdominal cavity, especially when these are pressed backwards by much inflated lungs, or when their weight is much increased by the addition of developed ova*. But in no Saurian, so far as we know at present, have they any relation to the external integuments; this we find to be the case in many Clupeoids, where their dilated centre protects the sharp abdominal edge.

As regards *Hatteria*, their increased number (exactly corresponding to that of cross series of external ventral plates) and their peculiar connexion with hæmapophyses dilated into a broad base for the attachment of muscles show plainly enough that this apparatus is subservient to some special function, viz. *to assist in locomotion.*

I have arrived at this conclusion from the following considerations:—

1. *Hatteria* lives on the ground, in rocky parts of the sea-shore, in sandhill-holes made by some other animal; its limbs, although muscular, are, compared with the majority of ground-lizards, short, especially the hind limbs. Its claws are comparatively very feeble, and acutely pointed†, showing that in a normal state they cannot be much used in dragging the heavy body, or even in burrowing.

2. The series of external ventral plates are not less imbricate than their homologues in Ophidians; they are covered with a very firm epidermis, much worn on the posterior edges.

3. Each transverse series of ventral plates, although consisting of fifteen or sixteen

* In the latter case they perform a service analogous to the ossified tendon of the *M. obliquus externus* of Marsupials.

† They are so in the specimens killed at the time of capture; in two examples kept for some time in captivity they are worn down to the base; this was doubtless caused by efforts to escape, as we observe in other lizards under similar conditions.

separate plates, is worked as if it were but a single piece, like an abdominal plate of a snake, all the plates being firmly united to the underlying abdominal rib.

4. When the fore part of the body is fixed, the action of the *M. rectus* (assisted by the *pectoralis major*) is to approximate the abdominal ribs and plates to one another, or to draw the hind part of the body forwards.

5. The end of the hæmapophysis is fixed in the *middle* of each branch of the abdominal rib—that is, exactly at the point where the greatest effect on the rib can be produced.

6. The action of the pair of hæmapophysial muscles is to draw the hæmapophysis, and with it the abdominal rib, backwards; the abdominal rib being attached to the base of the ventral plates, the edges of the latter must be raised, thus taking hold of any roughness of the ground with which they come in contact. The advantage derived therefrom, when the animal ascends a declivity, is evident.

I do not for a moment entertain the idea that an individual of *Hatteria* with the limbs disabled could glide from the spot where it lies, nor am I convinced that the action of the abdominal apparatus is constantly superadded to that of the limbs; but in the case of a lizard living on the rocks and sand-hills of the sea-shore the occasions must be frequent when the feebleness of its claws is complemented and assisted by its ventral plates. If the supposition should be confirmed that *Hatteria* lives in holes, where the free action of the limbs is naturally more or less impeded, the abdominal apparatus would be of material service. However, this habit has been attributed to it by Dieffenbach only on the authority of natives, and it is not in accordance with the feeble development of the claws and with the presence of a much developed dorsal crest; at all events it is obvious that the holes could not have been burrowed by the animal itself.

### *Bones of the Fore Limb.*

The cartilaginous portions of the *scapula* and *coracoid* (figs. 25 & 26) are very broad, that of the former being larger than the ossified portion. The osseous scapula (*g*) is, as usual, constricted in the middle, and there is in the concavity of the anterior margin a very distinct acromial tuberosity (*h*), to which the clavicle is attached by a strong ligament. The *coracoid has no notch whatever*; its osseous portion (*e*) is an irregularly subsemicircular disk, with a narrow foramen* (for the passage of blood-vessels) near the suture formed with the scapula, and with a second, less distinct, near its posterior extremity; its cartilaginous border is narrowest in its posterior half, where it is received into the glenoid cavity of the sternum; towards the front it widens to fill the angular space between episternum and clavicle. The glenoid cavity of the sternum is deep and long, like a slit, and the entire joint is formed by cartilage; the ligaments connecting these cartilages and fixing the coracoid to episternum and clavicle are strong, though very loose, allowing of great extent of motion. The *humerus* is very similar in form to that of *Varanus*, *Uromastyx* and others, being much expanded at its extremities, and

* It is also very distinct in the Crocodile and Varanus.

subcylindrical in the middle; the transverse axes of the expanded extremities intersect each other at an angle of about 40°; the entire bone is somewhat longer than the ulna. The *ulna* and *radius* do not exhibit any peculiarity; the facet of the former for articulation with the humerus is small, not larger than that of the olecranon, which is a well-developed, oblong, tetrahedral bone. The distal extremities of the bones of the forearm are not in contact with each other. The *carpus* is composed of ten bones, five in each series; most of them are broader on the inner (palmar) surface than on the outer. The ulna articulates with three: one (*pisiforme*) is, as it were, pushed out and attached to the outer side of the extremity of the ulna, where it forms a very conspicuous prominence for the attachment of the ligaments and muscles which will be mentioned subsequently; the two others (*triquetrum* and *lunatum*) are the largest of all these bones, making up for the shortness of the ulna, which does not reach so far downwards as the radius. The radius articulates with two bones, which together may be regarded as an *os naviculare*; the outer of them projects more than the other, and sometimes articulates with the fifth digit. The bones of the second series correspond to the metacarpals, that of the second digit (*capitatum*) being somewhat more prominent than the others. Variations of this arrangement of the carpal bones do not appear to be scarce; thus, for instance, the *os lunatum* has been found removed from the ulna, pushed between the two series. Of the *metacarpals* the third and fourth are longer, the first and fifth shorter, than, and the second as long as the two following phalanges together. The *phalanges* are two, three, four, five, and three in number, and the penultimate is never longer than the preceding (as, for instance, in *Monitor*).

### *Muscles of the Fore Limb.*

*Muscles of the humeral region and upper arm.*—The *MM. deltoideus* and *latissimus dorsi* do not show any peculiarity; the former arises only from the surface of the cartilaginous portion of the scapula. The mass of muscles nearest to the bone and passing over the humeral joint, homologous with the *MM. supra-* and *infraspinatus* et *teretes*, is only partially subdivided. All these muscles lift and approximate the limb to the trunk; and this is done in a direction more or less backward, according to the degree in which the *M. latissimus dorsi* is brought into action.

Two muscles take their origin from the bony arch formed by the transverse piece of the episternum and clavicle:—first, the *portio clavicularis* of the *M. pectoralis major*, which is elongate, hammer-shaped, its fibres converging into a very strong tendon, in advance of the middle of its outer margin, and inserted, as usual, into the *tuberculum majus*. Although its fibres are in perfect contiguity, and all tend to depress the limb towards the chest, they are so disposed that the anterior portion draws it forward, the posterior backward, and the middle vertically to the longitudinal axis of the body. The *second* muscle arising from the clavicle has no homologue in the fleshy part of Mammalia; it is the *claviculo-brachialis* of *Emys*, and situated immediately below the portio clavicularis of the *pectoralis major*, flat, arises from the whole posterior side of the clavicle,

and terminates in a tendon attached to the outer convex surface of the end of the humerus, immediately above the insertion of the *M. brachialis internus*. This muscle assists materially in drawing the limb *forward*.

The *M. biceps brachii* is situated as in higher animals, but divided into two distinct muscles running parallel to each other, being separated superiorly only by the insertion of the pectoralis major: the *inner* muscle (corresponding to the the *caput breve* of human myology) is the longer, indeed the longest muscle of the arm, extending from the sternal margin of the coracoid to the *upper end of the ulna*; above its middle, where it is crossed by the tendon of the pectoralis major, it is reduced to a narrow tendon, so that there is no friction between the two muscles although they cross each other; it is fleshy again towards the margin of the coracoid. The *outer* muscle, fixed to the upper end of the radius, is arrested in its course by the tuberculum majus humeri, to which it is attached by the side of the pectoralis major; a strong ligament running from this tubercle to the scapula may be regarded as the continuation of the tendon of this muscle, and as homologous with the "*caput longum*."

A very slender muscle accompanies this part of the *M. biceps*; and being also attached to the upper end of the radius, and passing uninterruptedly into the ligament described as homologous with the "corpus longum," it may properly be taken as a third detached part of the *biceps*.

The *M. coraco-brachialis* has, in accordance with the development of the coracoid bone, become a very powerful muscle; it may be incompletely divided into two portions, the *anterior* of which is broad, flat, arising nearly from the entire surface of the bone, and inserted all over the *concave* surface of the end of the humerus, downwards to the middle of its length, where it is confluent with the *inferior* portion; this is a more slender muscle, taking its origin from the lower posterior angle of the coracoid, and inserted at the condylus externus humeri. The function of the entire muscle is to depress the limb, and to effect a simultaneous rotatory movement of its longitudinal axis.

The *M. brachialis internus* is well developed, and does not essentially differ from that of Emys or of Mammalia.

The *M. triceps* is very peculiar; it consists of two strong portions, an inner and a superficial, both confluent with each other and with the *M. brachialis internus* near the olecranon: the inner portion is attached along the body of the humerus between olecranon and tuberculum minus. The superficial stronger portion passes between the tendinous ends of the Latissimus dorsi and Deltoides, and is inserted into the contracted part of the scapula; however, before it passes the tendon of the Latissimus dorsi, it emits another, long, slender tendon, crossing the plexus nervorum brachialis and attaching itself to the inside of the posterior angle of the coracoid. This tendon appears to serve merely as an additional point of attachment to the muscle.

*Muscles of the lower arm.*—The *flexor* muscles arise chiefly from the inner condyle of the humerus and the inner surface of the bones of the lower arm. Their number is reduced to three, viz.:—First, the *M. flexor digitorum profundus*, terminating in five tendons for the

extreme phalanges of the fingers, without a trace of *MM. lumbricales*; a *M. flexor digitorum sublimis* appears to be represented in a rudimental condition by a flat and thin muscle, which takes its origin from the fascia covering the lower part of the *profundus*, and chiefly from a ligament extended between the *os pisiforme* and *os naviculare*; it is fan-like, expanded over the palm, and at the base of each digit the fibres diverge for the passage of the tendons of the *profundus*; for this reason I am inclined to regard this muscle as a rudimental *l. d. sublimis* rather than as a muscular *fascia palmaris*. A strong aponeurotic palmar fascia is not developed. Secondly, the *flexor carpi radialis* and *M. pronator teres* arise united into one muscle, the latter becoming distinct towards its insertion on the middle of the radius, and being entirely covered by the former, which, as usual, descends to the end of the radius and to the carpus. Thirdly, the *flexor carpi ulnaris*, subdivided by a broad aponeurosis along the middle of its interior; a part of it passes the carpus and forms an *abductor digiti minimi*.

The *extensor* muscles arise chiefly from the outer condyle of the humerus and the outer surface of the bones of the lower arm; they are less powerful than the flexors, and more intimately connected with each other by intervening tissue, which disappears entirely towards their origins; so that their separation is artificial for a part of their length. Six can be distinguished, viz.:—The *M. supinator longus* and the *extensor carpi radialis brevis* and *longus*; they are slender, running parallel to each other, and inserted more distinctly into the carpal bones than into the metacarpals. The *extensor digitorum communis* is the most powerful muscle of this region, and in its lower part divided into two portions, the longer of which terminates in an aponeurosis attached to the metacarpal bones, whilst the shorter coalesces with the distal portion of the *extensor carpi radialis brevis*. Covered by the proximal portion of the muscle last described, and situated inwards of the olecranon, is a very distinct oblique muscle, arising from the condylus internus humeri, and attaching itself to the outer surface of the olecranal part of the ulna; it occupies exactly the position of the *Anconæus quartus*, and may be regarded as such, or as an *Extensor carpus ulnaris* (which otherwise could not be accounted for). Finally, the muscles which in Man are divided into the abductor and extensors of the thumb, form in *Hatteria* one flat layer covered by the *Extensor digitorum communis*, arising from the distal half of the ulna, and spreading over the bones of the carpus, to which they are attached; the fibres nearest to the radial margin are collected into a tendon which is inserted at the metacarpal bone of the thumb.

There is no *ligamentum osseum* between ulna and radius; it is replaced by a strong muscle, the fibres of which are transverse, either vertical to the longitudinal axis of the arm, or obliquely descending from the ulna to the radius.

Of the *muscles of the hand*, besides those mentioned above and the *interossei*, one remains to be noticed, lying below the *flexor digitorum communis* on the metacarpal bones; it arises from the second series of carpal bones, its fibres diverging chiefly from the carpal bone of the fourth digit (os hamatum) to the base of the proximal phalanges of the digits; it essentially assists in adduction and flexion of the digits.

### *Bones of the Hind Limb.*

The *pelvis* (figs. 27 & 28) represents the Lacertian, and not the Crocodilian type, and differs from the former in the subvertical direction of the ascending part of the ilium, the axis of which forms with that of the vertebral column an angle of about 80°. Its lower surface is flatter and less convex than in the majority of Lizards. The ascending part of the ilium is about thrice as high as broad, and projects much beyond the transverse processes of the sacrum; its hinder margin is nearly straight, the anterior having two very slight tuberosities—the upper opposite the junction with the sacrum, and the lower on the ilio-pubic suture. The acetabulum is formed by the three pelvic bones, as in Lizards. The pubic and ischium are very similar in form, and form with their fellows moderately broad symphyses; the obturator vacuity is of the usual extent, divided into two by a strip of the symphysial cartilage. The pubic has a remarkably developed uncinate process (*c*) in the middle of its anterior margin; and still more prominent is the *tuberositas ischii* (*d*), the distance of the latter from the uncinate pubic process of the same side being quite equal to that between the two pubic processes. These processes serve for the attachment of ligaments and muscles, to be described subsequently. The pubic bone is perforated by a nerve and blood-vessels for the abductor muscles of the femur, about midway between the uncinate process and the foramen obturatorium. The symphysial cartilage shows scarcely a trace of ossification.

The bones of the *upper* and *lower* leg do not differ from the Lacertian type: there is one large trochanter (corresponding in position to the *trochanter minor*), no ossification in the *ligamentum patellare*, no sesamoid bone between femur and fibula, as in *Varanus*.

The tarsal and metatarsal bones agree with those of *Varanus* in number and arrangement; all are thin and flattened. The first series is composed of the two bones which may be briefly designated as *astragalus* and *calcaneum*; the suture between them is scarcely visible, but it may be distinctly seen that the fibula is articulated with the calcaneum alone, without coming in contact with the astragalus. The second series consists also of two bones (the inner being very small), intercalated between the first series and the three middle metatarsals. The fifth metatarsal has the proximal end dilated, this dilatation having the irregular form of a tarsal. A thick cartilage, without ossification, intervenes between the first metatarsal and astragalus. The first metatarsal is shorter, the three middle longer, than, and the fifth as long as the two proximal phalanges together. The number of phalanges is 2, 3, 4, 5, 4.

### *Muscles of the Hind Limb.*

The determination of the homologies of several of these muscles (fig. 34) is a matter of uncertainty; I am guided in it chiefly by the proximal point of their insertion, in the second place by their function, considering their development and extent a matter of but little importance. The powerful extensor known as *MM. rectus* and *vasti*, and properly regarded as one muscle by Hyrtl (*extensor cruris quadriceps*), is at once recognized; it

is formed by three heads: the broadest and most superficial arises from the os ilium*; the second is slender and comes from a slight tubercular prominence on the end of the ilio-pubic suture; the third is covered by the two former and attached to the anterior surface of the lower half of the femur.

The second muscle, which takes its origin from the ascending portion of the *os ilium*, may be regarded as *M. iliacus internus*; it is entirely covered by the *M. rectus*, and terminates along the outer and hinder surface of the upper half of the femur; it draws the limb backwards and upwards.

The *M. pectineus* arises from the ventral surface of *os pubis* and *ilium* and from their anterior edge, and, descending over the humeral joint to the antero-interior surface of the humerus, terminates just above the lower (third) branch of the *extensor cruris*; it draws the limb forwards†.

The *M. gracilis* runs parallel to the slender head of the *M. extensor cruris*; it arises from the uncinate process of the os pubis, and is inserted into the hinder side of the end of the tibia.

Immediately below the skin, the pelvic region is covered by a flat and thin muscle arising from the symphysis ossium pubis et ischii and the uncinate process of the os pubis; it is the hindmost part of the *M. obliquus abdominis externus*. On removing this muscle a strong ligament, stretched from the uncinate process to the hinder end of the symphysis ossium ischii, becomes apparent. From this ligament, as well as from the tuber ischii, arises a mass of muscles divided into more or less distinct portions, which, however, cannot well be designated by names taken from human anatomy. We can only determine it so far in a general manner, that, to judge from its origin and from its function of bending the lower leg, it corresponds to the *MM. semimembranaceus, semitendinosus, biceps* (and *glutæus*?) (fig. 34, *b c*). Its largest portion covers nearly entirely the lower side of the leg, and is inserted into the outer side of the end of the tibia; it emits a branch to the inner side of the distal half of the femur, which has the function of an *adductor femoris*. The other, more slender, portion is entirely on the hinder side of the leg; its fibres are confluent with those of the larger portion, near their origin; another portion

* This head is described by Dr. Haughton as *M. glutæus maximus*, in the Crocodile (Ann. & Mag. Nat. Hist. 1865, vol. xvi. p. 327). It has in the Crocodile exactly the same anterior position, the same origin, and the same extent as in *Hatteria* (and other Lizards); but it is somewhat more feeble. It passes into the tendon of the *extensor cruris* (*ligamentum patellæ proprium*), from which it can be severed only artificially; moreover it is one of the chief extensor muscles of the lower leg. For these reasons I cannot adopt Dr. Haughton's determination, but I regard it as the *rectus* portion of the *extensor cruris quadriceps*. A *M. glutæus magnus* is not developed in Saurians. Also Buttmann ('Dissert. de Musculis Crocodili,' dedicated to Meckel) describes it as a part of the extensor (*vastus externus*), and does not mention a *glutæus magnus*. What I have described as the second and slender head of the extensor, and has also by Buttmann been identified with the *vastus internus*, is named *M. rectus* by Dr. Haughton. In *Hatteria* it passes simply into the *ligamentum patellæ*, and there is no connexion with any of the muscles of the calf of the leg.

† This muscle might be taken for the *psoas*; but its origin is far distant from the vertebral column; indeed a psoas descending from the lumbar region to the humerus is not developed in *Hatteria*.

of its fibres arises from a strong ligament extended between tuber ischii and the root of the tail (*d*); its tendon is inserted into the inner side of the end of the tibia.

A very peculiar muscle (*a*) takes its origin from the eight or nine first inferior spinous processes of the caudal vertebral column and the lower surface of the corresponding vertebræ; it is very strong, imbedded between the caudal muscles proper*, compressed, with the lower margin rounded, and separated from the surrounding muscles by very loose cellular tissue. It tapers behind into a point; and becoming gradually stronger towards the trunk, it passes below and crosses the ligament extending from the tuber ischii to the root of the tail (*d*); the greater part of its fibres are here collected into a broad and strong tendon, which is attached to the inner trochanter-like protuberance of the femur; but another portion (*a'*), strengthened by additional fascicles from the broad ligament mentioned, passes into a slender chord-like tendon (*a''*) which runs along the entire length of the femur, and is inserted into the end of the fibula. This muscle draws the limb backwards and rolls it outwards; and with its slender tendon it assists in bending the lower leg†.

A very long and slender muscle (*h*) accompanies the ischiatic nerve; it arises from the os sacrum, passes along the outer side of the femur, and is inserted into the outer side of the fibula; it bends the lower leg (*M. agitator caudæ* of Dr. HAUGHTON).

A very short muscle, entirely hidden by the two muscles last described, reaches from the *tuber ischii* to that part of the femur where, in Mammalia, the *trochanter major* projects; it is an abductor muscle, and rolls the leg slightly outwards (*M. quadratus femoris*).

Finally, a broad muscle, arising from, and lying immediately on, the entire lower surface of the pelvis, is inserted into the prominent inner tuberosity of the head of the femur; it is covered by the hinder part of the *obliquus externus* and by the foremost part of the large flexor muscle; it is the principal *adductor* of the limb.

Of the *muscles of the lower* leg, those on the *anterior* side are readily distinguished as *tibialis anticus*, *extensor digitorum* (*communis longus*), and *peroneus longus*; the *extensor* terminates in two slender tendons only, inserted into the metatarsal bones of the second and third toes.

The muscles on the *hinder* side of the lower leg are disposed in several layers. The

* In a male *Grammatophora* it is situated above the penis and its muscle.

† Dr. HAUGHTON (*l. c.*) describes and figures this muscle in the Crocodile as *M. extensor femoris caudalis*, and expresses its use in the following words:—"The Crocodile, resting on mud, progresses chiefly by using his hind feet as paddles; and in this use of them the great caudal extensor of the thigh is the most powerful and important muscle employed." The fact that this muscle is developed in ground- and tree-lizards as well as in the Crocodile, prevents us from adopting the idea of its being an organ specially adapted for the mode of life of the latter. When in the progress of ordinary locomotion one of the hind limbs is advanced, and its sole becomes the fixed point, this muscle lifts the hind part of the body forwards, in which it is materially assisted by having a double insertion, viz. on the upper end of the upper leg, and by its slender tendon on the lower. When the trunk is fixed, this muscle simply draws backwards the limb, effecting at the same time a slight flexion of the lower leg.

*outermost layer* consists of two muscles equivalent to the *MM. soleus* (*l*) and *gastrocnemius* (*k*), although their origins are somewhat modified*. The *M. soleus* occupies the tibial side, and arises from the upper end of the tibia, emitting a tendon which is confluent with the slender portion of the great flexor (*biceps*?) of the femoral region; it terminates in a broad tendon thinned into a fascia extending across the tarsus, and chiefly attached to the astragalus on one side and to the metatarsus of the fifth toe on the other. In order to have a full view of the *M. gastrocnemius*, it is necessary to remove the *soleus*; it is much larger than the *soleus*, and occupies the fibular half of the lower leg; it has two heads; the larger arises from the femur and long tendon of the *extensor femoris caudalis*, and passes, in the tarsal region, uninterruptedly into the *flexor digitorum communis brevis*, the division between both muscles being faintly marked by a transverse tendinous inscription. The second head is very slender, and arises from the tendon of the great *flexor* of the femoral region, being confluent at its origin with the *M. soleus*.

The *second* layer consists of the *flexor digitorum communis longus* only; but this muscle has three heads, which are united into an exceedingly strong tendon in the tarsal region, which, as usual, is split into five branches, each perforating the corresponding branch of the short flexor. The longest and most superficial of the three heads is confluent with the large head of the *gastrocnemius*, both having the same origin on the femur. The second head is situated below, and covered by the first, and arises from the upper ends of tibia and fibula. The third head is additional, and may be regarded as a separate muscle, running and working in a quite different direction from the others; it is flat, rhomboid, and arises from the outer edge of the lower end of the fibula and of the tarsus, runs transversely to the longitudinal axis of the limb, and meets the common tendon at an angle of 45°: whilst the two longer heads of the muscle act and bend the toes in the direction of the longitudinal axis of the bone, the action of this short head crosses it obliquely, and bends the three inner toes only.

The *third* layer consists of the *M. tibialis posticus*; it arises from the entire posterior side of the tibia, its fibres descending obliquely, and converging into a broad tendon inserted into the metatarsals of the three inner toes; it draws the foot backwards and inwards.

The space between tibia and fibula is filled by a muscle the fibres of which run transversely from one bone to the other.

The *muscles of the foot* agree perfectly with those of *Grammatophora* and *Iguana*, of which latter lizard they have been described by Meckel†.

I have given a detailed description of the muscles of the extremities, not in the hope of finding any peculiarity by which *Hatteria* might be distinguished, or which might assist in determining its affinities, but because little attention has been paid to the special myology of Saurians, and because the muscles of the hind limb of the Crocodile have

* Dr. Havekron appears to have named the *M. gastrocnemius Plantaris*, and the *M. soleus Gastrocnemius*.

† System d. vergl. Anat. vol. iii. p. 285.

been recently treated of in a separate paper*. I have chosen for collateral examination *Grammatophora*, *Iguana*, *Varanus*, and *Crocodilus* (*americanus*, young, 18 inches long), and found the general arrangement of the muscles remarkably uniform. Dr. HAUGHTON describes as a peculiarity of the Crocodile an interlacing of the tendons of various muscles, and says that the effect of it must be to produce simultaneity of action among them. Such a connexion of muscles by means of tendons has been described above as existing between the *Extensor femoris caudalis* and *Gastrocnemius*, and between a portion of the *Flexor cruris* (*biceps*) and *Soleus*; but I have failed to find the connexion between the "*M. rectus* and *plantaris*" (so named by Dr. HAUGHTON).

Generally speaking, and not taking into consideration the numerous instances which might be adduced to the contrary, the actions of the limbs are less diversified in Saurians than in the higher classes of Vertebrata. The energy of the contractions of their muscles, although it may be momentarily great, is less enduring, so that the disproportion between their feebly developed limbs and the size of the body appears only the greater. In accordance with this the muscles are found reduced in number, and simplified in their arrangement, inasmuch as muscles the origins of which are in close proximity are frequently partly confluent, or a fascicle of one muscle passes into the substance of its neighbour. There is no doubt that such muscles act simultaneously; but this want of separation refers to *collateral* muscles only.

A want of separation of muscles belonging to *different regions*, such as the interlacing of the tendons of the muscles of the upper and lower legs, does not prove the simultaneity of their actions,—first, because the connexion between them is effected by tendons more or less intimately attached to the bone, which *interrupts* the continuity of contraction of the upper and lower muscles. Secondly, the simultaneity of action could not be produced without the simultaneous influence of the nerves entering those muscles; and as it is dependent on the nerves, the tendinous connexion is not needed to produce it. It is a circumstance worthy of notice that all these interlacements are in the *fossa poplitea*, behind the knee-joint, which in Saurians is almost always in a state of flexion, and that these animals are able to draw the lower leg so far upwards as to lie alongside of the upper. This will readily account for the unusually high insertion of a part of the tendinous terminations of the lower muscles, at a place above and somewhat remote from the end of the femur, and more especially into the tendons of the upper muscles, there being no room on the bone itself.

In *Hatteria*, as well as in other lizards, I have observed that parts of one and the same muscle are often so loosely united that it may be easily, though artificially, split from one end to the other, thereby tempting one to adopt a nomenclature created for higher Vertebrates with a more diversified action of the limbs and a greater multiplicity of distinct muscles. Thus the muscular system of the limbs of lizards appears to be characterized not only by the partial confluence of the fascicles and tendons of different muscles, but also by the loose connexion of the fascicles of the muscles generally.

* "On the Muscular Anatomy of the Leg of the Crocodile. By the Rev. SAMUEL HAUGHTON, M.D.," in Ann. and Mag. Nat. Hist. 1865, vol. xvi. pp. 326–331.

*Integuments.*

The external modifications of the integuments are fully noticed in the diagnoses and figure published by Dr. GRAY; and the external arrangement of the ventral plates has been mentioned above; so that I have but little to add. No part of the cutis contains any ossification. There are no cutaneous glands in any part of the body, except the pair of large anal glands, which may be more properly described with the organs of propagation. So-called femoral and præanal pores are entirely absent, with the subcutaneous follicles.

*Organs of Sense**.

A simple turbinal bone occupies the bottom of the entrance of the *nasal cavity*, and an undulated cartilage projects far into it from its roof; the latter may be distinctly seen from the palatal opening.

The *eye* is protected above by an upper very short eyelid; the lower shuts the eye entirely, and contains a cartilaginous subsemiglobular disk, as in Agamoids generally; a membrana nictitans and the lachrymal gland are present. The sclerotic ring is composed of seventeen bony lamellæ. The iris is divided into two lateral halves by an upper and a lower strip of accumulated elastic fibres, covered with an intensely black pigment on the inner surface; the pupil appears, in preserved specimens, nearly round, but is slightly contracted vertically. The lens is, as in other lizards, globular, flattened in front; but *Hatteria* differs from them in not having a *pecten* (falciform process).

With the *tympanum* a *tympanic cavity* is entirely absent. The only remaining portion of this sphere of the ear is the *stapes* (*c* in figs. 2 & 5), 11 millims. long; it lies in a groove of the exoccipital, imbedded in cellular tissue between other soft parts immediately below the membrane of the auditory recess of the pharynx, and terminates at its outer extremity in a subsemicircular cartilaginous disk, to which the outer horn of the hyoid bone is attached by a fibro-cartilaginous ligament. At its inner extremity it is thickened into a knob fitting into the *fenestra ovalis*.

After removal of the bony part of the exoccipital and basisphenoid, which forms the bottom of the *labyrinth*, a cartilaginous capsule becomes apparent; its thickness is half a millimetre; the membrane coating the walls of the cavity is of a deep black colour. The *saccus vestibuli* contains a single pear-shaped otolith 3 millims. long and 2 millims. broad at its widest (inner) end. The cochlea is more developed than in other lizards, showing the commencement of a spiral turn; the membrane at its base contains the terminations of the cochlear nerve, which is abruptly split into four dichotomically divided and fan-like branches. The three semicircular canals lie behind and somewhat outward of the cochlea, and are also membranaceous, otherwise well developed, being from 12 to 15 millims. long.

* The following remarks on the anatomy of the soft parts are of necessity incomplete, as, besides a perfectly emaciated and badly preserved one, only a single example (and this preserved in spirits for nearly twenty years) has been available for dissection; and I have not thought myself justified in destroying parts of the head of so rare a specimen, in order to investigate points in which *Hatteria* may reasonably be expected not to differ from the Lacertian type.

*Organs of Digestion.*

The *teeth* have been described in connexion with the bones of the jaws.

The *tongue* (fig. 16) is fleshy, elongate-triangular, posteriorly with the base entirely grown to the bottom of the mouth, and angularly incised to receive the glottis*. Its surface is densely covered with soft pointed papillæ, without any tubercles or scales, and divided into two lateral halves by a shallow median furrow.

The *salivary* organs are but little developed; the mucous membrane on the outer side of the middle of the mandible has a spongy appearance, the surface being irregularly reticulated. Along the side of the base of the tongue there are simple solitary glands in small number, nowhere aggregated into a larger mass. Their openings are minute, and most distinct on the side of the root of the tongue.

The *hyoid* does not show any peculiarity, except as regards its attachment to the skull. The body is arrow-shaped, but little ossified, tapering into a long point in front, and split into a pair of accessory horns behind. The anterior horn consists of two pieces, nearly entirely cartilaginous, the inner much longer than the outer, which is attached by a ligament to the terminal cartilage of the stapes (see p. 26). The posterior horn consists of a long, arched, entirely osseous piece, to which a short terminal cartilage is joined. No part of the horns is dilated.

The *œsophagus* is wide, and passes, without distinct separation, into the elongate, spindle-shaped *stomach*; the muscular layer of the latter is nowhere conspicuously thickened, and its mucous membrane is raised into only a few longitudinal folds. No *curvatura major* can be distinguished. The pylorus is indicated by the cessation of the longitudinal folds; the duodenum is 9 millims. long, and separated from the small intestine by a circular valve only about 1·5 millim. deep, and consequently not entirely shutting the duodenum†. No part of the small intestine is provided with valvulæ conniventes, all the folds running in a longitudinal direction; they are numerous and very low and narrow in the upper fourth, broad and less numerous in the middle, and disappear entirely towards the rectum. There are no patches of accumulated glands anywhere. The passage into the very wide rectum is narrowed by an incomplete valve. The cloaca is separated from the rectum on the dorsal side only by a fold of the mucosa.

The walls of the intestines are throughout very thin; the small intestine makes two complete circumvolutions, and is 180 millims. long, the rectum with the cloaca 100 millims.

The *liver* is conspicuously divided into two lateral portions, each of which is subdivided in a curious and apparently irregular manner. The bridge between the two portions crosses the lower surface of the posterior part of the stomach. The *left* portion lies in the lower part of the abdominal cavity, is thin, twice as long as broad, and fixed by an exceedingly strong and long filament to the pubic bone. This portion is at some places

* This posterior incision is still deeper in *Grammatophora*, in which, moreover, each posterior angle of the tongue is produced into a short, scaly, pointed lobe.

† Cf. a similar arrangement in *Regenia* and *Monitor*, Proc. Zool. Soc. 1861, pp. 60, 110.

very thin; and at others the parenchyma is entirely interrupted, so that the peritoneal coverings of both sides are in immediate contact with each other. Several small lobes project from the dorsal surface; and one larger, long, spindle-shaped lobe (about 1 inch long) adheres to the body of the liver by a thin style only. The *right* portion is extremely irregular in shape, with several projecting appendages, and considerably thicker (although, on the whole, not larger) than the left*.

The *gall-bladder* is of the size of a currant-berry, and imbedded in the right lobe close to its transition into the left. The *spleen* and *pancreas* are very elongate and narrow. No trace of *corpora adiposa*.

*Heart.*

The hearts of two examples were examined; but in both the substance had become so deteriorated that it was impossible to obtain a clear insight into their structure. The muscular part is extremely thick, the ventricular cavity being very small and apparently simple. The two atria are entirely separate, of nearly equal size. The aortæ and arteriæ pulmonales are externally united into one stem; and although there is only one ostium for the aortæ, the truncus aortæ is extremely short.

*Organs of Respiration.*

The *larynx* (fig. 16) is composed of a complete posterior cartilaginous ring and a pair of separate anterior cartilages. The ring is broadest laterally and narrowest anteriorly; it has a slight protuberance (*c'*) in the middle of its lateral posterior margin for the insertion of the *M. dilatator glottidis* (*c*). The anterior cartilages are separated in front by a wide interspace and united posteriorly by a ligament; they are produced to form the glottis, which, compared with Agamoid Lizards', is very wide. Two muscles are attached to the larynx on each side: the *M. dilatator glottidis* covers the outer lateral surface, being extended from the margin of the *glottis* to the posterior tubercle, as mentioned above. The *compressor glottidis* lies within the glottis, and extends from its antero-interior margin transversely to the hyoid bone.

The *trachea* (fig. 16) has none of the cartilaginous rings closed on the dorsal side; and many reach only to the median line in front: they are very irregularly arranged, and soft and flexible like membrane. The two bronchi are very short, and terminate immediately behind their entrance into the lungs. The *lungs* of both sides are of nearly the same capacity, rather large; they are simple bags with large cells in small number, more resembling the lung of a Batrachian than of a Lizard.

*Uropoëtic Organs.*

The *kidneys* are situated in the hindmost part of the abdominal cavity, viz. within the pelvis; they are flat, entirely separate from each other, and the left is considerably larger and different in shape from the right. Each is incompletely divided into four or five lobes very irregular in form. The right is of an elongate-ovate shape, without any pro-

* In a specimen kept for some time in captivity, the entire liver was shrunk to the thinness of a membrane.

jecting parts; the left is of nearly twice the extent of the right, and has a lobe-like appendix in front, situated in the cavity between the transverse processes of the two sacral vertebræ. As the kidneys lie in the closest proximity to the cloaca, the ureters are extremely short, each opening, together with the vas deferens of its side, in a papilla situated in the recess behind the fold of the mucous membrane which separates the rectum from the cloaca. The urinary bladder is large, elongate, subcylindrical, opening at the usual place into the cloaca.

The *testicles** are elongate-ovate, situated in the anterior half of the abdominal cavity, nearly opposite to each other. The *vas deferens* lies at some distance outwards from the testicle, receiving the "efferent canals" at nearly right angles; its course is but slightly undulated. *There is no trace of an intromittent copulatory organ in Hatteria*, either at the root of the tail, or in the anterior wall of the cloaca,—a peculiarity quite unique among Saurians. The pair of lateral *anal glands*, however, which open into the outer corner of the anal cleft, are quite as much developed as in other lizards. They have been but little noticed by anatomists, although they appear to be present in most lizards. They lie, quite free or surrounded by very loose cellular tissue, in a hollow on the lower side of the root of the tail, behind the caudo-ischiadic ligament. They are of a globular form, and open into the base of the exsertile male organ. In *Grammatophora* and *Gecko* a portion of the corpus cavernosum extends to the base of the gland, whilst it is perfectly isolated in *Uromastyx*. I have not found it in *Amphisbæna*, which has a paired penis. Stannius designates the secretion of this gland as *smegma*. In *Hatteria* the gland (fig. 34, *o*) has a diameter of about four lines, and is quite free in the cavity behind the ischium: its opening is closed by a perforated membrane (fig. 29, *b*); and the contents are discharged by about eight small foramina. The interior has a spongious appearance, being divided into many irregular smaller and larger compartments, separated by septa with partly free or floating margins (figs. 30, 31). They are small and short towards the periphery of the gland, imbedded in dense fibrous tissue, becoming larger and more elongate in the middle; the larger have a tendency to radiate towards the external opening, behind which they empty into a common vacuity. All these compartments were found to contain a finely granular secretion, which of course was much altered by the action of the spirit in which the specimens were preserved.

### *Concluding Remarks.*

Before concluding this paper with a few words on the position of *Hatteria* in the system, I shall briefly review the more noteworthy peculiarities by which it is distinguished from other Saurians. Those presented by the skull have been already recapitulated (p. 10).

1. Above all, the amphicœlian structure of the vertebræ must be mentioned, a character which does not occur in any of the recent Saurians, except in the degraded type Geckotidæ. The vertical division of the centra of the caudal vertebræ observed in

* Unfortunately both specimens examined were males.

the majority of lizards occurs also here, but each centrum is split into two equal halves. The property of reproducing lost portions of the tail is indicative of low organization; yet it is evidently much less pronounced in *Hatteria* than in other lizards of a similarly low type. In other respects the vertebral column of *Hatteria* does not deviate from the Lacertian type, such points as the uniform development of strong neural spines, or the singularly oblique direction of the caudal pleurapophyses, which point forwards, being of minor importance.

2. The modifications of the costal hæmapophyses:—*a*, into a series of appendages identical in position with the uncinate processes of birds; and, *b*, into a double terminal series connecting the ribs with the thoracic and abdominal sterna, the distal pieces being much dilated to form the base of a system of muscles.

3. The development of a system of abdominal ribs, neither floating (Crocodile) nor mere continuations of the true ribs (Chamæleon), but standing in functional relation to the ventral integuments.

4. The continuity of the ossifications of the coracoid, the presence of an acromial tuberosity of the scapula, and the subvertical direction of the os ilium are very remarkable deviations from the Lacertian type; whilst the arrangement of the bones of the limbs does not show any peculiarity, except in the articulation of the *fibula*, which is joined to the *calcaneum* only.

5. Absence of the pecten of the eye. Entire absence of the tympanic cavity; attachment of the hyoid bone to the terminal cartilage of the stapes; commencement of a spiral turn of the cochlea.

6. Although the details of the structure of the heart are, for the present, unknown, it has been ascertained to be of the Lacertian, and not of the Crocodilian type.

7. The organs of respiration and digestion adhere closely to the Lacertian, and more especially to the Agamoid type, as does the greater portion of the uropoëtic organs. However, the kidneys are entirely within the pelvis; and the absence of a copulatory organ is a character by which *Hatteria* is distinguished from all other Saurians.

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There can be no doubt that *Hatteria* must be removed from the family of *Agamidæ*, and that it is the type of a distinct group; but the question is whether this group should be subordinated to the Saurians as a family like the *Agamidæ*, *Iguanidæ*, &c., or whether the characters pointed out are deemed of sufficient importance to assign to it a more distant position from the other Lizards. In a system founded upon external characters only, the former course would be taken, and a family *Hatteriidæ* would be placed near the *Agamidæ*, with which it agrees in the majority of its external characters. However, there is scarcely a systematist of the present day who does not pay more or less attention to anatomical characters in establishing the higher divisions. Thus Dr. Gray admits into the diagnosis of his section *Squamata* (= Lizards and Ophidians) the articulation of the tympanic to the skull and the paired copulatory organs as principal

characters, by which it is distinguished from the second section *Cataphracta* (= Tortoises and Crocodilians + Amphisbænians). The division proposed by STANNIUS into *Streptostylica* (= Snakes and Lizards + Amphisbænians) and *Monimostylica* (Tortoises + Crocodilians) is founded essentially on the same characters as that of GRAY. *Hatteria* would not enter any of the divisions defined in those two systems. Professor OWEN refers the recent Reptiles to four coordinate orders—*Chelonia*, *Lacertilia*, *Ophidia*, and *Crocodilia*; *Hatteria* would probably enter the second, inasmuch as the character of "procœlian vertebræ" assigned to this order, is the only one by which *Hatteria* would be excluded, but the Geckos, which are Lacertilians in Professor OWEN'S system, also have amphicœlian vertebræ. In the Crocodilia, likewise, this is not considered an ordinal character, but is used for the distinction of the suborders.

In these three systems the Crocodiles are removed from the Lizards, into a distinct order or section, on the ground of osteological characters as well as on account of the higher organization of their soft parts. Now in *Hatteria* the modifications of the Lacertian skeleton extend to the same parts as in the Crocodiles (except the anterior ribs and thoracic sternum), although they are frequently of a different nature; and the repetition of Lacertian characters in its soft organs is in some measure counterbalanced by the absence of copulatory organs. Therefore we cannot hesitate to claim for it a rank higher than that of a family. The presence of a double bar across the temporal region, the intimate and firm connexion of the os quadratum with the skull and pterygoids, the erect ilium, and the uncinate processes of the ribs are characters by which a tendency towards the Crocodilians is manifested; but here the resemblance ceases; and the affinities of *Hatteria* with the Lizards are far more numerous and of greater importance. I need only mention the structure of the heart, of the organs of respiration and digestion, the absence of a diaphragm and of peritoneal canals, the transverse anal cleft, the absence of an external ear, the free tongue, &c. Yet were we to associate it with the Lizards in one group, the unity of this group would be entirely destroyed. I propose, therefore, the following modification of STANNIUS'S division of recent Reptilia, adding a few of the characters which appear to be of special importance in the determination of the affinities of *Hatteria*:—

RECENT REPTILIA.

I. SQUAMATA. Anal cleft transverse. Copulatory organs paired, if present. All the ribs single-headed, sacral vertebræ two or none.

First order: *Ophidia*. Quadrate bone articulated to the skull; brain-capsule entirely osseous; rami of the mandible united by ligament. Copulatory organs present.

Second order: *Lacertilia*. Quadrate bone articulated to the skull; parts of the ali- and orbito-sphenoid regions fibro-cartilaginous; rami of the mandible united by suture; temporal region without, or with only one horizontal bar. Copulatory organs present.

Suborder A. *Amphisbænoidea*. Vertebræ procœlian. No posterior orbital ring or temporal bar; no columella.

Suborder B. *Cionocrania*. Vertebræ procœlian. An orbital ring with a temporal bar more or less complete. Columella present.

Suborder C. *Chamæleonoidea*. Vertebræ procœlian; a bar crossing from the parietal to the mastoid; temporal bar complete. No columella.

Suborder D. *Nyctisaura*. Vertebræ amphicœlian; orbital ring and temporal bars not developed. A columella.

Third order: *Rhynchocephalia**. Quadrate bone suturally and immoveably united with the skull and pterygoid; columella present. Parts of the ali- and orbito-sphenoid regions fibro-cartilaginous; rami of the mandible united by a short fibrous ligament. Temporal region with two horizontal bars. Vertebræ amphicœlian. Copulatory organs none.

II. Loricata. Anal cleft longitudinal; copulatory organ simple. Anterior ribs bifurcate; sacral vertebræ two.

Fourth order: *Crocodilia*. Quadrate bone suturally united with the skull; parts of the ali- and orbito-sphenoid regions fibro-cartilaginous; rami of the mandible united by suture. Choanæ formed by pterygoid and palatine bones.

III. Cataphracta. Anal cleft longitudinal; copulatory organ simple. Trunk-ribs and sternum dilated, more or less completely united by suture.

Fifth order: *Chelonia*.

The skeleton of *Hatteria*—with its amphicœlian vertebræ and abdominal sternum on the one hand, and its highly developed osseous skull and uncinate apophyses of the ribs on the other—presents a strange combination of elements of high and low organization; and this is the more significant as this peculiar animal occurs in a part of the globe remarkable for the low and scanty development of Reptilian life. The New Zealand of the present period is inhabited by only a few (about nine) small species of the cosmopolitan Geckos and Skinks and by a single species of frog; and it is not probable that this small list will be considerably increased by future researches. With more confidence may we look forward to discoveries of remains of extinct forms, of which one *Plesiosaurus* only, the *Plesiosaurus australis* of Owen, is known at present; but whether they will be of such a nature as to afford a better insight into the history of development of the Rhynchocephalian type, whether they will show that *Hatteria* was at one time not its only representative, and whether such evidence will be found in New Zealand at all, the future must decide.

* In this way the name *Rhynchocephalus* may be preserved, which, otherwise, must give way to the prior *Hatteria*.

## Explanation of Figures.

### PLATE XXVI.

Figs. 1–7. Skull. In fig. 3 the temporal and zygomatic arches are removed to show the side of the base of the skull.

*a*. Exoccipital.
*b*. Alisphenoid.
*c*. Stapes.
*d*. Paroccipital.
*e*. Posterior hypapophysis of basisphenoid.
*f*. Anterior hypapophysis of basisphenoid.
*g*. Parietal.
*h*. Mastoid.
*i*. Prefrontal.
*k*. Lacrymal.
*l*. Postfrontal.
*m*. Os quadrato-jugale of Stannius.
*n*. Zygomatic.
*o*. Os quadratum.
*p*. Columella.
*q*. Vomer.
*r*. Pterygoid.
*s*. Palatine.
*t*. Ectopterygoid.
*u*. Dentary.
*v*. Splenial.
*w*. Articular.
*x*. Coronoid.

Figs. 8–15. Dentition.

Fig. 8. Front view of the intermaxillary teeth of a very old example.

Fig. 9. The same of an example of less age, or of one in which the teeth had not been used for some time.

Fig. 10. Front view of the intermaxillary teeth of a young example.

Fig. 11. Lateral view of the dentition of the same example.

Fig. 12. Lateral view of the dentition of an adult *Uromastyx hardwickii*.

Fig. 13. Front view of the same. *i*. Intermaxillary. *n*. Nasal opening. *m*. Dental process of mandible.

Fig. 14. Front view of the anterior teeth of a young *Uromastyx hardwickii*.

Fig. 15. Front view of the anterior teeth of an adult *Uromastyx spinipes*.

Fig. 16. Tongue and trachea, the latter slit open behind the larynx.

*a*. Anterior cartilage.
*b*. Posterior cartilage; *b'*, its tuberosity for insertion of the dilatator-muscle.
*c*. Musculus dilatator glottidis.

### PLATE XXVII.

Fig. 17. First nine vertebræ. Development of hæmapophysis into uncinate process.

*a*. Sternum with lateral slit for articulation with coracoid.
*b*. Episternum.
*c*. Clavicle.
*d*. Hæmapophysis of first sternal rib.
*e*, *e'*. Hypapophyses.

Fig. 18. Front view of epistropheus.

Fig. 19. Back view of the same.

Fig. 20. Inner view of the middle portion of the abdominal sternum.
*a*. Anomalous joint between central and lateral pieces of abdominal rib.
*b*. Intermediate (floating) abdominal rib.
*c*. Dilated lower piece of hæmapophysis.

Fig. 21. The five anterior caudal vertebræ, lower view. *a*. Hæmapophysis.

Fig. 22. Vertical section of three dorsal vertebræ.

Fig. 23. Vertical section of four (7th to 10th) caudal vertebræ. *a*. Epiphysial line passing through the middle of centrum.

Fig. 24. Isolated rib of 17th vertebra.
*a*. Upper (dorsal) hæmapophysial piece.
*b*. Lower (ventral) hæmapophysial piece.
*c*. Right half of abdominal rib.

Fig. 25. Separate view of scapula and coracoid.
*d*. Cartilaginous; and *e*, osseous portion of coracoid.
*f*. Cartilaginous; and *g*, osseous portion of scapula.
*h*. Acromial tuberosity.

Fig. 26. Sternum with abdominal ribs. Bones of the shoulder in their natural position.
*a*. Sternum. *b*. Episternum. *c*. Clavicle. *d*–*h*, as in fig. 25.

Fig. 27. Lateral; and fig. 28, lower view of pelvis.
*a*. The two sacral vertebræ.
*b*. Os ilium.
*c*. Uncinate process of os pubis.
*d*. Tuberositas ischii.

Fig. 29. Vent with the reticulated openings of the paired anal gland. A scalpel is introduced on one side to press the gland outwards.

Fig. 30. Vertical; and fig. 31, transverse section of the anal gland (3 × natural size).
*a*. Vacuity behind external opening.
*b*. An unusually broad septum.

## PLATE XXVIII.

Fig. 32. Muscles of ventral region.
*a*. Musculus pectoralis major.
*b*. Part of outer layer of musculus obliquus externus.
*c*. Subcutaneous fibrous tissue.
*d*. Musculus rectus.
*e*. Ninth; and *e'*, eleventh abdominal rib with corresponding hæmapophyses.
*f*. Abdominal fascia.
*g*. Three of the transverse series of ventral plates.

Fig. 33. Lateral muscles of trunk (the Latissimus dorsi is removed).

*a*. Longissimus dorsi confluent with Sacrolumbaris.
*b*. Small muscles for the spines of the dorsal crest.
*c*. Obliquus externus; *c'*. outer layer; *c''*. middle layer.
*d*. Hinder portion of pectoralis major, partly intercalated between the outer and middle layers of obliquus externus.
*e–h*. A portion of the obliquus externus has been removed to show:—
*e*. Upper anterior angle of the dilated costal hæmapophysis.
*f*. Musculus intercostalis.
*g*. Muscle connecting the bodies of two costal hæmapophyses.
*h*. Muscle connecting the body of a hæmapophysis with the upper posterior angle of the succeeding hæmapophysis.
*i*, *i*. Uncinate processes of the ribs.

Fig. 34. Muscles of hind limb. The hind limb is drawn forwards, showing its posterior side.

*a*. Extensor femoris caudalis. *a'*. Continuation of the same with additional fascicles from ligament *d*. *a''*. Long tendon of the same muscle.
*b*. Part of the great Flexor cruris (Semimembranosus and Semitendinosus).
*c*. Slender portion of the great Flexor cruris (Biceps). The two latter muscles are drawn downwards, the margins of *c* and *h* being collateral.
*d*. Transverse ligament between tuber ischii (*r*) and root of tail.
*e*. Terminal portion of Musculus gracilis.
*f*. Adductor-branch of the great Flexor cruris.
*g*. Musculus iliacus internus. *g'*. Its origin, covered by the M. rectus (*i*).
*h*. Musculus agitator caudæ.
*i*. Principal head of the Extensor cruris (M. rectus).
*k*. Gastrocnemius.
*l*. Soleus.
*m*. Nervus cruralis.
*n*. Trochanter-region of femur.
*o*. Anal gland.
*p*. Musculus peronæus longus.
*q*. Musculus tibialis anticus.
*r*. Tuber ischii.

## XX. *On the Development and Succession of the Teeth in the Marsupialia.* By William Henry Flower, *F.R.S., F.R.C.S., &c., Conservator of the Museum of the Royal College of Surgeons of England.*

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Although the dentition of adult individuals of the animals which constitute the remarkable order, or, rather, subclass Marsupialia, has been repeatedly subjected to examination, and described with exhaustive minuteness of detail, it is a singular circumstance that most of those peculiarities of the succession of their teeth which distinguish them from other mammals appear hitherto to have escaped observation.

Professor Owen has, indeed, established the fact that those posterior teeth of each side of each jaw which have no deciduous predecessor are, as a general rule, four in number, instead of three, as in most placental mammals*, and has further contributed some important observations upon the later stages of the dentition of one family, the *Macropodidæ*†.

Beyond this, I have not been able to find any information upon the subject. Indeed it is remarked by the author just mentioned, that "an interesting field of observation still remains open in regard to the period and order of development of the deciduous and permanent teeth, in the different carnivorous, omnivorous, insectivorous, and frugivorous marsupials."

To supply this blank is the object of the present communication. Fortunately the materials contained in the Museum of the Royal College of Surgeons, if not quite so complete as might be desired, are amply sufficient to illustrate the main aspects of the question, and to furnish a result as interesting as it was unexpected.

### Family Macropodidæ.

From many observations upon the early dentition of the Kangaroos (genus *Macropus*). I will only select for description certain well-marked stages, which are sufficient to illustrate the successive steps of the process.

1. The object of the first observation which needs recording was a marsupial fœtus of a *Macropus* of uncertain species, but probably belonging to one of the larger forms. Its entire length was 6·5 inches, of which the head occupied 1·4 inch, and the tail 2 inches. The surface was destitute of hair, the margins of the lips were still adherent except in front, and the eyelids were completely closed.

* "Outlines of a Classification of the Marsupialia," Trans. Zool. Soc. vol. ii. pp. 315–333 (1839).

† See especially Cyclop. Anat. and Physiol. Art. "Teeth," and Cat. Mus. Roy. Coll. Surgeons, Osteol. Series, vol. i. (1853).

On examining the mouth, no traces of teeth were visible through the gums. The appearances seen after dissecting the alveoli were as follows (Plate XXIX. fig. 1):—

In the upper jaw the capsule of the first incisor was very large, and contained an uncalcified pulp, showing distinctly the form of the crown of the future tooth; behind this was a small capsule containing the rudiment of the second incisor. Further back were two capsules containing distinct pulps of two teeth of the molar series, but as yet showing no trace of calcification. In the lower jaw a large procumbent incisor, $\frac{3}{10}$ of an inch long and partially calcified, occupied the anterior portion of the ramus. Above and behind this were the uncalcified pulps of two molars; as in the upper jaw, the second was considerably larger than the first. Although these pulps were very soft, they showed distinctly the form of the summit of the crown of the future tooth. No other tooth-germs could be detected.

2. A more advanced specimen of the same genus, in which the eyes were open, and the lips separated backwards to their normal extent, measured from muzzle to end of tail 11 inches, of which the head occupied 2·6 inches, and the tail 5 inches. No teeth had protruded through the gum, but the region of the upper incisors presented a bulbous prominence; the apices of the lower incisors were almost visible through the thin covering membrane, and prominences on the alveolar borders indicated the situation in which molar teeth were shortly about to break through their superficial investments.

On dissection (Plate XXIX. fig. 2), in the upper jaw, the crowns of the first two incisors were found to be advanced in calcification, the first more forward than the second. Both presented the characteristic form and also size of the extremity of these teeth in an adult animal. The pulp of the third incisor occupied a capsule placed in the premaxillary bone at some distance from the alveolar margin. It represented only the extreme summit of the tooth, and was not calcified. Behind the premaxillary suture was a minute conical calcified canine. Behind this the calcified germs of the teeth of the molar series. These were three in number, the first having the form peculiar to the premolars of the genus, compressed, and narrower in front than behind; the second (coloured red in the figure), which was the furthest advanced both in position in the alveolus and in development, had the quadrate form and characteristic anterior and posterior transverse ridges of a true molar. The third was also a true molar, but only the tips of the cusps were calcified. Immediately above the anterior part of the second was lodged a small rounded capsule (coloured blue in the figure), of a yellowish colour, perfectly distinct, though not more than $\frac{1}{10}$ inch in diameter.

In the mandible, the large procumbent incisor, corresponding in form and size to the crown of that of the adult animal, was calcified to the length of $\frac{3}{10}$ inch. The molar series consisted of three calcified crowns, corresponding in general characters to those of the maxilla, *i. e.* having the form respectively of a premolar and two true molars, but they were rather more advanced in their growth. The capsule of a fourth was also visible; and underneath the fore part of the second tooth and to the outer side of the base of the incisor, was a small capsular germ similar to that observed in the upper jaw.

3. In the next stage of dentition, the crowns of the first and second upper incisors

project considerably from the premaxillary bone. The crown of the third, though completely calcified, is still retained in its alveolus. The canine has disappeared, but its socket is visible, and as this was a macerated skull, it may have been lost accidentally. The two anterior teeth of the molar series are in place in both jaws, and the summit of the third is on a level with the alveolar border. The tip of the lower procumbent incisor was free. The germs of the reserve teeth of the molar series were not calcified.

4. In a skull, 5 inches long, belonging to a great Kangaroo (*Macropus major*), probably about half grown (Plate XXIX. fig. 3), the first and second upper incisors are in place, the third just appearing beyond the alveolar margin; all traces of the canine and its socket have disappeared. The lower incisor projects $\frac{1}{8}$ inch beyond the front of the mandible. The three anterior teeth of the molar series described above, viz. one premolar and two molars, are in place and in use in both jaws. The crown of a posterior molar is just visible in its socket. The crown of the single reserve tooth in each jaw is completely calcified, and shows the compressed character of a premolar, having two cusps behind and one in front. It is a slightly larger tooth than the premolar which is in place.

The remaining changes in the dentition of the great Kangaroo have been described so fully by Professor Owen that it is unnecessary to follow them here in detail. They amount, however, to this—the gradual evolution of the reserve premolar tooth in each jaw, which displaces the first tooth having the character of a true molar; the concomitant shedding of the first premolar; and the subsequent shedding of the second or reserve premolar, followed ultimately by the loss of the two anterior true molars.

The foregoing observations on the earlier stages of the development of the teeth of *Macropus* are not in accordance with the description of Professor Owen, which runs thus. "The deciduous dentition of the great Kangaroo (*Macropus major*) is *i.* $\frac{3-3}{1-1}$, *c.* $\frac{1-1}{0-0}$, *d.m.* $\frac{3-3}{1-1}=18$. The canines are rudimental, and are absorbed rather than shed. The deciduous incisors are shed before the young animal finally quits the pouch; when this takes place, the dentition is *i.* $\frac{1-1}{1-1}$, *d.m.* $\frac{3-3}{1-1}=12$, the upper incisors being *i.* 1, the molars *d.* 3, and *d.* 4 of the typical dentition"*.

I have not been able to find any trace of deciduous incisors in any specimen of *Macropus* examined, and I do not think that if they had advanced to the calcified stage, in which alone the term "shed" would be applicable to their disappearance, they could have escaped observation. Further, the analogy with other marsupial genera, presently to be shown, gives me greater confidence in the belief that *Macropus* is, at all events as regards the incisors, a monophyodont.

I am also not disposed to regard the first tooth of the molar series as representing one of the deciduous molars of the typical diphyodont dentition; but the proofs upon which this view rests will appear after an examination of the dental succession in other genera.

As the general characters of the teeth of the other members of the family *Macropo-*

* Cyclop. Anat. and Phys., Art. "Teeth," vol. iv. p. 933.

*didæ* agree in all essentials with those of the type, differing only in the form of the crowns of the molars and premolars and the longer or shorter persistence of the rudimental maxillary canine, we may conclude that their development follows the same rule, and such observations as I have been able to make fully bear out this conclusion. Differences, however, occur in the order of the evolution of the teeth among themselves. Thus in *Hypsiprymnus*, the reserve premolar is relatively later in acquiring its position in the jaw than in *Macropus*, being still in germ, at least in some species (see Plate XXIX. fig. 4), after the last permanent molar is in place and use. This probably has relation to the extraordinary size of this tooth, and the length of time consequently required for its development. Moreover this tooth, as well as the anterior true molars, are not commonly lost during the lifetime of the animal as in the great Kangaroo.

### Family Phalangistidæ.

*Phalangista vulpina.*

1. A marsupial fœtus, nearly destitute of hair. Entire length 8 inches, of which the head occupied 1·7, and the tail 3·5. Eyelids open. Lateral margins of the lips not adherent (see Plate XXIX. fig. 5).

There were no teeth above the gums in either jaw; but the prominent apices of the first and second upper incisors, the lower incisors, and the two anterior teeth of the molar series raised the overlying mucous membrane.

On dissection, the crowns of the three upper incisors were found to be calcified, and corresponding in form and size to those of the adult animal. The conical crown of the canine was calcified, but lay deep in the alveolus. The large temporary molar (coloured red) and the first and second true molars had also solid crowns. Above the former was a slightly calcified germ of the reserve premolar (coloured blue). The capsule of the third true molar was distinct. In the lower jaw, the large procumbent incisor was half an inch in length. An extremely minute canine, with its apex calcified, lay close to the alveolar border; the molar series resembled those of the maxilla in their development, but the germ of the reserve premolar was still a soft papilla.

2. An older specimen, well covered with hair, measured from the nose to the end of the tail 13·5 inches, of which the head occupied 2·1 and the tail 6. In the upper jaw (Plate XXIX. fig. 6), the apices of the first and second incisors protruded through the gum; as did the cusps of two molar teeth (the temporary and the first permanent molar). In the mandible, the broad, flat, cutting extremity of the incisor projected about $\frac{1}{10}$ inch beyond the membrane; the rudimentary canine was also seen, and the cusps of two molars as in the maxilla.

On dissection the appearances found resembled those described in the last specimen; but all the teeth were in a more advanced condition of development. The crowns of both reserve premolars and of the third true molar were largely calcified. No trace of the fourth true molar could be detected. Between the canine and the temporary molar was a small conical germ of one of the rudimental premolars.

A comparison of the dentition of a specimen at this age with that of an adult animal, will be quite sufficient to enable us to fill in the intermediate stages. The change which takes place is only the substitution of the reserve premolar for the temporary molar, and the complete evolution of the other teeth, including often a second rudimental premolar*.

*Phascolarctos.*—The skull of the youngest Koala that I have been able to examine is 3 inches in length, that of the adult being 5. In this the permanent incisors and canines are in place, though not quite so prominent as in the adult. The single permanent premolar, and the first and second true molars are also in place, and the germs of the remaining two molars are calcified. We may infer from this, that if the premolar replaces a temporary molar, which according to analogy with the allied genus *Phalangista*, and, as will be shown, with all other marsupials the succession of whose teeth is known, is most probable, this must take place at a very early age.

### Family Peramelidæ.

I have not had an opportunity of observing the earlier stages of dentition in any member of this family, but when the animal is not quite full grown, the teeth are in the condition shown in Plate XXX. fig. 1. The permanent incisors, canines, and two anterior premolars are in place. Behind these in each jaw is a very minute, rather compressed tuberculated tooth, succeeded posteriorly by the true molars of the permanent series. In the alveolus above this minute tooth, which is the temporary or deciduous molar, is lodged the germ of the posterior permanent premolar, a tooth having a large compressed, pointed triangular crown, with small anterior and posterior basal tubercles, seen in *situ* in the figure of the adult dentition, Plate XXX. fig. 2, coloured blue.

In an immature specimen of *Chœropus* the dentition is exactly in the same stage as shown in the *Perameles* (Plate XXX. fig. 1), but the temporary molar is of still smaller relative dimensions.

### Family Didelphidæ.

In a young Opossum of one of the larger species (probably *Didelphys virginiana*), which measured 7·5 inches long, the tail being 2·25 and the head 1·75, the extreme points of the incisors, canines, premolars, and first molars had just pierced the gum. The state of development of these teeth is shown in Plate XXX. fig. 3; the incisors, canines, and first two premolars corresponded exactly with those of the adult animal. The succeeding tooth above and below, which in the adult is a compressed triangular premolar, was here a low-crowned, broad tooth, bristling with cusps like the true molars. Beneath it, in the lower jaw only, a minute yellow capsule of a successional tooth was found.

In a half-grown Virginian Opossum the incisors, canines, and first two premolars were fully developed, but more crowded together in the jaws than in the adult (see Plate

* I have confirmed these observations by the examination of a large series of skulls of Phalangers of various ages in the Leyden Museum.

XXX. fig. 4). The broad deciduous molar teeth were still in place (coloured red). On removing part of the alveolar wall, the calcified crowns of their successors (coloured blue) were seen, the upper one less advanced than the lower, and, as in the other genera described, placed rather anteriorly to the tooth it was destined to succeed. The form of the crowns of these reserve teeth was quite different from their molariform predecessors, being in fact that of the third premolars of the adult (Plate XXX. fig. 5).

The true molars had begun to take their place in the jaw, two in the maxilla and four in the mandible on each side, having their crowns calcified. Except those mentioned above no traces of reserve or successional teeth were seen.

We thus find in this American family of Marsupials precisely the same peculiarity in the succession of the teeth as noticed in their Australian congeners. I have been careful to verify these observations upon several other members of the genus, being unable to reconcile them with the figure given by Professor OWEN of the lower jaw of a *Didelphys* with two reserve premolar teeth, below the crowns of teeth, which, though showing the triangular pointed form of the permanent premolars, are indicated as the molar teeth of the deciduous series*.

## Family DASYURIDÆ.

Genus *Thylacinus*.—In a young female Thylacine, very scantily covered with hair, and the entire length of which was 13 inches, the head being 2·8 and the tail 4, the gums were entirely edentulous (see Plate XXX. fig. 6). There was no appearance even of teeth raising the mucous membrane, except a small sharp prominence rather behind the middle of each alveolar border. On raising the membrane this was seen to be caused by a small tooth which was just elevated above the level of the bone. This tooth is the deciduous or temporary molar (coloured red in the figure). In the upper jaw it had a trihedral obtusely pointed crown $\frac{1}{10}$ inch in length from before backwards, flat externally, and having an angle projecting inwards. The fang is absent, being either not developed or absorbed; the corresponding tooth of the mandible is slightly smaller and more compressed, also rootless. The crowns of these teeth, by their hardness and whiteness, contrast with those next to be described, which have all a brown colour, and are evidently the germs of the persistent teeth. Such parts as are calcified correspond precisely in size and form with those of the adult animal; they are consequently much crowded in the jaw.

In the upper jaw the four incisors have their crowns calcified; the second and fourth lie superficially, nearly concealing the first and third. The apex of the canine is calcified to the extent of ·35 inch. The apices of the crowns of the three premolars are calcified. That of the third (coloured blue) is least developed in proportion to its ultimate size; its apex is just above and rather in front of the minute deciduous molar. The crown of the first molar is in great part calcified, that of the second to a less extent. The third could not be recognized.

* Reade Lecture "On the Classification and Geographical Distribution of the Mammalia" (1859), p. 18.

In the lower jaw the first and third incisors lie superficially, the second being deeply placed between them,—a relation which may also be observed in the fangs of these teeth in the adult animal. The canine is deeply placed in the bone, its crown partially concealed from the outer side by the third incisor and first premolar. The third premolar is placed lowest in the mandible, its apex is nearly in contact with the inner side of the base of the rudimentary temporary molar. The crown of the first molar is completely calcified, that of the second and third partially so; no germ of the fourth could be traced.

The dentition above described is one of the most interesting and instructive in the whole series. It would be desirable to examine the teeth of some member of the allied genus *Dasyurus* at a corresponding age, but of this I have as yet had no opportunity.

### Family Phascolomyidæ.

I have placed this family at the end of the series, only because it is the one upon the dentition of which there is the least satisfactory information.

In a young Wombat (*Phascolomys vombatus*) 11 inches in length, the tips of the incisors, and of the premolars and first two molar teeth, were just appearing through the gums. The extent to which calcification had taken place in these teeth and the third molar is shown in Plate XXX. fig. 7. A trace of the germ of the fourth molar was discernible. There were no vestiges of successional teeth.

It is stated of this genus by Professor Owen that "the incisors and the first molar tooth are shed when the animal is young; the latter is superseded by the premolar tooth"*. Without further evidence, the statement as to the first may well be questioned, for even in the Rodents it has never been shown that the scalpriform incisors have deciduous predecessors; while, on the other hand, the analogy with the remaining marsupial families fully bears out the second. No details as to the mode or period at which this change occurs are given. It remains, therefore, for those who have opportunities of examining young Wombats at various ages to ascertain whether the first tooth of the molar series shown in the figure has already replaced, or is about to be replaced by another. Judging from its characters, which resemble those of the so-called "premolar" of the adult and not one of the true molars, I have very little hesitation in pointing to the former conjecture as the one most likely to prove correct.

From the foregoing observations, embracing members of each of the six natural families of the Marsupialia, it may be safely concluded that the animals of this order present a peculiar condition of dental succession, uniform throughout the order, and distinct from that of all other mammals.

This peculiarity may be thus briefly expressed. The teeth of Marsupials do not vertically displace and succeed other teeth, with the exception of a single tooth on each side of each jaw. The tooth in which a vertical succession takes place is always the

* Odontography (1840–45), p. 394.

corresponding or homologous tooth, being the hindermost of the premolar series, which is preceded by a tooth having the characters, more or less strongly expressed, of a true molar*.

Considerable differences occur in the various genera as to the relative period of the animal's life at which the fall of the temporary molar and the evolution of its successor takes place. In some, as *Hypsiprymnus*, it is one of the latest, in others, as *Thylacinus*, and probably *Phascolomys*, one of the earlier phenomena of dental development. Further observations on this point as opportunities occur will be interesting.

As before stated, I regard the first tooth of the molar series of the young Kangaroo as one of the premolars of the permanent series, and not (like the tooth placed immediately behind it in the first stage of dentition) as a molar of the deciduous or milk-series. It is in fact the homologue of the penultimate premolar of *Phalangista*, *Perameles*, *Didelphys*, *&c.* The circumstance of its being shed at a comparatively early period is in relation to the general conditions which, in this genus, cause the early loss of all the teeth between the incisor and true molar series, including the canine, and even the successional premolar. In *Hypsiprymnus* a still more potent reason prevails for its early removal, in the immense size of the successional premolar, which requires so much more space than is occupied by its diminutive actual predecessor.

It has been usual to divide the class Mammalia, in regard to the mode of formation and succession of their teeth, into two groups—the *Monophyodonts*, or those that generate a single set of teeth, and the *Diphyodonts*, or those that generate two sets of teeth; but even in the most typical diphyodonts the successional process does not extend to the whole of the teeth, always stopping short of those situated most posteriorly in each series.

The Marsupials occupy an intermediate position, presenting, as it were, a rudimentary diphyodont condition, the successional process being confined to a single tooth on each side of each jaw. This position is, however, by no means without analogy among the mammals of the placental series. In the Dugong, and in the existing Elephants, the successional process is limited to the incisor teeth. It is questionable whether the first premolar of many of those animals which have four teeth of this group, as the Dog and Hog (mandible), ever has a deciduous predecessor, at all events so far advanced as to have reached the calcified stage.

The closest analogy with the marsupial mode of succession is found among the Rodents. Here the incisors appear to have no deciduous predecessors; and in the Beaver, Porcupine, and others, which have but four teeth of the molar series, *i. e.* three true molars and one premolar, the latter is, exactly as in the Marsupials, the only tooth which succeeds a deciduous tooth. The analogy, however, no longer holds in those Rodents which have more than one premolar, as the Hare and Rabbit; for in this case each of these teeth has its deciduous predecessor.

* The convenient distinction between false molars or premolars and true molars, is well marked in the form of the crown, especially in the upper jaw, in all Marsupials.

In the preceding account I have used the term "permanent" for those teeth which remain in use throughout the animal's life, or, if they fall out (as with the rudimentary canines and the premolars of the *Macropodidæ*), do not give place to successional teeth, and I have therefore assumed that the milk or temporary dentition of the typical diphyodont mammals is represented in the Marsupials only by the deciduous molars.

It may be held, on the other hand, that the large majority of the teeth of the Marsupials are the homologues of the milk- or first teeth of the diphyodonts, and that it is the permanent or second dentition which is so feebly represented by the four successional teeth. This view is supported by many general analogies in animal organization and development, such as the fact that the permanent state of organs of lower animals often represents the fœtal or transitional condition of the same parts in beings of higher organization.

Looking only to the period of development of the different teeth in some of the marsupial genera, we might certainly be disposed to place the successional premolar in a series by itself, although, indeed, all its morphological characters point out its congruity with the row of teeth among which it ultimately takes its place, the reverse being the case with its predecessor.

It is, however, almost impossible, after examining the teeth of the young Thylacine (Plate XXX. fig. 6), to resist the conclusion originally suggested. The unbroken series of incisors, canines, premolars, and anterior true molars of nearly the same phase of development, with posterior molars gradually added as age advances, form a striking contrast to the temporary molars, so rudimental in size and transient in duration. I can scarcely doubt but that the true molars of this animal would be identified by every one as homologous with the true molars of the diphyodonts, which are generally regarded as belonging to the permanent series, although they never have deciduous predecessors. Now, if the homology between the true molars of the Thylacine and those of a Dog, for instance, be granted, and if the anterior teeth (incisors, canines, and premolars) of the Thylacine be of the same series as its own true molars, they must also be homologous with the corresponding permanent teeth of the Dog.

It may be objected to this argument, that the true molars of the diphyodonts, not being successional teeth, ought to be regarded as members of the first or milk-series; but, in truth, the fact that they have themselves no predecessors does not make them serially homologous with the predecessors of the other teeth, while their morphological characters, as well as their habitual persistence throughout life, range them with the second or permanent series.

We have been so long accustomed to look upon the second set of teeth as an after development or derivative from the first, that it appears almost paradoxical to suggest that the milk or deciduous teeth may rather be a set superadded to supply the temporary needs of mammals of more complex dental organization. But it should be remembered that, instead of there being any such relation between the permanent and milk-teeth as that expressed by the terms "progeny" and "parent" sometimes applied to

them*, they are both (if all recent researches into their early development can be trusted) formed side by side from independent portions of the primitive dental groove, and may rather be compared to twin brothers, one of which, destined for early functional activity, proceeds rapidly in its development, while the other makes little progress until the time approaches when it is called upon to take the place of its more precocious *locum tenens*.

Many facts appear to point to the milk-teeth as being the less constant and important of the two sets developed in diphyodont dentition. Among these the most striking is the frequent occurrence of this set in a rudimentary and functionless, or, as it were, partially developed state. The milk-premolars of some Rodents (as the Guinea-pig), shed while the animal is *in utero*, the simple structure and evanescent nature of the milk-teeth of the Bats, Insectivores, and Seals, the diminutive first incisors of the Dugongs and Elephants, all appear to be cases in point.

On the other hand, examples of the commencing or sketching out, as it were, of the successors to a well-formed, regular, and functional first set of teeth are rarely, if ever, met with. Occasional instances of the habitual early decadence, or, perhaps, absence of some of the second or so-called permanent teeth occur in certain animals; but these are rather examples of the disappearance or suppression of organs of which there is no need in the economy, and chiefly occur in isolated and highly modified members of groups in the other members of which the same phenomenon does not take place, as the *Cheiromys* among the Lemurs, *Trichecus* among the Seals, and the recent Elephants (as regards the premolars) among the Proboscideans. They form no parallel to the cases mentioned above of the rudimentary formation of an entire series of teeth of the temporary or milk-set.

To return to the Marsupials:—If this view be correct, I should be quite prepared to find, in phases of development earlier than those yet examined, some traces either of the papillary, follicular, or saccular stages of milk-predecessors to other of the teeth besides those determinate four in which, for some unexplained reason, they arrive at a more mature growth†. Such proof as this would alone decide the truth of these speculations; and I have not at present either the requisite leisure or materials for following out so delicate an investigation. I trust that the facts already elicited are sufficiently novel and important to justify my bringing them, as they now stand, before the Society.

* Cyclop. Anat. and Phys. Art. "Teeth," vol. iv. p. 901.

† It may be remarked that the milk-tooth which alone is developed in the Marsupials corresponds homologically with that which, as a general rule, is most persistent in the typical diphyodonts, including Man, viz. the posterior milk-molar, replaced by the posterior permanent premolar.

### Postscript, *October* 31*st*, 1867.

Since the foregoing was sent to the Society I have met with the following observation in Waterhouse's 'Natural History of the Mammalia,' vol. ii., Rodentia, 1848, p. 4, footnote:—"*I have sought in vain for deciduous incisors in young Marsupialia; if they exist they must be shed at a very early period in these animals.*" This important statement appears to have passed quite unnoticed by all subsequent writers.

Burmeister, in his "Erläuterungen zur Fauna Brasiliens" (1856), at p. 59, has given a somewhat detailed account of the supposed "milk dentition" of the South American Opossums. He describes the teeth of young animals (corresponding in age to that shown at Plate XXX. fig. 4), taking it for granted that they are changed as in ordinary mammals.

### Description of the Plates.

The figures are all of the natural size, and drawn from specimens in the Museum of the Royal College of Surgeons. The teeth which give place to vertical successors are coloured red; the replacing teeth are blue. Those teeth which neither succeed, nor are succeeded by others, are uncoloured.

### PLATE XXIX.

Figs. 1, 2 & 3. Different stages of the dentition of *Macropus*, described at pp. 632 & 633.

Fig. 4. *Hypsiprymnus murinus*. To show all the true molars in place before the evolution of the permanent premolar.

Figs. 5 & 6. Two stages of the dentition of *Phalangista vulpina*, described at p. 634.

### PLATE XXX.

Fig. 1. Immature dentition of *Perameles fasciata*.

Fig. 2. Adult dentition of the same animal.

Figs. 3 & 4. Two stages of immature dentition of *Didelphys*, described at pp. 635 & 636.

Fig. 5. Adult dentition of *Didelphys virginiana*, for comparison with the above.

Fig. 6. Early condition of dentition of *Thylacinus cynocephalus*. The germ of the first upper incisor is concealed by that of the second.

Fig. 7. Dentition of young *Phascolomys vombatus*. The premolar is coloured blue, in accordance with the conjecture expressed in the description at p. 637, although no absolute proof that it replaces another tooth has been given.

# XXI. *On the Genera* Heterophyllia, Battersbyia, Palæocyclus, *and* Asterosmilia; *the Anatomy of their Species, and their Position in the classification of the* Sclerodermic Zoantharia.

*By* P. Martin Duncan, *F.G.S., Secretary to the Geological Society. Communicated by Professor* Huxley, *F.R.S.*

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1. The admirable classification of the *Zoantharia* by MM. Milne-Edwards and Jules Haime*, although very elaborate and natural, presents several very decided "breaks" in the continuity of the generic succession. The great distinction between the important families of the *Turbinolidæ* and *Astræidæ*† renders the classification rather more artificial than it really is; and at first sight this classification may appear not very practical, on account of there being several genera which either cannot be placed at all or are arranged provisionally amongst certain divisions or families.

The objects of this communication are to describe the species of a genus which unites the family of the *Turbinolidæ* to that of the *Astræidæ*, to describe some very peculiar *Heterophylliæ* and *Battersbyiæ* and to classify them, and to remove the genus *Palæocyclus* from the family of the *Fungidæ* into that of the *Cyathophyllidæ*.

2. The genus *Heterophyllia* was established by M'Coy, who described two species of it from the Carboniferous limestone of Derbyshire. He was so struck with the anatomical peculiarities of the species that he had no hesitation in asserting that they were totally unlike those of any other recent or fossil group.

MM. Milne-Edwards and Jules Haime placed the genus amongst the "incertæ sedis."

Several specimens from the Carboniferous beds of Ayrshire, Fifeshire, Lanarkshire, and Stirling have lately been found to contain no less than six well-marked species of the genus *Heterophyllia*; and some of them are more paradoxical in their anatomy than those described by M'Coy. Still the generic peculiarities are strongly marked in all the species.

The genus may now be admitted to contain the following species.

* Histoire Naturelle des Coralliaires; Paris, 1860. † Brit. Foss. Corals, 2nd series, p. 34, 1866, pt. 1.

Genus HETEROPHYLLIA, *M'Coy*.

Ann. Nat. Hist. 2nd Series, vol. iii. p. 126; Brit. Palæoz. Foss. plate 3 A. figs. 1 & 2.

1. *Heterophyllia ornata*, M'Coy.
2. —— *grandis*, M'Coy.
3. —— *mirabilis*, sp. nov.
4. —— *granulata*, sp. nov.
5. —— *angulata*, sp. nov.
6. —— *M'Coyi*, sp. nov.
7. —— *Lyelli*, sp. nov.
8. —— *Sedgwicki*, sp. nov.

The new species come from highly fossiliferous beds which are very low down in the Carboniferous series.

The generic diagnosis of M'COY is as follows; and it is proposed to modify it slightly after the description of the new species.

*Gen. Char.*—"Stem elongate, subcylindrical, irregularly fluted longitudinally; *horizontal section*, few distinct lamellæ, destitute of any order of arrangement, but irregularly branching and coalescing in their passage from the solid external walls towards some indefinite point near the centre where the few main lamellæ irregularly anastomose; *vertical section*, showing about the middle an irregularly flexuous line (the edge of one or two of the radiating vertical lamellæ), from which on each side a row of thin, distant, sigmoidally curved plates extends obliquely upwards and outwards, forming a row of large rhomboidal cells on each side."

According to the terminology now used, the stem is the corallum, the flutings are intercostal spaces, the lamellæ are septa, and the curved and oblique plates are endothecal dissepiments.

*Description of the Species.*

1. HETEROPHYLLIA GRANDIS*, M'Coy.

The corallum is tall and slightly flexuous. The costæ are few in number, and project as large polygonal unequal ridges. The intercostal spaces are grooves, and are deep as well as unequal. The horizontal section has an irregularly angular outline. The surface of the corallum is smooth. The septa are very irregular in disposition and number. The diameter of the stem is five lines.

The species is an uncommon form in the Mountain-limestone of Derbyshire.

2. HETEROPHYLLIA ORNATA, M'Coy.

The corallum is subcylindrical, long, and flexuous. The costæ are narrow, subequal, and are ornamented with small and rounded tubercles. The costæ are about sixteen in number. The intercostal spaces are flat, and are rather wider than the costæ. The

* These descriptions of M'Coy's species are altered so as to meet the requirements of the terminology of the received classification.

surface of the corallum is finely granular. The septa are fourteen in number. The diameter of the corallum is about $1\frac{1}{2}$ line.

The species is found with *H. grandis*.

3. Heterophyllia granulata, sp. nov. Plate XXXI. figs. 1 *a*–1 *d*.

The corallum is long, slightly flexuous, and more or less cylindrical; it is rendered irregular in shape from alternate constrictions and swellings of the wall. The costæ are numerous, flexuous, rounded, unequal, and coriaceous; they bifurcate and project but slightly. The intercostal spaces are grooved, shallow, narrow, and wavy: they are narrower than the costæ. The horizontal section is nearly circular. The surface of the corallum is finely granulated. The septa are nineteen in number, thin, and often unite, to form five principal sets. The diameter of the corallum is $\frac{4}{10}$ inch at the swellings and $\frac{3}{10}$ inch at the constrictions.

In the Carboniferous series of Fifeshire.

4. Heterophyllia angulata, sp. nov. Plate XXXI. figs. 2 *a*–2 *d*.

The corallum is long, flexuous, and rather angular in its transverse outline. The costæ are numerous, unequal, projecting, close, faintly granular, and coriaceous; they are usually eighteen in number, are rather sharp at the free edge, and smaller there than at their base. The intercostal spaces are unequal, concave, rather deep, narrow, and coriaceous. The horizontal section of the corallum is angular and irregular. The surface of the corallum is finely granular. The septa are seventeen in number, thirteen large and four small; they are arranged in five groups: the largest septa are nearly straight, and one is solitary and larger than the others. The small septa reach and become united to those nearest to them. The longest septa are joined by a lamina which stretches across the axial space*. The endotheca is very abundant, and stretches across the interseptal spaces, the concavity being outwards. The wall is thin and partly composed of very close endothecal cells. There is some vesicular endotheca. The diameter is $\frac{3}{10}$ inch.

The species is found in the Carboniferous strata of Brockley, Lesmahagow, Lanarkshire.

5. Heterophyllia M'Coyi, sp. nov. Plate XXXI. figs. 3 *a*–3 *c*.

The corallum is tall, nearly straight, and is hexagonal in its transverse outline. The costæ are small, projecting, rounded, six in number, without ornamentation, and are grooved here and there. The intercostal spaces are very wide, shallow, slightly concave, nearly equal, and are marked occasionally with faint linear and festoon-shaped depressions. The horizontal section shows the wall to be stout and hexagonal; it is concave between the angles. The surface of the corallum is smooth. There are six septa, which unite centrally to a linear septal columella. The endotheca is very abundant. The diameter of the corallum is $\frac{9}{10}$ inch.

The species is found in the Carboniferous strata of Brockley, Lesmahagow, Lanarkshire.

* See Anatomy of *Sclerenchyma*, Brit. Foss. Corals, 2nd series, pt. 1, P. M. Duncan, Pal. Soc. 1866.

6. Heterophyllia Lyelli, sp. nov. Plate XXXI. figs. 4 *a*–4 *c*.

The corallum is very long, very slender, and is slightly bent. The costæ are large, smooth, and rounded; they project and are marked with occasional tubercles, pits, and grooves. The intercostal spaces are wide, shallow, and equal; they are slightly concave, and are marked with festoon-shaped ridges or lines. The horizontal section of the corallum is hexagonal in outline; the wall is stout and thick, and only very slightly concave between the costæ. The surface of the corallum is smooth and plain. There are six septa, which are united by a linear septal columella. The endotheca is tolerably abundant. The diameter of the corallum is $\frac{1}{10}$ inch or less.

In the Carboniferous limestone of Craigenglen, Stirling, and Brockley, Lesmahagow, Lanarkshire.

7. Heterophyllia mirabilis, sp. nov. Plate XXXI. figs. 5 *a*–5 *h*.

The corallum is tall, very slender, and nearly straight. The costæ are narrow, rounded, smooth, and slightly projecting; they have tubercles at regular and frequent intervals. These tubercles are rounded and oblique, and project slightly. To each of them is articulated a curved hook-shaped process, which stands out from the costa and the tubercle, its concavity being directed inwards and downwards. The intercostal spaces are shallow, wide, and usually slightly convex, but occasionally concave; they are marked with three longitudinal delicate shallow grooves with very slightly rounded longitudinal eminences between them. A groove is central. The horizontal section of the corallum is nearly circular; there are projections which correspond with the costæ; and the wall is moderately thick. The surface of the corallum is smooth. There are six septa, which are united by a linear septal columella. The endotheca is scanty, and the dissepiments are wide apart. The diameter of the corallum is rather more than $\frac{1}{10}$ inch.

From the Carboniferous limestone of Craigenglen and Brockley.

8. Heterophyllia Sedgwicki, sp. nov. Plate XXXI. figs. 6 *a*–6 *e*.

The corallum is tall, thin, and very flexuous, except near the calice, where it becomes straight. The costæ are numerous, small, very slightly projecting, rounded, unequal, distant, and flexuous; they are grooved, and vary in number, eighteen or nineteen being the maximum. The intercostal spaces are shallow, rather wide (but occasionally narrow), slightly convex, and grooved longitudinally; and the appearance of false costæ is thus produced. The horizontal section of the corallum is nearly circular in outline; the wall is thin, and the costæ project very slightly. The surface near to the calicular end is coriaceous, and over the rest of the corallum it is smooth. The septa are twenty in number. There are ten large and ten small septa in the calice. In sections there are eighteen or nineteen very irregular septa; all the larger radiate to the centre. The calice is regular, and there is a small columella (a septal). The reproduction is partly by gemmation around the calice at the end of the stem-like corallum. The buds have many septa. The endotheca is very abundant, and the wall is thin. The diameter of the corallum is from $\frac{1}{10}$ to $\frac{3}{10}$ inch; and the calicular end, with its buds, measures in diameter $\frac{4}{10}$ inch.

In the Carboniferous limestone of Brockley, Lesmahagow, Lanarkshire.

It will be very readily observed that the stout wall, the costæ, and the absence of epitheca render this genus rather exceptional amongst those of the Palæozoic fauna. The irregular septal arrangement of some of the species, and the presence of one septum larger than the others in certain forms, are *rugose* peculiarities; but the scanty and simple dissepimental endotheca is very differential.

In *Heterophyllia mirabilis* the endotheca is very sparingly seen, and it exists in the form of simple dissepiments very wide apart, which, as in the *Astræidæ*, close more or less the interlocular spaces. They do not close in the whole of the visceral, axial, and interlocular cavities in a horizontal plane, and they are not tabulæ in the strict acceptation of the term. In other species the dissepiments are closer, and, although curved, simulate tabulæ. True vesicular endotheca is rare, but it exists in most of the specimens. The columellary lamina, curved in some species, but straight in others, is probably produced by the soldering of the septal ends, and is not really a true lamellar columella. In the specimen of *Heterophyllia Sedgwicki*, whose calice is preserved, the formation of the septal columella can be seen. The costæ, so variable in their structure, afford admirable specific distinctions; and those of *H. mirabilis* are totally unlike any others from any madreporarian family. The septa, irregular in number in some species, are then rather numerous; but when they are regular in their arrangement they never number more than six.

The species may be distinguished by comparing the specimens with the following Table:—

| | |
|---|---|
| *Heterophyllia grandis* . . | Costæ few, large, as polygonal ridges.<br>Septa very irregular in number. |
| —— *ornata* . . . . . | Costæ 16 in number, narrow with round tubercles.<br>Septa 14 in number. |
| —— *granulata* . . . . | Costæ numerous, flexuous, rounded, bifurcate, granular.<br>Septa 19 in number, grouped in 5 series. |
| —— *angulata* . . . . | Costæ projecting, sharp, unequal.<br>Septa 17 in number, in 5 series and 1 large septum. |
| —— *M'Coyi* . . . . . | Costæ 6 in number, small, rounded, grooved.<br>Septa 6 in number. |
| —— *Lyelli* . . . . . | Costæ large, smooth, rounded, projecting, tuberculate.<br>Septa 6 in number. |
| —— *mirabilis* . . . . | Costæ narrow, rounded, tuberculate and spined.<br>Septa 6 in number. |
| —— *Sedgwicki* . . . . | Costæ 18 in number, distant, flexuous, small, grooved.<br>Septa 18 to 20 in number. |

The genus may be subdivided into a group with numerous septa and a group with six septa.

In the first subdivision the *rugose* type is faintly, and in the second the hexameral

arrangement is well observed. The dense wall and the dissepimental endotheca prove that the type of the mesozoic coral-fauna was foreshown.

The generic characteristics may be admitted to be as follows.

Genus *Heterophyllia*, M'Coy.

The corallum is simple, long, and slender. The gemmation takes place around the calicular margin, and is extracalicular. The septa are either irregular in number and arrangement, or else are six in number and regular. The costæ are well developed, and may be tubercular, spined, and flexuous. The wall is thick, there is no epitheca, and the endotheca is dissepimental.

The position of *Heterophyllia* in the classification of the Madreporaria will be more easily comprehended after the following examination of the genus *Battersbyia*.

3. Genus Battersbyia.

The genus *Battersbyia* was founded by MM. Milne-Edwards and Jules Haime in order to include a very anomalous species which is very plentiful in the Devonian limestones of Torquay.

Since that species was described I have discovered two others, whose anatomy renders the generic determination of MM. Milne-Edwards and Jules Haime capable of some extension, and the position of the genus in the classification of the Madreporaria more decided. The great French zoophytologists at first considered the species *Battersbyia inæqualis* so anomalous as to form the type of a new division of fossil corals; but subsequently they classified it amongst the *Milleporidæ*, but evidently only provisionally. The study of the new species proves that the genus cannot be retained amongst the *Milleporidæ*, and that the first suggestion of MM. Milne-Edwards and Jules Haime was correct.

The reproduction of *Battersbyia* can be studied in the new specimens, and its peculiar character, and the evident absence of a cœnenchyma around the corallites, prove that the so-called cœnenchyma of *Battersbyia inæqualis* is of *stromatoparous* origin and is parasitic.

"Genus *Battersbyia*, Milne-Edwards and Jules Haime*.

"The corallum is massive, and the gemmation is lateral. The walls are thick. The septa are small but well developed. The 'planchers' are vesicular. The cœnenchyma is very lax and spongy."

1. Battersbyia inæqualis, Ed. and H.*

The corallum is massive; the corallites are very unequal, they have thick but non-costulate walls. The calices are circular. The septa are readily seen, but they are small and unequal. They are twenty six in number in the largest calices. The "planchers" are vesicular and fill up the visceral chamber. The cœnenchyma is scanty and spongy.

* Monogr. des Polyp. des terr. Palæoz. 1851, p. 151.

† Hist. Nat. des Corall. vol. iii. p. 244. Brit. Foss. Corals, Pal. Soc. p. 213. pl. 47, fig. 2.

The specific determination is given more fully in the Monograph of the Devonian Corals (p. 213). Thus the septa are said to be somewhat unequal in size alternately, to be rather thick towards the wall, but very thin inwardly. The diameter of the largest corallite is pronounced to be above $1\frac{1}{2}$ line.

2. Battersbyia grandis, sp. nov. Plate XXXII. figs. 1 *a*–1 *d*.

The corallum is in fasciculate masses; and the corallites are tall, and very close in some parts of it and rather distant in others; they are very unequal in size. The calices are wanting; but transverse sections prove the corallites to be generally circular in outline; but they are often deformed, oval, and elliptical. The outline of the larger corallites is rendered irregular by the pressure of the growing buds; and where the wall of the bud and that of the parent corallite touch, one is often absorbed. The wall is dense and not costulated, but it is thin in comparison with the diameter of the largest corallites. The septa are very thin, are alternately long and short, and are often wavy or curved; they spring from the wall by wide wedge-shaped processes, and none of them reach the centre of the calice. The septa vary in number; in the smallest corallites there is a confused tissue formed of vesicular endotheca, and a few rudimentary septa; in other corallites, and as they increase in length, there are 12 to 16, 36 to 40, 46 to 52 septa. The wedge-shaped origins of the septa are equal in perfect corallites; and the axial space varies in diameter. The endotheca is greatly developed; it is often as stout as the septa, and it forms a series of small vesicles placed one over the other, and side by side in the interlocular space. Each interseptal loculus contains many vesicles, so that a transverse section which cuts across the vesicles presents a series of concentric lines. There are no horizontal tabulæ, nor do the vesicles cut off all below them on the same plane. There is no columella, and its space is occupied by a vesicular endotheca. There is no cœnenchyma. The diameter of the largest corallites is $\frac{1}{3}$ inch, and of the smallest, with 12 to 16 septa, $\frac{3}{10}$ inch. The height of the corallites is several inches.

Locality. Devonian limestones, Torquay.

3. Battersbyia gemmans, sp. nov. Ptate XXXII. figs. 2 *a*–2 *d*.

The corallum is fasciculate, and the corallites are tall, very close and crowded; they are very unequal in size. The wall is thin, and not always double when two corallites impinge, but in the buds and young corallites it is very thick; in transverse outline the wall is oval, circular, and even polygonal. The septa are very thin, wavy, and arise from small wedge-shaped processes; the septa are alternately very long and short, and they vary in number. In the largest calices there are thirty-two, and in the smallest there are six. The endotheca is highly developed; the vesicles are small, and their tissue is thicker than the septa. The budding is of two kinds:—1. From the walls of corallites. These buds have either five septa or more; and in the first instance budding takes place as follows. 2. From the interseptal spaces, so that five buds arise from each lateral bud whose septa are not more than six in number. The septa and the wall of each interlocular space develope other septa, and thus a very rapid budding is

completed. When the first kind of bud has more than five septa the resulting corallite grows like the parent. Diameter of corallites from $\frac{1}{10}$ inch to $\frac{1}{4}$ inch.

Locality. Teignmouth. Devonian limestones.

The generic characters of *Battersbyia* may be altered in consequence of the specific characteristics of the new forms:—

Corallum fasciculate and branching. Corallites tall, cylindrical, unequal in size and distance. Septa numerous and following no apparent cyclical order.

Endotheca very abundant; it is vesicular, and there are no tabulæ. Epitheca, costæ, and cœnenchyma wanting. The wall is stout; and the septa spring from wedge-shaped processes. The columellary space is occupied by vesicular endotheca. Gemmation extracalicular, and calicular from buds having only five septa.

Being satisfied that there is no cœnenchyma in *Battersbyia grandis* and *Battersbyia gemmans*, and that it is simulated in *Battersbyia inæqualis*, Ed. & H., by an investing and incrusting *Stromatopora*, the removal of the genus from the *Milleporidæ*, where it had been placed somewhat provisionally by MM. Milne-Edwards and Jules Haime, is absolutely necessary.

The corallum in every species is fasciculate; and the unequal size of the corallites is determined by the method of gemmation. In the case of *Battersbyia gemmans* the difference in the size of the corallites is excessive; and it is very probable that its peculiar gemmation is witnessed in the other species. The buds which develope more than five septa appear to grow into corallites, which are destined to bud again from the external wall; and the buds which develope five septa soon produce other buds from their interseptal loculi, the buds thus developed resembling the multiseptate corallites. This curious alternation of gemmation has not been observed in any other genus; but it is remarkable that the *Heterophylliæ* with six septa should be related to *Heterophylliæ* with numerous septa.

The genera *Heterophyllia* and *Battersbyia* have much in common. They have a stout wall, a vesicular and dissepimental endotheca, delicate septa, very irregular in their number, and neither tabulæ, epitheca, nor a quaternary septal arrangement. The genus *Battersbyia* has nothing to ally it to the *Rugosa*; and *Heterophyllia* has in some of its species the solitary large septum or a vacancy where a septum should be, which is so often observed in the *Cyathophyllidæ*. Its costæ and endotheca connect the genus with the mesozoic and recent *Astræidæ*; and that this family, unrepresented in palæozoic strata, is foreshadowed by the genera now under consideration is very evident.

The costæ of *Heterophyllia mirabilis* are the most extraordinary ever recorded, and they are unlike those of any other species of coral.

There is nothing unusual in the irregular septal development of *Battersbyia*; for it is noticed in the Liassic fasciculate *Astræidæ*; and whilst this genus points to the genera *Pentacœnia*, *Calamophyllia*, and *Thecosmilia*, *Heterophyllia* may be likened, faintly it is true, to *Rhabdophyllia*.

It would appear necessary to associate the genera *Battersbyia* and *Heterophyllia*

together in a division of the *Astræidæ*, and to place it near the above-mentioned mesozoic genera under the name of *Palastræaceæ*.

### 4. Genus Palæocyclus*. Plate XXXII. figs. 6 *a*–6 *e*.

This genus of Silurian corals has hitherto been regarded as belonging to the *Fungidæ*, and as the only representative of that great family in the palæozoic rocks. It is rather anomalous that a number of species of a highly organized coral genus should be found in the Silurian rocks of Dudley, Wenlock, and of Gothland, and that not a species of any other genus of the family of the *Fungidæ* should have been found in the Devonian or in the Carboniferous and Permian strata. The doubt whether *Palæocyclus* could belong to the *Fungidæ* was rendered more worthy of consideration by the discovery in the Australian tertiaries of a simple fungoid coral which had all the generic attributes of the Palæozoic genus†.

Careful sections of specimens of *Palæocyclus porpita*, Linnæus, sp., and *Palæocyclus rugosus*, Milne-Edwards and Jules Haime, were made, and it became evident that, although the external characteristics of the corals were those of the simple *Fungidæ*, the internal structure was analogous to that of the Palæozoic family of the *Cyathophyllidæ*.

The generic diagnosis of *Palæocyclus* is as follows.

The corallite is simple, short, and in general discoid, it is free and subpedicellate. The wall is covered with a complete epitheca. The calice is circular, and has a well-marked central fossa. The columella is rudimentary. The septa are moderately numerous, stout, straight, slightly exsert, and free internally. They are granular laterally, and dentate superiorly.

In order to admit the genus into the family of the *Fungidæ*, the septa of all its species must have synapticulæ upon them, and the corallites should not possess curved dissepiments or tabulæ.

Transverse and longitudinal sections of *Palæocyclus porpita* and *Palæocyclus rugosus* were prepared, and the absence of synapticulæ was proved, as was also the presence of an inclined dissepimental endotheca at the sides, and of tabulæ in the centre of the corallites.

The taller the specimen the more numerous were the tabulæ, and the uppermost of them formed the base of the septal fossa. In the discoidal species no tabulæ could be distinguished, but there were some marked rugose peculiarities and no synapticulæ. The septa of *Palæocyclus* are ornamented laterally with ascending rows of granules or blunt dentations. When the septa are worn from above downwards, these ornamentations stand out on either side of the laminæ and give the appearance of synapticulæ. Transverse sections through the corallites show the adhesion of the dissepiments to the septa; but the cross-bar-like appearance is produced by the section passing through a

* Milne-Edwards et Jules Haime, Compt. Rend. de l'Acad. des Sc. t. xxix. p. 71, 1849. Hisinger's *Cyclolites*.

† Ann. and Mag. of Nat. Hist. Sept. 1864, plate vi. fig. 2.

dissepiment. In the longitudinal sections the obliquity of the dissepiments is seen, and it explains the appearance in the transverse view. The dissepiments are numerous and curved, and they unite with the central tabulæ. There is no columella in the species. The septa of some of the specimens of the discoidal species are not of the same size on both halves of the calices, and very often one septum is longer than the others. The cyclical arrangement of the septa in six or in four systems is never distinguishable. It must be remembered that the discoidal species may be in some instances (for there are several new discoidal species) the young of the taller and tabulate corallites.

The removal of the genus *Palæocyclus* from the family of the *Fungidæ* is necessary, and it is very evident that the species classified under it belong to two divisions of the family *Cyathophyllidæ*. The *Cyathophyllidæ* with large tabulæ and short septa have been separated from the genus *Cyathophyllum*, whose species have the septa passing to the axis of the corallum, and have been arranged under the genus *Campophyllum*, Ed. et H.; but it is too specific a distinction to be of generic value. It is therefore proposed to place all the species of *Palæocyclus* in the genus *Cyathophyllum*, which, like the genus *Montlivaltia*, now possesses simple corallites of every form.

The genus *Palæocyclus* may therefore be abolished altogether, and its species will be named as follows:—

1. *Cyathophyllum porpita*, Linnæus, sp.
2. —— *præacutus*, Lonsdale, sp.
3. —— *Fletcheri*, Ed. et H., sp.
4. —— *Edwardsi*, the *rugosus* of Ed. et H.

### 5. Genus Asterosmilia.

Whilst investigating the fossil corals of the Nivajé shale* of San Domingo, my attention was strongly attracted by the very unusual combination of structural elements presented by a species named *Trochocyathus abnormalis*, nobis. Mr. Lonsdale had noticed the peculiarities of the specimens in his manuscript notes, and had proposed to include the species in a new genus.

Lately other specimens have been examined, and the necessity of forming a new generic division which will include these species has become evident.

The new genus unites the great families *Turbinolidæ* and *Astræidæ*, between which a great break had existed previously. It allies the *Trochocyathi*, *Placocyathi*, and the *Paracyathi* with the simple *Troschosmiliaceæ*, and contains simple corals with endothecal structures, columellæ, and pali.

The correct position of the new genus *Asterosmilia* will be comprehended after the study of the characteristics of the families it allies.

The *Madreporaria aporosa* are divided into two great groups. In the first, which includes the great family of the *Turbinolidæ*, the interseptal loculi, and the visceral cavities, generally speaking, are open from the base to the calice; and in the last, which

* Foss. Corals of the West Indies, Proc. Geol. Soc. 1863.

includes amongst others the family of the *Astræidæ*, the interseptal loculi and visceral cavities are not open from the base to the calice, but are more or less completely shut off from below upwards by endothecal dissepiments.

This endotheca is absent in the *Turbinolidæ*, although it is very faintly foreshadowed in the well-known *Paracyathi* of the London clay, and it is invariably present in the *Astræidæ*. There are two divisions amongst the *Turbinolidæ*: in one the septa reach inwards from the wall and have no appendages between their inner ends and the columella; and in the other these appendages or pali always exist before certain definite septa. The first division, the *Turbinolinæ*, has, then, neither pali nor endotheca; and the second division, the *Caryophyllinæ*, has pali but no endotheca.

The second great group of the *Madreporaria* has endothecal structures but no pali.

The new genus *Asterosmilia* has species which possess pali and endotheca.

Endothecal dissepiments are secreted by the innermost of the tissues which line the visceral cavity of recent corals; and the portion of the corallum shut off by the formation of the dissepiment is no longer filled with any of the soft parts, but is dead to all intents and purposes. As the coral grows in height, these dissepiments are formed, so that the soft tissues are really restricted to a space a very little lower than the calice. Where there are no dissepiments the visceral cavity reaches to the base of the corallum.

The pali are structures which spring from the base of the corallum, are secreted by the soft tissues, and determine the existence of a row of tentacles between the mouth and the innermost of the rows of the tentacles of the septal part of the disk.

The corals contained in the *Turbinolidæ* are simple and not aggregate. The species of the new genus are simple; and it is interesting to observe that the form of the corallites closely resembles that of the elongate *Trochocyathi*. The position of *Asterosmilia* in the classificatory scale is between the *Turbinolidæ* and the simple *Astræidæ*.

### *Description of the Species.*

1. Asterosmilia anomala, Duncan. Plate XXXII. figs. 3 *a*–3 *d*.
   *Trochocyathus abnormalis*, Proc. Geol. Soc. 1863, p. 26.

2. Asterosmilia cornuta, sp. nov. Plate XXXII. fig. 4.

The corallum resembles *Asterosmilia anomala* in shape, but is often longer. The costæ are wavy and are slightly prominent, especially those of the principal septa. There are crests inferiorly. The endotheca is abundant. Height of corallum 1½ inch.

Locality. Nivajé shale. In the collection of the Geological Society.

3. Asterosmilia exarata, sp. nov. Plate XXXII. fig. 5.

The corallum is small, pedunculate, nearly straight, and it enlarges suddenly. All the costæ are prominent and nearly equal. There are no crests. Height ¾ inch.

Locality. Nivajé shale. In the collection of the Geological Society.

From the characteristics of the species the generic diagnosis may be thus given.

*Asterosmilia*.—The corallum is simple, long, and more or less cornute. The costæ are

irregular in their development, and crests are occasionally present. The septa are numerous and exsert. The columella is more or less solid, essential, and compressed. Pali exist. The endothecal dissepiments are distinct, tolerably numerous, and are curved.

The genus forms the link in the classificatory chain between the genera *Placosmilia*, *Parasmilia*, and *Trochosmilia* and the genera *Placocyathus*, *Paracyathus*, and *Trochocyathus*. As it cannot be correctly associated either with the *Turbinolidæ* or the *Trochosmiliaceæ*, it must be included in a new division—the *Asterosmiliaceæ*.

6. It has been attempted to prove the necessity of classifying the genera, the anatomy of whose species has been described in this communication, as follows.

*Heterophyllia*, M'Coy, must be associated with *Battersbyia*, Ed. et H., in a division of the *Astræidæ*—the *Palastræaceæ*.

The genus *Palæocyclus*, Ed. et H., must be abolished and its species added to the genus *Cyathophyllum*, Goldfuss.

The genus *Asterosmilia*, nobis, forming the link between the simple *Astræidæ* and the *Turbinolidæ*, should form the division *Asterosmiliaceæ*, and should be placed between the *Trochosmiliæ*, *Placosmiliæ*, and *Parasmiliæ* and the *Trochocyathi*, *Placocyathi*, and *Paracyathi*.

By the absorption of the genus *Palæocyclus* a representative of the tertiary coral-fauna is removed from the Palæozoic; and by the separation of *Battersbyia* from the *Milleporidæ*, and its association with *Heterophyllia*, formerly a genus incertæ sedis, in a division of the *Astræidæ*, two genera with mesozoic affinities are introduced into the Palæozoic coral-fauna.

*March* 1867.

On April 20, 1867, the author added the genus *Pentacœnia* to those allied to *Battersbyia*, and corrected the description of the gemmation of *Battersbyia gemmans*.

## Explanation of the Plates.

### PLATE XXXI.

Fig. 1 *a*. A portion of the corallum of *Heterophyllia granulata*, sp. nov.
*b*. Magnified view of a transverse section, showing the septa and the wall.
*c*. Magnified view of the granular costæ.
*d*. The wall magnified, with a septum and an endothecal dissepiment.
*e*. The curved arrangement of the costæ.

Fig. 2 *a*. A portion of the corallum of *Heterophyllia angulata*, sp. nov.
*b*. A transverse section: magnified.
*c*. The wall, septa, and dissepiments: magnified.
*d*. A longitudinal section showing the dissepiments: magnified.

Fig. 3 *a*. A portion of the corallum of *Heterophyllia M'Coyi*, sp. nov.: magnified.
*b*. A transverse section: magnified.
*c*. The wall, septa, and dissepiments: magnified.
Fig. 4 *a*. A portion of the corallum of *Heterophyllia Lyelli*, sp. nov.: magnified.
*b*. A transverse section: magnified.
*c*. A portion of the wall and some costæ: magnified.
Fig. 5 *a*. A portion of the corallum of *Heterophyllia mirabilis*, sp. nov.: magnified.
*b*. A transverse section: magnified.
*c*. The costæ without the spines: magnified.
*d*. Side view of the costæ with the spines: magnified.
*e*. Magnified view of the process of the costæ which supports a spine.
*f*. The intercostal line: magnified.
*g*. Longitudinal section: magnified.
*h*. Restoration of the corallum (diagram).
Fig. 6 *a*. A portion of the upper part of the corallum of *Heterophyllia Sedgwicki*, sp. nov.
*b*. A transverse section: magnified.
*c*. A magnified view of the upper part of a corallum.
*d*. A longitudinal section: magnified.
*e*. Magnified calice.

## PLATE XXXII.

Fig. 1 *a*. A transverse section of part of the corallum of *Battersbyia grandis*, sp. nov.: natural size.
*b*. A corallite, magnified, transverse section.
*c*. A young corallite, magnified, transverse section.
*d*. A portion of a large corallite: magnified.
Fig. 2 *a*. A transverse section of part of the corallum of *Battersbyia gemmans*, sp. nov.
*b*. A group of buds, transverse section: magnified.
*c*. A bud commencing to develope five buds, each of which will resemble the original corallum or parent stock, transverse section: magnified.
*d*. Magnified view of a transverse section of a corallite, the result of one of a series of five buds. Such corallites produce buds like unto themselves, as well as others which develope five buds.
Fig. 3 *a*. The corallum of *Asterosmilia anomala*, Duncan.
*b*. A corallum with the wall partially removed, showing the endothecal dissepiments, slightly magnified.
*c*. A dissepiment: magnified.
*d*. A view of a worn calice, showing the central columella, the pali attached to it, and the septa.
Fig. 4. The corallum of *Asterosmilia cornuta*, sp. nov.

Fig. 5. The corallum of *Asterosmilia exarata*, sp. nov.: slightly magnified.

Fig. 6 *a*. Magnified view of the calice of a species of the genus *Palæocyclus* (*Cyathophyllum*).

*b*. Magnified view of the septa: transverse section, showing the absence of synapticulæ.

*c*. Magnified view of the dissepiments between the septa.

*d*. A magnified longitudinal section of a discoid species.

*e*. Magnified view of a longitudinal section, showing tabulæ and a vesicular endotheca, proving the genus to belong to the *Cyathophyllidæ*.

# XXII. *Researches into the Chemical Constitution of Narcotine, and of its Products of Decomposition.*—Part II.

*By* A. MATTHIESSEN, *F.R.S., Lecturer on Chemistry in St. Mary's Hospital Medical School, and* G. C. FOSTER, *B.A., Professor of Physics in University College, London.*

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ABOUT four years ago we had the honour of communicating to the Royal Society a paper entitled "Researches into the Chemical Constitution of Narcotine, and of its Products of Decomposition"*, and we now desire to lay before the Society some results obtained in the further prosecution of the same inquiry. We are fully aware that our present communication is in many respects very incomplete, but as we have no prospect of being able to resume the investigation conjointly, we venture to present the results already obtained as they are.

In the previous paper it was shown that narcotine and its principal derivatives, opianic acid, meconin, hemipinic acid, and cotarnine, are decomposed when heated with hydrochloric acid or hydriodic acid into iodide or chloride of methyl, and one or more other products. With the exception, however, of those obtained from hemipinic acid and cotarnine these second products had not been examined: the present memoir relates principally to the further study of these reactions.

## 1. *Action of Hydrochloric and Hydriodic Acids on Opianic Acid.*

When opianic acid is heated to 100° or 110° with three or four times its weight of strong hydrochloric acid, either in a sealed tube or in an open flask, under a layer of paraffin† for about twenty hours, or with about twice its weight of fuming hydriodic acid, and evaporated to dryness on a water-bath, chloride or iodide of methyl is produced, and at the same time a crystalline acid containing

$$C_9 H_8 O_5.$$

The reaction may be represented by the equation

$$C_{10} H_{10} O_5 + H Cl = C_9 H_8 O_5 + C H_3 Cl.$$

For reasons indicated in our former paper‡, and more fully developed in the Journal of the Chemical Society§, we regard opianic acid as the dimethylized derivative of a

* Philosophical Transactions, 1863, p. 345; for abstracts see Proc. Roy. Soc. vol. xi. p. 55, and vol. xii. p. 501.

† When preparing a large quantity of this or any other substance by the action of hydrochloric acid this method is very advantageous, as there is no danger of loss by bursting as often happens with sealed tubes. In preparing a new base from narcotine, where we employed 200 grms. at each operation, the saving of time and expense by using this method was very great.

‡ Philosophical Transactions, 1863, p. 365.

§ Vol. xvi. p. 342.

hitherto unknown normal opianic acid,

$$C_8H_6O_5,$$

between which and opianic acid itself the product above-mentioned is exactly intermediate:

| | |
|---|---|
| Opianic acid . . . . . | $C_{10}H_{10}O_5$ |
| New compound . . . . | $C_9H_8O_5$ |
| Normal opianic acid . . | $C_8H_6O_5$. |

For want of a better name we therefore propose to call the compound $C_9H_8O_5$ *monomethyl-normal opianic acid*, or if the contraction is admissible, *methyl-noropianic acid*.

Dried at 100° C. the substance gave the following results on analysis*:—

I. 0·4486 grm. gave 0·9036 grm. carbonic acid and 0·1694 grm. water.
II. 0·4135 grm. gave 0·8365 grm. carbonic acid and 0·1600 grm. water.
III. 0·4435 grm. gave 0·8970 grm. carbonic acid and 0·1680 grm. water.

| | Calculated. | | Found. | | |
|---|---|---|---|---|---|
| | | | I. | II. | III. |
| $C_9$ . . . . . | 108 | 55·10 | 54·94 | 55·16 | 55·16 |
| $H_8$ . . . . . | 8 | 4·08 | 4·20 | 4·29 | 4·21 |
| $O_5$ . . . . . | 80 | 40·82 | —— | —— | —— |
| $C_9H_8O_5$ . . . | 196 | 100·00 | | | |

Methyl-noropianic acid crystallizes with $2\frac{1}{2}$ molecules of water, which it gives up at 100° C.

I. 8·509 grms.† lost 1·548 grm. at 100° C.
II. 9·422 grms. lost 1·772 grm. at 100° C.

| | Calculated. | | Found. | |
|---|---|---|---|---|
| | | | I. | II. |
| $C_9H_8O_5$ . . . . | 196 | 81·33 | —— | —— |
| $2\frac{1}{2}H_2O$ . . . . | 45 | 18·67 | 18·2 | 18·8 |
| $C_9H_8O_5$, $2\frac{1}{2}H_2O$ | 241 | 100·00 | | |

The crystallized acid, when heated, first melts in its water of crystallization, and then, as the water evaporates, solidifies to a white crystalline mass. Hence it appears that the acid dissolves in less than a quarter of its weight of hot water; in cold water, however, it is only sparingly soluble. It is easily soluble in alcohol, but almost insoluble in ether.

Like hypogallic acid it strikes a dark blue with perchloride of iron, but, on addition

* All the combustions given in this paper were made with oxide of copper and oxygen.

† For these and other like determinations the substance was first drained on filter-paper and then pressed between two pieces of wood in a strong vice, the paper being renewed until it was no longer wetted.

of ammonia in excess, a light-red solution is produced, whereas the hypogallic-acid blue becomes blood-red with ammonia.

To determine the basicity of the acid, the ammonium salt was precipitated by nitrate of silver, which forms a gelatinous precipitate becoming crystalline on standing. It is soluble in hot water, from which it crystallizes on cooling.

This salt, dried at 100° and heated to redness, gave the following results:—

I. 0·521 grm. gave 0·185 grm. metallic silver.
II. 0·549 grm. gave 0·196 grm. metallic silver.
III. 0·544 grm. gave 0·194 grm. metallic silver.

| | Calculated. | | Found. | | |
|---|---|---|---|---|---|
| | | | I. | II. | III. |
| $C_9H_7O_5$ . . | 195 | 64·36 | — | — | — |
| Ag . . . . | 108 | 35·64 | 35·51 | 35·76 | 35·66 |
| $C_9H_7AgO_5$ . | 303 | 100·00 | | | |

Salts I. and II. were made from the acid obtained from the action of hydrochloric acid, and III. from that obtained by the action of hydriodic acid.

It therefore appears that methyl-noropianic acid is monobasic. The fact that opianic acid when heated with excess of strong caustic potash splits up into meconin and hemipinic acid, leads us to hope that methyl-noropianic acid would with the same reagent undergo a similar decomposition. It was, however, found that the acid remains unaltered; for after treating it with strong caustic potash, a silver-salt was made, which yielded on ignition 35·5 per cent. silver, methyl-noropianate of silver requiring 35·64 per cent.

The reduction of opianic acid to meconin by the action of sodium-amalgam caused us to try this reagent on the new acid, but here again we could not produce the corresponding reduction.

When methyl-noropianic acid is dissolved in cold water, and about a sixth of its volume of strong nitric acid added, an action immediately sets up, and the solution becomes dark from nitric oxide and afterwards again light, when a nitro-acid crystallizes out. To prevent the action going on too far the solution must be kept cold.

This new nitro-acid contains

$$C_9H_7NO_7 = C_9H_7(NO_2)O_5.$$

We have called this acid nitromethyl-noropianic acid.

Dried at 100° C. it gave the following results:—

0·451 grm. gave 0·736 grm. carbonic acid and 0·127 grm. water.

| | Calculated. | | Found. |
|---|---|---|---|
| $C_9$ . . . . | 108 | 44·81 | 44·51 |
| $H_7$ . . . . | 7 | 2·90 | 3·13 |
| N . . . . | 14 | 5·81 | —— |
| $O_7$ . . . . | 112 | 46·48 | —— |
| $C_9H_7NO_7$ . . | 241 | 100·00 | |

Nitromethyl-noropianic acid crystallizes with one molecule of water, which it loses at 100° C.

1·62 grm. lost 0·115 grm. at 100° C.

| | Calculated. | | Found. |
|---|---|---|---|
| $C_9H_7NO_7$ . . . | 241 | 93·05 | —— |
| $H_2O$ . . . . . | 18 | 6·95 | 7·09 |
| $C_9H_7NO_7, H_2O$ . | 259 | 100·00 | |

2. *Action of Hydrochloric and Hydriodic Acids on Meconin.*

When meconin is treated with hydrochloric or hydriodic acid as above described in the case of opianic acid, it is resolved into chloride or iodide of methyl and a new compound containing

$$C_9H_8O_4.$$

The reaction which takes place is

$$C_{10}H_{10}O_4 + HCl = C_9H_8O_4 + CH_3Cl.$$

This substance may be regarded as a monomethylized derivative of a hypothetical normal meconine,

$$C_8H_6O_4,$$

and we therefore propose to name it *monomethyl-normal meconin*, or shorter, *methyl-normeconin*.

It gave the following results on analysis:—

I. 0·4623 grm. gave 1·0152 grm. carbonic acid and 0·1904 grm. water.

II. 0·4300 grm. gave ·9105 grm. carbonic acid and 0·1775 grm. water.

| | Calculated. | | Found. | |
|---|---|---|---|---|
| | | | I. | II. |
| $C_9$ . . . . . | 108 | 60·00 | 59·89 | 59·65 |
| $H_8$ . . . . . | 8 | 4·44 | 4·58 | 4·59 |
| $O_4$ . . . . . | 64 | 35·56 | —— | —— |
| $C_9H_8O_4$ . . . | 180 | 100·00 | | |

Methyl-normeconin crystallizes without any water of crystallization; it is soluble in cold, but much more so in hot water; it is easily soluble in alcohol, and slightly so in ether.

With perchloride of iron it behaves exactly in the same manner as methyl-nor-

opianic acid. It reduces salts of silver in the cold, so that to determine its basicity we employed its barium-salt, from the analysis of which it appears that this new acid is monobasic.

0·240 grm. gave 0·113 sulphate of barium.

| | Calculated. | | Found. |
|---|---|---|---|
| $2(C_9H_7O_4)$ . . . . | 358 | 72·32 | —— |
| Ba . . . . . | 137 | 27 68 | 27·50 |

3. *Action of Hydrochloric and Hydriodic Acids on Hemipinic Acid.*

In our former paper we have (p. 355) described the action of hydriodic acid on hemipinic acid. We there stated that when hemipinic acid is treated with hydriodic acid the following reaction takes place:—

$$C_{10}H_{10}O_6 + 2HI = CO_2 + 2CH_3I + C_7H_6O_4.$$

The acid $C_7H_6O_4$ we called hypogallic acid.

We also mentioned (p. 359) that when hemipinic acid is heated with strong hydrochloric acid, the reaction is

$$C_{10}H_{10}O_6 + HCl = CO_2 + CH_3Cl + C_8H_8O_4.$$

The further investigation and analyses confirm this formula,

$$C_8H_8O_4,$$

for this acid; and as it contains one molecule of methyl more than hypogallic acid, and may be converted into that body by the prolonged action of hydrochloric acid, it may be called *methyl-hypogallic acid.*

One of the simplest modes of preparing this acid is to digest the hemipinic acid with strong hydrochloric acid on a water-bath at 100° C. under a layer of paraffin for about three days. The purification of the acid is very simple, owing to its being almost insoluble in cold and sparingly soluble in hot water, whence it crystallizes out on cooling in long transparent prisms. The crystals contain no water of crystallization.

I. 0·4020 grm. gave 0·8460 grm. carbonic acid and 0·1786 grm. water.

II. 0·5140 grm. gave 1·0700 grm. carbonic acid and 0·2276 grm. water.

| | Calculated. | | Found. | |
|---|---|---|---|---|
| | | | I. | II. |
| $C_8$ . . . . . | 96 | 57·14 | 57·39 | 56·79 |
| $H_8$ . . . . . | 8 | 4·76 | 4·93 | 4·92 |
| $O_4$ . . . . . | 64 | 38·10 | —— | —— |
| $C_8H_8O_4$ . . . | 168 | 100·00 | | |

The basicity of the acid was determined by precipitating the ammonium salt with nitrate of silver; the silver-salt is white, crystalline, rather insoluble in cold, soluble in hot, and decomposes in boiling water.

0·3596 grm. silver-salt gave 0·1408 metallic silver.

| | Calculated. | | Found. |
|---|---|---|---|
| $C_8H_7O_4$ . . . . | 167 | 60·73 | —— |
| Ag . . . . . | 108 | 39·27 | 39·15 |
| $C_8H_7AgO_4$ . . . | 275 | 100·00 | |

The properties of this acid are given in our former communication (p. 358).

When methyl-hypogallic acid is treated with dilute nitric acid (1 part acid to 3 parts water) and gently heated till the acid is dissolved, it is converted into a nitro-acid which separates out on cooling. Its composition was found to be

$$C_8H_6N_2O_8 = C_8H_6(NO_2)_2O_4.$$

I. 0·5760 grm. gave 0·7850 carbonic acid and 0·1216 grm. water.

II. A nitrogen determination made by LIEBIG's method gave the ratio of carbonic acid to nitrogen as 8·07 to 1.

| | Calculated. | | Found. | |
|---|---|---|---|---|
| | | | I. | II. |
| $C_8$ . . . . . | 96 | 37·21 | 37·17 | —— |
| $H_6$ . . . . . | 6 | 2·33 | 2·35 | —— |
| $N_2$ . . . . . | 28 | 10·85 | —— | 10·74 |
| $O_8$ . . . . . | 128 | 49·61 | —— | —— |
| $C_8H_6N_2O_8$ . . | 258 | 100·00 | | |

This acid may be called dinitromethyl-hypogallic acid, as it contains $(NO_2)_2$ in place of $H_2$.

It crystallizes with one molecule of water.

2·133 grms. of the crystallized acid lost at 100° C. 0·138 grm. water.

| | Calculated. | | Found. |
|---|---|---|---|
| $C_8H_6N_2O_8$ . . . . | 258 | 93·48 | —— |
| $H_2O$ . . . . . | 18 | 6·52 | 6·47 |
| $C_8H_6N_2O_8, H_2O$ . | 276 | 100·00 | |

4. *On the different crystalline Forms of Hemipinic Acid.*

Whilst experimenting with hemipinic acid we found that this acid may crystallize in different forms. The crystals were found to contain different amounts of water; thus (I.) when crystallized from a dilute solution by spontaneous evaporation, the crystals contain half a molecule of water; (II.) when from a supersaturated solution, they contain one molecule; and lastly, (III.) when crystallized in the ordinary way by cooling a hot solution, they contain two molecules.

I. 11·281 grms. acid lost at 100° C. 0·448 grm. water.
II. 3·130 grms. acid lost at 100° C. 0·230 grm. water.
III. 7·646 grms. acid lost at 100° C. 1·0576 grm. water.

| | Calculated. Water per cent. | Found. Water per cent. |
|---|---|---|
| I. $C_{10}H_{10}O_6$, $HO_j$ . . | 3·83 | 3·97 |
| II. $C_{10}H_{10}O_6$, $H_2O$ . . | 7·33 | 7·35 |
| III. $C_{10}H_{10}O_6$, $2H_2O$ . . | 13·74 | 13·83 |

In the following Table the acids &c. are tabulated which have been and probably may be prepared from opianic acid.

| | | |
|---|---|---|
| $C_{10}H_{10}O_4$ | $C_{10}H_{10}O_5$ | $C_{10}H_{10}O_6$ |
| 1. Dimethyl-normeconin (ordinary meconin). | 2. Dimethyl-noropianic acid (ordinary opianic acid). | 3. Dimethyl-norhemipinic acid (ordinary hemipinic acid). |
| $C_9H_8O_4$ | $C_9H_8O_5$ | $C_9H_8O_6$ |
| 4. Methyl-normeconin. | 5. Methyl-noropianic acid. | Methyl-norhemipinic acid. |
| $C_8H_6O_4$ | $C_8H_6O_5$ | $C_8H_6O_6$ |
| Normeconin. | Noropianic acid. | Norhemipinic acid. |
| $C_8H_8O_4$ | | |
| 6. Methyl-hypogallic acid. | | |
| $C_7H_6O_4$ | | |
| 7. Hypogallic acid. | | |

Of the above, the following have been made, namely:—

1 and 3. $C_{10}H_{10}O_4$ and $C_{10}H_{10}O_6$ by the action of potash on opianic acid; thus,

$$2C_{10}H_{10}O_5 = C_{10}H_{10}O_4 + C_{10}H_{10}O_6.$$

4. $C_9H_8O_4$ by the action of hydrochloric and hydriodic acids on meconin; thus,

$$C_{10}H_{10}O_4 + HI = C_9H_8O_4 + CH_3I.$$

5. $C_9H_8O_5$ by the action of hydrochloric and hydriodic acids on opianic acid; thus,

$$C_{10}H_{10}O_5 + HI = C_9H_8O_5 + CH_3I.$$

6. $C_8H_8O_4$ by the action of hydrochloric on hemipinic acid; thus,

$$C_{10}H_{10}O_6 + HCl = C_8H_8O_4 + CH_3Cl + CO_2.$$

7. $C_7H_6O_4$ by the action of hydriodic acid on hemipinic acid; thus,

$$C_{10}H_{10}O_6 + 2HI = C_7H_6O_4 + CO_2 + 2CH_3I.$$

### 5. *Action of Hydrochloric and Hydriodic Acids on Narcotine.*

When narcotine is treated with strong hydrochloric acid for some time on a water-bath, in a flask under a layer of paraffin, a thick oily mass gradually separates out on cooling, which on examination was found to be the chloride of a new base. The best method of preparing this base is as follows:—

200 grms. narcotine are put into a large flask with 1000 cub. centims. strong hydrochloric acid (the pure commercial acid), and are digested together on a water-bath

under a layer of paraffin at 100° C. Much chloride of methyl is given off, and a thick oily mass separates out on cooling, and when no further quantity is formed the reaction may be considered finished*.

The reaction which takes place may be written thus—

$$C_{22}H_{23}NO_7 + 2HCl = C_{20}H_{19}NO_7 + 2CH_3Cl.$$

To purify the chloride, and to obtain the base from it, advantage is taken of the fact that it is comparatively insoluble in dilute hydrochloric acid, whereas in strong hydrochloric acid as well as in pure water it dissolves readily. After the reaction is finished the contents of the flask are allowed to cool. The liquid portion (strongly acid) is poured into a large beaker, and the oily mass dissolved in hot water, allowed to cool, and then poured into the strongly acid solution. This causes a precipitate, and water or hydrochloric acid is added in case either produce a further precipitate. The precipitated chloride is collected on a filter and washed with dilute hydrochloric acid (1 part acid, 9 parts water); after washing, the precipitate is dissolved in water, and carbonate of sodium added in excess, in which the new base is soluble, but narcotine insoluble; after filtering off any undecomposed narcotine, the solution is carefully neutralized with hydrochloric acid to precipitate the base, which becomes curdy on heating and may be filtered and washed with ease. After being well washed it is redissolved in hydrochloric acid, and fractionally precipitated with carbonate of sodium. The first portion precipitated contained most of the colouring-matter; the second portion was used for analysis. Dried at 100° C. in a LIEBIG'S drying tube, it gave the following results:—

I. 0·3026 grm. gave 0·6766 carbonic acid and 0·1354 grm. water.
II. 0·3400 grm. gave 0·7488 carbonic acid and 0·1519 grm. water.
III. 0·3698 grm. gave 0·8198 carbonic acid and 0·1594 grm. water.
IV. 0·4290 grm. gave 0·9535 carbonic acid and 0·1990 grm. water.
V. 0·3660 grm. gave 0·8170 carbonic acid and 0·1690 grm. water.
VI. 0·5168 grm. gave 0·1218 platinum.
VII. 0·6024 grm. gave 0·1422 platinum.

| | Calculated. | | Found. | | | | | | | Mean. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | I. | II. | III. | IV. | V. | VI. | VII. | |
| $C_{20}$ | 240 | 60·91 | 60·98 | 60·06 | 60·46 | 60·62 | 60·88 | —— | —— | 60·60 |
| $H_{20}$ | 20 | 5·08 | 4·97 | 4·96 | 4·79 | 5·15 | 5·13 | —— | —— | 5·00 |
| N | 14 | 3·55 | —— | —— | —— | —— | —— | 3·34 | 3·35 | 3·35 |
| $O_{7\frac{1}{2}}$ | 120 | 30·46 | —— | —— | —— | —— | —— | —— | —— | —— |
| $C_{20}H_{19}NO_7, \frac{1}{2}$ aq. | 394 | 100·00 | | | | | | | | |

* From two to six days are required for the completion of the reaction; it appears that the larger the quantities employed the shorter the time necessary for conversion. In an experiment made with the above quantities, the whole of the narcotine was converted into the new base in three days (*i. e.* in about twenty-four hours), whereas in another experiment made with 50 grms. narcotine and 250 cub. centims. hydrochloric acid the time required was six days.

I. and II. were of the same preparation, III., IV., V. were each of different preparations.

For reasons which will be clear from what follows, we have called the base methylnornarcotine. When freshly precipitated it forms an almost white amorphous powder, insoluble in water and ether, slightly soluble in alcohol, and easily soluble in carbonate of sodium, by which means it may be separated from narcotine.

None of its salts form crystalline compounds (the chloride, sulphate, and nitrate have been tried).

On determining the amount of chlorine and sulphuric acid in the chloride and sulphate, the following results were obtained:—

1·731 grm. chloride gave 0·594 grm. chloride of silver.

| | Calculated. Chlorine per cent. | Found. |
|---|---|---|
| $C_{20}H_{19}NO_7$, HCl . . . | 8·42 | 8·48. |

0·4235 grm. sulphate gave 0·1132 grm. sulphate of barium.

| | Calculated. $H_2SO_4$ per cent. | Found. |
|---|---|---|
| $2(C_{20}H_{19}NO_7)SO_4$. . . | 11·31 | 11·22. |

The chloride was prepared as follows: the base was dissolved in strong hydrochloric acid and fractionally precipitated by water, the middle portion collected, washed with dilute hydrochloric acid, and dried over sulphuric acid and lime, before drying it in the water-bath.

The chloride must not be washed with pure water, as it converts it immediately into a sticky mass.

The sulphate was precipitated by dissolving the base in sulphuric acid (one part acid, three parts water) and pouring the solution into water, collecting the precipitated sulphate and washing with water, redissolving the precipitate in hot water, and collecting the sulphate as it separates out in different portions. Like the chloride it must be first dried over sulphuric acid, and then in a water-bath.

It may be as well to mention that in the further prosecution of this research, one of us has already obtained two more bases from narcotine. The one by digesting it for a short time with hydrochloric acid, and the other as indicated in our former paper (p. 303), by the action of hydriodic acid on it. The reactions may be written—

I. $C_{22}H_{23}NO_7 + HCl = C_{21}H_{21}NO_7 + CH_3Cl$.

II. $C_{22}H_{23}NO_7 + 2HCl = C_{20}H_{19}NO_7 + 2CH_3Cl$.

III. $C_{22}H_{23}NO_7 + 3HI = C_{19}H_{17}NO_7 + 3CH_3I$.

The preparations and properties of two of these bases (I., III.) will form the subject of a future communication.

We propose to call them as follows:—

I. $C_{19}H_{17}NO_7$, nornarcotine.
II. $C_{20}H_{19}NO_7$, methyl-nornarcotine.
III. $C_{21}H_{21}NO_7$, dimethyl-nornarcotine
IV. $C_{22}H_{23}NO_7$, trimethyl-nornarcotine (or ordinary narcotine).

It need hardly be mentioned that endeavours will be made to reform ordinary narcotine from the above derivatives, and to make ethylated narcotine, as the decomposition of this substance may lead to the formation of ethyl-opianic acid, and a series of acids homologous to those described in this paper.

In conclusion we have much pleasure in thanking Messrs. Macfarlane and Co. of Edinburgh, for their liberality in presenting us with a large quantity of pure narcotine with which the experiments above described were carried out.

6. *On the Crystalline Forms of some of the abovementioned substance.*

By Professor Victor v. Lang.

(*a*) Acid derived from meconin, $C_9H_8O_4$.

System monoclinic:—

$$a:b=2{\cdot}7864:1,$$
$$ac=127°\ 56'.$$

Observed forms:—

(1 1 0), (0 0 1).

110, 110 = 49° 56′ observed.
110, 001 = 78 20 „

The observed forms do not completely determine the crystallographic elements. The crystals are perfectly cleavable, parallel to the plane (0 0 1).

(*b*) Acid derived by the actionof nitric acid on the hypogallic acid, $C_9H_6N_2O_8$, $H_2O$.

System monoclinic:—

$$a:b:c=1{\cdot}0122:1:0{\cdot}7156,$$
$$ac=104°\ 0'.$$

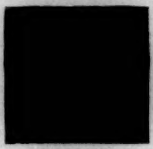
Observed forms:—

(0 0 1), (1 1 0), ($\bar{1}$ 1 1).

| | Calculated. | Observed. |
|---|---|---|
| | ° ′ | ° ′ |
| 110, 1$\bar{1}$0 = | 88 58 | |
| 110, $\bar{1}$10 = | 91 2 | •91 2 |
| 110, 001 = | 80 4 | •80 4 |
| $\bar{1}$11, 001 = | 49 40 | 49 38 |
| $\bar{1}$11, $\bar{1}\bar{1}$1 = | 65 40 | •65 40 |

Cleavage very perfect, parallel to the planes (1 1 0).

(*c*) Hemipinic acid, $C_{10}H_{10}O_6$, $HO_j$.



System monoclinic:—

$$a:b:c=2\cdot5210:1:2\cdot9597,$$
$$ac=92^\circ\ 40'.$$

Observed forms:—

(1 0 0), (0 0 1), (1 0 1), (1 1 2).

| | | Calculated. | Observed. |
|---|---|---|---|
| 1 0 0, 0 0 1 | = | 87° 20′ | 87° 30′ |
| 1 0 1, 1 0 0 | = | 39 18 | |
| 1 0 1, 0 0 1 | = | 48 2 | 48 2 |
| 1 1 2, 1 0 0 | = | 71 10 | |
| 1 1 2, $\bar{1}\,1\,\bar{2}$ | = | 79 20 | 79 20 |
| 1 1 2, 0 0 1 | = | 56 20 | 56 20 |
| 1 1 2, 1 0 1 | = | 52 42 | 52 42 |

Cleavage perfect, parallel (0 0 1).

(*d*) Hemipinic acid, $C_{10}H_{10}O_6$, $H_2O$.

System monoclinic:—

$$a:b:c=0\cdot5407:1:1\cdot2620,$$
$$ac=97^\circ\ 42'.$$

Observed forms:—

(0 0 1), (1 1 0), (0 1 1), ($\bar{2}$ 2 5), ($\bar{2}$ 2 7).

| | | Calculated. | Observed. |
|---|---|---|---|
| 1 1 0, 0 1 0 | = | 61° 49′ | *61° 49′ |
| 1 1 0, 1 $\bar{1}$ 0 | = | 56 22 | |
| 0 1 1, 0 1 0 | = | 38 40 | *38 40 |
| 0 1 1, 0 0 1 | = | 51 20 | |
| 1 1 0, 0 0 1 | = | 83 13 | |
| 1 1 0, 0 1 1 | = | 63 44 | 63 28 |
| $\bar{2}$ 2 5, 0 1 0 | = | 68 33 | |
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| $\bar{2}$ 2 7, 0 0 1 | = | 39 30 | |
| $\bar{2}$ 2 7, $\bar{1}$ 1 0 | = | 58 12 | 58 40 |
| $\bar{1}$ 1 0, 0 0 1 | = | 96 47 | *96 47 |

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POISSON (S. D.) Recherches sur le Mouvement des Projectiles dans l'Air. 4to. *Paris* 1839.

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———— Mémoire sur le Mouvement d'un Corps Solide. 4to. *Paris* 1834.

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RITT (G.) Problèmes d'Application de l'Algèbre à la Géométrie, avec les solutions développées. 2 parts. 8vo. *Paris* 1836–37.

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YOUNG (J. R.) On the Theory and Solution of Algebraical Equations. 12mo. *London* 1835.

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# Diurnal Inequality.

Fig. 1.
TIME.

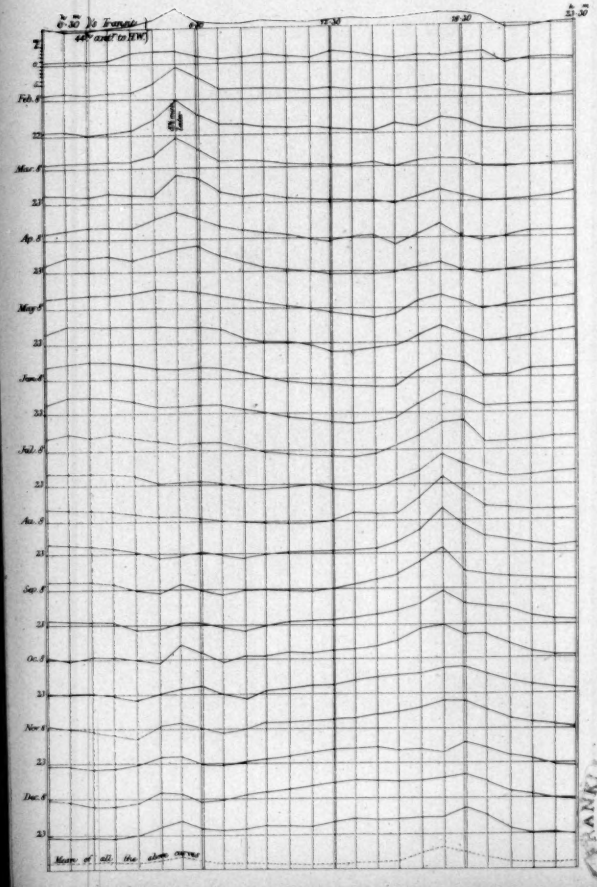


Fig. 2.
HEIGHT.

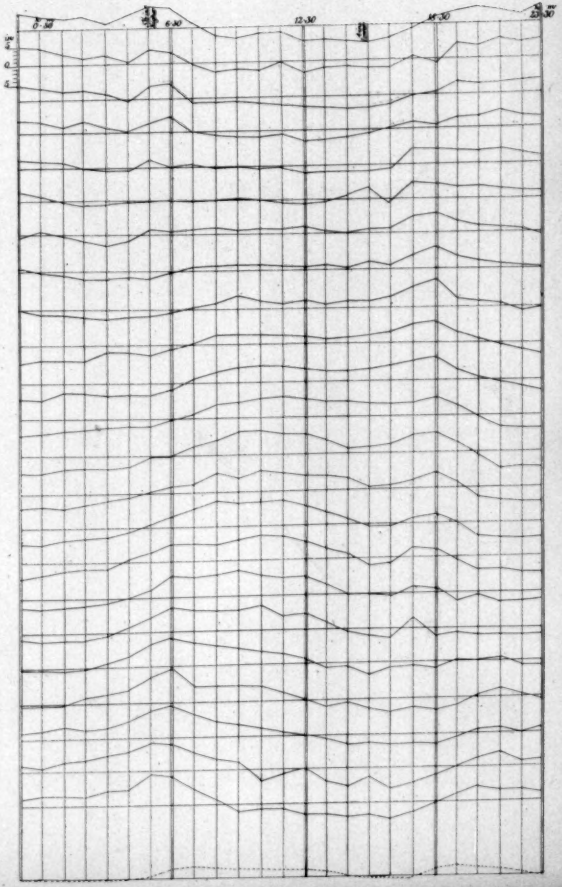


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Fig. 3.

*Solar Inequality.*

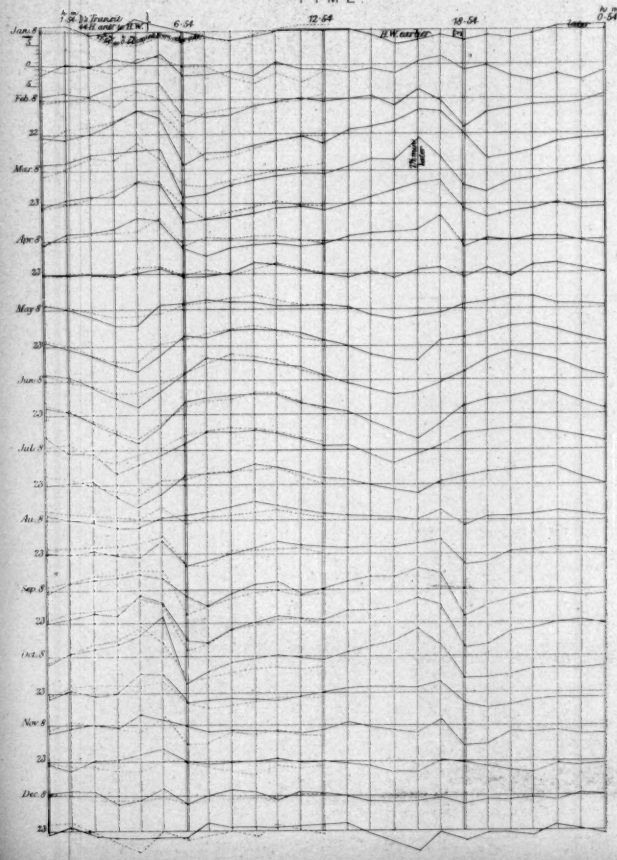


Fig. 4.

*Parallax.*

Fig. 5.

*Declination.*

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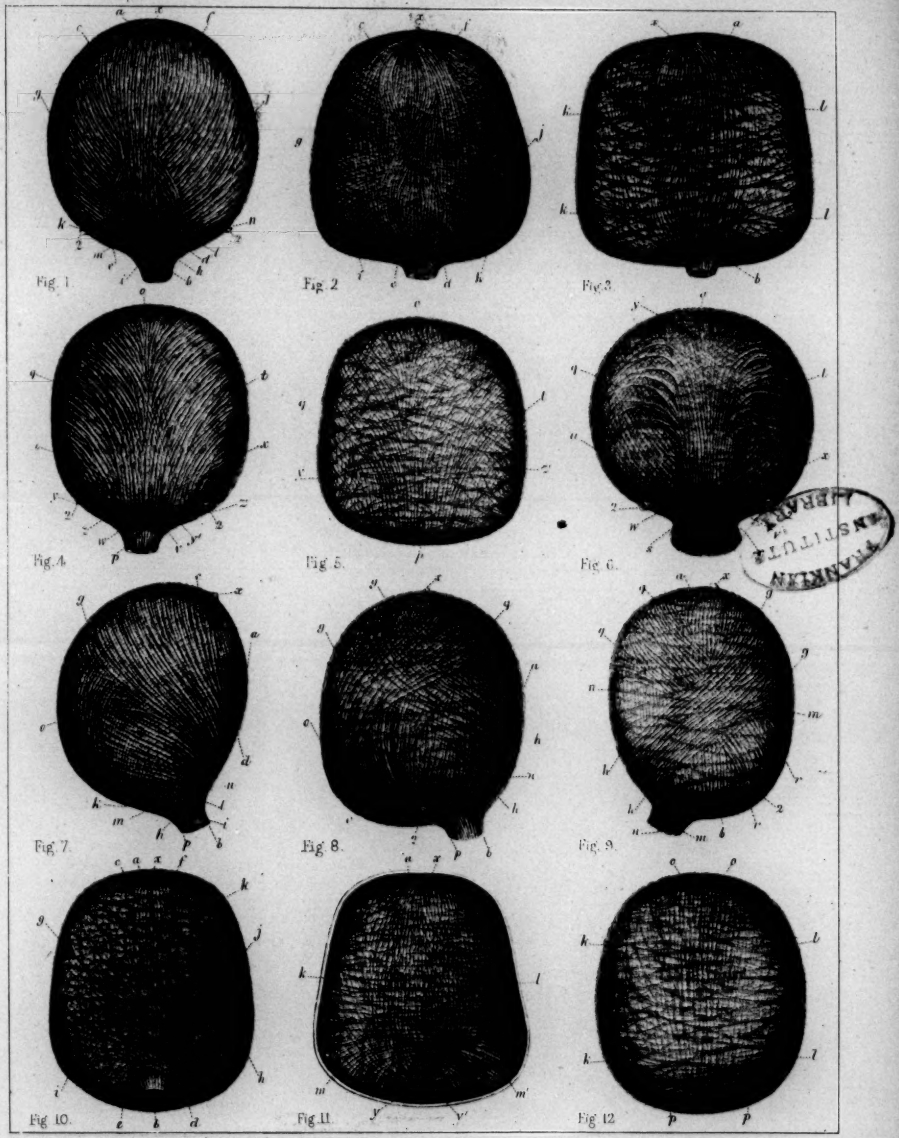

Fig. 1. Fig. 2. Fig. 3.

Fig. 4. Fig. 5. Fig. 6.

Fig. 7. Fig. 8. Fig. 9.

Fig. 10. Fig. 11. Fig. 12.

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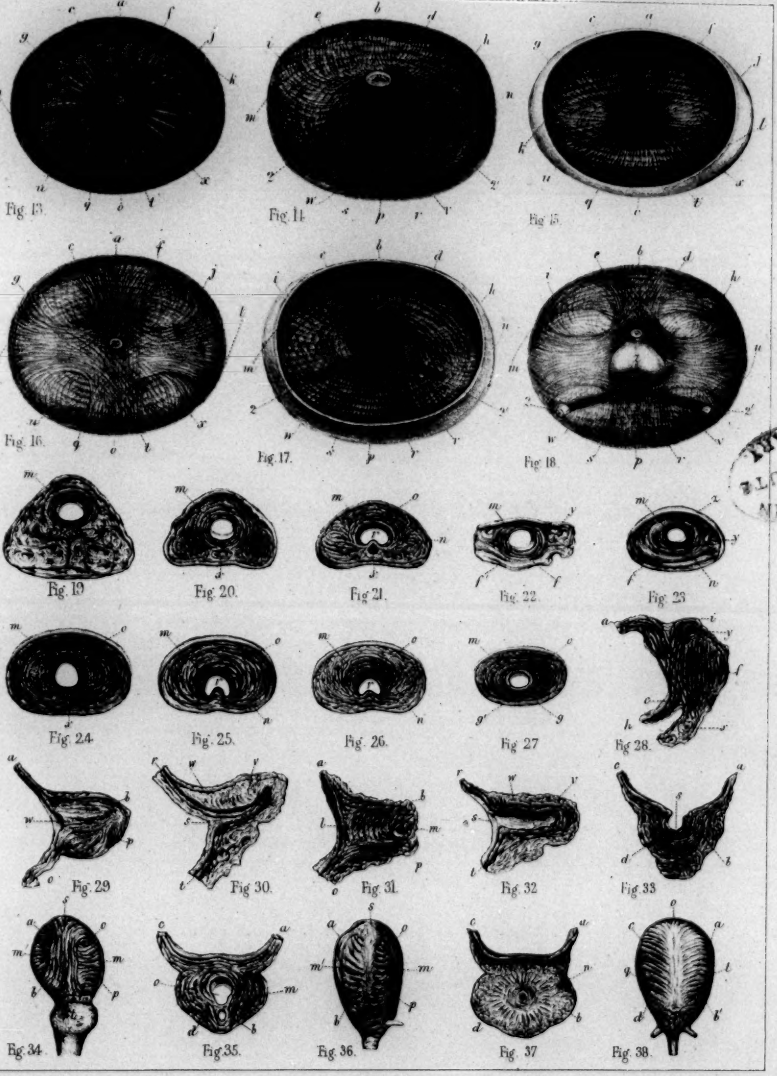

Fig. 13. Fig. 14. Fig. 15.

Fig. 16. Fig. 17. Fig. 18.

Fig. 19. Fig. 20. Fig. 21. Fig. 22. Fig. 23.

Fig. 24. Fig. 25. Fig. 26. Fig. 27. Fig. 28.

Fig. 29. Fig. 30. Fig. 31. Fig. 32. Fig. 33.

Fig. 34. Fig. 35. Fig. 36. Fig. 37. Fig. 38.

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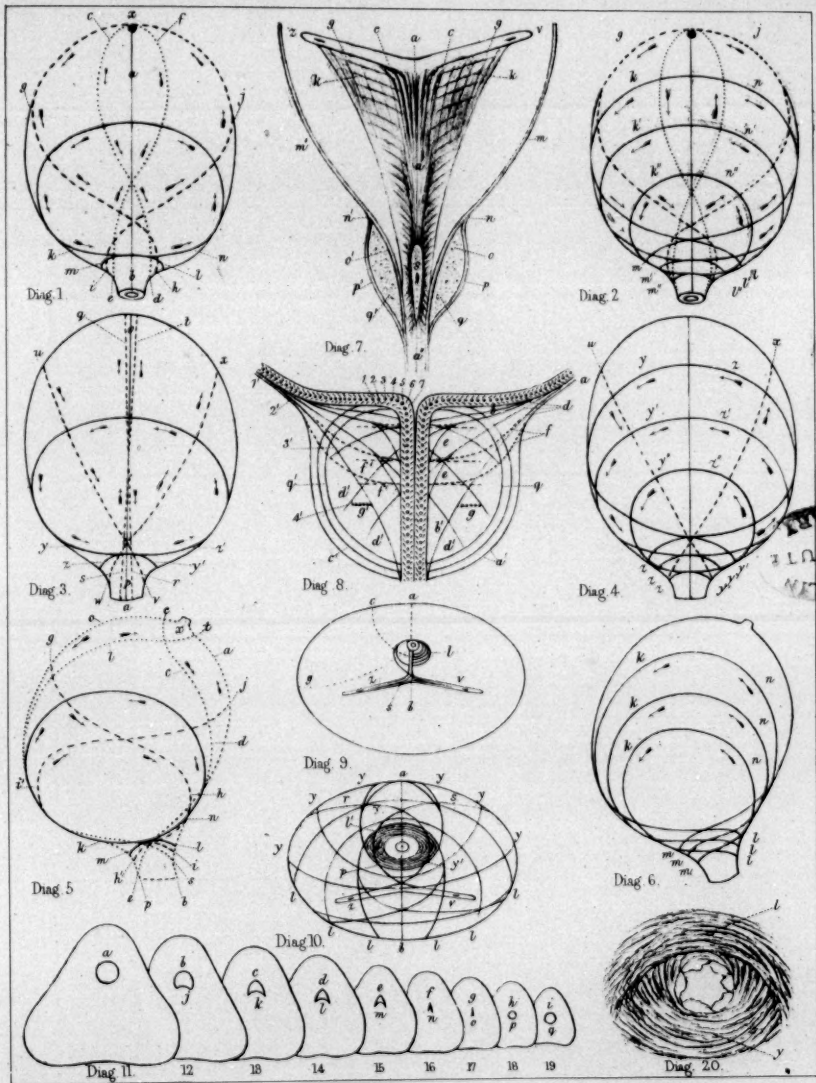


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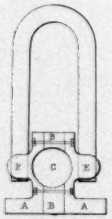
Fig. 1.

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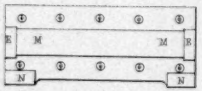


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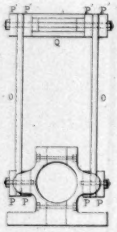


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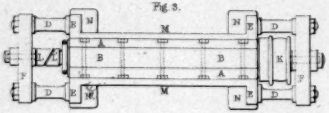



Fig. 5.

Fig. 4.

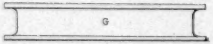
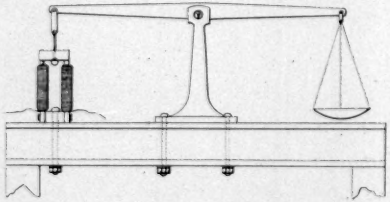



Fig. 7.

Fig. 6.

Fig. 8.

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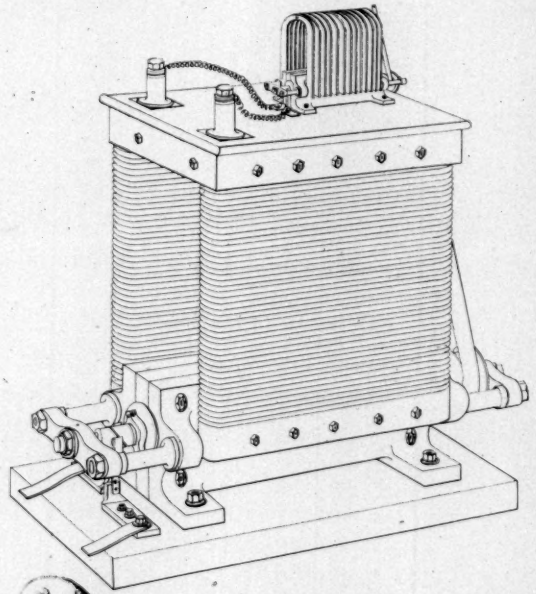

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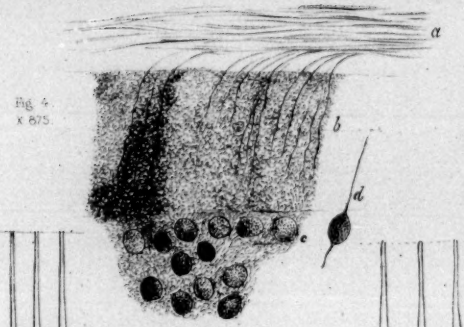
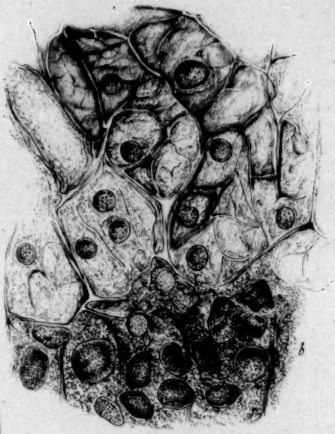



Fig. 4
x 875.

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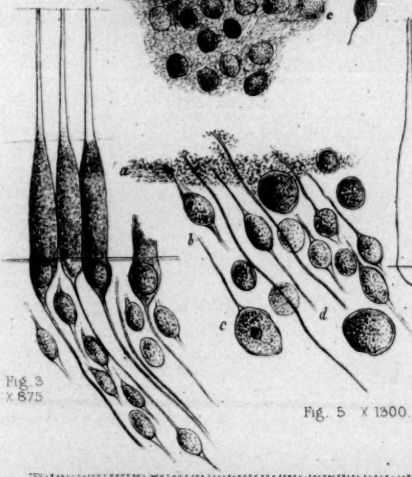


Fig. 3
x 875

Fig. 5 x 1300.

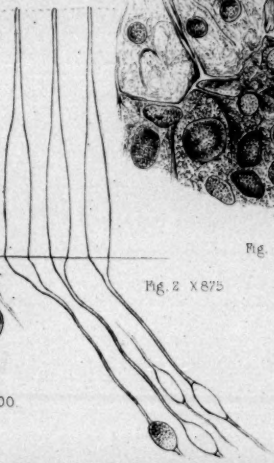

Fig. 2 x 875

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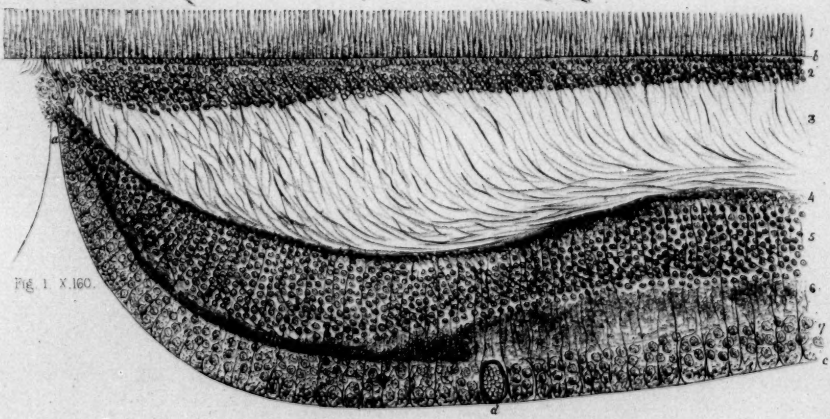


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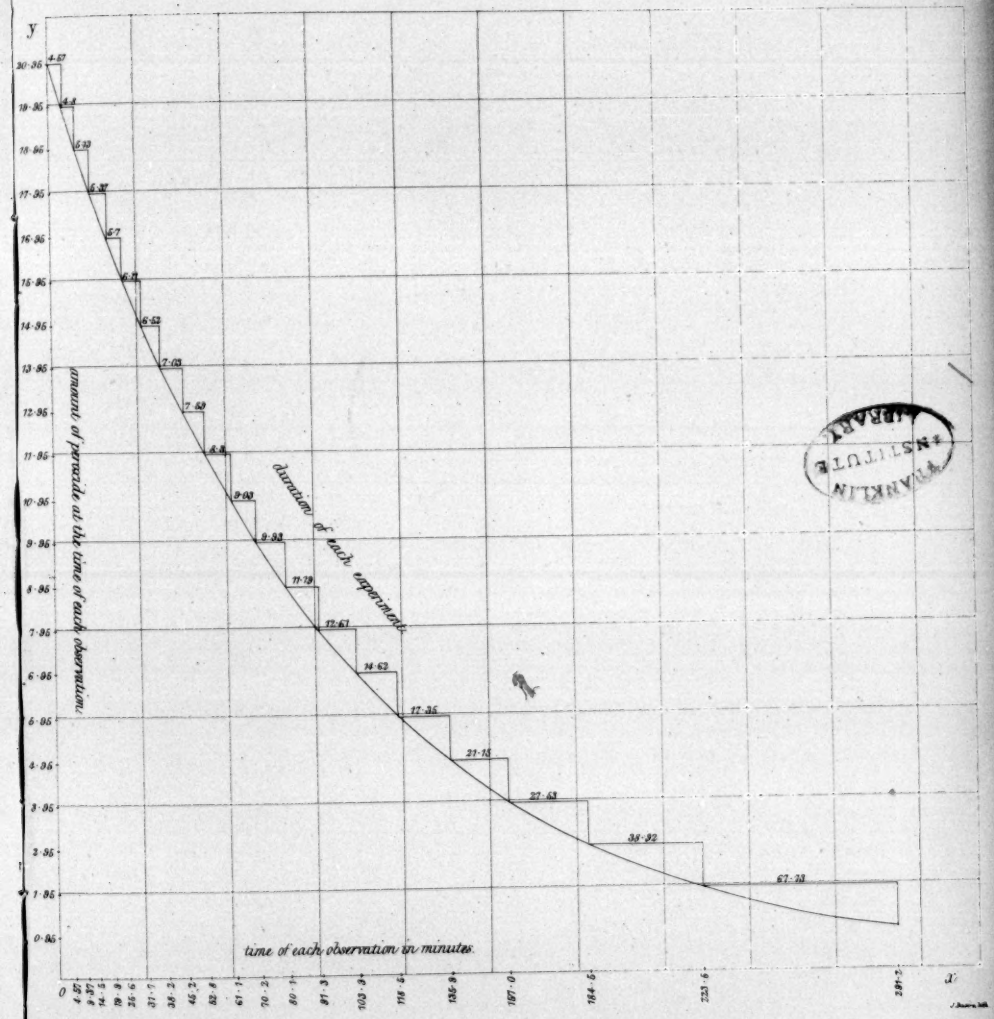


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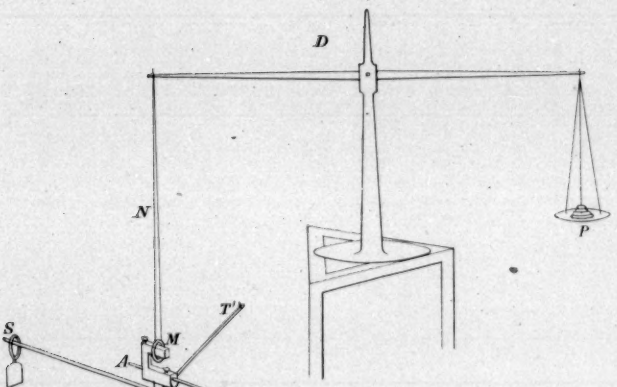


Fig 1.

Fig 5.

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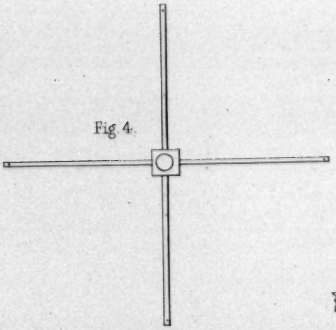

Fig 4.

Fig 3.

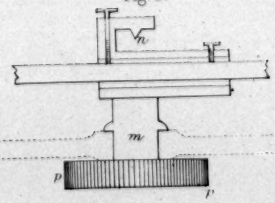

Fig 2.

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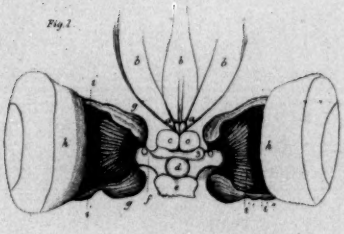

Fig. 2.

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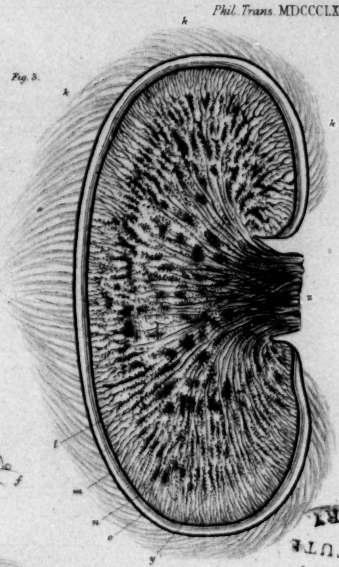

Fig. 10.

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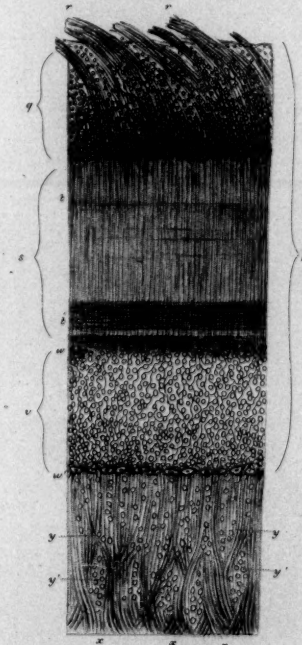

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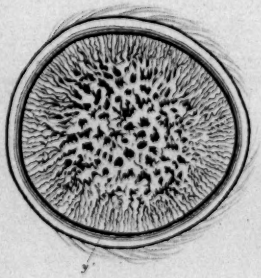

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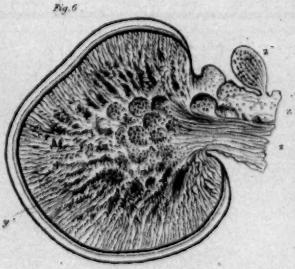


Fig. 7. × 60

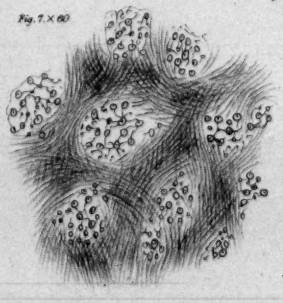
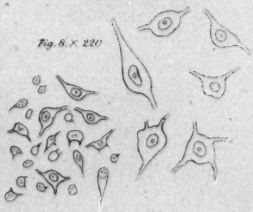


Fig. 8. × 220



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J. Basire sc.

Fig. 1

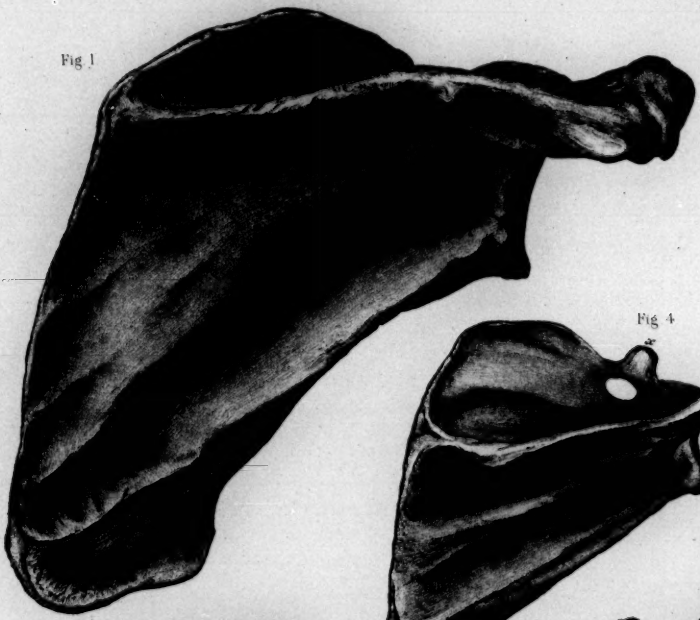

Fig. 4

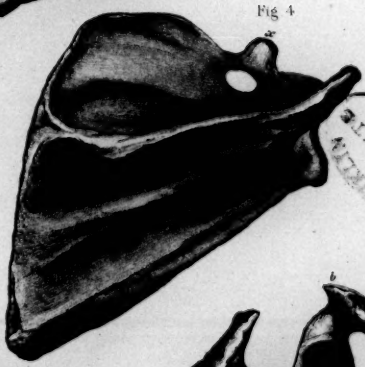

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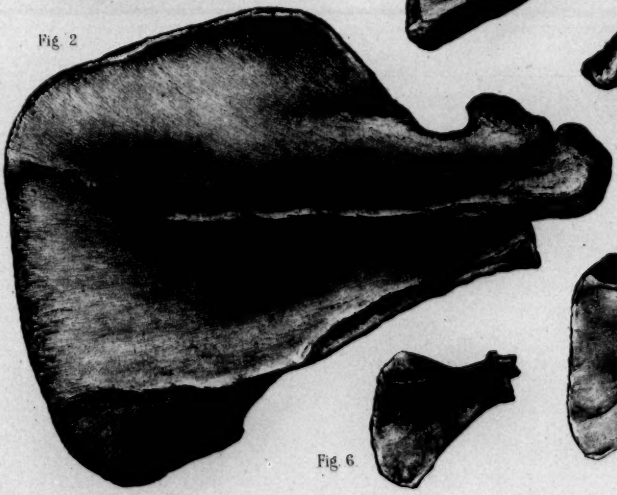
Fig. 3.

Fig. 2





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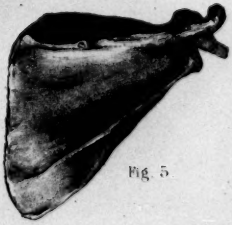

Fig. 5.

Fig. 1.

Fig. 2.

Fig. 3.

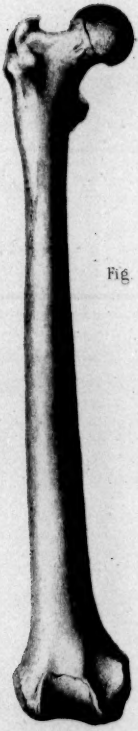

Fig. 4.

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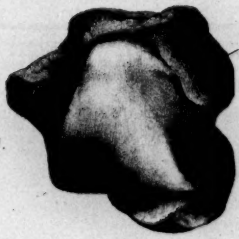


Fig. 6.

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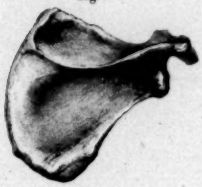

Fig. 2.

Fig. 3.

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Fig. 4.

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Fig. 1.

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Fig. 8.

Fig. 9.

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Fig. 13.

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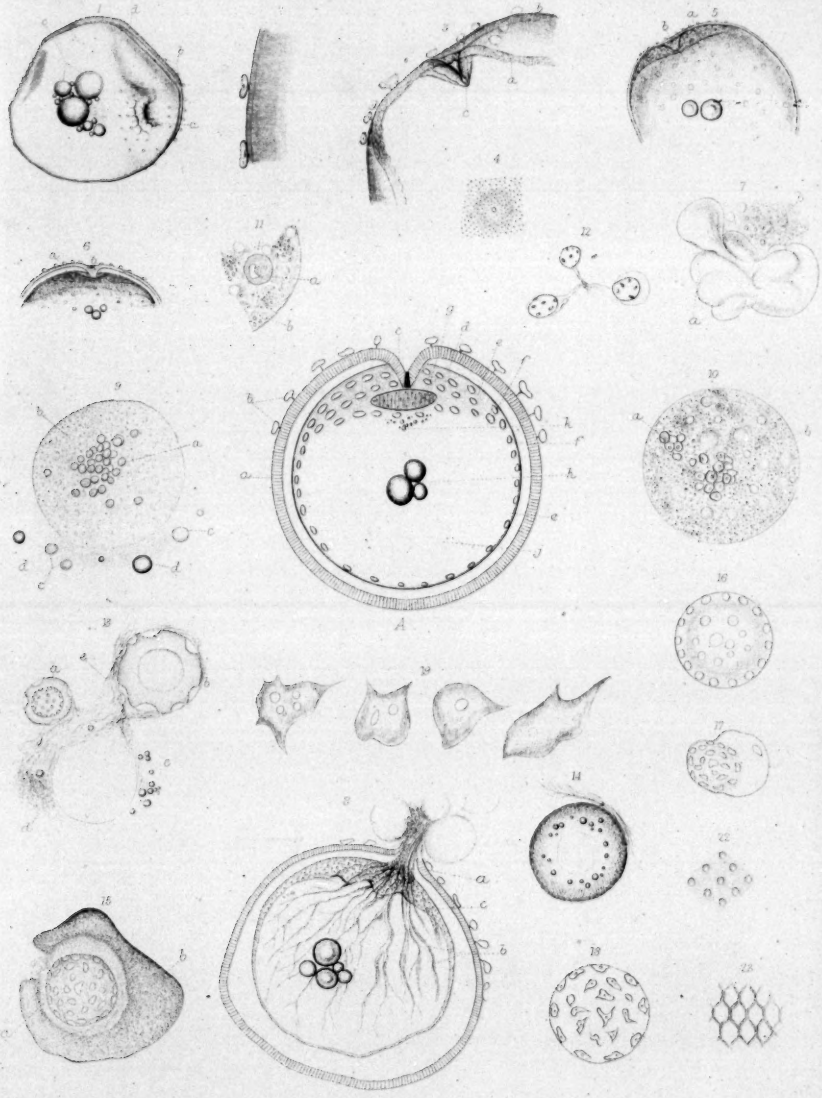
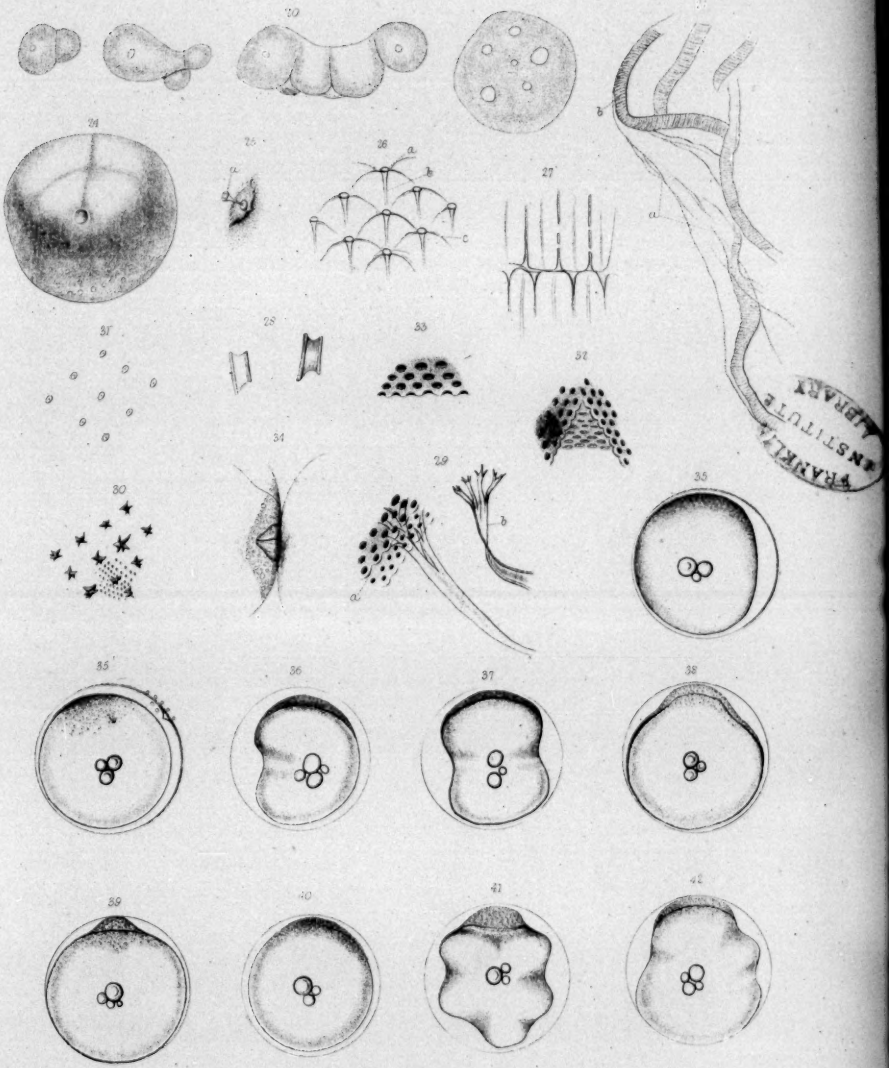
Fig. 18.



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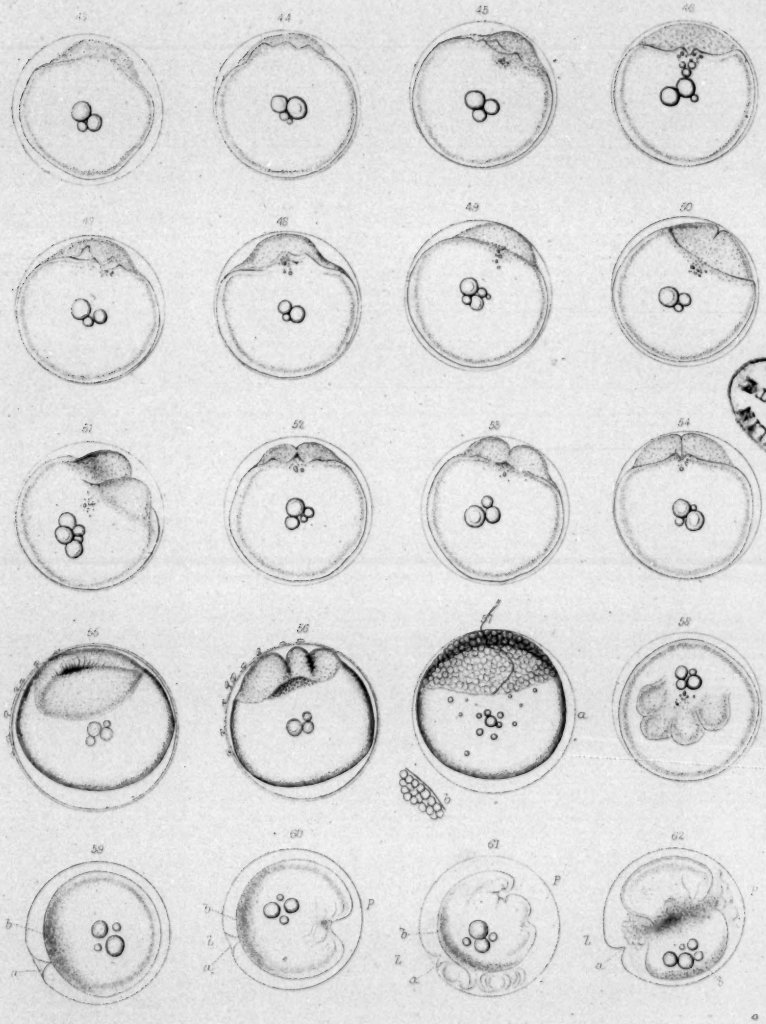
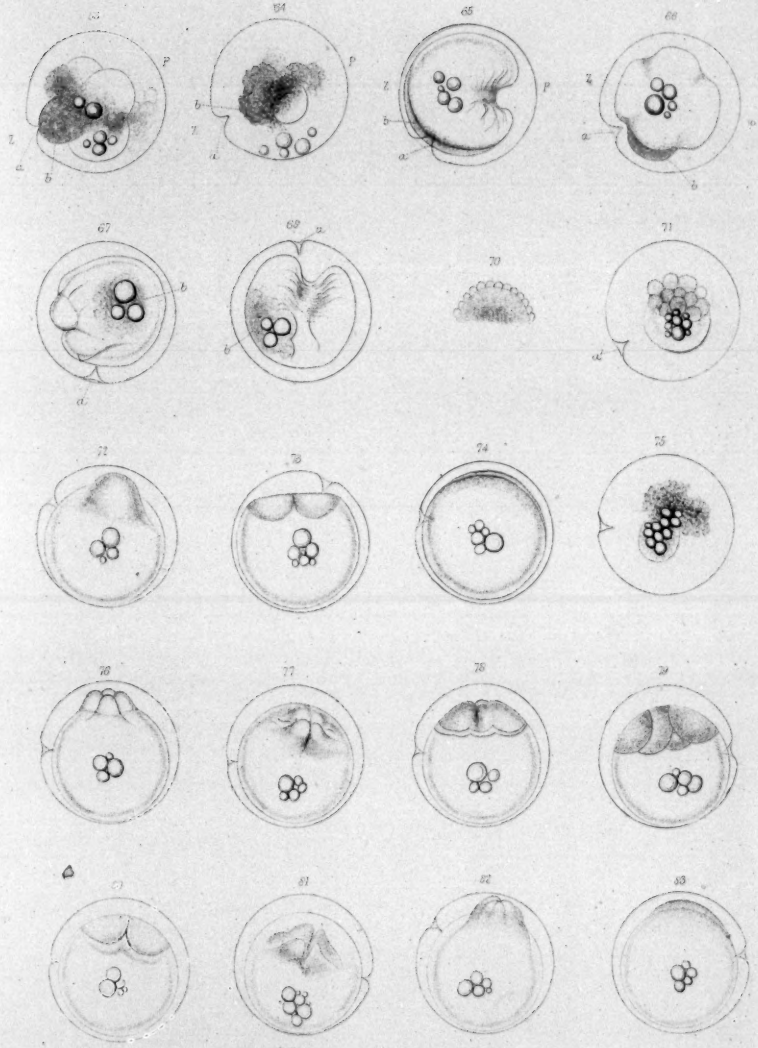



W.H. Evans lith.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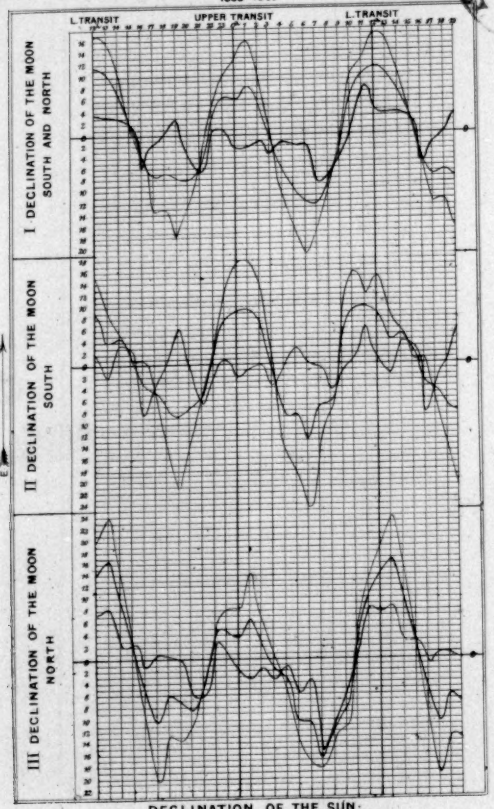
CURVES SHEWING THE
LUNAR DIURNAL VARIATION OF THE MAGNETIC DECLINATION
1858–1863.

TABLES II & III.

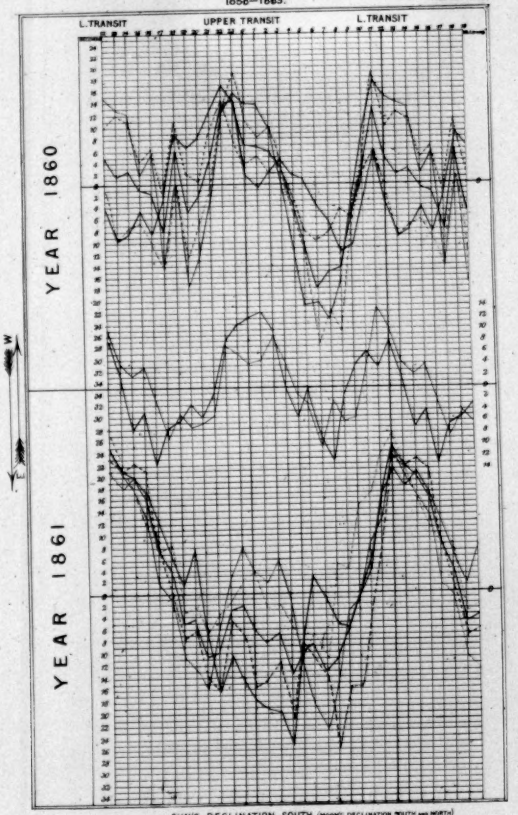



DECLINATION OF THE SUN:
SOUTH AND NORTH (YEAR)
SOUTH (OCTOBER TO MARCH)
NORTH (APRIL TO SEPTEMBER)

J. Basire, Lith.

CURVES SHEWING THE
LUNAR DIURNAL VARIATION OF THE MAGNETIC DECLINATION
1858–1863.

TABLES II & III.




SUN'S DECLINATION SOUTH (MOON'S DECLINATION SOUTH AND NORTH)
MOON'S DECLINATION SOUTH (SUN'S " " " " )
SUN'S DECLINATION NORTH (MOON'S " " " " )
MOON'S DECLINATION NORTH (SUN'S " " " " )
MEAN FOR THE YEAR 1858-59 (MAY TO APRIL) AND FOR THE YEARS 1860 AND 1861.
MEAN FOR THE YEAR 1860-61 (MAY TO APRIL). (in black)

Fig. 1.




Fig. 2.

*The lower cusp of the Bicorn is drawn out of its true position, which is much further off along the asymptote, the co-ordinates in fact are* $x=-76\frac{19}{20}$, $y=-41\frac{1}{2}$ *(the co-ordinates of the upper or node-cusp being −1, 1.*

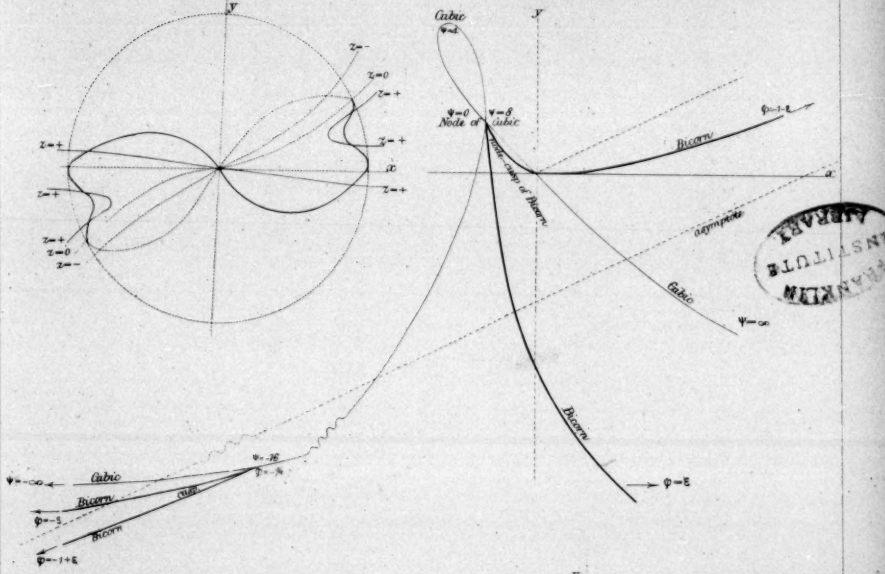



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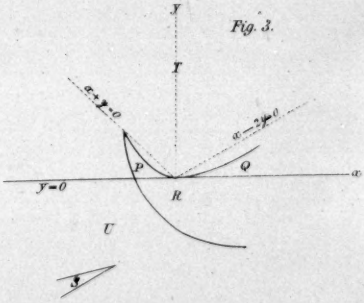
Fig. 3.

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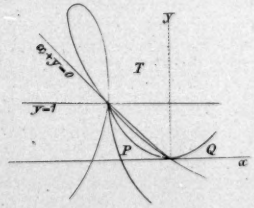


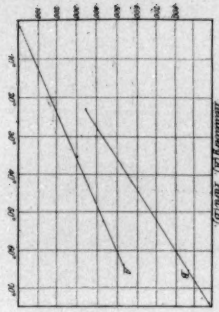



Fig. 1.

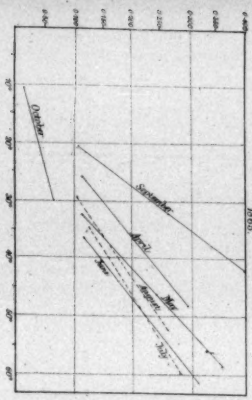



Fig. 2. 1866.

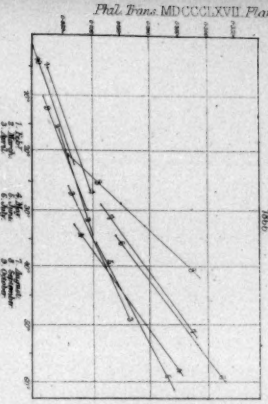


Fig. 3. 1866.

Fig. 9.
April 18th 1866.

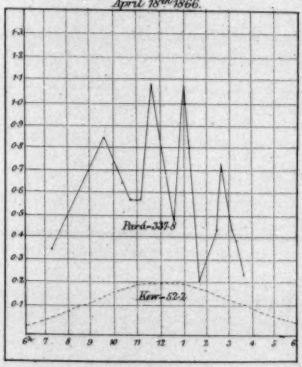



Fig. 10.
April 23rd 1866.

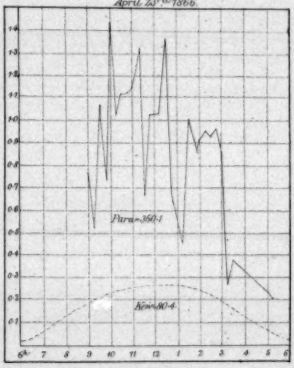



Fig. 11.
April 24th 1866.

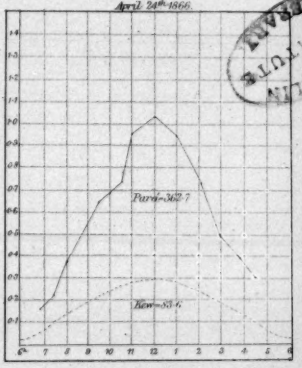



Fig. 12.
April 25th 1866.

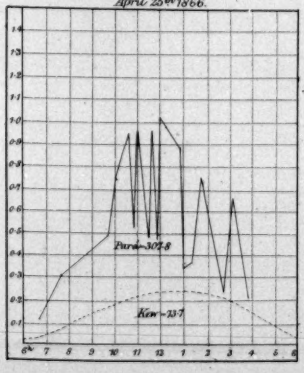



Fig. 13.
April 26th 1866.

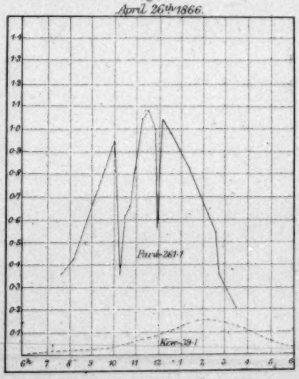
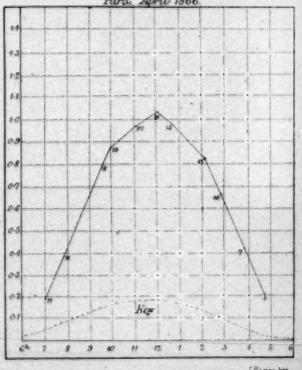



Fig. 14.
Curve showing March of Total Chemical Intensity for Sunshine.
Pard. April 1866.



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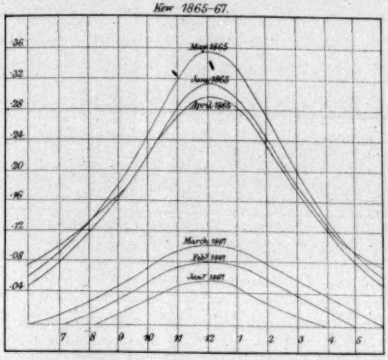
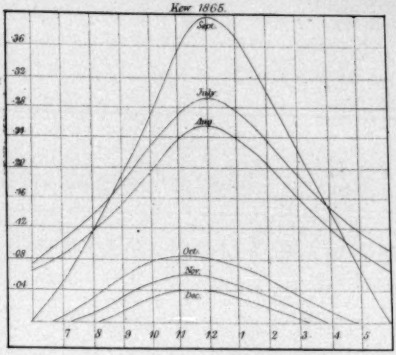



Fig. 5.

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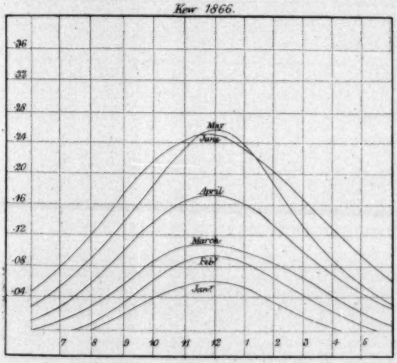


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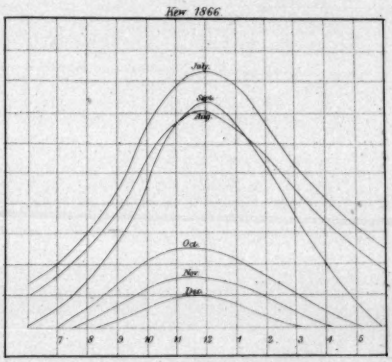


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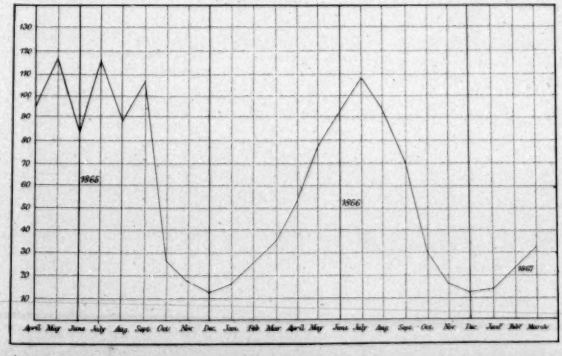
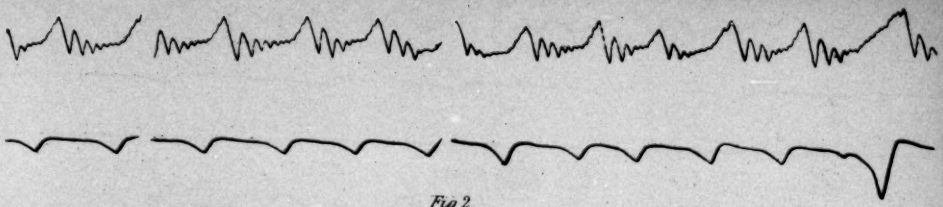



J. Basire lith.

Fig 1



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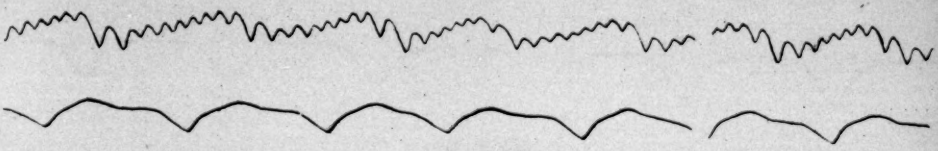

Fig 3.

Fig 4.

Fig 5.

Fig 6.

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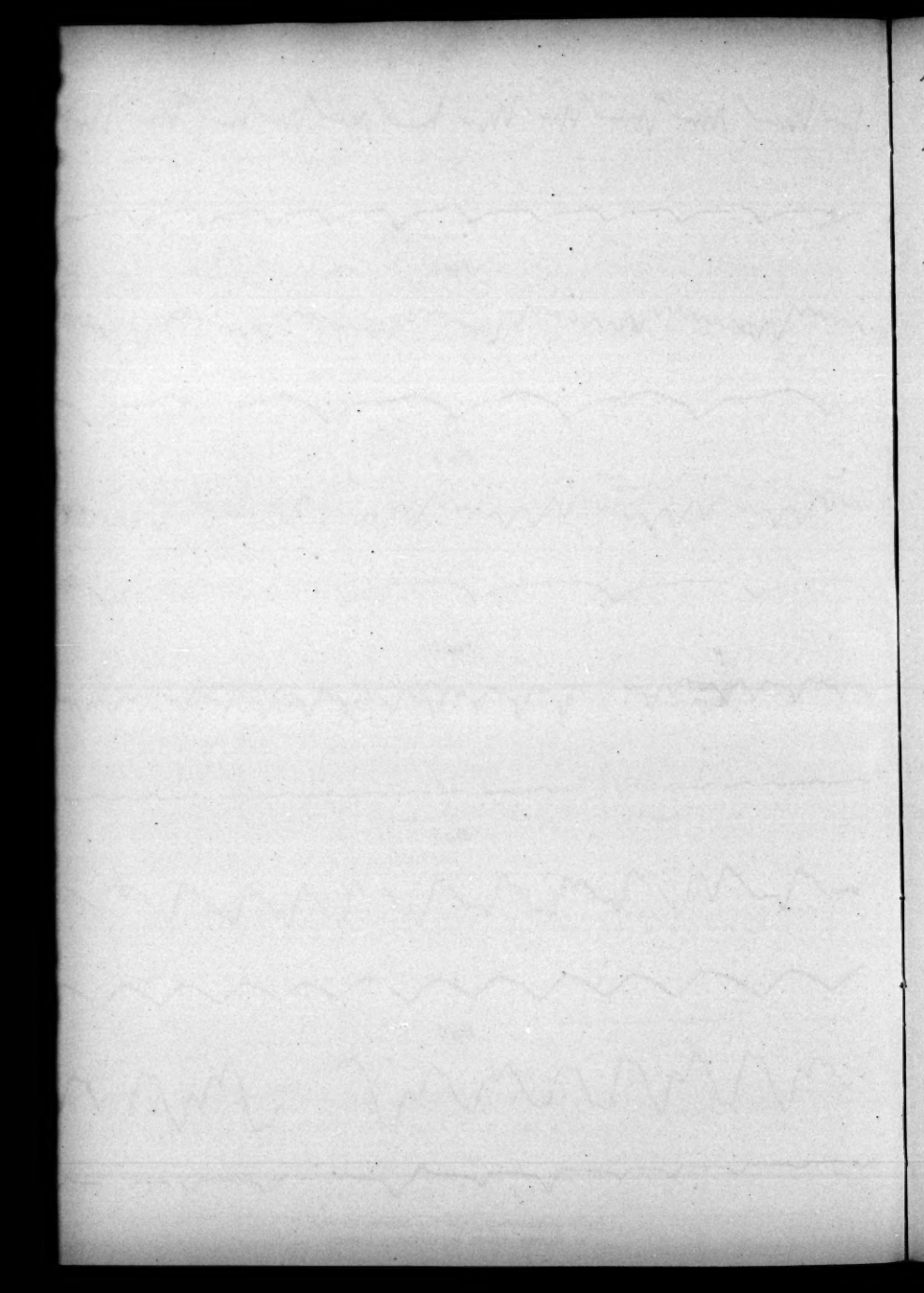
BY STEPHEN AYLING, 493, OXFORD ST. LONDON.



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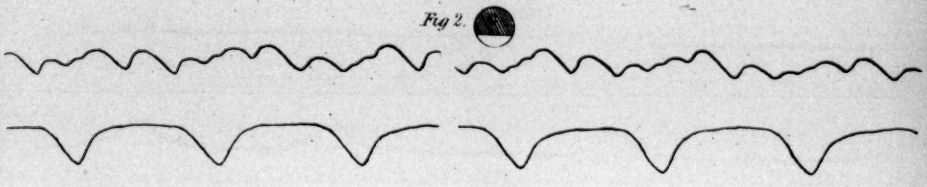

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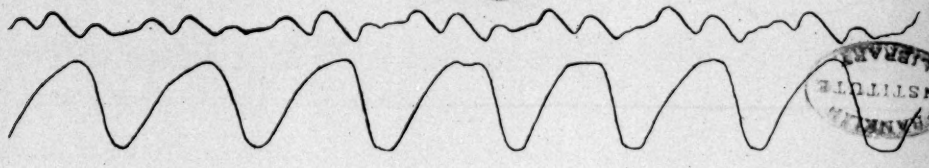

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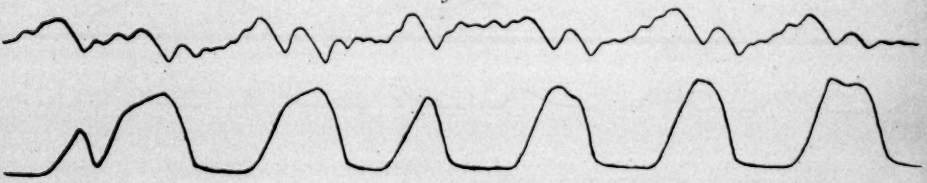

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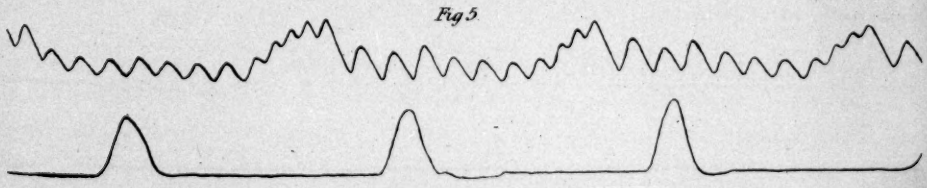

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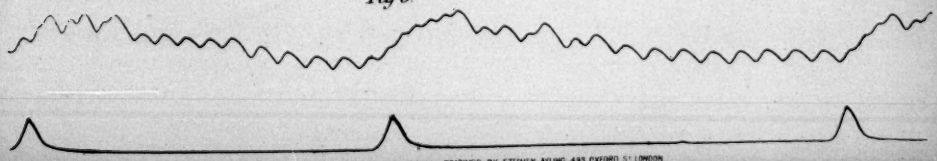

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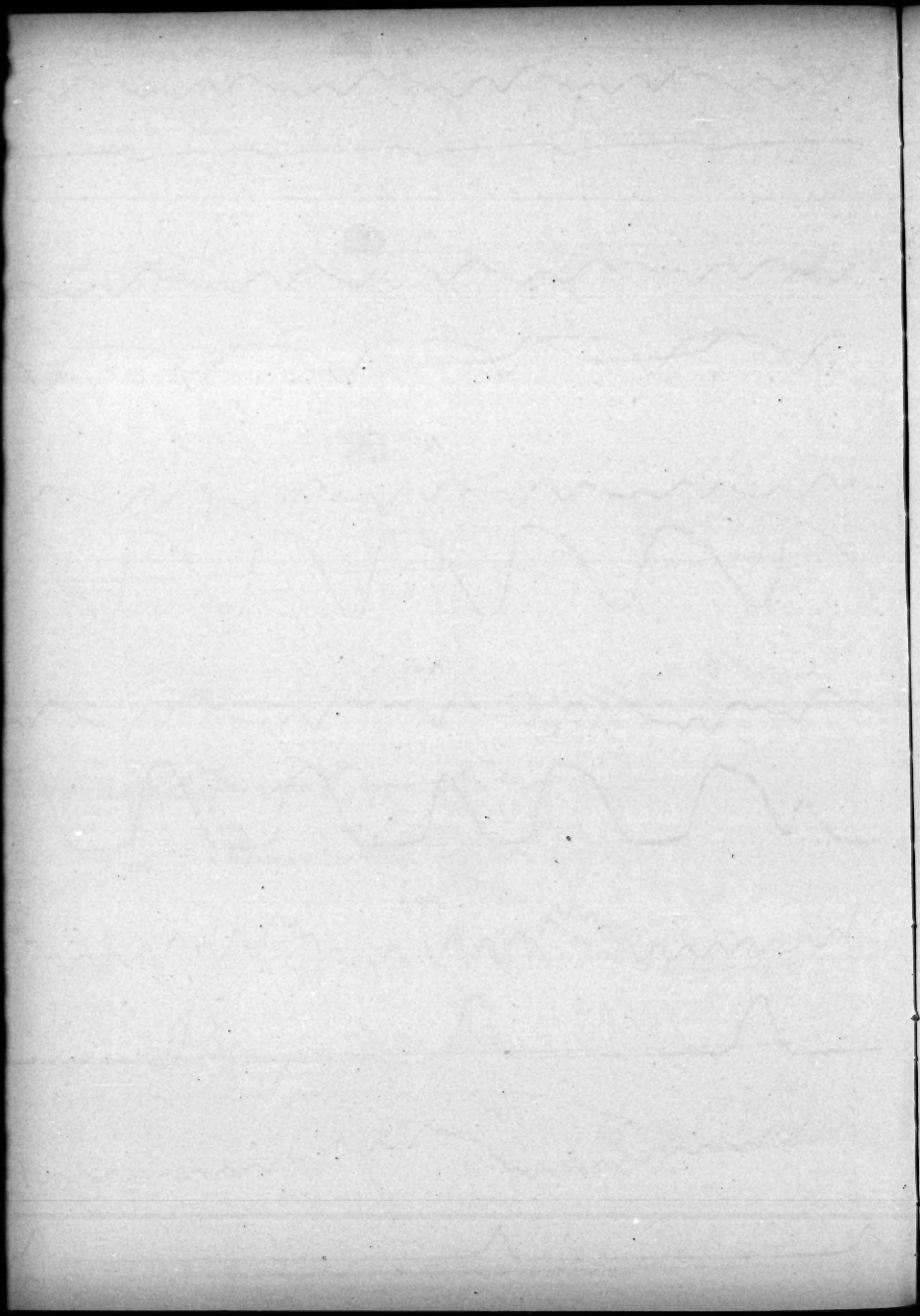
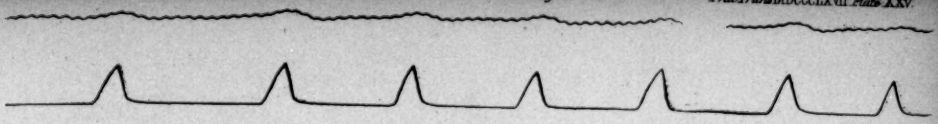


Fig 1



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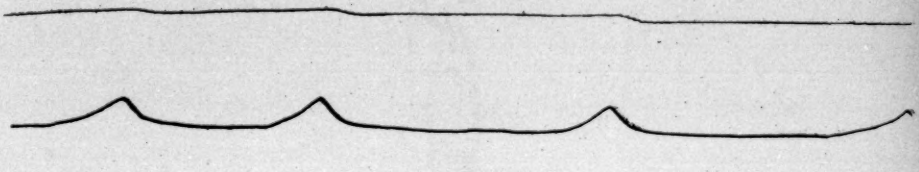

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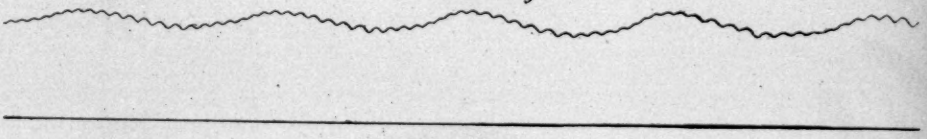

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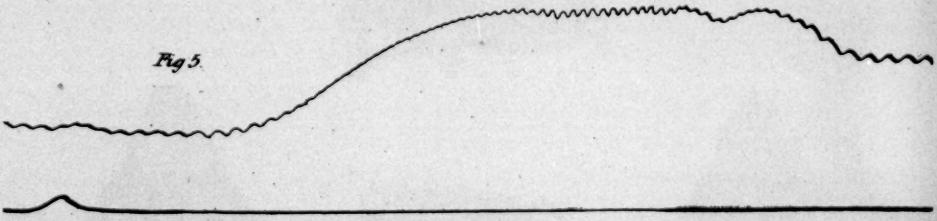

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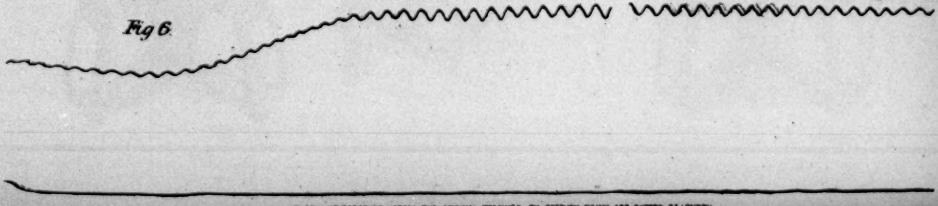

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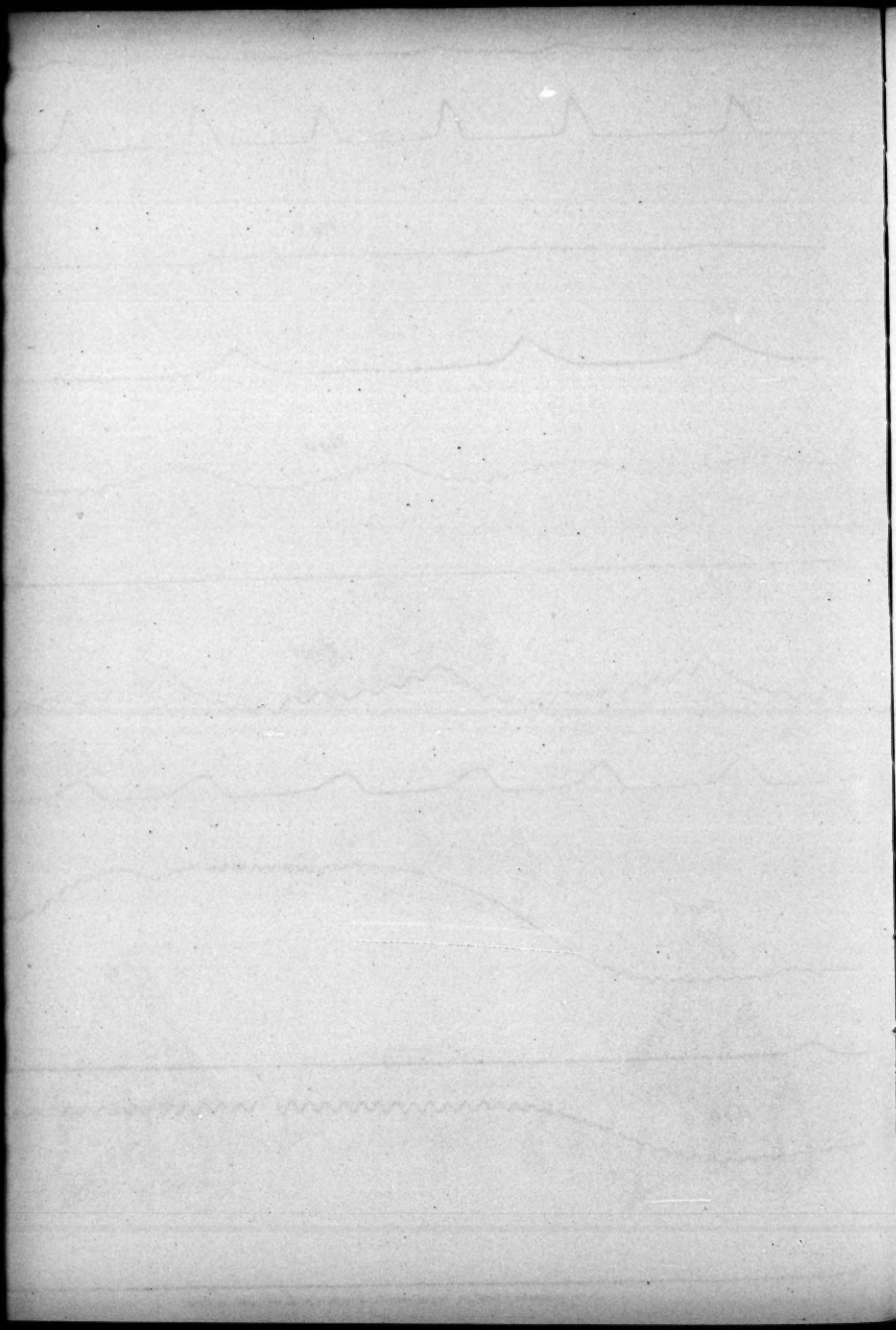
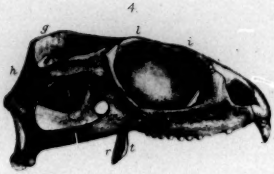
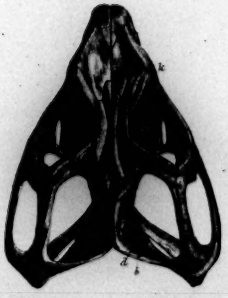
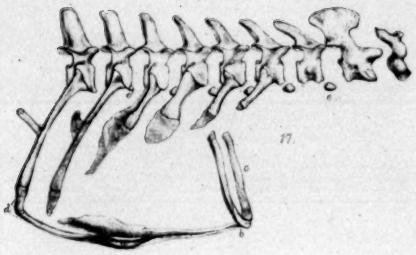


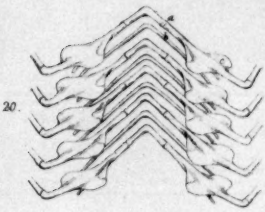


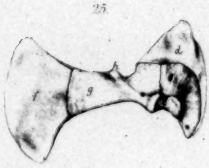



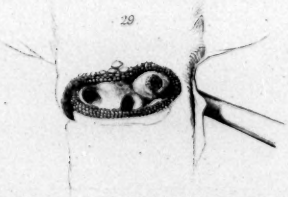
G.H. Ford.

W. West. imp.









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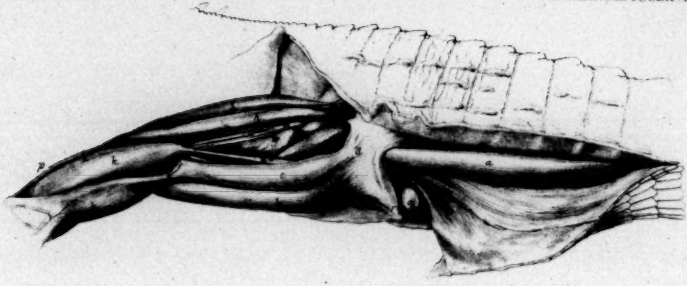

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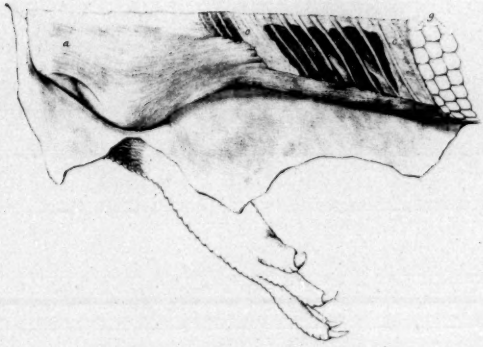

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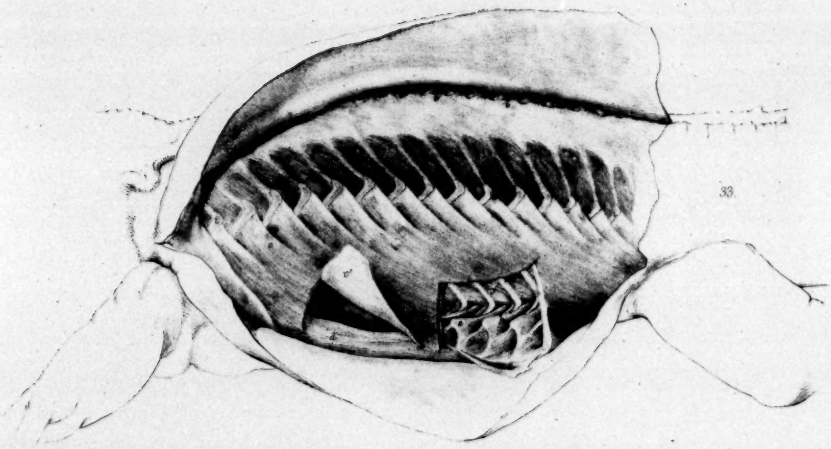

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Fig. 4

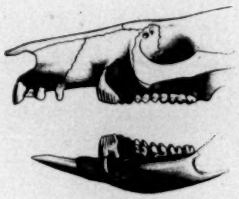

Fig. 1

Fig. 5.

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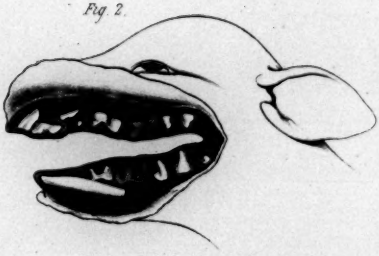

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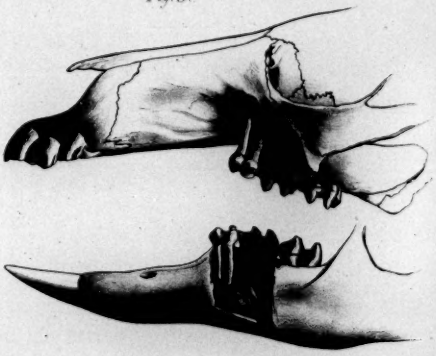

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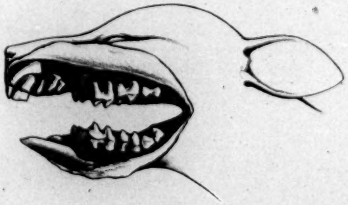


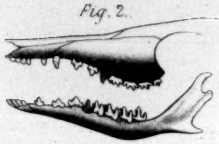
J. Gould, del.

M&N Hanhart, imp.

Fig 1 2 & 3. MACROPUS. Fig 4 HYPSIPRYMNUS.
Fig 5 & 6. PHALANGISTA.

Fig. 1.

Fig. 2.



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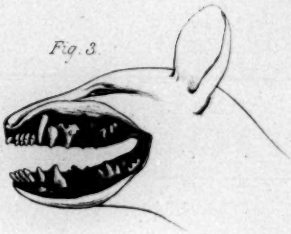

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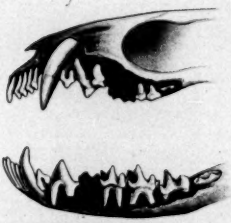
Fig. 4



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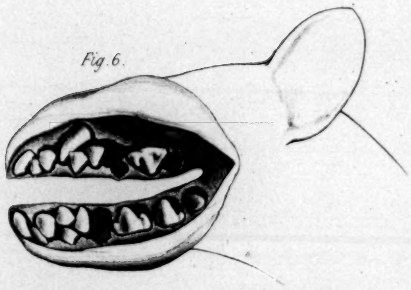

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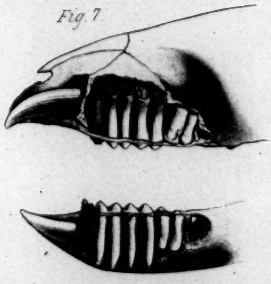

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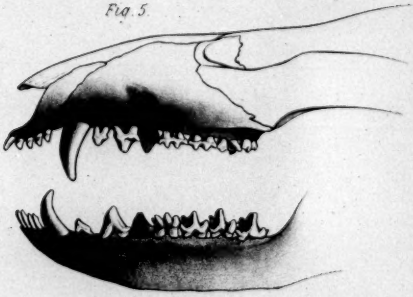
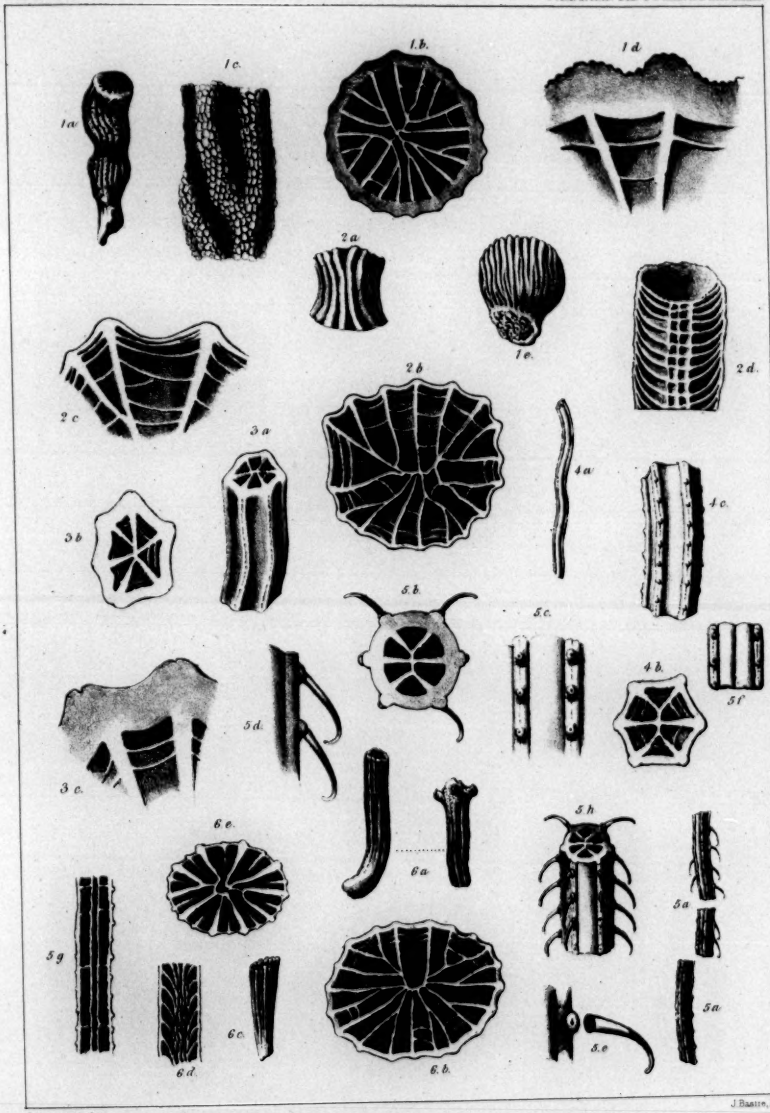


J. Smit. lith.

M & N Hanhart, imp

Fig 1 & 2. PERAMELES. Fig 3. 4 & 5. DIDELPHYS.
Fig. 6. THYLACINUS. Fig. 7. PHASCOLOMYS.

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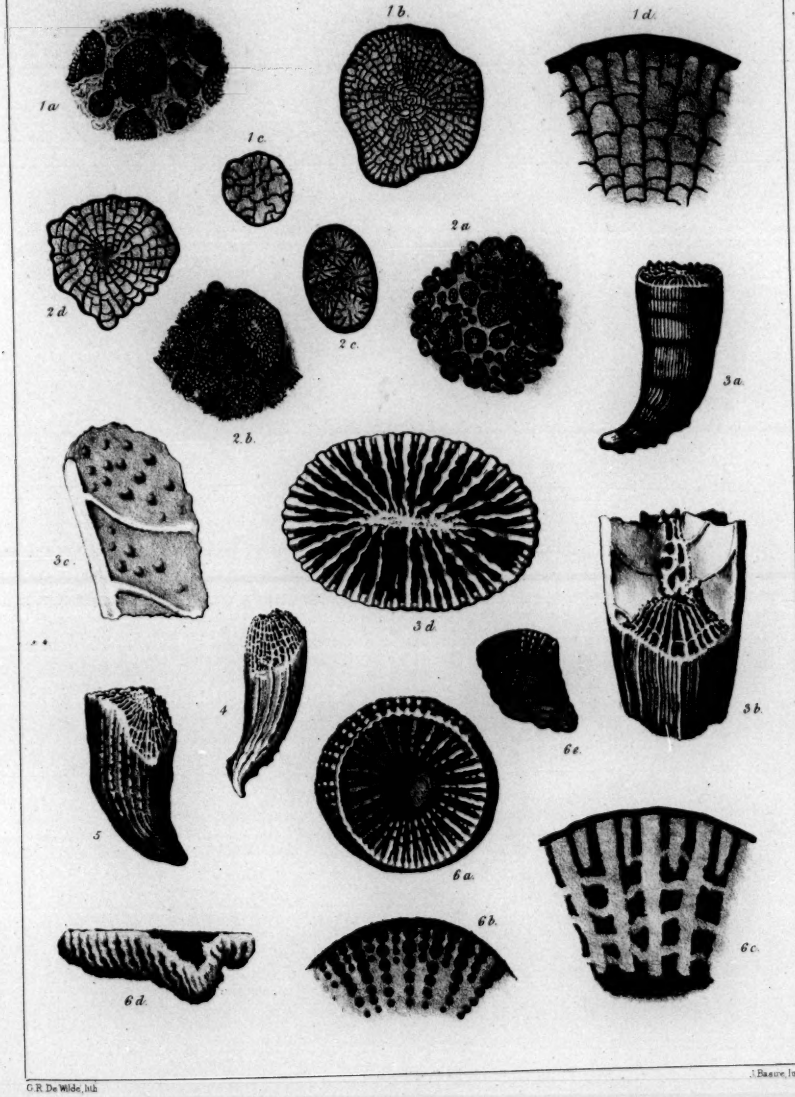
1a.
1b.
1c.
1d.
2a.
2b.
2c.
2d.
3a.
3b.
3c.
3d.
4
5
6a.
6b.
6c.
6d.
6e.